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ESTIMATION OF DOWNWIND CONCENTRATION
OF AIRBORNE EFFLUENTS DISCHARGED IN
THE NEIGHBOURHOOD OF BUILDINGS

by

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Synopsis

The air flow in the neighbourhood of buildings is briefly described and compared with that assumed for the usual atmospheric diffusion equations. The literature is reviewed and empirical formulae which have been proposed are listed and compared.

Talk presented to the Advisory Committee on the Safety of Particle Accelerators of the Atomic Energy Control Board. September, 1963.
The behaviour of waste gases from stacks after discharge to the atmosphere can be divided into two distinct phases. First there is the aerodynamic phase, where the behaviour pattern is determined primarily by the specific properties of a particular discharge; for example excess momentum, excess temperature and the shape and size of the stack which determines the nature of the air flow. The second phase begins when the excesses of temperature and momentum have been dissipated and at a down wind distance sufficient to eliminate the disturbance of air flow caused by the presence of the stack. The subsequent behaviour of the plume would then be determined by the turbulence structure of the free air and would be identical for all plumes injected into an identical atmosphere irrespective of the initial conditions. The popular diffusion equations used to estimate the effects of atmospheric pollution apply only to the second phase of the discharge behaviour. They are based upon an assumption that the turbulence structure of the atmosphere can be generalised with respect both to time and space for a given meteorological condition. They cannot therefore be expected to apply where roughness on the earth's surface introduces additional turbulence of a local nature. They do not, for example, apply very near to a stack nor where the stack is surrounded by other buildings or topographic features of purely local significance such as trees or hills. Similarly, they assume that the point
of interest down wind of a stack is free from obstacles to air flow. Thus it is possible to estimate the concentration of a pollutant at a point down wind of a stack situated in relatively flat and open country, but if a building or hedge is upwind of the sampling point the calculation might no longer be valid. It is important to preserve a sense of scale. In general, a plume expands down wind as its contents are dispersed throughout larger masses of air. When the plume occupies a volume which is large compared with the size of a building or other obstacle, the departures from predicted concentrations are insignificant.

Similarly, it is possible to generalise about the air flow over cities provided that the points of interest are sufficiently high above the ground for those unique features which distinguish one city from another to be small and if the air flow is affected by roughness elements which are essentially similar in size and density. Under these conditions a diffusion equation may be applicable if suitable parameters are used. Similarly, if a stack is situated among a cluster of buildings or in a city, and projects high enough above the rest of the buildings, it may be regarded as discharging into free air, and a diffusion equation may be expected to give reliable estimates of pollution levels.

There is a general rule of thumb used by engineers to the effect that a stack mouth should be $2 \frac{1}{2}$ times the height of surrounding buildings in order that the plume behaviour should be independent of local aerodynamic features. For stacks which are built shorter than this, the plume must be expected to exhibit, to a greater or lesser extent of the features
of both phases of behaviour. When a stack is about the same height as surrounding buildings, generalizations become difficult and, although some guidance may be available from experienced engineers, it is usually necessary to resort to tests made at the specific location. Unfortunately, such tests are often prohibitively expensive, particularly when alternative locations and designs are to be compared.

It is customary when dealing with this type of problem to use models to represent the local features and to conduct the tests in a wind tunnel. Although many such tests have been carried out, the results are of limited general interest. Most tests have been qualitative, in the sense that the experiment results depend upon visual observation of the plume behaviour under various test conditions (primarily stack height and stack exit velocity) until conditions are found which appear to the experimenter to give a satisfactory behaviour of the plume. Such tests have fulfilled their function since they have been used as design guides, but they have contributed little to a better understanding of the processes involved. They have often been carried out with little attention to, or perhaps without understanding of, what may be referred to as the Model Law, i.e. choosing the conditions in the wind tunnel so that the air flow is exactly similar to that in the prototype.

During the last few years, some quantitative studies have been made where a tracer gas was released and its concentration (or the time integral of the concentration) measured in the space around the model. There has still been virtually no subsequent comparison of the wind tunnel results with
tests in the field to see how closely they corresponded.

Up to this point my approach to the problem has been negative. This has been deliberate because it is important to realise the limitations of existing working formulae, including those which I shall give later.

It would be useful now to describe briefly the nature of air flow in the neighbourhood of obstacles such as buildings (Slide 1). As the stream lines approach the face of the building they are displaced laterally and vertically. The pressures and velocities in this displacement zone are related through Bernoulli's Law. At the centre of the face there will be a stagnation point where the pressure is at a maximum and the velocity is zero. Immediately behind the building there is a region in which velocities and pressure are reduced and the turbulence is very high. This has been called the "cavity" by Halitsky (4). The flow lines in the cavity are as shown in the diagram. There are two station points, B and C, where B is at the downstream building face and C about 2 building lengths downstream of the leeward face. Surrounding the cavity is the wake, which extends a considerable distance down wind of the cavity. The wake has velocities below those in the free stream. This picture is idealised, and in a city where buildings are close to one another it is obvious that such ordered air flow cannot be expected.

The stack is itself an obstruction to flow. Vortices are formed at the edges of the cylinder and, breaking away from the surface, flow down stream with the wind (Slide 2). These "trailing" vortices have an axis of rotation parallel to that of the stack. In addition "tip vortices" are formed
at the top edge of the stack orifice. These have an axis of rotation parallel to the air flow. Because these vortices are formed at the level of discharge of the gases from the stack there is a tendency for the gases to be drawn into the open end of the vertical vortices and discharged at a lower level. This causes the familiar "downwash" from stacks in high winds shown by the blackening of a chimney. If the discharge level is within the wake behind the building, pollution may be brought rapidly to ground level. This situation is normally overcome by the "2 1/2 times rule" for stacks, while the primary tendency for "downwash" is corrected by ensuring a high exit velocity or high temperature excess in the stack air. This ensures that the effective discharge height is well above the zone of turbulence created by the stack.

**PRACTICAL DIFFUSION FORMULAE**

The simplest form of an equation relating rate of discharge of a pollutant to the concentration down wind of the stack is

\[ K \cdot \frac{Q}{L^2 u} \]

where \( Q \) is the concentration of the pollutant emitted at the rate \( Q \) units per sec., \( u \) is the wind speed, \( L \) is a length so far unspecified and \( K \) is an unspecified coefficient, which is probably variable depending upon the prevailing meteorological conditions. There are several ways of looking at \( K \) and \( L \). If \( L^2 \) is equated to the projected area of the building normal to the mean wind, \( K \) may be looked upon as the fraction of \( Q \) entering the volume defined by \( L^2 u \) per second. Alternatively \( L \) may be the distance
from the stack to the point of interest, when \( K \) becomes a dispersion coefficient. In a third form, \( L^2 \) may be the product of two dispersion parameters, one for the lateral and the other for the vertical displacements. In this case, \( K \) becomes a building or site coefficient where a characteristic length or distance appears in nondimensional form e.g. the ratio of the distance to the building width or height. In the diffusion formulae which follow, examples of all three interpretations will be found. It is important to realise the interdependence of \( K \) and \( L \). Thus, in a formulae where \( L \) appears as a characteristic length, there is nothing unique in the choice and any other length characteristic of the building might have been used with a corresponding change in the value of \( K \). A summary of working diffusion formulae which have appeared in the literature is as follows:


In this, as in all the other formulae listed, the symbols and units appearing in the original document have been changed to conform to a uniform presentation.

In this formula it is assumed that all the stack effluent enters the turbulent wake behind the building. According to the authors, at a distance of three times the height of a cubic building, the effective cross section is three times the projected area of the building in the direction of flow and the mean wind velocity over the cross section lies between
0.5u and u, where u is the free stream velocity.

Thus their equation becomes

$$C = 0.67 \frac{Q}{L^2u}$$

(1)

Where \(L^2\) is the projected area of the face of the building in the direction of the wind. The distance of observation is considered to be about 3L away from the stack.

**Formula 2.** R.S. Scorcer and C.F. Barrett "Air and Water Pollution" 6, 49 (1962) present a simple model in which the effluent is contained within a cone of half angle 12° "bent" around the building.

$$C = \frac{10Q}{L^2u}$$

(2)

where \(L\) is the length of the trajectory round the building to the place where the pollution is measured.

Scorer and Barrett suggest that (2) is a maximum at \(u = 5\) m/sec and they actually give the maximum short term concentration as

$$C = \frac{2Q}{L^2}.$$  No details are given as to what they mean by short term.

**Formula 3.** Quoted by F.A. Gifford "Nuclear Safety" Dec 1960 p 56 and ascribed to Fuquay (personal communication)

$$C = K \frac{Q}{L^2u}$$

where \(\frac{1}{2} \leq K \leq 2\) and \(L^2\) is the cross sectional area of the building. It is stated that \(K\) must be determined by experiment. Taking the worst case i.e. \(K = 2\)

$$C = \frac{2Q}{L^2u}$$

(3)

This approach is more detailed and is based upon the results obtained in a wind tunnel. The experimental results were presented in the form

\[ K = \frac{C_m A_m U_m}{Q_m} \]  

(4)

where the symbols have the same meaning as previously indicated, the suffix m referring to the model. \( A \) is the cross sectional area of the building. A large number of experiments were performed and values of \( K \) plotted on a chart, the points of equal \( K \) values then being joined to give a distribution map.

From the maps, maximum values of \( K \) were located and the greatest distance \( S \) away from the building at which this value of \( K \) was found was noted. A graph showing maximum value of \( K \) against \( S/D \) where \( D \) is the diameter of the building (circular) was constructed. It was possible to draw a line given by

\[ K = \frac{20}{(S/D)^2} \]  

(5)

which would include all the experimental points. Assuming exact similarity between model and full scale the maximum value of \( K \) in (5) can be put in equation (4) and, revert.ing to our terminology, (i.e. \( A = L^2 \)),

\[ C = \frac{20 Q}{(S/D)^2 L^2 u} \]  

(6)
If \( D^2 \) may be equated to \( L^2 \) (as in a cube) then

\[
C_{\text{max}} = \frac{20Q}{S^2u} \tag{7}
\]

where \( S \) is the distance from the down wind face of the building.

However, \( S \) may not necessarily be measured at ground level, and in general it will not be unless the release from the building is at this level. The authors also point out that a line given by \( K = \frac{10}{(S/D)^2} \) would give a better fit of the experimental points. The average concentration down wind then becomes

\[
C_{\text{ave}} = \frac{10Q}{S^2u} \tag{8}
\]

which is very similar to Scorer and Barrett’s equation (2) when \( S \) is large compared with the dimension of the building measured in the direction of the wind.

**Formulae 5. J. Halitsky - "Gas Diffusion Near Buildings"**


In this more recent paper of Halitsky, the work described above has been extended and the results presented in a slightly different form. The experimental results were obtained using a duct exhaust from the building.

The dilution ratio \( D \) is the ratio of the concentration in the duct aperture \( (C_e) \) to the concentration at the point of observation in the field \( (C) \). The results fit the curve

\[
\frac{C_e}{C} = D = [a + 0.11s (1 + a/5)]^2 \tag{9}
\]
where \( s \) is the non-dimensional length \( S/\sqrt{A_e} \) where \( S \) is the distance from the aperture and \( A_e \) is the cross sectional area of the aperture. \( a \) is stated to be a function of the "configuration". For minimum dilution Halitsky gives a value of \( a \) between 1 and 2. Maximum dilution \((10 < a < 20)\) occurs with buoyant plumes and chimneys. Low effluent momentum, corner orientation and upwind emission produce low dilutions \((2 < a < 5)\).

For values of \( S > 100 \), the difference due to changing \( a = 0 \) to \( a = 2 \) is less than a factor 2 and we may reduce (9) to the standard form of the preceding formulae by taking \( a = 0 \) and remembering that \( C_e = \frac{Q}{V_e A_e} \) where \( Q \) is the source strength and \( V_e \) the efflux velocity. Then

\[
C = \frac{8.3Q}{V_e S^2}
\]  

(10)

Since down wash of the chimney effluent is only a problem when the wind speed approaches the efflux velocity then \( V_e = u \) and

\[
C \approx \frac{8Q}{US^2}
\]  

(11)

It should be remembered that equation (9) applies to the plume centre line and not to ground level.

Formual 6. An entirely different approach is that given by M. Jensen and N. Frank ("Model-Scale Tests in Turbulent Wind" Part I, The Danish Technical Press, Copenhagen 1963). Again wind tunnel studies have been made where a trace gas was released and the concentration was measured at various points in space around the model. The results are expressed in the form of a non-dimensional coefficient \( b \) which indicates the variation of concentration from one place to another in a given arrangement.
The terms \( Z_0 \) and \( U^* \) correspond to the \( L \) and \( u \) terms in our previous equations, but have a special significance. They are, in fact, parameters characteristic of the general terrain rather than a specific building. Under certain specific conditions, the variation of wind speed with height above the "surface" may be described by the logarithmic profile law which gives the speed at height \( Z \) as

\[
U_Z = \frac{U^*}{k} \ln \frac{Z}{Z_0}
\]

where \( Z_0 \) is a constant of integration termed the roughness length, \( U^* \) is called the friction velocity and \( k \), von Kármán's constant---usually given the value 0.4. \( Z_0 \) is assumed to be a characteristic of the roughness of the surface and \( U^* \) depends upon both surface roughness and gradient wind speed. It is interesting to note here that both Halitsky and Jensen regard scaling of \( Z_0 \) as important in achieving similarity between model and prototypical experiments. Jensen retains both \( U^* \) and \( Z_0 \) as independent variables, thereby hoping for wider applicability of his data in different terrain, whereas Halitsky standardizes the terrain (and thus \( Z_0 \) and the ratio \( U^*/U \) where, \( U \) refers to a wind velocity at a fixed reference height) and considers the building characteristics as the independent variables. It is not possible at this stage to decide which is the more fundamental approach.

It is apparent that in the form of equation (12), Jensen and Franck's results are difficult to apply in practice. There are two ways to reduce...
(12) to more useful form. Firstly we may take the value of $Z_o$ given by them for the city of Copenhagen and calculate from their published data the ratio of $U_*/U$ where $U$ is taken to be at some convenient reference height. Unfortunately they give wind velocities at only three heights over Copenhagen and the three points do not lie on a straight line in their log $Z$ against $U$ plot. However, they estimate $Z_o$ to be 7.5 m and, using their same method, it is possible to estimate that $U_* = 0.15 u$ when $U$ is measured at 100 m. We thus obtain

$$C = \frac{Q}{8.4U} \cdot b$$

We shall consider the value of $b$ later.

The second approach is to take Jensen and Franck's experimental condition that $h \approx 3Z_o$. Then, using the relationship above that $U_* = 0.15 U$ we obtain

$$C = \frac{60Q}{h^2U} \cdot b$$

where $h$ is the height of the building of interest down wind from the source. The significance of this unexpected measure of a characteristic length will be evident later.

Two cases were considered in the experiments. The first was that of a single isolated stack of height $H$ and a house of height $h$ down wind. The distance $X$ of the house was varied and samples were collected on the wind ward and leeward walls separately. The results are presented graphically. The abscissa is the nondimensional distance $x/h$ from the source and the ordinate is the non-dimensional stack height $H/h$. A series
of curves is obtained, each with a constant parameter \( b \). The second case was that of a chimney of height \( H \) on top of a house. Again measurements were made at a house of height \( h \) situated at a distance \( x \) from the stack.

The same graphical presentation of the results was given. This case is more relevant to the problem under review, and we may take the case where the stack height \( H \) is comparable with the house height i.e. \( H/h \approx 1.2 \).

A value of \( b \) equal to \( 8 \times 10^{-3} \) can be found from the published results for a house situated 6 house heights away.

Putting this value into equation 14 and 15 we obtain respectively

\[
C = \frac{Q \times 10^{-3}}{U} \\
C = \frac{0.5Q}{h^2u}
\]

Summary and Conclusions

The general formula for calculating concentrations of pollutants discharged from a short chimney is

\[
C = K \cdot \frac{Q}{L^2u}
\]

1. Where \( L^2 \) is the projected area of the building \( K \) has a value of 0.5 to 2 (Cave and Phillips 0.67, Fuquay 2.0; and Jensen and Franck 0.5).
2. Where \( L \) is the distance from the source, \( K \) has a value equal to 8 - 20 (Scorer and Bartlett 10; Halitsky (a); Halitsky (b) 8).

All the formulae given above, except perhaps that of Jensen and Franck, can be reduced to a common form with a surprising measure of agreement.

It is to be noted that in reducing the original formulae to this common
form, I have sometimes ignored differences in the details of models used. For example, I have ignored the differences between a square faced and an oblong faced building. Whether or not these differences are significant is not clear but the uncertainty introduced is probably not important.

The other thing I have done is to rob some formulae of their "sophistication" in order to make them conform to the least sophisticated of the set. This of course can be rectified by consulting the details from the original sources but the story becomes longer and more detailed. However, one wonders at this stage how much has really been lost by doing this. With the present limited experimental data, too little is known about the effects of other variables.

The first four formulae more or less apply to single isolated buildings with the surroundings largely unspecified except for the formula of Halitsky. The experimental work of Jensen and Franck appears to be more directly applicable to the problem under review, i.e. diffusion amongst buildings in a city. However, this work is the least useful, in that the parameters used are not easily measureable.

Bibliography


2. Scorer, R. S. and Barrett, C. F. Int. J. Air and Water Pollution 6, 49 (1962).

SLIDE 1  Diagram showing flow patterns near a building.
   (Based on the figure on page 9 of ref. 4)

SLIDE 2  Downwash of stack gases.
   (Based on Sherlock R.H. and Stalker E.A.
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