This paper is an explication of the concept of a network of consultants to help people obtain answers to day-to-day questions for which answers are known. Each member of such a network is viewed to have three capabilities: 1) answering questions directly from his own memory, 2) answering questions with the aid of library resources, and 3) referring the question to a member of the same network or to an expert outside it. Conditions are derived which involve each member's ability to choose appropriately among the three alternatives. If he judges correctly concerning when and where to refer a question, such a network, suitable organized, has greater net utility than does a reference librarian by himself. Mathematical models are used to formulate and analyze designs for the referential consulting function. It is argued that the existence of such a network makes possible a much broader range of information service to the community than is afforded by traditional reference librarianship. (Author/KB)
Referential Consulting Networks

Manfred Kochen
Mental Health Research Institute and
Department of Library Science
University of Michigan

1. Introduction

Who has not discovered treasure in a library? An exquisite, bewildering variety of treasures. A few come in the form of answers to riddles or urgent questions, explicitly stated or covert. Most forms take the shape of books, and traditional libraries as well as their users value the physical form alongside the content it embodies. But how can we better think of the use of libraries as analogous to treasure hunting? We discuss this question by helping to explicate "reference service" as a theoretical concept, and by seeking conditions for excellence of such service. Few of us expect libraries to help solve our most common or most urgent day to day problems. Seldom does a voter who just moved into a community, for example, think of the local librarian as his most promising source of help in evaluating the candidates or the issues in an impending election.¹ But, it is conceivable that the local reference librarian -- or rather his modernized counterpart, the community's professional information-please officer, for whose existence we argue in this paper -- could help the newcomer, perhaps better than any other source to which he, on his own, would think to turn. Certainly the library contains copies

The author's work on this paper was partially supported by grant NSF-GN-T16.

¹For some typical questions with which libraries deal see the excellent compilation: Case Studies in Reference Work by Grogan (7).
of at least the local newspaper which might have sketched the qualifications of the candidates and summarized the pros and cons of the issues; it might also contain more detailed readings on the issues as well as biographies.

The traditional definition of a library is: a "collection of books organized for use". But the word, "use", is explicated no further. To maintain a collection organized for use, the library performs three traditional functions: (a) book selection, (b) bibliographic control, (c) reference.

It is also traditional to complain that library resources are under-utilized. De Solo Pool (17) estimated that less than one-quarter of Americans, when in need of information, regularly use libraries. Of these regular university and college library users, the majority do known-item searches, and less than 30% do subject searches, according to Brooks and Kilgour (2), Lipetz and Stangl (13) Palmer (15), and a current catalog use study by us (21). A recent study directed by Swanson (20) also showed that experimental subjects recall the title or author of a book with sufficient accuracy to locate the book through the catalog only one quarter of the time. Finally, even if a book is located in the catalog — in either a known-item or a subject search — it is likely to be found in the stacks in slightly less than half of the cases for a number of large libraries where such studies were made.

It has been said that library sources are under-utilized because potential users have dismal expectations. These figures do little to help those who argue that such expectations are unjustified. If we
use libraries rarely to acquire knowledge, understanding or wisdom when we can't specify a book likely to help us, then we are under-utilizing library resources even more than is traditionally supposed.

These depressed conditions may be due to an overly narrow and outdated conception of libraries and librarianship rather than to poor performance of the three library functions. Shera and Egan (5) challenged the traditional concept of a library when they proposed that its function should be "to maximize the effective social utilization of the graphic records of civilization". This redefinition is a vast step forward, because it did not confine the librarians' responsibility to books, nor to a specific collection usually delimited by funds and space. Above all, it replaced the vague term "use" by the more meaningful term "maximize the effective social utilization".

The Shera-Egan proposal, however, implies that if a member of the U.S. Congress acts on an important social issue on the basis of wrong or missing knowledge when the correct information exists in the Library of Congress, then the Librarian of Congress is responsible. He should have seen to it that the legislator was given the option of using, not using, or misusing, relevant knowledge in the Library.

But the Librarian can hardly be expected to share the entire responsibility for the legislator's misuse or failure to use such knowledge. Misuse of relevant knowledge does not maximize its "effective social utilization". ²

²For example, when Stalin ignored the well documented, encyclopedic compilation of Soviet intelligence by Richard Sorge, which showed that the Germans would attack Russia on June 22, 1941, Sorge was in no way to blame for such minimizing of the effective social utilization of graphic records.
The Librarian (of Congress, or Sorge, in the above examples) is only partially responsible. Perhaps the Shera-Egan definition should be revised to: "to maximize the greatest potentially attainable effective and efficient social utilization of documented knowledge."

Note that we also substituted the more abstract term "documented knowledge" for the "graphic records of civilization", to include non-graphic embodiments of documented -- i.e., validated -- truths, such as magnetic tape recordings. Note also the insertion of "and efficient" to suggest that this is to be done with a reasonable or minimum expenditure of necessary resources; in particular, the cost of information overload on the user due to lack of fine selectivity is to be kept within bounds.

How can librarianship change to fulfill most effectively the new demands of such a revised definition? Some thought has already been given to this question. At a conference on reference and information services held at Columbia University in 1966, Kilgour (9) stressed the need for a more intellectual approach to librarianship. The development of the "knowledge industry" has placed libraries in a much more central and responsible position in our society, which demands recognition of new, more viable techniques in librarianship.

Because of these increased responsibilities to the community of users, the reference function has become especially important. Thus far, however, reference librarians have no established definition of the scope and method of their work.* Wynar has defined reference

*This is not meant to imply that there are no definitions at all. The ALA Glossary, for example, has defined reference work as "that phase of library work which is directly concerned with assistance to the readers in securing information and in using the resources of the library in study and research."
service as "any activity related to providing information as well as guidance and instruction in the use of library resources (a necessary compromise)". Unless one recognizes, however, that 'information' and 'library resources' have become vastly broader in scope, such a definition could easily be applied to a very conservative exercise of the reference function. The vastness and variety of resources available to the reference librarian make necessary the development and use of a real information and referral network if libraries are to achieve "the greatest potentially attainable effective social utilization of documented knowledge".

In this paper we stress such an expanded scope and depth for reference librarianship. If mathematical library scientists can be stimulated to explore and develop the various lines of investigations opened up by the models sketched later in this paper, perhaps the concept of indirect referral will come to be recognized as a very important part of any good theory of reference service.

2. The Reference Function

The concept of "reference" is probably the most basic in library science. Although the word "refer" has several meanings (e.g., the librarian referred him to an encyclopedia; or to a consultant; the book referred to baseball scores; the user referred to an encyclopedia), they all involve the action of pointing, directing, "passing the buck".

In order to perform this referral function well, the reference librarian must be a generalist. While he does not have to know where to look up the latest and most reliable measurement of the velocity
of light, he should at least know whom to ask, or whom to ask for a name of someone to ask. Like an executive, he needs more to know to which source to assign the responsibility for solving a problem than to solve the problem himself.

Consider the following sample of possible information needs on the part of average people in their daily lives, at work or at home.

P1. What is the address of the Bolger Laboratory, in the vicinity of Boston? (The client may be a doctor and on questioning reveal that Bolger is a drug testing lab).

P2. A herpetarium attendant is showing symptoms of snake poisoning, though he hasn't recently been bitten. Has anyone ever published reports of delayed snake poisoning? (The client may be a local physician or the afflicted patient; he might even settle for unpublished reports or the names of experts, or the names of people who have encountered this).

P3. We just moved into the city of Jonesville which has no local hospital, and our child seems to have heart trouble from eating too much animal fat. What kind of physician should we call? And how can I get reliable help in selecting one? (The client might wish a "non-obsolete cardiologist, hematologist or internist", though he may not realize how to ask for or spot one.)

---

3 This example is due to D. Dennis, then head of the Health Sciences Library at the University of Michigan, now at the National Library of Medicine (private communication). This request was forwarded from a small town to the regional Medlars post in Ann Arbor, thence to NLM in Washington. The Medlars search revealed one relevant paper which was supplied a day later (rather than the usual 2 weeks) because of possible urgency of the case.
P4. I need a light, strong, rust-resistant material for a toy I wish to produce and market. What should I consider? (The client may be happy to learn of the existence of a materials information center, or even of a directory which informs him of its existence).

P5. Mr. Green is being considered for a very responsible public position, and I must decide whether or not to endorse this appointment. Where can I get pertinent information about him?

P6. Gross sales and morale in our company have been declining steadily since last January. What should we do?

P7. Lately everyone seems to be taking advantage of me and I feel very uncomfortable with other people. It has reached the point that I can't even show my face to my fellow workers anymore. I need help desperately.

P8. Our university is considering increasing its investment in computers to $10 million/year by 1970. How can we make sure of getting an optimal return on this investment, especially so as not to aggravate our key problems?

P9. What would be the economic consequences of establishing and enforcing national standards in the metric system, to be in effect by 1975?

P10. How have the attitudes of Southern whites toward integration changed from 1900 to 1968.

Consider next the following sample of "answers" for which the
questions may have to be found, sought out, motivated:

A1. A new serum which is extremely effective against many kinds of snake poisoning was recently tested by the Bolger Labs. (The author of P2 may be interested).

A2. Dr. Jones, who has been practicing cardiology in Jonesville for the past 30 years, was found guilty in a malpractice suit in 1950; after one year in jail, he returned to his practice. (The author of P3 may be interested).

A3. Action is urgently needed on the long-delayed discussions about U.S. arms aid for Israel.

A4. The second law of thermodynamics implies the impossibility of perpetual motion machines. (The inventor of a perpetual motion machine ought to be interested).

A5. Task forces in which the members' personalities are most alike on their need to give and get affection have higher morale and productivity than do task forces in which the members' personalities are dissimilar in this respect, regardless of other personality factors. (The author of P6 may be interested).

Statements such as A1–A5 usually appear in newspapers and journals. Libraries, at best, publish a periodic accession list of book titles, and regard anything beyond this as the responsibility of information centers.  

Few users would think of using a library for an answer to P1,  

4 Eloquent refutations of this viewpoint have been made by Lorenz (14) Freiser (6), and Rees (18).
when they could ask a telephone operator or a colleague to help them. While many librarians may not consider themselves responsible for helping a user with such a problem, most librarians could with great facility find the answers while the user is still on the telephone; if they could, they would probably be eager to do so.

No one in the public library of the small town receiving query P2 may have the materials, the time or the expertise to do the search, but he should give the user the telephone number of, or better yet, switch the call directly to the nearest MEDLARS search center.

There is hardly any source to which a person can turn with P3 except simply picking a hospital or physician from the classified telephone directory. A person with problems like P4 or P6 would normally seek the services of a consultant rather than a librarian; for P4 a consultant might consult literature and look up a material with specified properties, though some librarians could do this, too. If he cannot handle P4, and certainly for P6, the information officer could reasonably be expected to refer the client to an appropriate consultant.

A personnel investigation such as called for in P5 is usually started by personal contacts who know people who know people ..., etc. The librarian does not now expect, or is not now expected, to contribute vitally to such an investigation, but his 1980 counterpart will have the opportunity to do so.5

5A unique, early experiment in using libraries as community information centers is the Sheffield Free Public Library System, Sheffield, England. For details see (19).
Suppose that, in addition to "socially acceptable or neutral" questions like P1 - P10, the information officer receives questions that lead him to suspect antisocial, criminal, revolutionary or otherwise destructive intent by the questioner. Of course, the information officer, as an individual, employs his personal value system in judging these questions and interests. If his value system is not consistent with that of his supporters, and the general social group in which it is embedded he is likely to be replaced, or the information service will not remain viable. Conflicting social groups might develop competitive information systems, and through a natural system of checks and balances a larger information system emerges.

This important question requires separate discussion, which was already partly begun by H. G. Wells (23). It should be kept in mind that, in general, there exists more than one referential consulting network. This gives a user options about where to turn first.

A person who can give users with any question resembling the above minute sample some useful first lead toward an answer would be a most important professional in the community. Is it suggested that librarians of the future be expected to discharge these great responsibilities? Yes, though they should perhaps, no longer be called librarians to dispel any association with users' and librarians' own past images of their profession. They might be called "information officers", "general community advisors" or something like that. To discharge such responsibilities at a high level and standard, they
would have to be selected and educated to possess adequate qualifications, mainly an advanced liberal arts background; they would have prestige and pay scales comparable to those of other professional consultants in law, medicine, engineering, etc.

Just what, however, is the nature of this referential consulting task? How can its performance be evaluated? What resources are necessary to perform it well? In what follows we try to answer the first two questions, deferring the last to other papers.

Let us simplify our discussion by restricting the referential consultant's role entirely to question-answering. This includes question-negotiation. But it postpones for another study the even more significant role of helping selected people seek the questions to which he can provide answers. The referential consultant, henceforth denoted by R, can draw on three main resources in answering questions:

(I) his own understanding and memory

(II) his auxiliary memories and means of access to them

(III) his colleagues who are themselves referential consultants.

To fix ideas, imagine a community, such as all residents of a township on a given date, from which arises a stream of questions of unlimited variety. Select any question from this stream at random. Call it q. Suppose that it is forwarded to R. If q falls into R's expertise, he may try to answer it directly, drawing on resource (I). In this regard R acts not only as a referential consultant but as an
expert consultant. In other words, every R is also an expert consultant to some degree for some class of questions.\footnote{Although the problem of expertise might raise several interesting questions, we do not stress it in this paper. We treat R as a specialist only secondarily.}

If R cannot rely on resource (I) alone but believes that he can find (or check) the answer with the help of resource (II), he does so. By auxiliary memories we mean an abstraction of the resources consisting of what is now his filing cabinet, a collection of reference books at his fingertips, his personal library including old notes, address books, all kinds of directories, tests, papers, etc.; also included is the nearest non-personal library, such as the departmental library (in a university, research center or other organization) located within 100 yards of his office; the next larger "regional" library (of which his departmental library may be a branch). Indeed the entire international network of library resources to which he has access (at least through conventional inter-library loans) is indirectly part of the resource (II). For any particular R, the Library of Congress, of course, can hardly be called his auxiliary memory; he shares it with millions of others. And we are justified in calling it an aid to memory only to the extent that he recalls enough of the title, author, or subject-headings of a book he remembers to contain the answer to q. In other words, viewing the use of a library as an aid to memory is consonant with a large number of uses to which libraries are put: known-item searching for items other than those recommended by colleagues or cited in other documents. Even the latter
can often be interpreted as an aid in recall. So, every R is to some degree also a reference librarian as well as a literature searcher/analyst for some class of questions. It is this use of resource (II) that we stress most in this paper.

If it takes R too long, too much effort, or if R judges it unlikely that he will answer q with the help of resources (I), or (II), he resorts to "buck-passing". This is most important. It is not meant to have any negative connotation this colloquialism may imply. It takes considerable wisdom by R to exercise good judgment about whether and just when he ought to refer the question to someone else. Thus, every R is also to some degree a buck-passer for some class of questions.

We assume that no R engages in research. If the answer to q is "not known" either in the recorded literature or within the community of R's, then q may be referred to an outside community of researchers. By saying that the answer to q is known we mean that it can be looked up, recalled, retrieved -- that q has previously been answered -- and that it need not be deduced or inferred.

What about questions asking for advice, opinion, stimulation, for education, decisions, for service, like P6? Or questions revealing (or concealing) illness, confusion, ignorance, malice, need, like P7? Responses to such questions differ sharply from "answers" in the sense meant above. Let us suppose, as in the case of questions requiring research, that such questions are referred by any R in the community

7For more comments on the effectiveness of question referral, see (3).
of referential consultants to someone in an outside professional community of doctors, lawyers, teachers, ministers, public servants, social workers, businessmen, etc. The important qualification required of an R is the ability to recognize when to refer a question and good judgment about where to refer it.

This means that the referential consultant is a very responsible professional. Though he may also be a specialist he must be, first and foremost, a generalist. He should be broadly and deeply educated in "advanced liberal arts", experienced in and dedicated to public service with mature, sound judgment concerning the wise use of resources (I) – (III). He need not be a scholar, researcher, innovator, teacher, nor what is now a professional librarian. He would constitute a new breed of professional, a pillar of his community, a highly valued (and paid), esteemed and essential leader of that community.

3. A First Mathematical Model\(^8\) for an Idealized Referential Consulting System: A Chain Organization

Consider a community of \(n\) referential consultants. Label them \(R_1, R_2, \ldots, R_n\). Suppose that the randomly selected question \(q\) always

\(^8\) We use the term "model" not to describe more simply an observed entity, nor to depict an ideal way of performing a function, but to formulate and analyze designs for the referential consulting function. The virtues and limitations of mathematical thinking are well known: clarity at the cost of oversimplification, insight at the price of exact applicability, stimulation for all kinds of further investigation, experimental and theoretical, in place of minutely cataloged observations and data describing existing reality. Theories can be better analyzed and compared if expressed mathematically, though existing mathematics imposes limitations on the complexity of theories. Sometimes mathematics is inappropriately used in a merely decorative manner. Candidly, we create and analyze models because it is exciting; models are the breeding ground of intellectual problems.
reaches $R_i$ first. This assumption is weakened in more refined models. To reduce verbiage, let $K_i$ denote the event that $R_i$ knows the answer to $q$, using resources (I) or (II). Let $A_i$ denote the event that $R_i$ produces an answer to $q$ within a certain time after $q$ reaches him, having used resources (I) or (II). Also, $\overline{K}_i$ and $\overline{A}_i$ stand, respectively for the events of $R_i$ not knowing and not answering $q$.

Next, we characterize $R_i$ by the following variables: $a_i$ is the probability of $K_i$; $b_i = 1 - a_i = \text{Prob}(\overline{K}_i)$; $p_i$ is the conditional probability of $A_i$ given $K_i$, and

\[
p_i = 1 - p_i = \text{Prob}(\overline{A}_i | K_i)
\]

\[
p'_i = \text{Prob}(\overline{A}_i | \overline{K}_i)
\]

\[
q'_i = \text{Prob}(\overline{A}_i | \overline{K}_i)
\]

We characterize the system as follows: $v$ is the utility of an acceptable answer to $q$, averaged over all $q$; $c$ is the cost to the querist of an unacceptable* answer to $q$, averaged over all $q$; $V$ is the total net utility of the system per question, averaged over all questions.

Assumption 1. $a_i = a + (i-1)e$, $i=1,...,n$. If $R_i$ does not answer $q$, he refers it to $R_{i+1}$ for $i=1,...,n-1$.

Assumption 2. $p_i = p'_i = p$ for $i=1,...,n$.

Assumption 3. The conditional events $A_1 | K_1$, $A_2 | K_2$, ..., $A_n | K_n$ are statistically independent.

*An unacceptable answer is one which is either false or insufficient. Answers which are made unacceptable due to extensive delay are not dealt with here.
The first assumption implies a linear chain organization shown in figure 1.

\[ q \rightarrow R_1 \rightarrow R_2 \rightarrow R_3 \rightarrow \cdots \rightarrow R_n \]

Figure 1: A chain network

If \( R_1 \) "passes the buck", it is only to \( R_2 \), who is more likely to know the answer to \( q \) than is \( R_1 \) by an amount \( e \). Similarly, \( R_2 \) passes the buck to \( R_3 \), and \( R_n \), the smartest of the group, is at the end of the line.

Given:

\[
\begin{array}{c|c|c}
A_i & A_i & \bar{A}_i \\
\hline
K_i & p_i & 1-p_i \\
\bar{K}_i & 1-p'_i & p'_i \\
\end{array}
\]

Figure 2: The Contingency Table
"Knowing" and "Responding" to Questions

The assumption is best seen in the contingency table of fig. 2. The top left cell is the event that \( R_1 \) answers \( q \) (using resources (I) and (II)), given that he "knows" the answer. Its probability should be fairly high perhaps 0.90. The bottom right cell is the event that \( R_1 \) doesn't answer \( q \), given that he doesn't know the answer. Its probability should also be quite high, and setting it equal to \( p_i \) is not implausible. This assumption simplifies the mathematics.

The third assumption states that \( R_1, R_2, \ldots, R_n \) do not influence one another in their abilities to answer questions and in their referral judgments.

\(^9\text{Recall that this means his being able to answer the question from memory or by looking in the library resources available to him. We stress, of course, the latter in this paper.}\)
\[ b_1(1-p) = P(\overline{R}_1)P(A_1|\overline{R}_1). \] Note that if \( R_1 \) does not know the answer and responds, the answer is taken to be unacceptable to the querist, with penalty \( c \).

The events \( A_1K_1 \) and \( A_1\overline{R}_1 \) thus terminate buckpassing, while either event \( \overline{A}_1K_1 \) or \( \overline{A}_1\overline{R}_1 \) means that \( R_1 \) refers the question to \( R_2 \). The probability of \( \overline{A}_1\overline{R}_1 \) or \( \overline{A}_1\overline{R}_1 \) is:

\[
P(K_1)P(A_1|K_1) + P(\overline{R}_1)P(\overline{A}_1|K_1) = a_1(1-p) + (1-a_1)p = a_1(1-p) + b_1p
\]

This probability, or something like it, occurs so often that we call it \( P_1 \), the probability of passing the buck, with \( k=1 \). If the buck-passing sequence terminates with no answer, the utility of that state is taken to be 0, with \( V=0 \).

**Acceptable answer**

- Utility = \( a_1pv \)

**Unacceptable answer**

- Utility = \( b_1(1-p)c \)

**No answer, refer; utility = 0**

\[ P_1 = a_1(1-p) + b_1p. \]

**Acceptable answer**

- Utility = \( a_2pP_1v \)

**Unacceptable answer**

- Utility = \( b_2(1-p)P_1c \)

**No answer, refer; utility = 0**

\[ P_2 = (a_2(1-p) + b_2p)P_1. \]

**Acceptable answer**

- Utility = \( a_3pP_2P_1v \)

**Unacceptable answer**

- Utility = \( b_3(1-p)P_2P_1c \)

**No answer, refer; utility = 0**

\[ P_3 = (a_3(1-p) + b_3p)P_2P_1. \]

**No answer, refer; utility = 0**

\[ P_4 = (a_4(1-p) + b_4p)P_3P_2P_1. \]

**No answer, refer; utility = 0**

\[ P_5 = (a_5(1-p) + b_5p)P_4P_3P_2P_1. \]

**Figure 4**

A Diagram for the Buck-Passing Process
Let \( k \) be the number of times the "buck has been passed" before the querist gets a response, \( k=0,1,2, \ldots, n-1 \). Thus if \( R_1 \) answers \( q \), \( k=0 \). If he does not, but \( R_2 \) does, then \( k=1 \). Let \( t \) be the total average time (say in hours) elapsed between \( R_1 \)'s receipt of \( q \) and the delivery of an answer. If \( T \) is the average time it takes \( R_i \), for any \( i \), to consult resources (I) and (II) until he provides an answer or "passes the buck" to \( R_{i+1} \) then \( t=(k+1)T \). It would be eminently reasonable to assume that the utility of the answer decreases with \( t \), perhaps as \( v/t \), or as shown in fig. 3; and the cost may increase with \( t \), perhaps as \( ct \).

![Utility of answer with delay of t](image)

Figure 3
A possible relation between utility and response time

To make a first analysis mathematically tractable however, we make the following assumption.

Assumption 4: The utility of an acceptable answer and the cost of an unacceptable answer does not vary with \( t \), the time it takes to deliver it.

We can now derive a simple expression for \( V \). The expected net utility of an answer from \( R_1 \), with \( k=0 \), is \( a_1pv - b_1(1-p)c \), because \( a_1p \) is the joint probability of \( A_1 \) and \( K_1 \), being \( P(K_1)P(A_1|K_1) \), and
Figure 4 shows the general calculation procedure. The solid circles stand for terminal states and the hollow circles for "pass the buck" or referral states. The probability $P_i$ of $R_i$ being in a referral state, meaning that $R_i$ "passes the buck" to $R_{i+1}$, is

$$P_i = a_i (1-p) + b_i p.$$ Hence, the probability of $R_{i+1}$ being the first to answer $q$ acceptably, without passing the buck, is

$$\prod_{j=1}^{i} P_{j}(A_{i+1}, K_{i+1}) = \prod_{j=1}^{i} P_{j}P(A_{i+1} | K_{i+1})P(K_{i+1}) = \prod_{j=1}^{i} P_{j}P_{i+1},$$ since $p_{i+1} = p$.

The (positive) expected utility is therefore,

$$a_1 p v + a_2 p P_1 v + a_3 p P_1 P_2 v + \ldots = p v \sum_{i=1}^{n} a_i \prod_{j=0}^{i-1} P_j,$$

with $P_0 = 1$.

Consequently,

$$V = p v \sum_{i=1}^{n} a_i \prod_{j=0}^{i-1} P_j - (1-p) c \sum_{i=1}^{n} b_i \prod_{j=0}^{i-1} P_j = p v \sum_{i=1}^{n} \left[ a_i - (1-p)(c/pv) b_i \right] \prod_{j=0}^{i-1} P_j. \qquad \text{(1)}$$

For assumption (1) we have

$$P_j = [a + (j-1)e] (1-p) + [1-a-(j-1)e] p$$

$$= a (1-p) + p (1-a) + (e-2ep)(j-1)$$

$$= P_1 \{ 1 + [e(1-2p)/P_1](j-1) \} \qquad \text{(2)}$$

This formula was evaluated with the help of a computer program. The results are plotted in Figures 6 and 7.
Figure 6
The coefficient of utility vs probability of providing acceptable service.

Figure 7
The coefficient of penalty vs probability of providing acceptable service.
The results show that for small values of $a$ and $e$, the coefficient of $v$ increases slowly with $p$ up to about $p = .5$, then rises sharply; the coefficient of $c$ decreases slowly with $p$ up to about .5, then drops sharply. For a value of $p$ sufficiently close to 1, therefore, the coefficient of $v$ exceeds the coefficient $c$ by enough to make $V$ positive. That is, in a larger network of referential consultants, under the conditions of this first model, the expected net utility is favorable if each consultant, though he may have a very small chance of answering questions, can very reliably refer it to a colleague whose chances are a little larger.

We express $V$, for any given values of $p$, $a$, and $e$ as $V = (CV) \cdot v - (CC)c$. It is instructive to examine the ratio $V = \frac{CV}{(CC)c} \cdot \frac{V}{c} - 1$. Clearly, $V$ is positive (and large) to the extent that $\frac{V}{(CC)c}$ is positive and large: i.e., to the extent that $\frac{CV}{CC}$ exceeds $\frac{c}{V}$. Let us therefore plot the ratio $\frac{CV}{CC}$ as a function of $p$ for a few values of $a$ and $e$. Note that the $\ln \frac{c}{V}$ is usually a positive quantity, because $c$, the penalty of a wrong answer, generally exceeds $v$, the utility of an acceptable answer. We can denote $\ln \frac{c}{V}$ by the dotted line in Figure 8.

The condition that the network results in useful service translates in Figure 8 into the condition that the curve representing the network must be above the dotted line. This can only happen when both $a$ and $p$ are sufficiently large. To satisfy this condition, the larger $a$, the less $p$ has to be, and the larger $p$, the smaller $a$ has to be.
Figure 8
4. A Second Mathematical Model: A Network of Referential Consultants

Even so simple a model as sketched in the previous section gives rise to some moderately complex formulas. These are more easily evaluated by computer than by analytic approximations, although the latter has been done for the formula of section 3. But the model is more interesting for the extensions to which it can easily lead than for its own sake.

The simplest and most interesting extension is to drop assumption 1. In its place we introduce a matrix of $n^2-n$ variables. Let $c_{ij}$ be the probability that if $R_i$ does not answer $q$, then he refers $q$ to $R_j$. In other words, instead of passing $q$ to a specific $R$, the choice of $R$ is now random. Clearly $c_{ii}=0$, for all $i$, and $\sum_{j=1}^{n} c_{ij}=1$. The query still goes to $R_1$ initially. The probability, $Q_1$, that it is answered acceptably after one referral, at $k=1$, is

$$Q_1 = \frac{c_1 P(K_2)P(A_2|K_2) + c_{13} P(K_3)P(A_3|K_3) + \ldots + c_{1n} P(K_n)(A_n|K_n)P_1}{P_1},$$

where $P_1$ is the probability that $R_1$ refers $q$; it is $a_1(1-p) + (1-a_1)p$.

This is

$$P_1 \sum_{j=2}^{n} c_{ij}a_jp_j = P_1 \sum_{j=1}^{n} c_{ij}a_j$$

under the remaining assumptions, namely that $p_j=p$ for $j=1, \ldots, n$.

The probability that it is answered unacceptably after one referral is $Q'_1 = P_1(1-p) \sum_{j=1}^{n} c_{ij}b_j$, where $b_j=1-a_j$.

The probability, $Q_2$, that $q$ is acceptably answered only after two referrals, with $k=2$, is the probability that $R_1$ refers it to $R_i$ for some $i$, which is $P_1 c_{ii}$ and that $i$ refers it on to $R_j$ for some
j, who answers it. The resulting probability is \( \sum_{i} P_{i} c_{i} P_{j} \sum_{j} c_{ij} p_{aj} \),

where \( P_{i} = a_{i}(1-p_{i}) + b_{i}p_{i} \).

Thus,

\[
Q_{2} = P_{i} \sum_{j} c_{ij} p_{i} \sum_{j} c_{ij} a_{j} ; \quad Q'_{2} = P_{i} (1-p) \sum_{j} c_{ij} p_{i} \sum_{j} c_{ij} b_{j} .
\]

Next,

\[
Q_{3} = P_{i} \sum_{j} c_{ij} p_{i} \sum_{j} c_{ij} p_{j} \sum_{k} c_{jk} a_{k} ; \quad Q'_{3} = P_{i} (1-p) \sum_{j} c_{ij} p_{j} \sum_{k} c_{jk} b_{k} .
\]

The probabilities that q is acceptably and unacceptably answered after k referrals are, respectively, \( Q_{k} \) and \( Q'_{k} \), \( k = 0, 1, 2, \ldots, n-1 \).

and

\[
V = v \sum_{k=0}^{n-1} Q_{k} - c \sum_{k=0}^{n-1} Q'_{k} .
\]

With the help of a simple computer program it is now easy to study the effect of different referral matrices, C. Because the computer program is simpler than the mathematical formulas which explicitly express \( Q_{k} \) and \( Q'_{k} \), we present the listing of a PIL program. This is the "Pittsburgh Interpretive Language" available on the Michigan Time-Sharing System, and its commands are self-explanatory. There is, of course, no need, with the use of a computer program, to assume that \( a_{1} = a + ie \), and \( a_{1}, \ldots, a_{n} \) can be arbitrarily specified. The effects of different vectors \( a \) can thus also be investigated.
Figure 9

Listing of PIL program for computing the coefficient of utility, CV and the coefficient of penalty, CC as a function of the number of times the buck is passed, BP = 0, 1, ..., N-1, in any network of N referential consultants.
Notes: Symbol Corresponding Symbol Used Herein Text

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Corresponding Symbol</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>N</td>
<td>n</td>
<td>Nr. of referential consultants</td>
</tr>
<tr>
<td>A(I)</td>
<td>a_i</td>
<td>Prob. (R_i knows answer to q)</td>
</tr>
<tr>
<td>C(I,J)</td>
<td>c_ij</td>
<td>Prob. (R_i refers q to R_j</td>
</tr>
<tr>
<td>P</td>
<td>p</td>
<td>Prob. (R_i answers q</td>
</tr>
<tr>
<td>R(I)</td>
<td>p_i</td>
<td>Prob. (R_i doesn't answer, refers)</td>
</tr>
</tbody>
</table>

This program was run for a number of networks with N=5 and N=7.
The ratio of the value coefficient CV to the cost coefficient CC was calculated in relation to C/V. The results are presented in Figure 9.

They supported the following general remarks:

1) High ratios are achieved only with relatively high a_i's. Thus the vector \( \mathbf{a} = (.2, .4, .6, .8, 1) \) yielded consistently high ratios. It proved to be the most flexible for all networks.

2) The arrangement of the R_i's in the organization seems to be very important. When the "smartest" R_i was placed first in the network and was supported by a number of 'less smart' consultants, a substantially higher ratio is achieved than if he were placed further along, thus receiving q at a later stage [e.g., (.8, .03, .03, .03, .03) is better than (.03, .03, .8, .03, .03)].

3) Although [as stated in (1)] high a_i's seem to indicate greater ratios, even low ratios can be increased by increasing the size of the network.
4) Note that in many cases, for example, in the case where $a_1 = .8$, $a_i = .03$ for $C = 2, 3, 4$, $V = .743v - .104c$, which is less than $V_0 = .72v - .02c$ the net value for a network with the same $R_i$ but no one else to whom to refer. Here $V - V_0 = .023v - .084c$, which is positive if $\frac{v}{c} > \frac{.084}{.023} > 3$. This shows that if the benefit of an acceptable answer is more than three times the penalty of an unacceptable answer, then the network of referential consultants, gives greater net utility than would the smartest consultant standing alone. Of course, the utilities and penalties of an answer can hardly ever be quantitatively estimated, so that these results are to be used only as qualitative indications of the relations among key variables.

Figure 10
Tabulation by Network

I. The situation in which each $R_i$ refers $q$ to $R_{i+1}$, $i=1, ..., n$ with the exception of $R_n$ who can refer it to $R_{n-1}$. No $R_i$ may refer to himself. Two networks, one with 5 $R_i$'s and the other with 7, are considered.

<table>
<thead>
<tr>
<th>Values of $a_i$</th>
<th>CV(p) - CC(p)</th>
<th>Ratio $\frac{CV(p)}{CC(p)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>.8, .03, .03, .03, .03</td>
<td>.743v - .104c</td>
<td>7.14</td>
</tr>
<tr>
<td>.2, .4, .6, .8, 1</td>
<td>.851v - .145c</td>
<td>5.86</td>
</tr>
<tr>
<td>.03, .03, .8, .03, .03</td>
<td>.613v - .234c</td>
<td>2.62</td>
</tr>
<tr>
<td>.01, .01, .02, .03, 1</td>
<td>.605v - .334c</td>
<td>1.81</td>
</tr>
<tr>
<td>.01, .01, .05, .05, .4, .4, .8</td>
<td>.561v - .388c</td>
<td>1.44</td>
</tr>
<tr>
<td>.01, .01, .01, .02, .02, .03, 1</td>
<td>.500v - .451c</td>
<td>1.11</td>
</tr>
<tr>
<td>.01, .01, .02, .02, .03, .03, .04</td>
<td>.093v - .493c</td>
<td>.188</td>
</tr>
<tr>
<td>.01, .02, .03, .02, .01</td>
<td>.064v - .391c</td>
<td>.164</td>
</tr>
<tr>
<td>.01, .01, .02, .02, .03</td>
<td>.061v - .395v</td>
<td>.159</td>
</tr>
<tr>
<td>.01, .01, .01, .01, .01</td>
<td>.036v - .399c</td>
<td>.090</td>
</tr>
</tbody>
</table>
II. A. The situation in which \( R_1 \) passes \( q \) directly to \( R_n \)

Values of \( a_i \) | CV(p) - CC(p) | Ratio \( \frac{CV(p)}{CC(p)} \)
--- | --- | ---
.01, .01, .02, .02, .03 | .033v - .186c | .177
.2, .4, .6, .8, 1 | .918v - .082c | 11.20

B. In this variation of (A) \( n=7 \), and \( q \) is passed with probability \(.7\) to \( R_6 \) and with probability \(.3\) to \( R_7 \). The only exceptions are that \( R_6 \) always passed to \( R_7 \), and \( R_7 \) always passes to \( R_6 \).

Values of \( a_i \) | .CV(p) - CC(p) | Ratio \( \frac{CV(p)}{CC(p)} \)
--- | --- | ---
.01, .01, .01, .02, .02, .03, 1 | .831v - .168c | 4.93

III. Random buck-passing networks:

A. There is an equal chance of any \( R_i \) receiving \( q \)

Values of \( a_i \) | CV(p) - CC(p) | Ratio \( \frac{CV(p)}{CC(p)} \)
--- | --- | ---
.2, .4, .6, .8, 1 | .8675 - .118c | 7.40

B. There is an almost equal chance of any \( R_i \) receiving \( q \), but top \( R_i \)'s have slightly increased probability.

Values of \( a_i \) | CV(p) - CC(p) | Ratio \( \frac{CV(p)}{CC(p)} \)
--- | --- | ---
.01, .01, .01, .02, .02, .03 | .526 - .328 | 1.59
.01, .005, .005, .005, .005, .005, .005 | .029 - .510 | .057
.005, .005, .005, 01, .005, .005, .005 | .026 - .513 | .051
C. This is a non-uniform random referral situation according to the following given matrices:

1. \[
\begin{bmatrix}
0 & .3 & .2 & .2 & .3 \\
.3 & 0 & .3 & .2 & .2 \\
.2 & .3 & 0 & .3 & .2 \\
.2 & .2 & .3 & 0 & .3 \\
.3 & .2 & .2 & .3 & 0
\end{bmatrix}
\]

Values of \( a_{ij} \)  
\( .2, .4, .6, .8, 1 \)  
\[
CV(p) - CC(p) \quad \text{Ratio } \frac{CV(p)}{CC(p)}
\]
\[ .865v - .188c \quad 7.33 \]

2. \[
\begin{bmatrix}
0 & 1 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 \\
0 & 0 & 1 & 0 & 0
\end{bmatrix}
\]

Values of \( a_{ij} \)  
\( 2, 4, .6, .8, 1 \)  
\[
CV(p) - CC(p) \quad \text{Ratio } \frac{CV(p)}{CC(p)}
\]
\[ .671v - .193c \quad 3.49 \]

3. \[
\begin{bmatrix}
0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 \\
.6 & .4 & 0 & 0 & 0 \\
.5 & .3 & .2 & 0 & 0 \\
.4 & .3 & .2 & 1 & 0
\end{bmatrix}
\]

Values of \( a_{ij} \)  
\( .2, .4, .6, .8, 1 \)  
\[
CV(p) - CC(p) \quad \text{Ratio } \frac{CV(p)}{CC(p)}
\]
\[ .18v - .08c \quad 2.2 \]

IV. In this situation there is no buck passing at all.

Values of \( a_{ij} \)  
\( .01, .02, .03, .02, 1 \)  
\[
CC(p) - CC(p) \quad \text{Ratio } \frac{CV(p)}{CC(p)}
\]
\[ .009v - .099c \quad .091 \]
It is quite possible that in an organization, the people $R_j$ with higher values of $a_j$ also rank higher. An $R_i$ of a given rank in some organizations is more likely to pass the buck on certain questions to an $R_j$ of lower rank than to a superior. In this case, $c_{ij}$ would increase as $a_j$ decreases. This is quite unfavorable.

The model studied in the previous section is a special case of this model with $c_{i,i+1}=1$, for $i=1, 2, \ldots, n-1$, but $c_{nj}=0$ for all $j$. ($R_n$ could never refer, violating the condition $\sum_j c_{ij}=1$). In the present model, the question can get trapped in a bureaucratic cycle, and we can calculate the probability of this happening. For example, it is possible for $R_1$ to refer $q$ to $R_5$; $R_5$ could refer it to $R_9$, $R_9$ to $R_3$ and $R_3$ back to $R_1$, with $R_1$ referring it this time to $R_2$, etc. The question could eventually traverse through all the $R_i$ in all possible paths and stay in the network for an infinitely long time.

5. Toward Greater Realism

Some severe limitations of the models considered so far, beyond those formally stated before, are described next.

(a) The question was taken to be unchanged, as first presented by the querist. No provision for conversing with the querist has been made. Suppose now that any $R_i$ to whom $q$ is referred can converse with the querist. Indeed, we might include the querist -- let us call him $R_0$ -- in the network: $R_0$, $R_1$, $\ldots$, $R_n$. The question can occasionally be referred back to him. This would make sense only if we permitted each $R_i$ a fourth resource beyond (I) his memory,
his auxiliary memories (library items), (III) access to other R_j: namely, (IV) his ability to reinterpret, reformulate, substitute for q or to ask R_o questions in place of either providing an answer or "passing the buck". The latter would permit R_o to reformulate, reinterpret or substitute for q. This process may improve the quality of the question. Of course, we would need to characterize R_1 by an additional variable: d_1 = \text{Prob.}(R_1 \text{ converses with } R_o | A_1).

(b) The variables characterizing R_1, namely a_i, p_i, p'_i, d_i, were taken to be independent of the question q, and unchanging in time. Suppose that questions can be classified and graded, a job that R_1 might do. Suppose that R_1 works with m categories C_1, ..., C_m, such as specialties in which various R_i have expertise. Then, in place of a_i, we have a_{ij}, the probability that R_i can answer a question in category C_j. If R_i is an expert only in specialty C_j, then a_{ij} is high and a_{ik} for k \neq j is low. These question categories could not only aggregate questions by common subject matter, but by quality as well. Thus, a_{ij} may be low for all j if C_j is a category of poorly formulated questions, while a_{ik} may be higher for all k where C_k is a category of well-formulated questions.

We could now further specify the system by giving the a priori probabilities of a randomly chosen question falling into C_1, C_2, ..., C_m; call these r_1, ..., r_m. Let s_{ijk} = \text{Prob.} \{q is reformulated so that it is transferred from C_j to C_k, given that R_i converses with R_o\}.

For an interesting analysis of the structure of the negotiating process, see Taylor (22).
$R_0$. Hence, $d_{i\ j\ k}$ is the probability of $R_i$'s conversations with $R_0$ resulting in an answer to the revised question, given that $R_i$ did not answer it previously. Of course, there is a price for delay.

This permits us to compute the probability with which "buck passing" can upgrade the question. For lack of space this must be deferred for future investigations.

(c) The matrix $C$ embodies a referral strategy and organization. This might, however, change for questions in certain categories. The categories $C_1$, ..., $C_m$ could reflect priorities assigned to questions. A high-priority question might always immediately be referred to the $R_i$ with the highest $a_i$, and he might refer it, or fragments of it formed by him, down the line.

(d) The discussions so far have assumed only person-to-person messages (questions and answers), but no "to whom it may concern" messages. Certain questions reaching $R_1$, could be broadcast by $R_1$, with an instruction that whichever $R_i$ could readily answer $q$ should speak up. The motivation in such a system for "speaking up" would have to be at least that of the $R_i$ to whom a question was referred in the "buck passing" organization. If time is a very important factor, then this inherently parallel system may be preferable.

The models for analyzing such a system would be based primarily on a reinforcement function which greatly rewards the $R_i$ who spoke up, i.e. supplied the answer for a $q$ such that
his $a_{ij}$ is very high; moderately rewards the $R_i$ whose $a_{ij}$ is low and who does not speak up or the $R_i$ whose $a_{ij}$ is reasonable, but who converses with $R_o$; punishes the $R_i$ whose $a_{ij}$ is high and who does not speak up or the $R_i$ whose $a_{ij}$ is low and who does speak up. If no one speaks up, some reward might go to the $R_i$ who refers $q$ to an $R_k$ for whom $p_k a_{kj}$ is high.

The use of such a reinforcement schedule should result in learning, and the values of the $p_i, p_i'$ would change with time.

(e) Finally, there is the question of organizational design. We assume that requests originate randomly at various geographic locations in a community. The average time to forward a question to the nearest $R_i$ may therefore vary depending on where he is located. Perhaps there should be more than just one $R_i$, to each of whom all requests in his service area are initially forwarded.

(11) Or, perhaps having the service "areas" arranged by topic, rather than geographically, with the querist deciding upon the nearest topically specialized $R_i$ to whom to forward $q$ has high utility.*

Within the community of the $R_i$, similar questions arise. There is an additional question involving the number of $R_i$ with the same $a_{ij}$ to use redundantly, so as to handle expected loads. This is a problem in the analysis of querying networks such as studied by R. Disney (4), and an aim of organizational design is to balance idleness and congestion.

*This is currently the case for many of the questions cited earlier in this paper.
6. Some Problems for Further Research

(a) Even in the simplest model of section 3, what is the best point to enter a chain of R₁'s? If there is no basis for assigning questions to R₁'s, and if aₙ is largest, then why should not all questions go initially to Rⁿ? This would certainly be optimal from the user's viewpoint. Too, the smaller the x, where Rₓ is the one to whom q is initially submitted, the greater the probability of an unacceptable answer. The probability of the first acceptable answer occurring after the kth referral since Rₓ received q, is

\[ p \sum_{i=x}^{i-k} \prod_{j=0}^{i-1} \]

The average net utility, if q is initially received by Rₓ is

\[ V(x) = p \sum_{i=x}^{i-1} (a+(i-1)e) \prod_{j=0}^{i-1} c \sum_{i=x}^{i-1} (1-(a+(i-1)e) \prod_{j=0}^{i-1} \]

We can now seek the value of x which maximizes this expression.

(b) Suppose that a querist Rₒ poses several questions to R₁, each belonging to a subject category C_j, and each was referred by R₁ to R₁, and in each case R₁ provided an eminently satisfactory response. Rₒ will soon learn to pose any future questions in C_j directly and initially to R₁.

In practice, much more than half of all questions posed by Rₒ may fit into no more than a dozen categories, for each of which there may be an R₁ whom Rₒ has learned to contact initially, once he has classified the question himself. What advantages, then, does a referral network offer?
First, not all querists will have questions fitting into the same dozen categories. It may still be the case that well over half of the questions posed by anyone fall into a few dozen categories, and users may have learned to contact experts on these categories directly. The referral network then serves to teach users where to turn. This is important when the user population is constantly expanding, with an overwhelming and growing fraction of all users at any time being untrained, young newcomers.

Secondly, users may be less able to usefully categorize their own questions than could R\textsubscript{1}. This too, can be learned, but such learning is a continuing process for the less than half of the questions that do not fall into less than a dozen categories. Any system which will be able to handle these very many relatively rare and, hence unusual, questions is bound to be expensive, because it has to be customized. This is a very important area in need of investigation.

Third, and perhaps most important, knowledge, and, perhaps wisdom also (10), are constantly growing. This means that the categories constantly change and that the set of questions that could be answered is continually expanding. Conversations between R\textsubscript{0} and the R\textsubscript{1} he contacts, which can and should serve to teach R\textsubscript{0} which questions he might ask that he had not thought of or known to ask, play an increasingly important role. Here, a referral network offers the advantage of a multiplicity of potential teachers, and, therefore, a greater opportunity of finding treasure.
(c) The models (i.e., mathematical problems sketched and suggested here) are useful for clarifying essential concepts. They stimulate precise new ideas. They also serve as vehicles for developing methods of, and experience with, mathematical analysis. But they cannot lead to theories until they are connected with empirical or experimental data. A whole class of research problems suggested by this kind of mathematical thinking involves data acquisition.

To some extent, special libraries already perform an expanding reference function. By monitoring samples of the memoranda, telephone, and personal calls from R₁ to R₉ in such an organization, crude estimates of the matrix C can be obtained. A far better study, however, would be a controlled experiment in which a sample of questions is planted, i.e., submitted to an existing question-answering network of referential consultants R₁,..., Rₙ, such that the consultants cannot distinguish these planted questions from the ones they normally encounter. All the planted questions have definite answers. The experimenter may even know whether or not the answer to a given question is known to R₁ via library resources (II) [aids to memory]. That is, the experimenter may have chosen the question because he has seen the answer in the reference collection which is within R₁'s reach. Such an experiment may then permit us to estimate p₁, p'₁, a₁, c₁ and d₁ for i=1,..., n, and to test certain key predictions of one or another set of assumptions.

11 At least one origin-destination study of inter-office memoranda was done at IBM by Resnick et. al., but of course the content was not examined nor were many of the memoranda questions such as may be transmitted to a referential network.
(d) Another important line of experimental investigation involves categorization of the queries and specialization of the referential consultants. In our models, we have mixed three different bases for categorization: by specialty; by priority; and by question quality. There are undoubtedly more. Categorization by specialty is traditional (this is the sense of "special" in special libraries) and superficially the simplest, but it rests on a very weak theoretical foundation.

It is today no longer so important that \( R_i \) get only questions to which his library resources are specialized, because he has access to an apparatus for bibliographic control over resources beyond those which are literally within walking distance. The limitation lies in \( R_i \)'s ability to use this apparatus after the limits of his own expertise about the question are exceeded.

What probably matters most in a categorization of questions and \( R_i \)'s is the quality and priority of questions. In comparing two categorizations of a corpus of 100 questions, say \([C_1, \ldots, C_m]\) and \([C_1', \ldots, C_m']\), we might well ask which gives the greater value for

\[
\max \max_i \sum_j a_{ij} \text{ or } \sum_i \max_j a_{ij}
\]

where \( a_{ij} \) is the probability of an acceptable answer from \( R_i \) to a question in category \( C_j \) or \( C_j' \). This can be decided by data. One possible experiment to do this would be to categorize a sample of planted questions in two different ways and to broadcast with each question an appeal to all the \( R_1, \ldots, R_n \) for volunteers (to be rewarded) who can most easily and expertly answer it. The sample
of questions is carefully chosen so as to fit into a priori designed categories by definition; data about who responds to these questions, and how successfully, is then used to determine how consistently the same $R_i$ picks questions in $C_j$ and is characterized by very high $p_{i, a_{ij}}$.

(e) Another very important line of experimental investigation involves the utility and cost measures. Basically, $v$ is the amount a querist is willing to pay per question for an acceptable response delivered in the minimum possible time. This value was taken to be averaged over all questions and querists. In a categorization of questions by priority, however, each question class is characterized by a different value of $v$. Questions of the class with highest $v$ are of top priority.

Each question which enters the system is part of a submitted form with at least 3 parts:

(i) The initial formulation of the question, including at least some background and hints for the referential consultant as to what kind of answer is wanted

(ii) some indication of how much the querist values the answer, including how his utility for the answer decays with response time

(iii) data about himself, such as would relate to estimating $a_{0j}$, $j = 1, \ldots, m$

This may be done by experimenting with a sample of querists, asking them to allocate a certain sum of money given them by the experimenter over a given list of possible question-answering sources. The list might include elements like: (i) act as your
own referential consultant, using (1) and (2) for a particular library resource (II); (ii) same as (i) except for a different library resource (II); (iii) refer to R₁; (iv) refer to R₂; etc.

It is important to bear in mind that Ro can always choose between many competitive sources in getting an answer to his question. By going to R₁ he ought to be assured that R₁ could point him to at least those sources he would have known about himself. If Ro can do R₁'s job better by himself, he should, of course, do so.

(f) The cost of maintaining a referral network is likely to be high. The practicality of such a service hinges critically on the rate at which gross revenues grows relative to operating costs. Both will increase, though the service cannot be viable unless revenues grow faster than do costs. If they grow at the same rate, there must be a sizable constant difference of revenues over costs. This state can hardly be claimed to exist for current reference services. Budgets for library services are generally a small part of overhead and are the first to be cut if the total budget is reduced.¹²

Beyond the cost of maintaining the services of n referential consultants in a network are costs generated by the existence of the network itself. According to Parkinson's Law [16], the R₁,..., Rₙ will generate and send questions and answers to one another. Such messages would not have been generated if there were no network.

¹²The cost factor is one reason why industrial libraries have achieved such success. Not only do they have sufficient funds, but also they are able to assign a definite monetary value to correct information.
The volume of such message traffic may vary as $n^2$. This limits each $R_i$'s capacity to answer client-generated questions and may necessitate larger $n$ to handle a specified load.

Such internally generated communications are often considered unproductive. Many consider it unprofessional to "pass the buck". Yet, "buck passing" can be a sign of both very irresponsible or very responsible professional behavior. Referring a question is professionally very responsible when it reflects the professional's understanding of his own limitations; such a professional is much more valuable than one who never refers, unless the latter is omniscient. The very irresponsible buck passing professional can be easily discriminated from his opposite by noting that he answers very few questions adequately, and is valued so low as to be dropped.

The communications generated inside the network could be productive. They could help upgrade the organization by helping the $R_i$ teach, and learn from, one another. The measure of learning is the number of good questions $R_i$ can ask that he could not have known to ask before. Conditions for such learning to occur could be derived; a cost-effectiveness model, backed by data, can readily be set up and used to contribute to arguments for the economic feasibility of referential consulting.

7. **Conclusions**

We have argued, in this paper, for the significance of "referential consulting." This is a new type of service to be performed by a new breed of reference librarian. It resembles expanding reference functions now practiced to an extent in some
special libraries. But it goes as much beyond contemporary reference service concepts as these go beyond book delivery service concepts. In essence, referential consulting is a means of providing some kind of useful response to almost any question of importance to people in their daily lives, either by having a member of an organization of referential consultants rely on his expertise, on the library resources at his command, or on his ability to refer the question to a colleague in the organization or outside. The response may be either a direct answer, a document likely to contain the answer, or advice to go to a document or another source. A referential consultant is a very mature, learned, responsible "information officer", an essential, highly valued professional in the community.

Some investigators (1) claim that today's reference librarians already have the status of professionals like doctors, engineers, lawyers, etc. The fact that some librarians already believe this, is a hopeful sign that referential consulting is feasible. A small number of practicing librarians who were casually interviewed stated that questions such as the ten examples of section 2 reach them frequently. Though they were not prepared in library schools to answer them, on the ground that such questions are outside a reference librarian's responsibility, they rarely turn them down. Indeed, they can often give the user acceptable answers.

The very fact that libraries do contain the necessary resources to provide such services reaffirms the need for expanded referential consulting in order to utilize these library resources more effectively and to perform library functions more satisfactorily. We have redefined
the function of a library to be:

to maximize the greatest potentially attainable effective and efficient social utilization of documented knowledge.

Hopefully the models of referential networks presented here will stimulate ideas and actions to be more nearly consonant with this definition.

We have assumed that there is a latent need for such a referential consulting service. This need will be made manifest if people will use, request and pay for the referential consulting service if it is offered. We therefore recommend -- urge -- the creation of such a service. We predict that it will create demand and, in time, pay for itself. Part of this recommendation is addressed to library science educators, to educate some of the high-level professionals capable of serving as referential consulting services, to educate innovators, scholars and scientists who can advance and develop the concept and the underlying rationale and discipline.

We have begun an explication of the "referential consulting" concept. Though crude, it has proved capable of clarification, of leading to further ideas and of providing some results. We have derived conditions under which various forms of the referential consulting organization lead to maximum or specified expected net utility.

Even our first step toward an explication of the concept of referential consulting service reveals a number of exciting intellectual puzzles. To investigators inclined toward mathematical
thinking, they can be a challenge inviting further exploration. To investigators inclined toward observation, they can suggest useful, empirical or experimental studies. One study, if done well, can lead to another and, in time, toward a theory of an important aspect of librarianship.

Acknowledgment: Many of the ideas in this paper benefited from stimulating discussions with members of our information science research team: C. Drott, D. Koson, R. Palmer, H. Quenemoen, B. Segur R. Tagliacozzo, M. Thall, Wm. vanLoo, H. Hamburger, and T. Slavens, a colleague in the Library Science Department. Special thanks go to W. Lehmann for her assistance in the preparation of the paper and in checking the calculations, and to A. Tars for his valuable assistance in running the computer programs.
References


ABSTRACT

We explicate the concept of a network of consultants to help people obtain answers to day-to-day questions for which answers are known. Each member of such a network is viewed to have three capabilities: 1) answering questions directly from his own memory; 2) answering questions with the aid of library resources; 3) referring the question to a member of the same network or to an expert outside it. We derive conditions involving each member's ability to choose appropriately among the three alternatives. If he judges correctly concerning when and where to refer a question, such a network, suitably organized, has greater net utility than does a reference librarian by himself.

We argue that the existence of such a network makes possible a much broader range of information service to the community than is afforded by traditional reference librarianship.