Under the pentagonal principle, a class is divided into groups of five (or six) to complete a discrete learning task, which is divided into five bits. Students take turns presenting the bits; if one fails, the group takes over. In this rotation, each student leads at least once in the session; the group verifies all steps. The instructor acts mainly as consultant. The principle is illustrated here with a course in math or philosophy. The outline allows for varied student interests; shows progress from strict first objectives to more flexible ones (through small group rather than lecture sessions); gives direction in the use of the textbook (Copi's SYMBOLIC LOGIC); sets up relationships among the instructor and students as individuals and as learning team members. Expected outcomes were: various interests would be generated; four to six student leaders would emerge; students would help each other; they would use the library more; their anxiety about grades would lessen; some would reach an A grade before end of term; behavior would be reinforced with little pressure from the teacher; average learning team size would stabilize at five; ten or fewer would have trouble adapting to the learning style; some would settle early for a C or B and become less attentive. At quarter's end, some groups met most expectations; some held stable and highly motivated; others were distracted by other classwork; many opted for extra sessions; students exhorted each other to study; they learned to function as group members. (HH)
The Pentagonal Principle for Self-Oriented Classes

by Jerald T. Ball
This report is an outline for a course in mathematics or philosophy. The student may elect to take the course under either title.

The following paragraphs contain (1) some of the reasons for developing such an outline, (2) some of the expected outcomes, and (3) comments about the experiences of the Spring 1969 quarter.
PART I

A. Rationale for Developing Course Outline

First, a student may take the course either under the mathematics or philosophy heading. This would indicate—and experience confirms—that a wide variety of student interests and attitudes must be taken into account if the course is to be relevant. The key idea of the outline is a minimum core of strict objectives followed by objectives that are increasingly flexible. This concept is further facilitated by the use of small groups rather than a single lecture session. In fact, a general lecture is rarely useful within this context. (See Part II).

Second, it must be admitted that the curriculum really is the textbook (Copi's *Symbolic Logic*, MacMillan Company 1968). This is stated so that the student has some idea of how he is supposed to use the textbook. Books may be read in many different ways for many different reasons.

Third, the relationships of the following must be understood: (1) students as individuals, (2) students as participating members of a learning team, and (3) the instructor. Note that the instructor has some definite obligations.

B. Expected Outcomes

1. That many courses would be "taught"—not just one. Interests would be generated within learning teams and these would develop heterogeneously and at different rates.

2. That a number (four to six) of student leaders would emerge in the course of the quarter and that students would help each other.
3. That students would use the library more than previously in a "lecture" situation. To encourage this, answer books were placed on reserve.

4. That there would be less student anxiety about grades than in a lecture situation.

5. That some students would make an "A" grade before the quarter was over.

6. That it would take several weeks (two to three) before the students really believed that the instructor was serious about this method of teaching. In other words, in order to reinforce behavior, there must first be some behavior to reinforce. This developed with minimum authoritarianism on the instructor's part. Whenever a student had a question, the instructor tried to answer it as honestly and as fully as seemed appropriate at the time. This was done standing directly next to him in a one-to-one (or at most a one-to-eight) situation. Whenever the instructor was not immediately available, he expected groups (teams) to develop other resources than the instructor.

7. That the size of the learning teams would drop from about eight to about five.

8. That there would be a significant number (but less than ten) who would find it difficult to adapt to this method of learning.

9. That some students would "elect" a "C" or "B" grade and then turn their attention elsewhere.

C. Some Additional Comments

There is evidence that nearly all of the expectations were confirmed by some groups and not by others.
The major concern, at present, is the stability of the learning teams. Some have remained very stable and highly motivated while others have not. Although the reason is not known, one guess is that the pressure to hand in daily work in other courses tends to have students put off studying this course.

On the other hand, many groups have voluntarily scheduled extra study sessions.

One student was overheard admonishing another, "You had better be here on Friday! and you'd better be prepared!"

Two students have already completed the "A" objectives in the 8th week of the quarter. One has since disappeared, the other is still leading a learning team.

The crux of this project lies in teaching students how to behave as a functioning group member.
PART II

A. By Way of Introduction to Student Involvement

We have long understood the learning advantages of direct student involve-
ment in a meaningful discussion or task. At the same time, we have relied primari-
ly on the lecture as a means of instruction. The lecture is most certainly a
legitimate method, if it is purposeful.

The purpose of a lecture is:

1. To present information that is not available by any other form of
   presentation, including
   a. New information not published.
   b. A collection of information from a selected variety of sources.
   c. An interpretation of information not available elsewhere.
   d. A statement concerning the interrelationships of a-c with
      standard readings.

2. To entertain.

3. To present (visually) a process or sequence of steps that would
   otherwise be disconnected.

4. Initial familiarization, orientation, or to give instructions.

   A lecture is a creative presentation of a man that is unique to him
   and his relationship with his students.

B. Assumption:

   One person can efficiently supervise five others. The ratio of one to
   five optimum for efficiency and ease of interrelations. "No one can hide in a
   group of five." Six is acceptable. Seven, allows the existence of at least one
anonymous person. Four is acceptable.

C. **Question:**

In any classroom discussion on any single topic, what is the maximum number of participants? What happens to the discussion when more than 5 are involved?

D. **Empirical Observations:**

A human can remember seven or eight bits of information at any given time without memorization. (Ref.: Memory - Scientific American Reprint)

E. **And The Plan**

The pentagonal principle simply divides a class into groups of five or six students for the purpose of completing specific learning tasks. In groups where attitude change is the goal, groups may be somewhat larger to allow for a small degree of anonymity. The following is a general description of this micro-principle:

F. **Micro-Principle for Each Group**

1. Given a specific learning task within a time limit we use alternating or rotating responsibility.
2. Learning task is subdivided into modules of five.

3. Member one has responsibility for presentation of part one of learning task. Upon completion, number two takes over, etc. Method of presentation should be visually observable so instructor can easily monitor groups.

4. Instructions are as follows:

   Member one, you are to present task. If you cannot do so, then your group has the responsibility to teach you. If you can, then it is your responsibility to make sure that all the rest of your group is able to perform task 1. Once your task is completed, then member two becomes the leader. Every member becomes the leader at least once in every class session.

5. Assuming that at least an outline of the solution(s) to learning task(s) is available to all groups, each group should have a means of verifying that all steps are done correctly.

6. Groups should be considered to be semi-permanent for the duration of the course of study. Alternations in membership may be made by the instructor for reasons of leadership or common need.

   Compared to a class of 35, the pentagonal is at least three times as effective in involving students with a given learning task. At most, six people can be involved with a learning task in a 1:35 relation. Otherwise, the discussion strays from the point in a random fashion.

G. Question

   In a "lecture-discussion" situation, how many students actually contribute at any given time? Observe this!

   Using the pentagonal approach, by an actual count, there were 19 (of 24) students involved with a learning task in trigonometry at a single time - 19
students were explaining, answering, asking or visibly responding at the moment the observation count was made.

In a 1:35 fixed relation one of the main dynamics is that of working out a relationship with the leader. (Observe this!) "Success" in the 1:35 fixed relation depends on dispensation by authority figure. The opportunities for creativity are limited by the extreme stability of the situation. Also there is only one focus of thrust. Being creative in a 1:35 fixed group depends on anticipating what the leader wants and this may or may not have anything to do with a learning task. Also in a lecture type situation the student is "conveniently" occupied with notes and not necessarily with learning.

In a pentagonal relation the dynamics are more fluid and are less dependent upon recognition by authority. The "authority" behaves more as a consultant. Since the thrust of the group is decentralized, there are more chances for leadership and these are dependent more upon ability to perform that learning task. Also, since "answer outlines" are provided, then the instructor-figure becomes a means of help toward reaching solutions of learning tasks. Insofar as possible, the learning task should be stated in behavioral terms. This will minimize the need for "interpretation" by the instructor.

H. Some Experiences

The following are some anecdotes that have relevance to group techniques:

1. Winter Quarter, 1968. After using group techniques with a class in elementary algebra for two of three, the instructor called in sick. The Division Chairman was unable to find a substitute for the 8:00 a.m. class and therefore decided to cancel it. Upon hearing this, members of the class decided to proceed with the learning task in algebra. Without any specific directions, the class
divided into groups and worked for the full class hour.

2. Autumn Quarter, 1968. The instructor informed a trigonometry class that they would be working in groups next day. The class was scheduled at 8:00 a.m. The instructor deliberately arrived five minutes late and found that the class had already subdivided into groups. No one seemed to notice the instructor's arrival. The students were busy with the given learning task; it was not until 8:15 that the instructor was consulted.

3. Using group techniques, classes seem to become task oriented rather than time oriented. Classes will frequently continue with the learning task after the scheduled hour is over.

4. During this quarter (Autumn, 1968), the instructor conducted some action research using the pentagonal group method in two classes of trigonometry. One met at 8:00 a.m.; the other, at 9:00 a.m. Using 80 per cent as a minimum passing mark in an exam, the two classes performed as follows:

\[
\begin{array}{c|c|c|c|c}
\text{time} & 3 & 2 & 1 & 0 \\
\hline
8:00 & 4 & 6 & 22 & 2 \\
9:00 & 6 & 7 & 11 & 0 \\
\end{array}
\]

The exam was given on Friday; the following Monday, new material was introduced. In the 8:00 a.m. class the conventional 1:35 fixed relation prevailed. In the 9:00 a.m. class, there was a 20 minute 1:35 presentation followed by 30 minutes of group work.

On Tuesday an unannounced quiz of one problem (the same) was given to both classes. Scoring was on a 3, 2, 1, 0 basis. Results are as shown:

\[
\begin{array}{c|c|c|c|c}
\text{time} & 3 & 2 & 1 & 0 \\
\hline
8:00 & 4 & 6 & 22 & 2 \\
9:00 & 6 & 7 & 11 & 0 \\
\end{array}
\]

\[
\begin{array}{c|c}
\text{time} & \text{pass} \\
\hline
8:00 & 10/34 \text{ Pass} = 30\% \text{ Success} \\
9:00 & 13/24 \text{ Pass} = 54\% \text{ Success} \\
\end{array}
\]
Part III

General Introduction to Course

This study guide is divided into five parts:

1. General objectives of the course of study.
2. General description of instructional methods and materials used to meet these objectives.
   a. The "text".
   b. Role of the student as an individual.
   c. Role of the student as a member of a learning team.
   d. Role of the instructor.
4. Graded list of performance objectives.
5. Suggested list of assignments in basic text.

It is written to tell the student as precisely as possible the standards of performance in logic.

It is written with good will.

A. General Objectives of The Course of Study

When the student has finished this course of study he shall: (1) be able to read from a newspaper, magazine, or college text book and state correctly whether he has read argumentive material or not; (2) be able to identify premises and conclusion if the reading is argumentive; (3) be able to correctly determine
whether the argument is valid or invalid, or whether there is a missing or suppressed premise by using the structure of the predicate calculus, and (4) if there is a missing or suppressed premise he shall be able to state that premise and then determine whether the completed argument is valid or invalid.

Briefly the student shall be able to recognize and analyze arguments.

B. Relation of Students, Instructor, Instructional Methods and Materials.

1. The "Text".

There is a standard textbook for this course. Copi - Symbolic Logic, Macmillan 1968. Also there is an assignment list at the end of this study guide. An answer book is also available on reserve in the library. The student is urged to consult this whenever necessary - that is, try to get the right answer without spending undue time on a blind alley. Also a bibliography follows in Section C.

2. The Role of the Student as an Individual.

As an individual the student is expected to:

a. Read the standard text thoroughly
b. Attempt all problems on the assignment list
c. Develop a list of questions for use in group sessions
d. Be prepared to help other students when possible
e. Be prepared to help direct learning activities of his learning team

3. The Role of the Student as a Member of a Learning Team

Students will be grouped into learning teams of 8. The groupings at
first will be arbitrary and changes may be made. Within two weeks the teams should be semi-permanent. As a member of a learning team the student is expected:

a. to meet with his group during scheduled hours
b. to give instruction to other group members when possible
c. to receive instruction from other group members
d. to introduce questions to the group
e. to help plan a time schedule to meet the course objectives during the quarter
f. to request an examination on meeting any of the course objectives

4. The Role of the Instructor

He is responsible for the standards of performance and for the organization of the learning experience. He will act mainly as a consultant. As such he is required:

a. to be available in the classroom during assigned hours and in his office during posted office hours, to answer questions at any of these times from either individuals or learning teams
b. to diagnose and anticipate learning difficulties either of individuals or groups and recommend a plan to overcome them
c. given one day's notice, to lecture on any topic relevant to the course of study
d. given one day's notice, to write an examination of any course objectives, and to evaluate and record the examination results within two days
e. to award grades based upon the written standards listed in part 4 of this study guide

C. Bibliography

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</table>

D. Graded List of Performance Objectives

The following list of objectives is intended to be a study guide for students. The list is graded C, B and A. When you have met the objectives in each category then you will be awarded the grade. All tests will therefore be
of the "Pass/Fail" variety. You may request a test at any time during the quarter.

**C grade**

1. The student will be able to use all of the rules of inference stated by Copi. This means that: given a set of five or less premises and one conclusion, the student will be able to derive the conclusion when valid.

2. Using the method of truth assignment the student shall be able to show invalidity of a given argument when invalid.

3. The student shall be able to translate any paragraph from a newspaper into propositional symbols.

4. The student shall be able to derive conclusions in conditional form using the rule of conditional proof.

\[
\text{CP: If } \Pi \text{ and } A \vdash C, \text{ then } \Pi \vdash A \cup C
\]

\(\Pi\) = a set of premises

A = any additional premise

\(\vdash\) = yields

C = any conclusion

5. The student shall be able to derive any conclusion using the rule RAA =

\[
\text{RAA: If } \Pi \text{ and } \sim C \vdash A \rightarrow \sim A \text{ then } \Pi \vdash C
\]

6. The student shall be able to derive any of the tautologies listed on pages 67-68 of Copi using rules CP and RAA.

All of the above will be evaluated at the 90 per cent level of proficiency with no time limit. The student will be allowed the use of two pages 8½" x 11" of notes.
B grade

1. The student will complete all of the C grade objectives.
2. The student will translate ten sentences selected from a newspaper or a magazine into one- and two-place predicate logical symbols.
3. Given an argument of five or less premises, each in predicate-logic symbols, the student will:
   a. decide whether the argument is valid or invalid.
   b. show a deviation, if valid, using all of the rules EI, EG, UI, UG
   c. show invalidity by the method of truth assignment

All of the above will be evaluated at the 90 per cent level of proficiency with no time limit. The student will be allowed to use three pages of 8½" x 11" notes.

A grade

1. The student complete all of the B objectives.
2. The student will translate ten sentences from a newspaper or magazine into multipredicate logical symbols.
3. The student shall be able to derive all of the tautologies involving quantifiers listed on page 130 of Copi.
4. The student shall write a short paper (three to five pages) analyzing an argument from a newspaper, magazine or textbook. The paper shall consist of the following paragraphs:
   a. Introduction: a statement of the context in which the argument was found.
   b. A statement of the argument in English, noting which sentences are premises and which conclusion.
c. A translation of all sentences into predicate logical symbols
d. A derivation, if valid
e. A demonstration of invalidity, if invalid
f. A list of premises that are either suppressed or missing
g. A re-assessment of validity with the suppressed or missing premises added
h. Concluding comments: a personal statement (optional)

5. In place of the short paper, the student may take an examination of 16 problems selected from exercises on pages 175-176 in Copi. (90 per cent, two hours, one page of notes for reference)

In all of the above, 90 per cent is considered to be passing and unless otherwise stated there is no time limit.

ASSIGNMENTS FOR MATH 12B/PHIL. 12B

1. pg. 46-7 - all
2. pg. 49-51 - all
3. pg. 65 - all
4. pg. 67 = I 8, 9, 16, 17, 18, 19
   II 3, 4, 8, 9
B
5. pg. 79-8 - all
6. pg. 89 - all odd
7. pg. 95-7 I 9, 10 II i0, 11, 14, 15
8. pg. 102-3 - all
9. pg. 116-7 - all
10. pg. 121-2 - all
11. pg. 130 - all
C
Prop. Functions
And Quantifiers
12. pg. 144-147 all
13. pg. 149-151 1,2,3 16,11,12
14. pg. 157-7 1,2, 8,9,10
15. pg. 168-9 1,3,9,10
16. pg. 175-6 I all II all