In studying college effects, an input-output model is commonly used in which student input is controlled by using regression analysis to compute an "expected" output. The part correlation of the college environment variable and the output with input variance removed only from the output is interpreted as a measure of the college effect. However, this is not the most useful procedure that may be used since part (or partial) correlation may severely underestimate the magnitude of the true college effect. Interpreted within a causal model, partial regression coefficients appear to be a generally more satisfactory measure of college effects. Four models are used to illustrate the advantages of using partial regression coefficients in a causal framework. Another advantage in using these coefficients is that they have greater stability across different units of measurement. (Author)
Analyzing College Effects: Correlation vs. Regression

Charles E. Werts and Donivan J. Watley
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Abstract

In studying college effects, an input-output model is commonly used in which student input is controlled by using regression analysis to compute an "expected" output. The part correlation of the college environment variable and the output with input variance removed only from the output is interpreted as a measure of the college effect. However, this is not the most useful procedure that may be used since part (or partial) correlation may severely underestimate the magnitude of the true college effect. Interpreted within a causal model, partial regression coefficients appear to be a generally more satisfactory measure of college effects. Four models are used to illustrate the advantages of using partial regression coefficients in a causal framework. Another advantage in using these coefficients is that they have greater stability across different units of measurement.
A commonly used procedure in studying college effects involves an input-output model in which student input is controlled by using regression analysis to compute an "expected" output (e.g., Astin, 1963, 1964; Thistlethwaite and Wheeler, 1966). The correlation of a school environment variable with the residual output (i.e., actual minus "expected" output) is interpreted as a measure of the school's influence on the output. Although sometimes labeled a partial correlation, it is more accurately described as the part correlation (McNemar, 1962, p. 167) of the school with the output variable when the influence of the input variables has been removed from the output.

A potentially serious interpretational problem is that part correlations may severely underestimate the magnitude of the true college effect. This possibility was noted previously by Richards (1966):

"suppose that a real effect of small colleges is to encourage students to develop warm personal relationships with the faculty, and that the socio-economic status of college students has no inherent relationship to their tendency to develop warm relations with the faculty. Suppose further that there is a strong tendency for small colleges to attract rich students. Over a sample of colleges varying in size, the tendency of rich students to attend mainly small colleges will produce a positive correlation between socio-economic status and developing warm relations with the faculty, but the correlation between college size and developing warm relations will not be increased by the fact that small colleges attract rich students. Consideration of the basic formula for computing partial correlations makes it clear that, in these circumstances, controlling for differences in socio-economic status will tend to reduce the correlation between college size and the extent to which students develop warm relations with the faculty, and therefore to obscure the true causal relationship (p. 381)."

The logic of Richards' argument applies equally to part and partial correlations. How then should the problem of part correlations underestimating the size of the college effect be resolved? As Richards presents
the problem, a researcher seems to have two alternatives: he either controls for student input characteristics or he does not. Astin (1968) rejected Richards' implication that, given these alternatives, it might be better not to control for student input: "As long as the student is used as the unit of analysis in the control of input characteristics, any environmental effects...will not be 'obscured' by the statistical adjustments for input differences that are made in regression analysis or actuarial tables. It is true that the actual magnitude of the effect may be underestimated somewhat, but this is a necessary consequence of the partial confounding of student input and college environmental variables (p. 430)." However, even a moderate degree of underestimation may seriously obscure the college effect because the effect is likely to be relatively small and fragile across a wide sample of colleges. Only a small association attributable to the college influence is expected because: (a) students usually enter college with relatively stable attitudes and skills; (b) a single college variable seldom measures more than one aspect of the total college effect; and (c) any one aspect of the college may affect only a limited number of students.

The work of Blalock (1960, 1961, 1964, 1965, 1967) and Tukey (1954) indicates that a partial regression procedure is superior to part (or partial) correlation because controls for input may be introduced without underestimating the magnitude of the college effect. Their argument emphasizes the inherent need to interpret all statistics within a theoretical model that is relevant to the problem studied. The advantages of using regression coefficients, rather than part (or partial) correlations, to study college effects will be evaluated from the standpoint of four hypothetical models of "reality."

Model I

The situation presented by Richards is one that involves a developmental
sequence of the form \( A \rightarrow B \rightarrow C \). The variables corresponding to \( A \), \( B \), and \( C \) are socioeconomic status (SES), size of the college (SIZE), and warmth of the student relationship with the faculty (WARMTH). Specifically, the following relationships are implied: (1) SES directly affects SIZE, i.e., affluence influences the size of the college a student attends; (2) SIZE directly affects WARMTH, i.e. smallness produces warmer student-faculty relations; and (3) SES influences WARMTH, only indirectly through the mediating variable, SIZE. This model is shown in Figure 1.

![Diagram](image)

**Fig. 1** Input variable (SES) influences the college environment variable (SIZE), which in turn influences output (WARMTH).

In order to analyze these relationships in a causal model, it must be assumed that variables outside the system do not directly affect more than one of the three variables included. In essence, this assumption ensures that outside variables do not affect the correlations among SES, SIZE, and WARMTH. If it is known that an outside variable does influence more than one of the variables included, that variable should be brought into the causal model.

One of the advantages of using regression coefficients instead of part correlations in the \( A \rightarrow B \rightarrow C \) model is this: a control for \( A \) ordinarily reduces the magnitude of the partial correlation \( r_{BC,A} \), although a control for \( A \) does not affect the expected value of the corresponding regression coefficient, \( b_{BC,A} \). In order to illustrate this point, let us assume that the strengths of both the SES-SIZE and the SIZE-WARMTH relationships are
completely nonspurious correlations of +.50, and that all variances equal unity. Since assumption (3) necessarily (Simon, 1954) implies a zero partial correlation ($r_{AC,B}$) of SES with WARMTH when SIZE is controlled, the formula for partial correlation can be used to calculate the zero order correlation ($r_{AC}$) of SES with WARMTH. The zero order correlation, in turn, can be used to calculate the part correlation ($r_{B(C,A)}$) of SIZE with WARMTH when the influence of SES is removed from WARMTH as shown below.

$$(1) \quad r_{AC,B} = 0 = \frac{r_{AC} - r_{AB}r_{BC}}{\sqrt{(1 - r_{AB}^2)(1 - r_{BC}^2)}}$$

where $A = \text{SES}$

$B = \text{SIZE}$

$C = \text{WARMTH}$

$$\text{(2) } r_{AC} = r_{AB}r_{BC} = 0$$

$$\text{(3) } r_{AC} = r_{AB}r_{BC} = .50 \times .50 = .25$$

$$\text{(4) } r_{B(C,A)} = \frac{r_{BC} - r_{AB}r_{AC}}{\sqrt{(1 - r_{AC}^2)}} = \frac{.50 - (.50)(.25)}{\sqrt{(1 - .25^2)}} = .387$$

Removing the influence of SES from WARMTH reduces the correlation of SIZE with WARMTH from .50 to .39. In Model I the part correlation (.39) clearly underestimates the true strength (.50) of the SIZE-WARMTH relationship. If additional input variables not directly influencing the output were partialled out of the output, it would be expected that the part correlation might become even smaller. The relative reduction would depend upon the strength of the relationship between the input and the output variables (Blalock, 1964). It appears that the college effect is likely to be underestimated in the typical college effects study because many input variables usually are controlled. The corollary is that when a number of student input variables are controlled a small part correlation may not imply a small college effect.

On the other hand, Blalock (1964) has shown that regression coefficients
will provide a more interpretable estimate of college effects than will part or partial correlation. In Model I, the zero order regression coefficient ($b_{CB}$) for estimating WARMTH from SIZE is .50 ($b_{CB} = r_{BC} \frac{\sigma_C}{\sigma_B}$). Framed within this causal model, the coefficient signifies that if the size of the college decreases one size unit, then the warmth of student-faculty relationships will increase one-half warmth unit. The regression coefficient is a measure of the SIZE-WARMTH relationship that is interpretable in an if-then sense (if SIZE changes, then WARMTH will change in a determinate way); and it represents a hypothetical measure since it does not indicate how much SIZE actually changes. In practice, the researcher must give persuasive reasons for supposing a particular regression coefficient to be a measure of a particular if-then relationship. With SES controlled, the partial regression coefficient of WARMTH on SIZE is equal to the zero order regression coefficient ($b_{CB} = .50$):

$$b_{CB, A} = \frac{r_{BC} - r_{AC} r_{AB}}{1 - r_{AB}^2} \cdot \frac{\sigma_C}{\sigma_B} = \frac{.50 - .25(.50)}{1 - (.50)^2} \cdot \frac{1.0}{1.0} = .50$$

Thus the partial regression coefficient is numerically equal to the "true" relationship. The size of the regression coefficient $b_{CB, A}$ in Model I is not affected by controls for an antecedent input variable that does not directly influence the output.

Thus in a developmental sequence such as that shown in Model I, i.e. $A \rightarrow B \rightarrow C$, Richards' criticism of part or partial correlation as an estimate of the college effect is valid; however, his criticism does not apply to regression coefficients. The use of partial regression coefficients avoids ascribing to the college effect variance that may largely be due to input (Astin, 1963).
Regression coefficients are advantageous to an understanding of causal relationships because their behavior can be compared more safely than the behavior of correlation coefficients (Tukey, 1954; Blalock, 1964). Thus the mere reduction of a partial correlation is difficult to interpret. As Blalock (1961) noted: "The numerical value of a correlation coefficient may be reduced not only because a confounding influence has been controlled, but it may also be altered because we have decreased the total variation in the independent variable relative to that in other causes of the dependent variable (p. 87)."

In Model I, it can be shown, for example, that when SIZE is controlled the partial regression coefficient of WARMTH on SES is zero, the same as the "true" relationship:

\[
b_{CA,B} = \frac{r_{AC} - r_{AB} \cdot BC}{1 - r_{AB}^2} = \frac{.25 - .50(.50)}{1 - (.50)^2} = \frac{.00}{1.0} = .00
\]

Therefore, one can correctly deduce from this coefficient that SES does not directly influence WARMTH. In other words, granting the assumptions about linearity and outside variables, if there were a three-variable sequence in which A were antecedent to B, and A and B antecedent to C, and the regression coefficient of C on A with B controlled turned out to be zero, one could reasonably deduce that the total influence of A on C was mediated through variable B. It could be concluded, therefore, that the association of A with C in Model I is not spurious but results from the indirect (i.e., mediated) influence of A on C via B.

Model II

Although the part correlation yields misleading results if the true model is like Model I, part correlation, partial correlation, or partial regression coefficients will lead to correct deductions about the college
effect if the true model is like Model II. In this model, input influences both the college environment variable and the output, but the college itself does not influence the output.

**Fig. 2.** The input variable influences both the college environment and the output variable.

For the fictitious data in Figure 2, the correlation of the college with the output variable can be calculated since the partial correlation of college with output (input controlled) equals zero (Simon, 1954):

\[ r_{BC,A} = 0 = \frac{r_{BC} - r_{AB}r_{AC}}{\sqrt{1 - r_{AB}^2}\sqrt{1 - r_{AC}^2}} \]  
\[ \text{or } r_{BC} = r_{AB}r_{AC} = .50 \times .50 = .25 \]

The part correlation \( r_{B(C,A)} \) of college with output when the influence of input is removed from output will, like the partial correlation \( r_{BC,A} \) and the partial regression coefficient \( b_{BC,A} \), be zero.

However, the use of partial regression when attempting to build a complete causal model would lead to more accurate conclusions than would partial correlation. In Model II, for example, the partial regression coefficient of output on input with the college variable controlled is arithmetically identical to the zero order regression coefficient of output on input.

\[ b_{CA,B} = \frac{r_{AC} - r_{AB}r_{BC}}{1 - r_{AB}^2} \cdot \frac{\sigma_C}{\sigma_A} \]

\[ \text{but since } r_{BC} = r_{AB}r_{AC} \]
(3) \[ b_{CA.B} = \frac{r_{AC} - r_{AB}(r_{AC}r_{AB})}{1 - r_{AB}^2} \cdot \frac{\sigma_C}{\sigma_A} \]

\[ \frac{\sigma_C}{\sigma_A} = \frac{r_{AC}(1 - r_{AB}^2)}{1 - r_{AB}^2} \]

(4) and since \[ b_{CA} = r_{AC} \frac{\sigma_C}{\sigma_A} \]

(5) \[ b_{CA.B} = b_{CA} \]

Whereas the partial regression coefficient leads to the correct conclusion that no part of \( b_{CA} \) is spurious, the corresponding difference between \( r_{AC} \) and the part correlation \( (r_{A(C,B)}) \) is meaningless.

**Model III**

In an actual college effects study it is sometimes more reasonable to expect Model III (Figure 3), which is a combination of Models I and II. In Model III student input characteristics have direct influence on both the college environment variable and the output, and the college, in turn, has some influence on the output.

![Diagram of Model III](image)

**Fig. 3** Input variable influences both college environment and output variable; college variable also influences output.

For the fictitious data in Figure 3, the partial regression coefficient of output on college with input controlled is:

\[ b_{CB.A} = \frac{r_{BC} - r_{AC}r_{AB}}{1 - r_{AB}^2} \cdot \frac{\sigma_C}{\sigma_B} = \frac{.75 - .75(.50)}{1 - (.50)^2} = .50 \]

The use of part correlation, however, would overestimate the college effect.
(.50) in this example. With student input characteristics removed from the output, the obtained part correlation is .57:

\[
\rho_{B(c,A)} = \frac{.75 - .75(.50)}{\sqrt{1 - .75^2}} = .567
\]

The partial regression coefficient of output on input with the college variable controlled is again easily interpreted:

\[
b_{C.A.B} = \frac{r_{AC} - r_{AB}r_{BC}}{1 - r_{AB}^2} = \frac{.75 - .75(.50)}{1 - (.50)^2} = .50
\]

Further light can be shed on Model III by interpreting the correlations in terms of path coefficients (Duncan, 1966). In path analysis the zero order correlation of the college variable with the output \((r_{BC} = .75)\) in Model III consists of two parts: the association due to the direct influence of the college on the output, and some spurious association due to the common antecedent factor, student input characteristics. The correlation of input with output also consists of two parts: the association due to the direct influence of input on output, and the association due to the indirect, influence of input on output mediated through the college variable. The spurious component in \(r_{BC}\) is equal to \((r_{BC} - b^*_{CB.A})\) where \(b^*_{CB.A}\) is the standardized partial regression coefficient. The component of \(r_{BC}\) ascribed to the direct influence of the college on the output is \(b^*_{CB.A}\) (numerically equal to \(b_{CB.A}\) only because unit variances were assumed). That part of \(r_{AC}\) ascribed to the direct influence of input on output is equal to \(b^*_{CA.B}\); whereas the part due to the indirect influence of input on output mediated through the college variable is equal to \((r_{AC} - b^*_{CA.B})\). The equations with standardized partial regression coefficients are the "normal" equations of variance analysis:

\[
r_{BC} = b^*_{CB.A} + b^*_{CA.B} r_{AB} = .50 + .50(.50) = .75
\]
\[ r_{AC} = b_{CB,A}^* r_{AB} + b_{CA,B}^* = .50(.50) = .50 = .75 \]

The calculations shown are those used to compute \( r_{BC} \) and \( r_{AC} \) for the Model III example, which is a combination of the examples used in Figures 1 and 2.

Model IV

Often the investigator may not be justified in ascribing the input-college correlation solely to the influence of input on the college variable, as was assumed in Models I, II, and III. When this assumption is not warranted, Model IV (Figure 4) results; the double-headed arrow in Figure 4 indicates that the college and input variables are correlated for unknown reasons. For the fictitious data in Figure 4, the same partial regression coefficients are found as were previously calculated in Model III: \( b_{CB,A} = .50 \) and \( b_{CA,B} = .50 \). When input and college variables are correlated because they essentially measure the same underlying factor, any interpretation of \( b_{CB,A} \) and \( b_{CA,B} \) is unwarranted without further assumptions.

![Diagram](Fig. 4) Both the college and the input variable influence output; college and input variables correlated for unknown reasons (indicated by curved arrow).

Models III and IV can be distinguished by examining the normal regression equations for \( r_{BC} \) and \( r_{AC} \). In Model III the difference \( (r_{BC} - b_{CB,A}^*) \) is interpreted as spuriousness because input is antecedent to both the college and the output variables, whereas in Model IV this difference is uninterpretable.
because the causal relationship of input to college is unknown. In Model III the difference \( r_{AC} - b_{CA.B} \) is evidence of the indirect effect \( A \rightarrow B \rightarrow C \), whereas in Model IV this difference cannot be meaningfully interpreted. The point is this: in Model IV only the independent influence of college and input (as measured by the regression coefficient) is interpretable; the joint influence of A and B on C cannot be interpreted in causal terms. In Model III, however, the joint AB influence is ascribed to input. A generalized version of Model IV, in which standardized regression coefficients are used to compute the various components of the predictable variance was provided by Werts (1968).

**Overview**

Although part correlation is commonly used to study college effects, it may not be the most effective statistical procedure. For the four hypothetical models discussed, part correlation (i.e. the college environment variable with the output when the influence of input is removed from the output) correctly estimated the size of the college effect only in the trivial case of a zero college effect. On the other hand, partial regression coefficients appeared to be a generally more satisfactory measure of college effects.

Typically, college effects studies have not attempted to determine the causal relationships among variables. When causal relationships are not considered, however, the investigator usually lacks the framework he needs to interpret his results correctly. For example, it is common practice to partial all student input variables out of the output before correlating the available college environment measures with the residual output; any of the obtained part correlations that reach statistical significance are interpreted as evidence of college effects. If the "true" situation is like Model II and III, a zero part correlation means there is no college effect.
However, if the situation is like Model V (Figure 5), a zero part correlation means that the influence of college on output is mediated through the input variable. Since there are many other cases in which any interpretation of correlation or regression coefficients is unwarranted, the investigator must be able to show why his model is reasonable.

![Diagram](https://example.com/diagram.png)

Fig. 5 College variable influences input variable; input influences output

It would not seem wise, therefore, to adopt what might be termed a "shotgun" correlational approach to the study of college effects. The phenomenon is too complicated for reliance on such a blind procedure; and there is too much risk that incorrect interpretations will be made of the data.

A major reason that regression analysis appears more suited than correlation to the study of college effects is that regression coefficients are potentially more stable. Tukey observed that: "We are very sure that the correlation cannot remain the same over a wide range of situations, but it is possible that the regression coefficient might (1954, p. 41)." For example, Blalock (1961) pointed out that as one shifts units of measurement, e.g., from individual to class to school, the regression coefficient remains relatively stable, whereas the correlation coefficient usually increases markedly in a way that makes it hazardous to draw conclusions about individuals from correlation on grouped data (Robinson, 1950). Thus the stability of the regression coefficients makes it more appropriate for college effects...
research because, although often dealing with grouped data, such research frequently hopes to draw inferences about effects on individuals.

A question crucial to college effects studies concerns the analysis of multiple input or college variables with or without measurement error (Blalock, 1965). However, this problem is too complex to discuss here; this paper is intended only as an introduction to the use of structural equations (for more advanced treatments see Johnston, 1963; Wold and Jureen, 1953).

Consideration of the relative merits of correlation and regression coefficients for the study of college effects should not be construed as a rejection of the college effects studies conducted so far. The use of regression coefficients, framed within a causal model, may simply provide a more sensitive test of that model. The really pressing need is for more valid testable models.
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