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The materials in this booklet are designed especially for the low achieving student in mathematics. Containing some materials from a course in general mathematics, the booklet is intended to be used in conjunction with conventional textbook materials and is designed to serve as a source of new ideas for teachers instructional mimeographed materials. Mathematical concepts are drawn from such skills. This work was prepared under ESEA Title III contract. (RP)


## Central Iowa Low Achiever Mathematics Project

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# LAMP 

# (Low Achiever Motivational Project) 

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with<br>contributions<br>by<br>Terry Shoemaker<br>Dale Kennedy<br>Ron Huff

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## PREFACE

The following pages represent the efforts of a committee of four teachers, working during the 1965-66 school year, to improve the general mathematics course; particularly those classes composed of low- achievers. The committee was composed of Ronald Huff, East High, Dale Kennedy, Hat Junior High, Terry Shoemaker, Wilson Junior High, and Joe Zimmerman, Callanan Junior High. Mr. Zimmerman acted as editor for this supplementary text, plus being the author of many of the units.

The need for materials such as those presented here is acute in classes composed of low-achievers and potential dropouts. This type of student has usually failed, or we have failed him, in some of his previous mathematics courses and is not interested in seeing "more of the same" in another mathematics class. We need to change pace often, excite him, startle him, intrigue him. In other words, to catch his attention and to change his attitude toward mathematics in particular and toward school in general. Only when he reaches this stage can we teach him an appreciable amount of mathematics.

We feel that the material presented herein, with appropriate suggestions as to its use, will help the teacher to change pace frequently and to arouse the interest of most students. We stress the fact that this does not comprise a complete course in general mathematics but must be used in conjunction with some conventional textbook materials.

This is intended to be a Teacher's Guide with units for use with students. It is not expected to be used in its entirety by every teacher nor necessarily in the order presented.


## INSTRUCTIONS

This book was designed witil several purposes in mind. First of all it is intended to be a source of new ideas. It is hoped that teachers will read through the book occasionally and draw from it as they see fit. Secoddly, it's so constructed as to relieve the teacher of a lot of busy work in preparing attractive and instructional mimeographed materials. The pages can be used to make Spirit Masters when used with certain copy machines. Sections can be left out and new ones added by covering the old with a new sheet of paper, using scotch tape loops on the back of the paper to hold it in place. Additions and alterations can be made easily by using a lead pencil.

Your comments, suggestions, questions, etc., are welcome. Address all correspondence to:

## LAMP

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The Des Moines Project - A Mathematics Laboratory
During the past three years, the Des Moines Indedependent School District, under the leadership of its mathematics supervisor Mr. Wilson Goodwin, has been experimenting with a new approach to the teaching of Gencral Mathematics, centered around a laboratory classroom. The program began as a pilot project at Woodrow Wilson Jr. High, and was subsequently extended to James Callanan Jr. High, Amos Hiatt Jr. High, and East High. During the 1966-67 school year seven schools will have fully equipped laboratories.
(The materials of this book have been collected and tested by the teachers of the original four partici- ' pating schools.)

## What is a mathematics laboratory?

Primarily, a mathematics laboratory is a state of mind. It is characterized by a questioning atmosphere and a continuous involvement with problem solving situations. Emphasis is placed upon discovery resulting from student experimentation. The teacher acts as a catalyst in the activity between students and knowledge

Secondarily, a mathematics laboratory is a physical plant equipped with such material objects as calculators, overhead, opaque projector, film strips, movies, tape-recorder, measuring devices, geoboards, solids, graph board, tachistoscope, construction devices, etc.. Since a student learns by doing, the lab is designed to give him the objects with which he can do and learn.

## Goals

The primary goal of the lab approach is to change the student's attitude towards mathematics. Most students have become so embittered by habitual failure that they hate mathematics and everything connected with it. There is little possibility of this student learning matnematics until an attitude change has been effected. It is because of this goal that our approach is different. Some would label our approach as "fun and games", but I am sure that close examination will bring realization that everything in the program is oriented towards the twin goals of attitude change and mathematical improvement. Learning can be enjoyable and in fact, should be so. Once the student sees that he can enjoy mathematics, he will want to learn about it. Then, and only then, is learning possible. as a useful member of society. We desire to enable the student to cope with and not fear the use of mathematics in everyday life. This entails the ability to recognize the problem, select the best solution, and compute the answer accurately. The laboratory approach facilitates the problem solving technique and motivates the students to engage in it.

## The Student

The General Mathematics student is characterized by: a dislike of mathematics, a lack of desire to succeed, a short interest span ( 10 to 15 minutes), a reading problem, and a well developed inferiority complex with respect to his own ability. He is frequently antagonistic and a discipline problem, especially when treated with conventional tactics. Our objective is to capture his interest at the very start, and through an unconventional series of procedures to obtain a different reaction. Discipline problems usually are begging for attention or protesting their boredom. The students in General Mathematics fall into a very wide range of ability from 3 rd to 10 th grade. Because of this we face a very acute problem of talking over the heads of some and boring others. The new approach, then, must emphasize variety of treatment and level of difficulty. You cannot hit all the students all the time, but you should attempt to reach every student at some time. Our program allows for remedial work for the slower student, as well as enrichment work for the better student, in conjunction with the average approach.

## The Teacher

If the low-achiever is to be reached, a special type of teacher must be recruited and trained. The General Mathematics teacher must recognize the problems involved in this area of teaching and be willing to adapt his methods of teaching to fit them. It is essential that the teacher be motivated. If he is teaciing this course because he lacked seniority and "got stuck" with it, then the class will reflect this. The key virtue is patience and the password is creativity. The General Maihematics teacher must be willing to try new ideas, to discard those which do not succeed with this type of student, and to be continually sensitive to the needs of the student. If the teacher is not afraid to fail occasionally, the students will take a similar attitude toward success. Above all the teacher must be a willing student, able to learn from his students as well as from those in the profession.

## Content

The content of the program is real-life-oriented. Emphasis is placed upon usability and a special effort is made to approach problems in a practical context. Adding decimal numbers is certainly not an end in itself; however, in the context of dollar and cent economy it makes considerable sense. It is for this reason that such units as square root are deleted. If we can equip the student with the ability to add, subtract, multiply, and divide whole numbers, decimals, and fractions; to find per cent; to rename decimals as fractions, as per cent and vice versa; and to do some word problems we will have accomplished a great deal. The idea of our program is to do these things in a new way, different from the one they have repeatedly failed to grasp.

## Method

The key word on method is variety. Because the student has a relatively short interest span we advocate doing at least 3 different activities in a class period. The emphasis is on change of pee. In one class you might lecture, have a related class discussion, show a related film strip, and assign a couple of related problems. In another you might begin by the presentation of a problem by a student, have class discussion, then have small group experimentation on the problem concluding with observations by the student. There are many different activities which are possible. The teacher is encouraged to choose imaginatively from them. One point cannot be stressed too much; do not do anything longer than the student can follow (roughly 15 minutes).

The other key word on method is individual-difference. It is of the greatest importance to have something for everyone. Allowing students to choose from several different types of exercises is one way of allowing for individual difference. Since each student has different problems why not let him choose the exercise that helps him overcome it. Also the idea of having a test divided according to level of difficulty and giving the student his choice has much merit. Points could be given according to the difficulty of the problem. This encourages the student to be honest with himself about his own ability and also encourages him to improve.

## Special Features

Local Business Problems We have collected problems from actual businesses around Les Mines and have used them in the classroom. This brings mathematics down to earth for the student since he recognizes the businesses involved. A great aid to motivation.
Flow Charting

## Individual Files

## ESP

Student Reports

Games

An idea from IBM helps the student make a step-t. step instruction for solving a particular type of problem. It encourages logical thinking, because it gives an actual picture of the thought process involved.
A file containing all the student's work gives a day by day report of the students progress. It gives a feeling of worth to an assignment if it is filed instead of discarded. This also encourages the student to produce neater papers, since they are kept, even for parental viewing if requested. This also, gives the student experience in filing, and serves as a source of achievement since the student can look at this file and see what he has accomplished.
Enrichment-Student-Projects serve as a motivational force for the better student and at the same time offers a challenge suited to his ability. The projects also serve as a means of make-up for absentees.
Using the student as a teacher-aid is a proven teaching method. Not only does the student profit from the research and class presentation, but the class profits as well. They listen more intently when one of their own is speaking. The only pre-requisite for doing an extra-credit report is desire to do something out of the ordinary. This encourages the sincere slow student as well as the better student. Another way to allow for individual differences.
Students learn twice as fast when they consider it play; that is, when they can enjoy it. Many mathematical ideas can be taught using game techniques. The association between enjoyment and learning is one of the most important corner stones in the process of changing their attitude.

## Multi-Sensory Devices

Calculators

Overhead

Opaque

## Tape-recorder

Tachistoscope

Movies and Film Strips

Flow-chart Templates

## Overlays

This machine is one of the best motivational devices found in the laboratory. Its operation fascinates the student and helps remove a rather large block to learning. The calculator is used to check work, to eliminate long and tedious computations, to teach organization in preparing a problem, to discover errors in hand computations, and to reward work well done. The calculator is not used as an end in itself, but purely as a lever to motivate the student.

A variable way of presenting material which seems to catch the student's eye much more readily than writing on the board. Very effectively used for illustrating a lecture or introducing new ideas. Again the word is variety of approach.

Good for projecting charts and diagrams from books and for showing the student's work to the class.

The tape-recorder can be used in the classroum to record lessons that may be used for remedial students or for make-up work. Also, lessons can be prepared for students that need more of a challenge, thereby enabling the instructor to give a lesson that is meaningful instead of just "busy work" for that student. Pre-recorded drill problems are very useful for teaching listening skills and for giving the teacher the chance to give individual help.

The tachistoscope is good for teaching the solution of story problems. A different way of presenting something, which students pay closer attention to.

Another visual aid for strengthening a new idea. Excellent for summary and review. Care should be taken in their selection and they sho'ld be pre-viewed to determine the most applicable portions and appropriate points for discussion before and after viewing.

Tile students enjoy using this device to prepare a neat picture of their solution to a problem.

The student enjoys making overlays for the overhead, especially when it involves the exposition of the solution to a problem he has cracked. An especially good task for any artistic students in the class.

No single device makes the laboratory approach a success, but rather the overall use of many different devices to promote variety and learning.

## NOTES:

## GEOBOARD

## What is it?

Why use it?

## How to make it?

## How to use them?

A geoboard is a physical model for demonstrating perimeter, area, geometric shapes, locus of points, lines, etc. Essentially it is : square piece of wood which has been measured off into a matrix of squares and nailed at the vertices.
It is a novel approach to many topics which allows the student to use his hands in actually making what he is studying. Motor activity reinforces the learning.
Purchase low-grade lumber ( $1 \times 10$ ) and finishing nails 44 penny). Have the wood measured off into $91 / 2$ " lengths and then cut in the shop. Assign this as a project to each class, asking them to mark off their block into regular 1 " squares, with a $3 / 4 "$ border around the outside. This is a difficult task for most students so preface the assignment by work of a similar nature. If you have more than one class, have them mark both sides of the blocks. Choose the best side for nailing. A volunteer group after school doing the nailing completes the task.

Cost: per 30 geoboards - roughly $\$ 5.00$
Assign students to pass out and pick up the boards in each class.

1st day - Pass out the boards and one rubber band per student. Allow the studenis time to experiment with them before you begin using them. Begin by defining your unit of measurement - one square (inch). Ask students to make squares having areas of $4,9,16$. Take the unit and cut it in half so as to make a right triangle. Ask them what its area would be. Have them construct areas of
 $3,5,61 / 2$, etc. Then have them make as difficult a ingure as they like, challanging one another to find the correct area. To encourage this, have some students draw their regions on the board and ask the whole class to find the area. See if they can make a region whose area is $31 / 2,41 / 4$, etc. See if they can find a shorter way than counting squares.
2nd day - Work with perimeter of various shape regions. Start with simple regions and see if they can discover any short cuts. Triangles will bring up the problem of finding the length of the hypotenuse of a right triangle. This might be a good time to develop the Pythagorean theorem.

3rd day - Ask the students to make and name as many different kinds of triangles as they can. Group them according to angles, and sides and compile a list on the board. See if th.ey can suggest any relationships such as between angles and sides, angle bisectors, etc.

4th day - See how many regular geometric shapes and patterns the students can make. Explain the concept of symmetry about an axis (reflection). Have them use one rubber band as an axis, and with several others create symmetries.

## Geoboard (continued)

5th day - Use the geoboard to introduce graphing. Let the geoboard be the 1st quadrant. Locate for the students, the origin in the lower left hand corner. Using plastic-soda-straws cut into 1 " lengths, have the students locate and mark with the straws several points. By giving them only points on a line you can bring in the ideas of slope and equation of lines. Try giving them three points and have them give back several more that would be on the same line. By using two rubber bands you can convert the geoboard into a four quadrant graph-board.

## Suggestions:

The divisions above are merely suggested. This is not meant to be a five day sequence, but rather five different ways of using the geoboard. Best results will be obtained by careful and infrequent use of it. The geoboard can be abused both by the teacher and the student. Be prepared for a certain amount of noise - students drop them and play with the rubber bands. Have a good supply of rubber bands on hand, since they do break, especially when students stretch them too far. Colored rubber bands show up best if you can get hold of them. Feel free to find your own uses for it - be creative.

NOTES:

## CENTIGRID

What is it?

Why use it?

How to use it?

How to make it?

A set of grids and overlays to be used with the overhead in tearhing the meaning of many different mathematical concepts such as ratio, proportion, and per cent.

It is a new and effective way of presenting concepts which are especially difficult for low-achievers to grasp. It utilizes the advantages of the overhead projector in presenting an understandable picture of abstract ideas.

Centigrid can be used in different ways. One way is an ESP box (see page 88-89); however, its primary use should be before the entire class with active student participation. By placing the various grids on the overhead and putting different overlays on top of the grid, you can present a rather effective picture of the following:
Ratio: compare the areas of the shaded region and the unshaded region.
Fractions: compare the areas of the regions and the total region.
Per cent: using the grid with 100 squares use several overlays either separately or in combinations asking the following questions:

How many squares in the grid?
How many are shaded?
What is this as a fraction?
What is this as a per cent?
Proportion: starting with the per cent grid, go to the grids having 50, 25, 10, and finally 40 , and 81 showing how to convert to a region of 100 squares.

Area: count the number of square units in a region.
Perimeter: count the units around the border of a region.
The next two pages should be used with transparencies and a copy machine. Cut the grids and overlays apart so they can be used independently.

Note: Transparencies require a much higher setting than spirit masters do.

## NOTES:




## CIRCLE CHART AND QUIZZES

What is it?

Why use it?

When to use it?

## How to use it?

A circle chart is an arrangement of partitioned circles, shaded to represent different fractions.

The student enjoys translating from a picture to make the problem himself. It teaches the concept of fractions and is a novel approach to what, for most students of General Math, has been a painful and confusing problem.

Use when the four fundamental processes with whole numbers have been mastered and you are ready to review (and in many cases teach for the first time) the operations with fractions; probably during the first or second nine weeks.

Give each student a circle chart and stress that it is an aid for him and wili be used regularly, so he must save it to refer to (responsibility). Discuss the chart in class and encourage them to mark them with any helpful information. After they have become familiar with the chart then give quizzes which use it. It is recommended that the quizzes be given regularly, for example every Thursday, (roughly fifteen minutes).
Following this page you will find copies of the tests, the chart itself, and the answers to the tests. Feel free to expand this idea in your own direction.
The chart is equally useable with decimal numbers, in fact you may give the student the opportunity of arriving at a solution either by decimals or fractions, which ever he finds easiest.
An excellent student project is to have a large classroom circle chart made using tagboard. Be sure to make the circles large enough to be seen by those in the back, ( $5^{\prime \prime}$ to $6^{\prime \prime}$ in diameter).

ANSWERS TO CHART QUIZZES



## Circle Chart Quiz 1

Name: $\qquad$
Period _---
$\qquad$ 1. What is the sum of column 1 ?
2. What is the sum of column 2 ?
3. What is the sum of column 3 ?
$\qquad$ 4. What is the sum of column 4 ?

T F 5. Column $1+$ circle $1>11 / 2$.
T $\mathbf{F}$
6. Column $2+$ circle $4>11 / 2$.

T $\mathbf{F}$
7. Column $3+$ circle $1>21 / 2$.

T F
8. Column $4+$ circle $8>3$.

T F
9. Column $1>$ column 2.

T F
10. Column $2>$ column 3.

Circle Chart Quiz 2

Name: $\qquad$ Period $\qquad$
------- 1 . What is the sum of row $A$.
_-_-_-_ 2. What is the sum of row $B$.
------ 3. What is the sum of row $C$.
T F 4. Row $\mathrm{A}>$ row B .
T F 5. Row B $>$ row $C$.
T F 6. Row $C>$ rowA.
T F 7. Row A + circle $1 \neq$ circle $8+$ circle 3.
------- 8. What is row $\mathrm{A}+$ circle 2.
T F 9. Row $A+$ circle $8 \neq 21 / 2$.
T F 10. Row A + circle $1<13 / 4$.

## Circle Chart Quiz 3

Name: $\qquad$
Period $\qquad$
$\qquad$ 1. What is the sum of column 1 ?

T F 2. Column $1>$ row $A$.
--------
3. What is the sum of column 2 ?

T F
4. Column $2<$ row B.
$\qquad$ 5. What is the sum of column 3 ?

T $\mathbf{F}$
6. Column $3>$ row $C$.

T F
7. Column $1+$ column $2>$ column $1+$ column 1 .
T F
8. Column $2+$ column $3>$ column $1+$ column 1 .
$\qquad$ 9. What is column $1+$ row A ?
$\qquad$ 10. What is column $2+$ row $B$ ?

## Circle Chart Quiz 4

Name: $\qquad$
Period $\qquad$
_-_-_-_ 1. What is the sum of row $A$ ?
T F 2. Row A $>$ column 4.
_-_-_- 3. What is the sum of row B?
T F 4. Row B $>$ column 3.
$\qquad$ 5. What is the sum of row C ?

T F 6. Row $C>$ column 2.
T F 7. Row $A+\operatorname{row} B>\operatorname{row} A$ + row $A$.
T $\quad \mathrm{F} \quad$ 8. Row $\mathrm{B}+$ row $\mathrm{C}>\operatorname{row} \mathrm{A}$ + row A .
_-_-_- 9. What is column $2+$ row A ?
-------10. What is column $3+$ row $B$ ?

## Circle Chart Quiz 5

Name: $\qquad$
Period $\qquad$
T $\mathbf{F}$

1. Column $1>$ column 3.
$\qquad$ 2. Column $1 \div$ row $A=$ ?

T F 3. Row $\mathrm{A}<$ column 4.
$\qquad$ 4. Row $\mathbf{A} \times$ column $3=$ ?

T F 5. Column $2 \neq$ column 3.
$\qquad$ 6. What is the sum of the odd numbered columns?

T $\mathbf{F}$
7. Circle $1>$ (circle $10+$ Circle 11).
T $\mathbf{F}$
8. Circle $6<$ (circle $1+$ circle 2).
9. Column $1+$ column 2. $=$ ?
10. Column $3+$ column $4=$ ?

## Circle Chart Quiz 6

Name: $\qquad$ Period $\qquad$
_-_-_- 1. Column 1 - column $2=$ ?
_-_-_-_ 2. Column 3 _ column $4=$ ?
------- 3. Row $A+$ row $B=$ ?
_-_---_ 4. Row $C$ _ row $A=$ ?
T F 5. (Row $A+$ circle 8$)<$ row $B$.
T F 6. (Row B + circle 9$)<3$.
T F 7. (Column $1+$ circle 1) $>$ row A.

T F 8. (Row $\mathbf{A}+$ circle 9$) \neq 2$.
T F 9. (Column $3+$ circle 1 C$) \neq 3$.
T F 10. (Column $4+$ circle 1$)<2$.

## Circle Chart Quiz 8

## Name:

Period $\qquad$
_-_-_- 1. Circle 9: circle $1=$ circle 8: circle?
2. Row A: circle 9 = circle 3
: circle?
3. 15 : circle $1=45$ : circle?
4. 22 : circle $9=11$ : circle?
5. Row $\mathrm{A}+$ row $\mathrm{B}=$ ?
6. Row A : row $\mathrm{B}=3:$ ?

T $\mathbf{F}$
7. $\frac{\text { Row } A}{\text { circle } 12}>3$.

T $\mathbf{F}$
8. Circle 3 : circle $8=$ circle 6 : circle 9.
T $\mathbf{F}$
9. Circle 9 : circle $1=$ circle 11 : circle 6.

T $\mathbf{F}$
10. $\frac{\text { Circle } 10}{\text { circle } 9}>1$.

## Circle Chart Quiz 7

Name:
Period .-.-
_--_-- 1. Column 4 _ circle $4=$ ?
T F 2. Column $4+$ circle $8>21 / 2$.
3. Column $3+$ circle $1=$ ?

T F 4. Column $3+$ circle $9<21 / 2$.
T F 5. (Row $\mathrm{A}+7$ ) $>8$.
$\mathrm{T} \quad \mathrm{F} \quad$ 6. $(1 / 2$ row $\mathrm{A}+5)>6$.
T F
7. Column $3>$ row $A$.
8. Column $1+$ row $\mathbf{A}=$ ?
$\qquad$ 9. Column $1+$ column $4=$ ?

T F 10. Circle $5+$ circle $6>$ circle 8.

## NAPIER'S BONES

What is it?

## Why use it?

How to use it?

A mechanical device for doing multiplication by means of addition. Invented by John Napier (1550-1617), the rods serve many purposes. In form they are simply strips of paper or cardboard with a series of numbers on each.

Napier's rods are an interesting almost mysterious visual aid which the student finds quite interesting, especially in trying to figure out why it works. This is a good way to show the real meaning of multiplication as short cut addition. Also, it is a less obvious way of getting students to do drill work in multiplication (making the rods) and addition (using them).

Assign the making of the rods to a student as an extra credit project and have a good student make a report on their use.

Problem: Multiply 136x29

## Solution:

## Suggestions:

Take the 1,3 , and 6 rods and place them side by side in that order. Use the index rod to locate the 9 th row. Copy the numbers in that row, adding all diagonals within the row, carrying when necessary. Then copy the numbers in the second row in like manner, adding a zero at the end because 2 is in the tens place. Then add the two sums to get your final answer. Do not use too long or complicated problems, since they would only cloud the issue.

$4 \times 4$ inch squares show up best for class demonstration.
Make out of tagboard and use magic-marker to make the number and lines.
Use students as much as possible.


## PER CENT OF INCREASE AND DECREASE TESTSS

## What is it?

Why use it?

When to use it?

## How to use it?

A quick, easily corrected set of tests which are given regularly as an exercise in working with per cent.

They are short, easily administered and corrected. However, the primary advantage is that the student understands exactly what will be expected of him and as a result is able to approach this test in a confident manner. Also by taking a developmental approach, the student is able to learn from the test itself. Once the student knows what is required, the teste are a source of success for him.

Use one test each Friday during the second semester, after the operation of changing fractions to per cents has been introduced. There is an advantage to having a test at the end of the week, but the important thing is to be regular about it.

Simply put ten numbers on the board or overhead. To begin with you will have to explain the form. The first number is your starting point for an increase to the second number. You ask the question how much of an increase has taken place? What per cent is this change of your original number? To find the answer express the problem as a fraction, then express as a decimal and finally as a per cent. Likewise with decrease. What is your starting point? (the second number), how much does it decrease going to the first number? Express as a fraction, then decimal, then per cent. This exercise summarizes nicely the fraction-decimal-per cent type of problem with a minimum of teacher preparation.
Below, are four sample quizzes with their answers.

| SAMPLE: | $1-2)$ | $2 \longleftrightarrow 3$ |
| :--- | :--- | :--- |
| $3-4)$ | $2 \longleftrightarrow 4$ | BEST POSSIBLE |
|  | $5-6)$ | $2 \longleftrightarrow 5$ |
| $7-8)$ | $3 \longleftrightarrow 4$ |  |
| $9-10)$ | $3 \longleftrightarrow 6$ |  |$\quad$ SCORE = 10 POINTS


| FROM | TO | $\%$ INC. | $\%$ DEC. | FROM | TO | $\%$ INC. | $\%$ DEC. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 2 | 3 | 50 | $331 / 3$ | 5 | 6 | 20 | $162 / 3$ |
| 2 | 4 | 100 | 50 | 7 | 8 | $142 / 7$ | $121 / 2$ |
| 2 | 5 | 150 | 60 | 9 | 10 | $111 / 9$ | 10 |
| 3 | 4 | $331 / 3$ | 25 | 11 | 12 | $91 / 11$ | $81 / 3$ |
| 3 | 6 | 100 | 50 | 2 | 20 | 900 | 90 |
| FROM | TO | $\%$ | INC. | $\%$ DEC. | FROM | TO | $\%$ INC. |
| 4 | 5 | 25 | 20 | 5 | 8 | DEC. |  |
| 4 | 6 | 50 | $331 / 3$ | 6 | 9 | 50 | $371 / 2$ |
| 4 | 8 | 100 | 50 | 7 | 10 | $426 / 7$ | $331 / 3$ |
| 5 | 15 | 200 | $662 / 3$ | 8 | 11 | $371 / 2$ | $273 / 11$ |
| 5 | 20 | 300 | 75 | 3 | 27 | 800 | $888 / 9$ |

## FILL-IN PROBLEMS


#### Abstract

What is it?

Why use it?

When to use it?

A new way of presenting basic operation problems by which some numbers are left out and the student must fill in the missing numbers.

The students find this approach to what could be a boring drill, fun and interesting. The element of challenge changes the student's attitude towards such work. The end result is the same. To do each problem twice as much work is required, but the students seem to find the approach more enjoyable and as a result more profitable.

This is best used during the first nine weeks of school as a review of arithmetic. However it would be good to use problems of this nature throughout the entire year. Use sparingly and never as busy-work. When a student has successfully completed the assigned exercise, allow him to check his work at the calculator. This not only rewards him for his effort, but also gives him the chance to discover and correct his own errors.

How to use it? Have the following sheets prepared for the classes. After a sheet has been passed out, have the students try one of the problems. Then see if you can draw from them the method of solution. It might be best not to do this until after the second or third exercise. The key idea here is inverse operation. In order to find the addend you actually subtract. Problems of this nature serve as a smooth introduction to algebraic problems and form. The following pages are intended only as a starter or stimulus for you to make similar exercises yourself. Be creative!


## NOTES:

## ADDITION

I.)
2.)
3.)
4.)
35
74
$\begin{array}{r}-7 \\ 31 \\ \hline 190\end{array}$
5.)
6.)

7.)
8117
7930
753

8.) 81372
1750 5840
$\begin{array}{r}898 \\ 31331 \\ \hline 174115\end{array}$
9.)

| 10,862 | 10. |
| ---: | ---: |
| 51,016 | 1,599 |
| 43,610 | $43,-857$ |
| ,--- | 1,020 |
| 10,248 | 30,669 |
| 8,313 | 31,005 |
| 20,450 | 8,490 |
| 145,555 |  |

SUBTRACTION
I.)

$$
\begin{array}{r}
1394 \\
-\quad--- \\
\hline 839
\end{array}
$$

2.)

$$
\begin{array}{r}
411 \\
-\quad--- \\
\hline 21
\end{array}
$$

3.)
5.)

$$
\begin{array}{r}
6670 \\
-\quad---- \\
\hline 3556
\end{array}
$$

6.)

$$
\begin{array}{r}
1-\overline{1} \\
-\quad 78 \\
\hline 22
\end{array}
$$

7.)

$$
\begin{array}{r}
19,-0 \_ \\
-\quad 4,9 \_7 \\
\hline 14,208
\end{array}
$$

8.)

$$
\begin{array}{r}
6391 \\
-\quad-70 \\
\hline-121
\end{array}
$$

9.)

$$
\begin{array}{r}
6-2- \\
-\quad 282 \\
\hline 6,200
\end{array}
$$

10.) $1442-\ldots=1045$
11.) 2200-11-- = 1,089
12.) - $-3711=1227$

## - ADDING -

I.)
2.)

| 45 | 73 |
| :---: | :---: |
| 32 | 86 |
| $-\overline{64}$ | 49 |
| $\frac{48}{212}$ | 25 |
|  | $\frac{--}{289}$ |

3.)
4.)

| 123 | 92 |
| ---: | ---: |
| 45 | 86 |
| 284 | --- |
| --- | 49 |
| 66 | 222 |
| 676 | 1200 |

5.)

| 99 |
| ---: |
| 86 |
| 45 |
| -- |
| 280 |

6.)

| 42 |
| :---: |
| 44 |
| 46 |
| -- |
| 222 |

7.)

| 72 |
| ---: |
| 149 |
| 2249 |
| --- |
| 1370 |

8.)

67
49
$\frac{\text { - }}{22}$
9.)

| 29 |
| :--- |
| 31 |
| 37 |
| 46 |
| 52 |
| -- |
| 280 |

10.)
II.)

| 72 |
| ---: |
| 144 |
| 2352 |
| ---- |
| 196 |
| 44 |
| 4200 |

12.)
27
14
2922
146

- 666
4000

$$
\begin{aligned}
& \text { DIVISION }
\end{aligned}
$$

> (2.)
> (3).

## ADDITION

I.)

4.)

$$
21+\ldots-+367+254+147+360=1196
$$

5.)

$$
14,765+9-7+4056+3721=23,479
$$

6.)

| $\$ 1.97$ | 7.) 105 | 8.) $474+963=$ |
| :--- | ---: | ---: |
| .$--\overline{210}$ | 210 |  |
| $\frac{6.49}{\$ 11.20}$ | $\frac{111}{835}$ |  |

10.)

$$
\begin{array}{r}
61.30 \\
2.56 \\
4.10 \\
3.42
\end{array}
$$

FIND THE MISSING NUMBERS


$$
\text { (c) } \begin{array}{r}
\frac{27-2}{54-} \\
=--06
\end{array}
$$

## ALGEBRA SUBSTITUTION

## What is it?

Why use it?

## How to use it?

When to use it?

Suggestions:

A set of exercises in the basic operations hidden behind the algebraic concept of substitution.

The students feel greater success doing what they think is algebra. The challenge of the idea motivates the student to do what otherwise would be boring drill. This serves as an excellent pre-algebra orientation for those who may decide to take it.

Pass out form A and discuss the order of operations and the form of the problems. Then assign values for the variables, using the chalk board or overhead, i.e. $A=3$, $B=5, C=1$. Choose values according to the type of problems you war.t. These exercises are equally good for fractions, decimals, and whole numbers.
Emphasize the student's responsibility to keep this exercise for future uses. A large poster of the two forms could be made for the front of the room to be used by those who lose theirs. Explain that it is simpler to have and keep your own copy. When they have mastered form A, go on to B. Again regular use of this form builds confidence because the student knows exactly what is expected and yet the idea of doing algebra makes it seem enjoyable as well.

Throughout the year with each section of work, but especially as a section of prealgebra during the last nine weeks.

After using the first two sheets several times, try using short tests 5 to 12. Please note that tests $6,8,10,12$, require an understanding of sets and the use of the number line. Don't be afraid to introduce these ideas to them. They are not too advanced for them and in fact many students who have never succeeded before will find successs with this idea which does not require a background to work with.

## NOTES:

(1) $a+b$
(9) $\frac{a}{b}$
(2) $a-b$
(10) $\frac{a}{c}$
(3) $a+b-c$
(il) $\frac{a b}{c}$
(4) $a-b+c$
(12) $\frac{a}{b c}$
(5) $a+b+c$.
(13) $\frac{a b c}{a+b}$
(6) $a b$
(14) $\frac{a+b-c}{a c}$
(7) abc
(15) $\frac{a b}{a+b}$
(8) $a b+c$
(16) $\frac{a b-c}{a b c}$

## _ SUBSTITUTE

(1) $a b+a c+b c$
(2) $\frac{a b}{a+c}+\frac{a}{c}+b c$
(3) $\frac{a}{b} / \frac{\omega}{c}$
(4) $\frac{a+b}{c}+\frac{a b}{c}$
(5) $\frac{a c}{b}+\frac{a+c}{b}$
(6) $\frac{a+b}{c}+a+a b$
(7) $a b c-\frac{a+b}{c}$
(8) $a+a b+b c$

## TEST NUMBER 5

## TEST NUMBER 6

If $a=6 ; b=4$; and $c=2$, Find:

1. $a b=$ ?
2. $a c+b=$ ?
3. $\mathrm{bac}+\mathrm{bc}=$ ?
4. $\mathrm{ab}+\mathrm{bc}+\mathrm{ac}=$ ?
5. $a+b+b c=$ ?
6. $\mathrm{ab}+\mathrm{ac}=$ ?
7. $a b-b c=$ ?
8. $a+b-b c=$ ?
9. $\mathrm{ac}+\mathrm{b}-\mathrm{c}=$ ?
10. $\frac{a+b+c}{2}$
$\begin{aligned} A= & \{7,11,15, \ldots\}\{\mathrm{F}=2,4,8, \ldots\} \\ & \{\mathrm{C}=3,6,9, \ldots\}\end{aligned}$
11. in set $A$, show all numbers $>25$ and 40
12. in set $B$, show all numbers $\neq 8$ and 16 and $<20$
13. in set $C$, show all numbers $<30$ having a factor of 6 .
14. in set $A$, list all numbers $>25$.
15. in set $B$, list all numbers $<50$.
16. in set $C$, list all numbers $>9$ and $<40$

## TEST NUMBER 7

If $a=6 ; b=5 ;$ and $c=2$, find:

1. $a c+b$
2. $a b c+a a$
3. $a \mathrm{a}+\mathrm{bb}+\mathrm{cc}$
4. $a+b+c-b c$
5. $a b+b c-a c$
6. $a b c+b b$
7. $a a+b b-c c$
8. $a b a+b c b-c a c$
9. $a \mathrm{ac}+\mathrm{bbc}-\mathrm{ccc}$
10. $a b c+a b c+a b c$

## TEST NUMBER 8

$A=\{5,10,15,20\} \quad B=\{1,2,3,4,5\}$ $C=\{3,6,9, \ldots\}$

1. in set $A$, show all numbers having 4 as a factor
2. in set $B$, show all numbers $\neq 16 / 8$ and 20/4
3. in set $C$, show all numbers $<40$ and having 5 as a factor.
4. in set $A$, list all numbers which are factors of 120.
5. in set $B$, list all numbers $>2$ and $\nsubseteq 15 / 5$
6. in set $C$, list all numbers $>12$ and $<51$.

## TEST NUMBER 9

If $a=6 ; b=3 ;$ and $c=1 / 2$, find:

1. $a b c+a^{2}$
2. $a b+a c+c$
3. $b c+b^{2}+a c$
4. $a a c+a c-b^{2}$
5. $a c+b c+c c$
6. $a a b+b b a+a c$
7. $a^{2} b+b^{2} a+a^{2}$
8. $a b^{2}+a c$
9. $a c+b^{2} c+2 c$
10. $2 c+c^{2}+1 / 2 c$
$A=\{2,4,6, \ldots\} \quad B=5,10,15, \ldots\}$
$C=\{9,18,27, \ldots\}$
11. in set $A$, show all numbers $>5$ and $<60 / 5$
12. in set $B$, show all numbers $<$ and having a factor of 10 .
13. in set $C$, show all numbers $=(6 \times 3)$ and (6x6).
14. in set $A$, list all numbers having a factor of 5 .
15. in set $B$, list all numbers having a factor of 8 .
16. in set $C$, list all numbers $<100$ and having a factor of 12 .

## TEST NUMBER 11

If $a=3 ; b=4 ;$ and $c=2$, find:

1. $a^{2}+b^{2}$
2. $a^{2}+a c$
3. $c^{2}+a b$
C. $\frac{a b}{2}+c+a^{2} b$
4. $a^{2}+b+c^{2}$
5. $a^{2}+b-c^{2}$
6. $b^{2}-a^{2}-c^{2}$
$8 a+\frac{(b+c)}{2}$
7. $b^{2}-2 a c^{2}$
8. $1 / 2 \mathrm{ab}$

## TEST NUMBER 12

$A=\{7,14,21, \ldots\} \quad B=\{3,10,17, \ldots\}$ $C=\{1,2,3,4,5\}$

1. in set $A$, show all numbers $>12$ and $<16$.
2. in set $B$, show all numbers $\neq(6 \times 4)$ and $<35$.
3. in set $C$, show all prime numbers.
4. in set A , list all composite numbers.
5. in set B , list all prime $\mathrm{r}_{\mathrm{s}} \mathrm{immbers}<50$.
6. in set $C$, list all composite numbers.

## ANSWERS TO TESTS

Test Number 5 Test Number $6 \quad$ Test Number $7 \quad$ Test Number 8

1. 24
2. 16
3. 56
4. 44
5. 18
6. 36
7. 16
8. 2
9. 14
10. 6
11. $\{27,31,35,39\}$
12. $\{2,5,11,14,17\}$
13. $\{6,12,18,24\}$
14. $\{27,31,35, \ldots\}$
15. $\{2,5,8, \ldots, 47\}$
16. $\{12,15,18, \ldots, 39\}$
17. 17
18. 96
19. 65
20. 3
21. 28
22. 85
23. 57
24. 206
25. 114
26. 180
27. $\{20\}$
28. $\{1,3,4\}$
29. $\{15,30\}$
30. $\{5,10,15,20\}$
31. $\{4,5\}$
32. $\{15,18,21, \ldots, 4 \delta\}$

Test Number 9

1. 45
2. $211 / 2$
3. $13^{1 / 2}$
4. 12
5. $43 / 4$
6. 165
7. 198
8. 57
9. $8 \frac{1}{2} 2$
10. 2

## Test Number 10

1. $\{6,8,10\}$
2. $\{10,20,30\}$
3. $\{18,36\}$
4. $\{10,20,30, \ldots\}$
5. $\{40,80,120, \ldots\}$
6. $\{36,72\}$

Test Number 11

1. 25
2. 15
3. 16
4. 44
5. 11
6. 9
7. 3
8. 6
9. 4
10. 6

Test Number 12

1. $\{14\}$
2. $\{3,10,17,31\}$
3. $\{1,2,3,5\}$
4. $\{14,21,28, \ldots\}$
5. $\{3,17,31\}$
$6\{4\}$

## PREDICTIONS

What is it?

Why use it?

How to use it?

When to use it?

NOTES:

A sneaky way of getting the student to add, subtract, multiply and divide through number and letter sequences.

It is a different way of doing the basic operations, and the student does not realize that this is actually drill work. It helps the student to discover patterns in number sequences. Also a new problem-solving situation.

Give an example of a completed sequence. The students will catch on immediately. Ask them to formulate a rule for a few of the sequences and then use it to predict what the next five numbers will be. Students enjoy the mystery of the rule by which a sequence is formed.

Anytime a short exercise is desired, but especially during the first nine weeks while working with a review of the basic operations.

## Predictions


2) $3,6,9,12, \ldots, \ldots,-1-1$,
3) $5,10,15, \ldots, \ldots, \ldots, \ldots,-$
4) $1,3,5,7,9, \ldots, \ldots 1$,,
5) $a, c, e, g,-,-,-,-1$ -
6) $15,13,11,9, \ldots, \ldots,-1$.
7) $32,28,24,20, \ldots, \ldots,-, \ldots$,
8) $0,7,14, \ldots,-,-,-1$
9) $32,16,8, \ldots,-$
10) $1,2,4,8,16, \ldots, \ldots, \ldots$,
11) $7,11,15,19, \ldots, \ldots, \ldots$
(2) $1,3,9,27, \ldots,-$
(3) $18,16,14,12, \ldots, \ldots, \ldots,-$
14) $19,20,21, \ldots, \ldots,-1,-$
15) $5,8,11, \ldots, 17,-$
16) $1,2,4,7,11,16, \ldots, \ldots, \ldots$,
17) $75,66,58,51,45, \ldots, \ldots, \ldots$ ———
18) $0,7,0,14,0, \ldots,-,-$

Predictions
1.) $5,9,13,17, \ldots, \ldots, \ldots, \ldots$,
2.) $1,5,25,125$, $\qquad$
3.) $43,41,39,37$, $\qquad$
4.) $1, n, p, r,-\quad, \quad$
5.) $12,6,3, \ldots,-1$
6.) $97,90,83,76, \ldots,-,-$,
7.) $1,3,2,4,3, \ldots, \ldots,-$ -
8) $2,4,6,8, \ldots,-,-$,
9.) $51,54,57$ $\qquad$
10.) $28,29,30$ $\qquad$
II.) $32,8,2, \longrightarrow$,
12.) $6,12,24,48, \ldots, \ldots$,
13.) $y, w, u, \ldots,-$
14.) $12,24,36,48, \ldots, \ldots$,

ERIC

## CRYPTOGRAPHY

What is it? A coded message formed by setting up a one-to-one relationship between the let-

Why use it?

How to use it?

When to use it?

How to make it:

It is a very challenging way of teaching the ideas of equivalence and substitution of equals for equals.

Hand out the sheet of numbers and say nothing. As they look at the numbers some will ask what it is and others will try to guess. Usually they will be able to discover what it is. This is the first step in problem solving - seeing the problem. The next question they will undoubtedly ask is how do you decode it. Here you might explain the idea of a consistent one-to-one substitution and have them look at a regular English passage to see what letter is used most frequently, what letters can stand alone, which ones occur at the end of words, what are the most common double letters, what are the most common two letter and three letter words. With just this little bit of an introduction, turn them loose. You may choose to do the first message together as a class effort.
After the class has mastered the first code suggest the possibility of them writing their own coded messages as an extra credit project (the better of which you might use in class).
There is one hint you might want to give if they have trouble; the lower numbers are the first part of the alphabet and the larger numbers the latter part. The first two words of the first message are "if you..."
We were very skeptical of this idea at first, but it works, it really does work. Whether you doubt it or not, try it. You'll be pleased by their reaction.

Any time you want a boost in interest, but especially when pointing out the difference between number and numeral. If fits well with the use of symbols.

1. Make a code sheet composed of 2 columns. In the first column write dewn the letters of the alphabet. In the second column write any number or symbol so that each letter has a corresponding symbol.
2. Write the message out normally.
3. Substitute the proper symbol for each letter.

## NOTES:

CRYPTOGRAPHY

$$
\begin{aligned}
& 11-8 \quad 26-15-23 \quad 3-19-5 \quad 3-2-14-5 \quad 21-15 \\
& 6-5-1-11-16-4-5-19 \quad 21-4-11-18 \\
& 12-5-18-18-3-7-5 \quad 26-15-23 \quad 17-11-14-14 \\
& 19-5-1-5-11-24-5 \quad 3-10 \quad 3 \quad 8-15-19 \\
& 21-4-5 \quad 6-3-26 . \\
& 3-10 \quad 3-16-16-14-5 \\
& 13-5-5-16-18 \\
& 3-17-3-26 . \\
& 21-4-19-5-5 \\
& 18-5-24-5-10 \\
& 16-10-6 \\
& 16-14-3-26 \\
& 18-16-15-19-21-18-12-3-10-23-18-19 \\
& 18-15-19 \\
& 15-8 \\
& 21-4-5 \\
& 7-3-12-5-16
\end{aligned}
$$

3

## FRACTION-DECIMAL-GRAMS

What is it?

Why use it?

How to use it?

When to use it?

Suggestions:

A new approach to the form of traditional problems. By equating certain numbers with symbols, you present the problem as an operation on symbols. The student then actually constructs the problem as well as the answer.

The low-achiever is sick of the conventional form and reacts favorably to this new approach. He considers it fun to figure out what the problem is. This exercise teaches some rather fundamental algebraic concepts at the same time it is giving the student practice with the basic operations.

The fraction and decimal-grams are set up with a key or set of numbers in the upper-left-hand corner. These numbers are equated with symbols and are to be substituted for the symbols to make the problems. There are many possible combinations of symbols. Only one, or at most two sets of problems should be given at one time. Best results will be obtained by using just two on any given day, since this type of student is highly resentful of busy-work. Each page could bs used many different times. Each time the exercise is used the teacher should stipulate which sets of problems (1-9) are to be used. For example, use page 37 with set $8 \& 9$, have the student circle the numbers, and then write the symbols and operations on the exercise line. He will then proceed to write the problems and solve them.

Use when teaching fractions and decimals or when a review is desired. This student profits from repetion if it is not burdensome.

Run off enough copies so that each student will be able to receive 5 or 6 sheets over a period of time (a semester).

NOTES:

FRACTION-GRAM
$\bigcirc \square \diamond \square$
A) $\frac{1}{2} \frac{1}{4} \frac{1}{6} \frac{1}{8}$
B) $\frac{2}{3} \quad \frac{3}{5} \quad \frac{4}{9} \quad \frac{2}{7}$
C) $\frac{7}{8} \quad \frac{5}{6} \quad \frac{3}{8} \quad \frac{1}{12}$
D) $\frac{5}{8} \quad \frac{7}{10} \quad \frac{9}{14} \quad \frac{5}{16}$

1) $\square+O, \square+\diamond$
2) $\diamond+\square, O+\square$
3) $\mathrm{O}-\bigcirc, \mathrm{O}-\square$
4) $\quad \square-\square, \square-\bigcirc$
5) $\diamond x \square, \square \times \diamond$
6) $\bigcirc \div \diamond, \square \div \diamond$
7) $\square \div \square, \square \div \square$
8) $O \times \square \div\langle, \diamond \div \square$
A)
A)
B)
B)
c)
C)
D)
D)

## DECIGRAM

A) $\begin{array}{rr}6.72 \\ \text { B) } & 12.06 \\ \text { C) } & .78 \\ \text { D) } & 86.95\end{array}{ }^{2} 96$
126.7
19.23

1) $\square+O, O+\square$
18.8
1.19
2) $O-\square: O-\square$
3) $\square \times \bigcirc, \bigcirc \times \square$
4) $\square \div \bigcirc, \bigcirc \div \square$
5) $\square+\square \cdot \square-\square$
6) $\begin{array}{ccc}\square \times \square & \text { 7) } \square \times \square & 81 O \times 2 . \square \\ \square \div \square & O \times O & 9 . O\end{array}$
A)
A)
B)
B)
C)
C)
D)
D)

|  | DECIGRAMS |  |  |  |
| :--- | ---: | ---: | ---: | :---: |
|  | 0 | $\bigcirc$ | $\square$ | $\searrow$ |
| A) | 4.5 | 6.34 | 12.35 | 26.7 |
| B) | 19.3 | 21.04 | .35 | .39 |
| C) | .7 | .16 | 3.93 | 7.4 |
| D) | 22.3 | 6.50 | 9.11 | .88 |


| $\square+O+\square+Q$ | $\square \times \bigcirc$ |
| :--- | :--- |
| A) | A) |
| B) | B) |
| C) | C) |
| D) | D) |


| $\square \times O$ | $\square+\square \times \bigcirc$ |
| :--- | :--- |
| A) | A) |
| B) |  |
| C) |  |
| B) |  |
| D) |  |
| C) |  |
| D) |  |


|  | $\square \div \square$ |
| :--- | :--- |
| A) | $\square+\square^{-} O$ |
| B) | B) |
| C) | C) |
| D) | D) |

## DOT-TO-DOT PUZZLE

What is it?
Why use it?
Hew to use it?

A conventional drill exercise hidden behind a picture.
Motivation! The stuar $\cdot \mathrm{nt}$ en s doing them.

The answers to all the problems are hidden in the picture along with a lot of extra answers. The object is to find the first answer as a starting point and draw a line to the next dot which corresponds to the answer to the second problem.
Suggest the possibility of the student making one as an extra-credit project. Coloring books serve beautifully as a source of simple pictures.

## NOTES:

## DOT-TO-DOT PUZZLE

Complete the picture by connecting the points which correspond to the answers of the problems below.


1. $4.85+.0026$
2. $9.5+56$
3. $7.82-.003$
4. $.0037+3.4$
5. $6.0-3$
6. $53.52-23.16$
7. $5469-48$
8. $.58+.074$
9. $23.1+9.7$
10. $38.03+.65$
11. . $22-0.004$
12. $78.75-3.55$
13. $\$ 1-93 \mathrm{c}$
14. $35-1.661$
15. $6.86+.0505$
16. $32-1.6$
17. $.2+3.0$
18. $7.9+82$
19. $10-.084$
20. $53-6.116$
21. $650-.986$
22. $14.3+3.2$
23. $99-4.5$
24. $16-12.9$
25. $2.10+.640$
26. $2-.17$
27. $45.1+32.19$
28. . $2-.17$
29. $90-.77$
30. $.05+3$
31. $11-1.1$
32. $6.0+7.0$
33. $.846+.27$
34. $33-3.3$
35. $45+.71$
36. $33.2-4.2$
37. $4-.73$
38. $40-.73$
39. $22+16$
40. $8.6-.8$
41. $5-3$
42. $9.0+3.3$
43. $2.5+3.8$

## FLOW CHARTING

## What is it?

Why use it?
A student drawn picture of the step-by-step procedure used in solving a given type of problem.

It forces the student to think out the process which he has rather sloppily taken for granted before. Also the idea of using something connected with IBM gives an added incentive to their motivation. By means of flow charting many students are able to see for the first time some of the operations which have confused them for years.

## How to use it?

Introduce the idea of flow charting with a nonmathematical problem, such as getting a date. List the steps on the chalkboard or overhead. When the class is satisfied with the number of steps, connect tiem together with lines and boxes, and arrows to indicate the directions followed. Then gise them the IBM templates and see if they can construct a solution to a similar problem. Now whenever a new type of problem is studied you can use a flow chart as a summary of the problem solving procedure. To help in the process of familiarizing the students with flow charting we have used them as an introduction to the basic operations on the calculator. Be sure to go over the sheet before having the students try them at the machines. It is probably best to keep the number of symbols used to a minimum. So far, we have used the following:


When to use it?
to indicate the starting and stop-
ping points.
instruction box
decision point: used whenever there are two mutually exclusive alternatives.


At the very beginning of the year to introduce the use of the calculators and to establish a new method of problem solving to be used throughout the year.

Note: The following flow charts were made for use with the Olivetti-Underwood Divisumma 24 Printing Calculator, the machine being used in the Des Moines laboratories.

## NOTES:

## CALCULATOR OPERATION FLOW CHART FOR ADDITION



```
3859+8574 + 8574 + 4657 + 8673 =
4546 + 56 + 8675 + 2435=
7564+85614 + 9516 + 8594 +8=
```


## CALCULATOR OPERATION FLOW CHART FOR SUBTRACTION



Practice:

| 97364 | 63756 | 64736 | 374 | 56478 | 36475 | 3920 |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| 9756 | 57467 | 37465 | 48 | 37746 | 3464 | 468 |

```
6394-364=
78695-9748=
```


## CALCULATOR OPERATION FLOW CHART FOR MULTIPLICATION OF TWO NUMBERS



Sample:

| Find the product | (tape) <br>  <br> 6589 <br> 743 <br> $4,895,627$ |
| ---: | ---: |
|  | $743=$ |
|  | 4895627 T |

## Practice:

| 674859 | 36475 | 243068 | 564758 | 75859 |
| ---: | ---: | ---: | ---: | ---: |
| 72 | 73 | 640 | 35267 | 1183 |

Note: Observe the difference in the amount of time it takes the machine to get an answer when the least number is put in first. This is a crucial factor in economical computer programming.

## CALCULATOR OPERATION FLOW CHART FOR DIVISION



## CALCULATOR OPERATION FLOW CHART FOR MULTIPLICATION BY A CONSTANT



Practice: $C=\pi \mathrm{d}$ or $\mathrm{C}=2 \boldsymbol{T r}$
Find the circumference of the circles having the following diameters.

$$
\begin{array}{llllllllll}
2 & 8 & 12 & 15 & 20 & 35 & 124 & 62 & 81 & 7
\end{array}
$$

Find the circumference of the circles having the following radii.
$\begin{array}{ll}3 & 9\end{array}$ 18 2437 $\begin{array}{lll}59 & 70 & 78\end{array}$129

## CROSS-NUMBER PUZZLES

What is it?
Why use it?

## How to use it?

A crossword puzzle using numbers.
The puzzle idea helps motivate the student to do the drill problems. It serves as an excellent diagnostic exercise because the student is not worried about a grade.

We suggest you give it as an extra credit exercise that they don't have to do. The idea of doing something because they want to make this a new experience for most students. The problems are simple enough that even the slow student is able to acheive a measure of success with it. It also serves as a good review of the operations later on in the year.
The students quickly realize that they need only to do the down or across problems to fill in the puzzle, so encourage the use of the other as a check.

## NOTES:

## CROSS-NUMBER PUZZLE

Complete the puzzle by putting each digit of your answer in a separate square.
Down checks across.

## Across

1. $500 \times .03$
2. $22,200 \times .01$
3. $800 \times .04$
4. $.1 \times 10$
5. $.05 \times 40$
6. $3,500 \times .02$
7. $7,000 \times .05$
8. $550 \times .02$
9. $40 \times .5$
10. $200 \times .8$
11. $800 \times .4$
12. $40 \times .8$
13. $25,500 \times .02$
14. $500 \times .5$
15. $50 \times .06$
16. $500 \times .3$
17. $14,600 \times .05$
18. $.1 \times 20$
19. $500 \times 1.65$
20. $1,200 \times .4$
21. $900 \times .09$
22. $1,230 \times .5$
23. $3,970 \times .2$
24. $13,400 \times .05$
25. $100 \times .5$
26. $1,520 \times .3$
27. $8.9 \times 50$
28. . $04 \times 200$
29. $2.91 \times 300$
30. $1.734 \times 500$
31. $.10 \times 80$
32. $2.98 \times 200$
33. $220 \times 2.1$
34. $30 \times .8$
35. $21.8 \times 20$
36. $17.5 \times 50$
37. $5.6 \times 20$

Do

1. $40 \times .3$
2. $100 \times .05$


## CROSS-NUMBER PUZZLE

Complete the puzzle by putting each digit of your answer in a separate square.
Down checks across.

## Across

1. $20.16 \div .84$
2. $.672 \div .021$
3. $1.75 \div .025$
4. $34.1 \div 3.1$
5. $17.92 \div .16$
6. $16.5 \div .11$
7. $490 \div 1.4$
8. $8.4 \div .4$
9. $.0 \div 0$
10. $7.04 \div .32$
11. $18.81 \div .57$
12. $74.2 \div 1.4$
13. $28.2 \div .6$
14. $1.32 \div .12$
15. $45 \div 1.5$
16. $6.48 \div .08$
17. $3.75 \div .25$
18. $8.5 \div .17$
19. $4.0 \div .8$
20. $1.5 \div .5$
21. . $81 \div .09$
22. $1.44 \div .03$
23. $3.65 \div .05$
24. $46.9 \div .7$
25. $40.8 \div .8$
26. $126 \div 6.3$
27. $48.3 \div 2.1$
28. $9.45 \div .45$
29. $98 \div 2.8$
30. $15.36 \div .48$
31. $1.08 \div .18$
32. $12.1 \div 1.1$
33. $47.64 \div .06$
34. $59.28 \div .13$
35. $53.4 \div .12$
36. $24 \div 1.5$
37. $3.3 \div .05$
38. $94.12 \div 7.24$
39. $250.47 \div 2.53$

## Down

1. $2.52 \div .12$
2. $45.1 \div 1.1$
3. $.57 \div .19$
4. $29.4 \div 1.4$
5. $98 \div 1.4$
6. $0 \div .12$
7. $.9 \div .06$
8. $25 \div 2.5$
9. $4.4 \div .2$
10. $16 \div .32$
11. $54.4 \div 1.7$
12. $50.56 \div .32$
13. $11 \div .04$
14. $5.94 \div .18$
15. $2.16 \div .072$
16. $1.488 \div .048$
17. $6.15 \div .15$
18. $24 \div 1.6$
19. . $3 \div .03$
20. $2 \div .4$
21. $2.7 \div .9$
22. $3.6 \div .4$
23. $7.14 \div .17$
24. $4.8 \div .06$
25. $86.64 \div .12$
26. $1.32 \div .04$
27. $55.8 \div .9$
28. $220.41 \div .31$
29. $11.13 \div .21$
30. $9 \div .6$
31. $44.2 \div 1.3$
32. $6.5 \div .1$
33. $4.48 \div .32$
34. $2.84 \div .04$
35. $7.68 \div .08$
36. $7.36 \div .16$
37. $12.2 \div .2$
38. $34.3 \div .7$
39. $1.18 \div .02$
40. $1.8 \div .3$
41. $3.6 \div 1.2$

## CROSS-NUMBER PUZZLE

Complete the puzzle by putting each digit of your answer in a separate square.
Down checks across.

Across

1. $5 / 16$
2. $4 / 5$
3. $3 / 16$
4. $6 / 20$
5. $3 / 6$
6. $3 / 8$
7. $7 / 14$
8. $3 / 5$
9. $3 / 10$
10. $1 / 2$
11. $0 / 7$
12. $1 / 5$
13. $7 / 16$
14. $16 / 20$
15. $11 / 20$
16. $7 / 20$
17. $2 / 4$
18. $5 / 6^{*}$
19. $5 / 10$
20. $5 / 12$
21. $9 / 10$
22. $1 / 16$
23. $6 / 10$
24. $8 / 16$
25. $9 / 18$
26. $1 / 6^{*}$
27. $20 / 25$
28. $19 / 20$
29. $\quad 24 / 40$
30. $\quad 2 / 20$
31. $13 / 20$
32. $1 / 10$
33. $3 / 30$
34. $5 / 8$
35. $\quad 2 / 10$
36. $11 / 16$
37. $6 / 12$
38. $15 / 16$

## Down

1. $1 / 3^{*}$


## CROSS-NUMBER PUZZLE

Complete the puzzle by putting each digit of your answer in a separate square.
Down checks across.

## Across

1. $6 \%$ of 700
2. $3 \%$ of 700
3. $7 \%$ of 500
4. $6 \%$ of 300
5. $10 \%$ of 4,010
6. $5 \%$ of 4,500
7. $12 \%$ of 1,200
8. $5 \%$ of 1,220
9. $2 \%$ of 100
10. $6 \%$ of 800
11. $3 \%$ of 1,100
12. $4 \%$ of 800
13. $5 \%$ of 900
14. $11 \%$ of 700
15. $5 \%$ of 200
16. $5 \%$ of 500
17. $3 \%$ of 400
18. $9 \%$ of 700
19. $6 \%$ of 100
20. $10 \%$ of 40
21. $1 \%$ of 100
22. $9 \%$ of 300
23. $3 \%$ of 600
24. $2 \%$ of 800
25. $11 \%$ of 900
26. $6 \%$ of 1,100
27. $8 \%$ of 600
28. $5 \%$ of 1,000
29. $7 \%$ of 1,000
30. $6 \%$ of 900
31. $3 \%$ of 200
32. $6 \%$ of 600
33. $2 \%$ of 7,100
34. $4 \%$ of 8,200
35. $6 \%$ of 11,100
36. $4 \%$ of 1,200
37. $7 \%$ of 700
38. $3 \%$ of 500
39. $6 \%$ of 300

## Down

1. $4 \%$ of 1,100

2. $4 \%$ of 500
3. $1 \%$ of 200
4. $4 \%$ of 300
5. $7 \%$ of 500
6. $5 \%$ of 100
7. $1 / 5 \%$ of 7,000
8. $30 \%$ of 280
9. $2 \%$ of 800
10. $5 \%$ of 440
11. $6 \%$ of 300
12. $121 / 2 \%$ of 1,056
13. $25 \%$ of 1,808
14. $4 \%$ of 775
15. $3 \%$ of 1,000
16. $5 \%$ of 500
17. $10 \%$ of 410
18. $2 \%$ of 3,800
19. $331 / 3 \%$ of 219
20. $2 \%$ of 300
21. $1 \%$ of 400
22. $1 / 2 \%$ of 200
23. $4 \%$ of 650
24. $8 \%$ of 950
25. $12 \%$ of 1,200
26. $5 \%$ of 1,760
27. $3 \%$ or 500
28. $4 \%$ of 15,075
29. $25 \%$ of 388
30. $4 \%$ of 2,250
31. $8 \%$ of 650
32. $2 \%$ of 3,100
33. $3 \%$ of 2,200
34. $4 \%$ of 350
35. $3 \%$ of 1,600
36. $5 \%$ of 780

51 . $25 \%$ of 324
53. $2 \%$ of 3,050
54. $5 \%$ of 1,360
56. $4 \%$ of 100
58. $2 \%$ of 250

## DO - IT - YOURSELF - CROSSWORD

Make up a set of problems so that the correct answers will fill this puzzle.


## MAGIC SQUARE

What is it?

Why use it?

How to use it?

A magic square is an arrangement (matrix) of numerals in such a way that the sum of every row, column, and diagonal is exactly the same.

It is a novel and interesting phenomenon which serves as a sneaky exercise in addition as well as number patterns. Use for interest and motivation.

1st day — draw the empty $3 \times 3$ grid on the board and pose the $\mathfrak{l u}$ astion: is it possible to fill the squares in such a way that every row and column has the same sum, using the integers from 1 to 9 .

2nd day - Put the $3 \times 3$ magic square below on the board as a summary of the previous experience. Ask whether this is the only arrangement of these numerals that will work. There are several others, all of which are merely reflections and rotations of the original. There is essentially just one $3 \times 3$ magic square. Ask whether some other numbers might work as well, for example starting with 3 or 5 , or using the evens or odds. A one-to-one substitution yields many new squares.


3rd day - Put the $5 \times 5$ magic square on the board and ask the class to make a $7 \times 7$ square using the board as a pattern. Before the class is out see if they can formulate a set of rules for making squares having a odd number of rows and columns.


## MAGIC SQUARE (continued)

5th day - Give the students the opportunity of making as complicated a magic square as they wish, such as a $11 \times 11$, but stress that they must show it to be a magic square by adding several rows, several columns, and the diagonals showing the same result for each

6th day - Put on the $5 \times 5$ magic square below. They will observe that it is different. See if they can formulate the way in which it was formed. Give them the chance to make larger squares for extra credit.


7th day - Put an empty $4 \times 4$ grid on the chalk board. Ask them to see if they can make a magic square out of it using the integers from 1 to 16 . If they have trouble show them the one below. See if they can figure out how it was made. Are other arrangements possible?


Resources: Scientific American Puzzle Book, Vols. I, and II. Holt, General Mathematics, pages 28, 170-171, 195, 245-246, 257.

## FRACTION-DECIMAL-PER CENT CHART

What is in?

Why use it?

## How to use if?

The chart shows the relationship of the most common fractions to their decimal and per cent forms. it is a visual aid and an equivalence table when speed computation is desired.

Making it gives the student an exercise in renaming fractions, and once it is made the student can use it as a study aid. The finished product gives the student an added feeling of success.

As a class assignment have each student make a chart for his own use. Explain that he will be allowed to use it at times to speed up his work. As an individual project have one or several students make a larger poster for the front of the room. In using the chart show where it is best to use the fractional form and where either form can be used with the same degree of accuracy. Indicate also that the chart works as well for some other fractions which are nnt on the chart, such as $7 / 8=7 \times 1 / 8$, and $3 / 16=3 / 2 \times 1 / 8$.

| FRACTION | DEGIMAL | PER CENT |
| :---: | :---: | :---: |
| $\frac{1}{2}$ | .50 | $50 \%$ |
| $\frac{1}{3}$ | $.33 \frac{1}{3}$ | $33 \frac{1}{3} \%$ |
| $\frac{1}{4}$ | .25 | $25 \%$ |
| $\frac{1}{5}$ | .20 | $20 \%$ |
| $\frac{1}{6}$ | $.16 \frac{2}{3}$ | $16 \frac{2}{3} \%$ |
| $\frac{1}{7}$ | $.14 \frac{2}{7}$ | $14 \frac{2}{7} \%$ |
| $\frac{1}{8}$ | .125 | $12.5 \%$ |
| $\frac{1}{9}$ | $.11 \frac{1}{9}$ | $11 \frac{1}{9} \%$ |
| $\frac{1}{10}$ | 10 | $10 \%$ |

## SLIDE RULE

## What is it?

## Why use it?

How to use it?
A unit on making and using a basic slide rule.
It gives the student a different exercise in measurements as well as practice in mustiplication and division. The idea of using a slide rule serves as an excellent motivetonal device.

Prepare strips of tagboard $1 " \times 12$ " so that each student will have two. Paper cutter works best. Introduce simply as an exercise in following instructions and accurate measuring. Have them take one of the pieces and beginning at the lefthand side, mark off on the top edge the various points according to your directons. Using the diagram below instruct them as to the naming of points and the distance between; using either inches or (perhaps better) centimeters. When they finish have them mark corresponding points on the other piece which has been placed directly above it. Then tell them that they can test the accuracy of their work by placing the 1 of the top piece above the 2 of the bottom piece and reading the numbber 4 on the bottom piece, hopefully directly below the 2 of the top one. Some may jump to the idea that this is a multiplication table and in fact a basic slide rule Work several elementary problems using this method.

and day - Divide the slide rule they made previously into smaller units or get loggraph paper and make new rules which will of course be more accurrate. Work with discovering the values of the smaller units and perhaps finding larger numbers on this scale.
3rd day - Show the movie "Slide Rule", 30 minutes long, available from the Audio-Visual department.
th day - If possible, obtain a set of slide rules and try a few simple problems.

## NOTES:

## GRAPHING

What is it? A collection of ideas to be used as a unit on coordinate systems.
Why use it? General mathematics students are able to find success experiences with this type of work. They consider it sophisticated enough to be appropriate and challenging. It gives a picture of an abstract idea which helps them master the idea. It has many applications which are enjoyable to work with.

How to use it?

When to use it? Introduce the idea early in the year and then use as a thread of continuity throughout the year.
After the students have used a number line enough to be quite familiar with it. Extend this notion by rotating the number line 90 degrees to make a second line. Establish the idea of direction by such problems as coming to schoni. How would they tell a friend to find a mystery spot? Use the idea of traveling along the streets by blocks, that is, go 3 blocks west and then 2 blocks south. Now define negative and positive directions.

Graphing is best taught by limiting the range of the problens to the first quadrant, at least to start with. By using such games as submarine or 4 -in-a-row, establish the concept of an ordered pair. After the first quadrant has been mastered, the other quadrants could be used, building up to the use of all four quadrants.
Once the student has a fair idea of the names of points in all four quadrants, the learning can be reinforced by the use of picture graphs.
Students enjoy graphing most when they can come to the front of the room and locate a point for the class. For this purpose a $4^{\prime} \times 4^{\prime}$ peg-board serves ideally. The axis can be either painted on or taped on using masking tape. Having it painted a bright color also adds to the classroom's attractiveness. Plastic golf-tees work ideally as markers and colored yarn for lines. Several colors of golf-tees work best, especially for team competitions.
Once graphing has been introduced, use it to record the data of problems, especially when a function is involved, such as relation between radiue and diameter, radius and area, etc.

## NOTES:

## FOUR-IN-A-ROW

## What is it?

A classroom graphing game. Similar to the game of $\mathbf{X}$ and $\mathbf{O}$.

## Why use it?

How to use it?

It is an approach to graphing which the students really enjoy because it resembles a game and utilizes the idea of competition.

Divide the class into two teams, either two rows or the whole class. Without any introduction as to the mechanics of graphing instruct each team to name two small positive integers (less than the limit of your first quadrant). As a team gives its pair of numbers locate the point by positioning a golf tee, one color for each team. The object is to get four markers in a row, horizontally, vertically, or diagonally. Either team can block the other by getting in the way through careful selection of numbers. This method enables the student to discover that the first number tells you how far to move to the right, and the second how far up. After negative numbers have been introduced, the other quadrants can be used in the same way.
The students frequently get rather excited trying to name the right point, but this is the type of excitement we need in mathematics, so don't be disturbed.
You may want to choose a captain or spokesman for each team to provide for smoother selection of numbers.
After several games have been played, ask the students to verbalize the rules for locating points.
Emphasize the fact that the axes can be given any name that is convenient. The use of symbols like, $\square$ $\& \triangle$ has proved more successful than limiting the names to X and Y . One interesting method of naming is to call one axis "strikes" and the other "balls" and then locate the possible baseball calls. Conversely, you could choose points and inquire as to the batter's status, such as 3 balls and 2 strikes.



NOTES:

## SUBMARINE

What is it? A classroom graphing game, by which two teams attempt to locate and sink the enemy ships.

Why use it?

How to use in?

It is an enjoyable way of drilling the graphing skill, which students find most enjoyable.

Divide the class into teams by rows. Each team secretly locates two submarines ( 3 points in a row) and one battleship ( 5 points in a row).
A captain for each team is responsible to call out each time one of his fleet is hit. To make a hit an opposing team must name one of the points which make up a ship. To sink a submarine two hits are required; to sink a battleship, three hits.
Each team should try to locate their ships so as to be as hard to find as possible. To begin with, use a $10 \times 10$ grid in the first quadrant. As they catch on to the idea, other quadrants can be used.
Graphing is a very good way of introducing signed numbers. Simply define to the left, and down from selected axes as negative directions and show that points in the other three quadrants are named by one or two negative numbers.
Be careful not to let the area of play become too large. If it takes too long to make a hit, they will lose interest.
Use a different color of golf-tee to indicate each team's guesses and a special color for hits. The first team to sink the enemy navy wins. The game can be continued over a period of a week. If this is the case have the captains submit the location of their ships in writing and leave the graph board undisturbed after each useage.
Because the students enjoy this exercise it can be used as a reward or as a motivational force.

## NOTES:

## PICTURE-GRAPHS

What is it?
Why use it?

How to use it?

When to use it?

A listing of ordered pairs which when located and connected form a picture.
It is a different way of drilling the graphing skills which the student can check. A lack of accuracy results in a confused set of lines, while accurate plotting results in an identifiable picture. The students enjoy the mystery of not knowing what the picture is, and of seeing it take shape right before their eyes.

Explain the procedure and then put the list of ordered pairs on the board, or better yet, on the overhead. Now turn them loose. With everyone working the teacher is free to circulate and give individual help.
Listed below are a few pictures you might use to begin with. Give the students a chance to make their own pictures with corresponding ordered pairs for extra credit. You may want to use some of these for class work.

After graphing has been introduced and used. Use sparingly throughout the year.
(New York Sky-line)

| $\mathbf{X}$ | $\mathbf{Y}$ |
| ---: | ---: |
| 0 | 0 |
| 0 | 5 |
| 2 | 5 |
| 2 | 0 |
| 2 | 9 |
| 5 | 9 |
| 5 | 2 |
| 8 | 2 |
| 8 | 12 |
| 9 | 14 |
| 10 | 12 |
| 10 | 2 |
| 15 | 2 |
| 15 | 0 |

(The Face of a Girl)
see next page
(Automobile)

| $\mathbf{X}$ | $\mathbf{Y}$ |
| :--- | :--- |
| 1 | 1 |
| 0 | 1 |
| 0 | 3 |
| 2 | 3 |
| 3 | 4 |
| 6 | 4 |
| 7 | 3 |
| 9 | 3 |
| $91 / 2$ | $21 / 2$ |
| 9 | 2 |
| 9 | 1 |
| 8 | 1 |
| $71 / 2$ | 0 |
| $61 / 2$ | 0 |
| 6 | 1 |
| 3 | 1 |
| $21 / 2$ | 0 |
| $11 / 2$ | 0 |
| 1 | 1 |
|  |  |

NOTES:

## WHAT IS IT?

Locate each point drawing a line from the first point to the second, from the second to the third, etc.

| X | Y | $\mathbf{x}$ | X |
| :---: | :---: | :---: | :---: |
| -3 | -4 | -5 | 4 |
| -4 | -3 | -41/2 | $41 / 3$ |
| -51/4 | 0 | -3 | 4 |
| -53/4 | 6 |  |  |
| -5 | $71 / 2$ |  |  |
| -2 | $81 / 2$ | $\mathbf{X}$ | Y |
| 0 | $81 / 4$ | $\underline{ }$ |  |
| 2 | 8 | 0 | $41 / 2$ |
| 3 | $71 / 2$ | 11/2 | $43 / 4$ |
| 4 | 5 | $31 / 2$ | $41 / 2$ |
| 4 | 2 |  |  |
| 5 | 1 |  |  |
| 1 | -1 |  |  |
| 3 | -21/2 | X | $\mathbf{Y}$ |
| 0 | -41/2 | -5 |  |
| -21/2 | $-41 / 2$ | -4 | $31 / 2$ |
| -3 | -4 | -3 | $31 / 2$ |
| -51/2 | -2 | -21/2 | 3 |
| -7 | 0 | -3 | $21 / 2$ |
| -8 | $21 / 2$ | -41/2 | $21 / 2$ |
| -61/2 | 5 | -5 | 3 |
| -6 | 7 |  |  |
| -51/4 | 8 |  |  |
| -4 | $10^{1 / 2}$ |  |  |
| 0 | 12 |  |  |
| 3 | $111 / 2$ |  |  |
| 5 | 11 |  |  |
| 61/2 | 9 |  |  |
| 61/2 | 7 |  |  |
| 9 | $21 / 2$ |  |  |
| 8 | 0 |  |  |
| 8 | $-11 / 2$ |  |  |
| 6 | -3 |  |  |
| $41 / 2$ | -3 |  |  |
| 4 | -1 |  |  |


| $\mathbf{X}$ | $\mathbf{Y}$ |
| :--- | :--- |
| $-1 / 4$ | 3 |
| 1 | 4 |
| 2 | $31 / 2$ |
| $21 / 2$ | 3 |
| 1 | $23 / 4$ |
| $-1 / 4$ | 3 |


| $\mathbf{X}$ | $\mathbf{Y}$ |
| :--- | :--- |
| $-31 / 2$ | $-11 / 2$ |
| $-23 / 4$ | $-1 / 2$ |
| -2 | -1 |
| $-1 / 2$ | $-1 / 2$ |
| $1 / 2$ | $-1^{1} 1 / 2$ |
| $-31 / 2$ | $-1^{1} / 2$ |
| $-21 / 2$ | -2 |
| $-11 / 2$ | -2 |
| $1 / 2$ | $-1^{1} / 2$ |

## NOTES:

(

## MEASUREMENT

What is it? A section of ideas and exercises to be used in conjunction with the measurement sections of the various texts.

Why use it?
A convenient way to supplement the text. Easy to use.

## How to use it?

Page 65 - This is a different and more attractive way to present a measuring assignment. It can be used with centimeters as well as inches. By asking the students to find the difference between the measures of AB and BC it can be made into an exercise in using mixed fractions. By asking the students to find the total length we show the relationship between lengths and names for lengths.
Page 66 - This is an exercise in finding, as well as measuring, angles. Ask the students to measure $\angle \mathrm{AOC}$ and $\angle \mathrm{COA}$, to show the different possibilities in naming the same angle.

Page 67 - This is a novel way to present several ideas in an eye-catching form. First, it can serve as an exercise in recognizing triangles. Second, it can be used to determine the sum of the measures of the angles of a triangle (this idea has much more meaning if a student is allowed to discover it instead of being told the measure is 180 degress). Third, it can be used as an exercise in recognizing the different types of triangles (right, acute, obuse). Fourth, it can be used as an assignment in measuring the perimeter of triangles, or 'ast line segments.

Page 68 - Here is a list of suggested measuring assignments. They have been tried and proven successful. For the most patt they are different than anything now used.

NOTES:
$\qquad$
Measure the line segments below. Can you determine the length of $\overline{\mathbf{A Z}}$ ? Write your answer on the respective lines as you measure them.

E



| \# | NAME OF angle | angle MEASURE |  | $\begin{array}{\|c\|} \hline \text { NAME OF } \\ \text { ANGLE } \\ \hline \end{array}$ | $\begin{array}{\|c\|} \hline \text { ANGLE } \\ \text { MEASURE } \\ \hline \end{array}$ | \# | $\begin{array}{\|c\|} \hline \text { NAME OF } \\ \text { ANGLE } \\ \hline \end{array}$ | ANGLE | \# | $\begin{gathered} \text { NAME OF } \\ \text { ANGLE } \end{gathered}$ | $\begin{gathered} \text { ANGLE } \\ \text { MEASURE } \\ \hline \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1. | $\angle A O B$ |  | 6. | $\angle F O G$ |  | 11. | $\angle \mathrm{FOC}$ |  | 16. | LEOG |  |
| 2. | $\angle B O C$ |  | 7. | 二GOH |  | 12. | $\angle \mathrm{FOB}$ |  | 17. | $\angle A O C$ |  |
| 3. | $\angle C O D$ |  | - | $\angle \mathrm{HOI}$ |  | 13. | $\angle \mathrm{FOH}$ |  | 18. | $\angle \mathrm{DOI}$ |  |
| 4. | LDOE |  | 9. | LIOA |  | 14. | $\angle G O C$ |  | 19. | $\angle H O D$ |  |
| 5. | LEOF. |  | 10. | $\angle A O E$ |  | 15. | $\angle \mathrm{COI}$ |  | 20 | $\angle \mathrm{BOI}$ |  |

$\qquad$
How many triangles can you count'! Try naming as many as you can, giving the measure of each angle. How many right triangles are there? How many acute? How many obtuse? Use the space below to show your work.


## MEASURING ASSIGNMENTS

1. Find the center point of the paper by guessing, measuring, using diagonals, and folding. Repeat for $1 / 2$ and $1 / 4$ sheets of paper.
2. On one sheet of paper draw $1,2,4$, and 8 inch squares all having the same center point. Compare their perimeters and areas.
3. Draw a set of 4 lines, parallel to one of the margins, and then draw a second set of 4 lines not parallel to any margin, but still parallel to each other.
4. Draw a pair of perpendicular lines in the center of your paper and parallel to the margins, then draw another pair not parallel to any margin.
5. Mark and cut a piece of paper into $3,4,5$, or 6 congruent rectangles, each time using the entire sheet of paper.
6. Mark off a piece of paper into the greatest number of congruent 2 " squares, or 2 " $\times 3$ " rectangles, keeping the amount of wasted paper to a minimum. Practice in getting the fullest use of the material available.
7. Make $1,2,3$, and 4 inch borders on the same sheet of paper.
8. Make a geoboard pattern on paper, ( $71 / 2 \times 71 / 2$ inches having a $3 / 4$ " border and divided up into 1 " squares), (see page 6 and 7 ).
9. Make a chess or checker board out of tagboard. A checker board has 64 congruent squares alternating in color. If you wish to have it fold in half, cut it in half and use scotch tape as a hinge.
10. Make a cube, rectangular prism, parallellepiped, 3 or 4 sided pyramid, cone, circular cylinder, or any of the regular polyhedrons out of tagboard. (suggest the use of tat for smoother results), (see page 71).
11. Make a set of Napiers Bones, (see page 15 and 16).
12. Make a fundamental slide-rule, (see page 57).
13. Make a sketch with all measurements, of the teacher's desk, a filing cabinet, the front or side of the classroom, or a bookshelf. This would be most valuable if it was done to scale.
14. Find the wall area of the classroom or another room and then estimate the amount of paint or wood paneling it would require to cover it.
15. Make a floor-plan of the classroom or another room and then estimate the number of $9 "$ tile or square yards of carpet it would take to cover it.
16. Make a scale drawing of the front of your home.
17. Take a scale drawing and enlarge it or reduce it (multiply or divide all measurements by 2 or 3 ), (see page 81 ).
18. Make a large circle for the front of the room, (see page 12).
19. Make a set of angles for the musical-chairs angle quiz, drawing one angle on each sheet of plain white paper, using 30 different angles, (see page 69 ).
20. Make the plane figures for the geometric-shape mobile and put it together using needle and thread, wire, and dowel sticks, (see page 72 and 73).
21. Make a large magic square poster, (see page 54 and 55 ).
22. Make a fraction-decimal-percent chart for the front of the room, (see page 56).

## MUSICAL-CHAIRS QUIZ

What is it?

Why use it?

How to use it?

A moving quiz in which a question is placed on each desk and the students move from desk to desk with their own answer sheet and pencil.

It is an interesting way of bringing variety into the taking of tests. The students enjoy the moving from desk to desk. Also the time limit forces the slower student to work faster. It prevents a class from going to sleep on you during a test.

The test was originally design. $\perp$ io be used as ar exurcise in measuring angles. As an individual project one student would be given the task of drawing on plain white paper, 30 different size angles, some acute, some obtuse, some right, one angle per sheet of paper. Then at the time of the test these sheets are passed out one per desk. Each angle is numbered and given a name such as $\angle$ AOB. Each sturent is then given a protractor and an answer sheet (page 70). Note that it is necessary to put the angles in numerical order, in fact it works best when they have ben shuffled and the student must place the correct answer in the correct space. Explain to the class that they will have a limited amount of time to measure and name each angle. Emphasize the fact that they should measure the angle first, then name it, then tell whether it is acute, obtuse, or right. This way the slower student will at least get the measuring done, while the better student will be able to do all three in the same amount of time. Begin by allowing abc $\cdot \mathrm{t}$ d minute or two for the first angle. Then cut the time down to about 30 seconds, allowing about 10 seconds to change chairs. Before starting establish a motion-chart on the board to help each student to know where to go next. If you use a snake pattern they will follow quite easily.
When correcting, allow a 2 degree variance. Use a couple of different days to allow for improvement and to give them the feeling of success.
Although designed specifically for an angle quiz it is certainly adaptable to many different types of quizzes and exercises.
To operate smoothly set up a signal such as "STOP" or "MOVE", which they understand as being the signal to move on to the next desk. Watch the student and adjust the amount of time you take per question accordingly. After you have used it once it will go very smoothly with each additional use.

## NOTES:

## ANGLE QUIZ RESPONSE SHEET

$\qquad$
Record the measure, name, and type of each angle, in that order. You will be given a few minutes at the end of the quiz to fill in any missing names. Be sure that you record your answer in the correct space.

| NO. | ANGLE NAME | ANGLE MEASURE | OBTUSE ACUTE RIGHT | NO. | angle NAME | ANGLE measure | OBTUSE ACUTE RIGHT |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 |  |  |  | 16 |  |  |  |
| 2 |  |  |  | 17 |  |  |  |
| 3 |  |  |  | 18 |  |  |  |
| 4 |  |  |  | 19 |  |  |  |
| 5 |  |  |  | 20 |  |  |  |
| 6 |  |  |  | 21 |  |  |  |
| 7 |  |  |  | 22 |  |  |  |
| 8 |  |  |  | 23 |  |  |  |
| 9 |  |  |  | 24 |  |  |  |
| 10 |  |  |  | 25 |  |  |  |
| 11 |  |  |  | 26 |  |  |  |
| 12 |  |  |  | 27 |  |  |  |
| 13 |  |  |  | 28 |  |  |  |
| 14 |  |  |  | 2.9 |  |  |  |
| 15 |  |  |  | 30 |  |  |  |

PATTERNS FOR GEOMETRIC SOLIDS


ICOSAHEDRON



4-SIDED PYRAMID BEFORE CUTTING OUT, FOR SHARPER FOLDS.

## TRIANGLE MOBILE

What is it?

Why use it?
A mobile which shows the different types of triangles broken down according to sides, and angles. An interesting student project and an attractive addition to any math room. Use separately or add later to the larger geometric mobile.

It gives the student a chance to make and see that which they are talking about. Develops skill in balance.
Use tagboard for the figures, sections from coat hangers for the bars, and thread to hold it together. Have the girls (or boys) sew it together.
Once made the mobile serves as an excellent review of vocabulary. Do not label the different triangles, but instead ask different students to name as many of them as they can. As an additional artistic toucit have the students decorate them in different colors and patterns, perhaps using the angle-bisectors or medians,


## QUADRILATERAL MOBILE

What is it?

Why use it?

## How to use it?

A mobile which shows the relationship between the various four sided figures. From this a sudent could define a square as a quadrilateral having parallel sides and a right angle.

To show what you are studying. It serves as a constant reminder of what has been learned. It also identifies your room as a mathematics room the moment anyone comes in the door.

Have the students make it and put it together. There is a lot to be learned just in finding the balance points of the various figures.
Use separately, or add to the larger geometrical mobile.
The geometric mobile is completed by adding the other N -gons (pentagon,hexagon, heptagon, octagon, decagon) on another bar at the same level with triangles, and quadrilaterals, and perhaps have everything hanging from a multi-sided N -gon.


## BASEBALL QUIZ

What is it? A classroom quiz-game which is used to review and drill past material in an enjoyable way.

Why use it?

How to use it?

When to use it?

The students enjoy it so much they ask to play it. It associates their interest in baseball with mathematics and it lends a familiar form to a previously unpleasant task.

Explain the ground rules to the class. Two teams will be chosen arbitrarily. Each team will have a set batting order. Each correct answer earns a score and each incorrect answer earns an out. Three outs retire the team and the game will last for two or three innings.
Before starting, select some of the better students to officiate: an umpire - to judge the correctness of answers; a scorekeeper - to keep the score at the chalkboard; a pitcher - to ask the questions; a timer - to limit each student to a reasonable thinking time. Teams are determined by having the students put their name on a blank card and handing them in. Shuffle the cards and deal two stacks. Batting order is determined by the order of the cards. If desired, you can name the teams according to class consensus. Begin the game by letting one team be first. If necessary, explain that excessive noise or yelling of answers will be penalized by loss of team points. Questions should be made up beforehand either by the teacher or the students. Emphasize the importance of listening by reading a question only twice and by asking the same question until it is answered correctly.
A better, but more complicated, way of scoring is to grade the level of difficuity of each question and separate into divisions labeled single, double, homerun. Then allow each student to choose the level of difficulty he can handle. The students like this approach better because it requires strategy.
Allow students to use scratch paper (many will work every problem to make sure the correct answer is given).
Don't play too long or too often. Use it as a reward which must be merited.
Use questions which emphasize concept instead of lengthy computation, $.2 \times .3$ shows more than $67.34 \times 34.5$.

It works best during the world series, but don't restrict it to just then. Use it whenever a review is called for and especially when class spirit appears to be lagging.

NOTES:

## SIMPLIFIED STORY PROBLEMS

What is it?

Why use it?

How to use it?

A more realistic approach to story problems for the slow reade. Essentielly it is a method of presentation by which all unnecessary words are deleted so as to leave just the meat of the problem.

A large percentage of general mathematics students have a reading problem. Why complicate the teaching of mathematics by playing up this problem as well? Phraseproblems are easy to read and avoid the reading difficulty. Story problems have been a difficult topic for even algebra and geometry students. Granting that they are essential for all mathematics students, we need to try a special approach for the student who can't read and frequently will not even try to read a traditional story problem. Past failing experiences act as a mental block for this type of student. This new approach shows the student he can do a word problem and serves as an incentive to go on and try mor? "wordy" problems.

If there is a set of problems you want to use, rewrite them beforehand reducing their structure to short phrases having a minimum number of words.

Traditional form:

## Simplified form:

The Donaldsons had a friend in the lumber business who said he would sell him the lumber at a discount of $12 \%$. If the reguiar price of the lumber was $\$ 2200$, what could the Donaldson family save on this item?
A friend can get you a $12 \%$ discount on lumber. You would have paid $\$ 2200$ for your purchase. How much would you save?
As an assignment have each student simplify a story problem leaving out all unnecessary words. This teaches the real approach to story problems.
An alternative approach is to start from scratch and to create your own problems in a form midway between a story problem and a number problem.


NOTES:

1. Your body contains 206 bones. About $1 / 7$ of these are in your head. About how many bones are there in your head?
2. Muscles make up about $\mathbf{4}$ of man's body weight. What do the muscles of a $\mathbf{1 8 0}$ pound man weigh?
3. In one day, a man breathes in about 400 cubic feet of air. About $5 \%$ of this is oxygen that is absorbed into the blood stream. How many cubic feet of oxygen is this?
4. An eye blinks about 25 times each minute. About how many times does it blink in a day? (24 hours)
5. Your intestines (large and small) are about 25 feet long. Your small intestine is about $80 \%$ of this length. How long is your small intestine?
6. The body of an adult contains about 5 quarts of blood. A blood donor usually gives one pint of blood. What fraction of his blood does the donor give?
7. A man's brain is t tween $2 \%$ and $3 \%$ of his body weight. Between what two numbers is the brain weight of a 110 lb . boy?
8. Your heart beats about 80 times a minute. About how many times does it beat in a day?
9. The liver is the largest gland in your body. In a 140 lb . person, the liver is about $1 / 35$ his body weight. How many pounds would that be?
10. The human body is about $67 \%$ water. How many pounds of water are there in a 200 lb . man?

## MULTIPLICATION TABL

What is it? Why use it?

## How to use it?

When to use it?

An old fashioned multiplication table can be used effectively as a speed test.
At the beginning of the year it is advantageous to find out which students have problems in multiplication. Once this has been determined, remedial work can begin.

Pass out the blank table and take a few examples, showing that they write the product of a row-number and a column-number in the space common to both, such as 9 x 9 . Then time them. Give only enough time for the fastest student $t u$ finish. It is important to make the student understand that this is a way for him to find out where he is having problems. It is for this reason that it is important that all students stop together when time is called. If they realize that this is an ungraded exercise they will comply. Also, the incentive of a speed drill will motivate many students who would otherwise consider this pure boredom.
After the initial use, study the table for patterns. See if the students can discover the symmetrical character of the table and thus figure out a shorter way of completing the table. Point out that every row and column increases by a constant (stress the meaning of multiplication as short-hand addition).

At the very beginning of the year and whenever a review is desired. Use occasionally at first so that those students who have problems can see whether or not they are doing anything about them.

NOTES:


This is a speed test in multiplication. If there are any that you are uncertain about, skip over them. When time is called make a list of the errors and blanks in the space below. Use this as a guide for future study.

Problem Points

Do you notice any patterns in the squares above? What might you say is the roll of 1 in multiplication? of O ? Can you think of a shorter way of making a complete table without duplications?

## LEDGER SHEET

What is it?

Why use it?

## How to use it?

A regular business form which adapts nicely to many different mathematical problems.

The form is one which students can readily learn and as a result they know what is expected of them when using it. Once the method of solution is set up the ledger sheet provides the necessary drill to reinforce it. The idea of working a problem on a form used in business provides motivation.

Each column on the ledger sheet has a use. The first is for the list of names or items. The second set is for the day showing either amount of time worked or pieces proudced, etc.. The next is for the total of the week. The next for per-unit costs or hourly wage. And the last is for the total amount.
The student would ordinarily make out the sheet according to instructions. Data could be either mimeographed for them or put on the board or overhead.
Run off a good supply of the ledger sheets so that each student can be given 3 or 4 over the period of a year.

## Sample Problem:

English Leather Sale.

| ITEM | MONTHLY |  |  | SALES |  |  | total | $\begin{array}{\|c\|} \hline \text { PER UNIT } \\ \text { COST } \end{array}$ |  | TOTAL COST |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Jug | AUG | SEPT | Oct | NOV | DEC |  |  |  |  |  |
| STICK DEOD. | 11 | 19 | 6 | 9 | 23 | 35 |  | 1 | 37 |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |
| SPFAAY DEOD. | 6 | 2 | 42 | 11 | 48 | 56 |  | 1 | 28 |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |
| 40Z.ALL-PURPOSE | 16 | 19 | 8 | 35 | 59 | 32 |  | 2 | 17 |  |  |
| LOTION |  |  |  |  |  |  |  |  |  |  |  |
| 40Z.ALL-PURPOSE | 4 | 5 | 2 | 11 | 8 | 22 |  | 2 | 17 |  |  |
| TRAVEL BOTTLE |  |  |  |  |  |  |  |  |  |  |  |
| 80Z.ALL-PURPOSE | 2 | 0 | 7 | 26 | 23 | 20 |  | 3 | 86 |  |  |
| LOTION |  |  |  |  |  |  |  |  |  |  |  |
| 40Z.PRE-SHAVE | 3 | 2 | 5 | 10 | 7 | 4 |  | 1 | 44 |  |  |
| LOTION |  |  |  |  |  |  |  |  |  |  |  |
| 40Z. LOTION A | 22 | 1 | 4 | 29 | 9 | 28 |  | 2 | 91 |  |  |
| SPRAY DEOD. SET |  |  |  |  |  |  |  |  |  |  |  |
| 40Z.LOTION 8 | 15 | 6 | 28 | 46 | 47 | 24 |  | 2 | 24 |  |  |
| STICK DEOD. |  |  |  |  |  |  |  |  |  |  |  |
| 40Z.LOTION A | 6 | 6 | 8 | 9 | 2 | 1 |  | 1 | 19 |  |  |
| TALC. POWDER |  |  |  |  |  |  |  |  |  |  |  |


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## SCALE DRAWINGS

What is it?
Why use it?

## How to use it?

A good way to show the relationship between fractions and the length of lines.
Interest. Some students will find this type of work enjoyable who have been missed up until now. It is a clear application of mathematics which makes sense.

Introduce the idea by posing the problem of drawing a picture of a football field. Since you can't draw it as it is, you have to use a scale. Make up one. Show the class blueprints and let them see how scales are used. Then provide drill in operations performed when using scales, primarily converting from real measurement to scale and vice versa.
Have the students make some simple scaie drawings according to directions. Draw a picture of a square $60^{\prime} \times 60^{\prime}$, a rectangle $30^{\prime} \times 50^{\prime}$, etc..
Use scale drawing exercises such as those in Holt, General Mathematics page 374378, and page 381.
Have the students pick an object such as a desk or bookshelf and make a scale drawing of it.
Take a scale drawing from a book and have the student draw it to a different scale (enlarge or reduce).
As a special assignment make a scale drawing of the school grounds and building. The whole class could be involved in the project. Teams could be assigned to gather the data, and others to make the model. Have each small group take a specific job such as the football field. After the measurements have been taken and converted to scale, have them make their project out of tag board and then position it on the over-all plan. The final result then would, be a conglomerate of the small groups' work.
Another project is to have them draw their "dream house" to scale, showing its specifications. Then using old catalogs such as Sears or Pennys have them furnish a room, making out an order form as they go. Girls especially respond to this idea. Greater participation is obtained by asking them to bring any old catalogs they may have at home.

## NOTES:

## USING NEWSPAPERS IN CLASS

What is it?

Why use it?

How to use it?

A different approach to showing the practicality of learning mathematics. A way of impressing upon the student the useability of what they are learning. Also it is an extremely timely way of presenting up-to-date problems and of effecting "carryover" from mathematics to life.

Students are especially interested in anything that does not look mathematical. This is a good way of using their interest in sports and current events to effect numerical learnings.

It seems that the best approach is to keep your eyes open to this idea as you sip your morning coffee and skim the morning paper. Make a mental note of anything that involves numbers. Then take the paper to school with you, cut out the article and using scotch tape loops, arrange the article on a plain piece of paper. Newspapers copy beautifully or a copy machine. Be sure to use the lowest setting. Then have the spirit-master you have just made run off. This will probably take you between $5 \& 10$ minutes in the morning. Try it, you'll be pleased by your students' reaction.

After the article has been run off, look it over and decide what type of questions should be asked. If you use a write-up of last night's basketball game, you could have the students figure out some of the percentages involved. It is important to have in mind before hand the goal you desire to attain by means of this exercise.
Be sure to pick articles which the students will be interested in. Sports rate most highly. Perhaps best results would be to obtain by following the senior high most of them attend. The Oakleafs, Bulldogs, Hawkeyes, Twins, etc. are favorites of most students.
The next page is an example of the form such an exercise would take. This point should be emphasized, use recent articles.
This idea can easily be ruined by bringing the newspaper into the classroom and reading "at" the class. Student participation is essential. The student will rightly interpret such an act as a lack of preparation.
You may desire to get a classroom set and have the students find some articles. Such a service is offered free of charge by The Register and Tribune.

## NOTES:

## Injured Snook on Bench Most of First Half--

## NO. 1 MCH. STATE

## то BOWL, $35-0$

 ATTACK HELD TO ONE YARD

Spartans Bag Share Of Big Ten Title

By Maury White
(Sunday Register Staff Writar)
IOWA CITY, IA. - Convincing evidence was unreeled here Saturday, as Iowa was pulverized, $35-0$, that Michigan State is not flying false colors being rated the No. 1 team in college football.
The undefeated Spartans, stymied for a little more than 20 minutes by a defense that got plenty of exercise (and bruises) before the game ended, made it eight in a row as Iowa absorbed its eleventh straight Big Ten defeat.

Title Tie, Bowl
If memory serves correctly, the bare fact of winning is fun. That pleasure was compounded for the Spartans by

wrapping up a sure share of the league title and assuring a Rose Bowl invitation for Jan. 1.
Coach Duffy Daugherty's juggernaut is $6-0$ in the league, with only Indiana left to play. Even if the Spartans lose next week, which is unlikely, they go to Pasadena because other title hopefuls have been there more recently.
A Dad's Day crowd of 54,700 , and a regional telecast audience saw an old story repeated: The Hawkeyes, when it carne to moving the football, couldn't keep up with the Joneses.

Clinton Jones, a 206-pound junior from Cleveland, broke the scoring ice in the second quarter by cutting over tackle and scooting 19 yards for a touchdown.
That wasn't so bad but the swift Spartan decided to make it a habit.
Before the long, gloomy day had ended, Jones also cracked over from the six, three and four to become the twelfth man in modern league history to score four touchdowns in a conference game.
Modern history starts with the 1939 season in the Big Ten,


## ENRICHMENT STUDENT PROJECTS

What is it?

## Why use it?

How to use it?

How to make it?

An idea aimed at interesting the better student in general mathematics. Essentially the projects are a set of self-contained learning situations. The projects are designed to provoke interest and a spirit of inquiry.
One of the biggest problems of general mathematics is the wide range of abilities. Many students take general math, who could have taken algebra but were afraid, or simply didn't want to do the work. These students are bored stiff because they already know the basic skills and can do drill work quite rapidly. This type of student can finish an exercise in a fourth of the time it takes the rest, if he wants to. Our objective is to give this student the incentive to do the required work as quickly as he can and then go on to enrichment work at his own rate. Up until now the problem of teacher-guidance has prevented a lot of this enrichment work. The teacher simply hasn't the time to plan six different classes within the regular class period. All of the ESP ideas are completely self-contained. Because of this a student can plan his own enrichment according to his interest. Each ESP box contains instruction cards, blank response cards and some physical object (s).
The better student is especially challenged by a problem-solving situation resembling a puzzle. The majority of the boxes are puzzles of one type or another. All of them are mathematical.

Keep the boxes stored in a convenient place. From time to time bring out a box and show it to the class. Leave it out with this understanding; Any student, who finishes his work before the alloted time is up, may work with it; anyone else can work with it when the room is open (before or after school). If a student uses a box he should leave it as he found it. If he desires credit for his work, he must fill out a response card and put it in his file. It should be clear that the teacher will in no way help or advise the student as to the contents of a box. The majorty of the instruction cards contain questions which the student who works the project should be able to answer. All the responsibility is placed upon the student by this idea. Above all else, a teacher should never under any circumstance give the student a solution to any of the puzzles. This is primarily an exercise in discovery.
The joy of discoveiy is removed by giving answers. This student does not want just an answer, but rather he wants his answer. The majority of the questions are so designed as to be self-checking. When a student thinks he has the solution he can see for himself whether or not it is correct.

The next few pages are an attempt to show what each box is. Included with each idea is a copy of the instruction cards used. Suggestions are given as to sources of the objects used. The majority of the ideas can be made or purchased at a very small cost. However, this idea is of sufficient merit that most schools would allocate the necessary funds if requested.
Cigar boxes serve ideally as containers of the various ideas. Also shoe boxes can be used, but they are not nearly as sturdy. If neither of these are available to you, large manilla envelopes could be used instead. There is an advantage to hiding the contents of the box. This adds to the mystery.
Use two different size index cards, one for the instruction cards, the other for the response cards. Each box should contain a good supply of response cards for the

## E S P (continued)

students' usage.
Each box should be labeled as to name and number of pieces. Put only one project in each box.

The teacher may prefer to familiarize the class with the boxes by passing them out, one per student for a 15 minute period of time, allowing the students to look over the box and try the puzzle contained in it. This could be repeated many different times without duplication. Each time a student should have a different box.
It would be a good idea to keep track of which students are using which puzzles both for the sake of accurate evaluation and for the protection of the projects. This could be done by having a sign-out sheet which the student would use when he wanted to use a box.
Instruct the students to put the following information in the upper right hand corner of their response cards: Their Name, the date, The E S P name, and the card number as in the example below, for accurate reference.

Tommy Smith<br>May 3, 1966<br>Ten Men In a Boat<br>Card B 2

1. 
2. 
3. 

## E S P-(A) TOWER OF HANOI

What is it? A very ancient puzzle involving the moving of discs from one peg to another.

## How to make it?

It is easily made using difterent size washers and a wood block (1"x4"x6") and three $1 / 4 "$ dowel sticks. Drill three holes in the block for the dowels to set into.

## E S P

TOWER OF HANOI
A 1
Object: Move the pyramid of discs from one peg to another.
Rules: 1. You may only move one disc at a time.
2. You may never put a large disc on top of a smaller one.

Try to do it in as few moves as possible.
It is recommended that you start out with just four discs. As you perfect your system add more discs.

E S P
TOWER OF HANOI
A 2
There is a formula that will tell you the minimum number of moves it takes to move all the discs from one peg to another.
It is:

$$
2-1=\Delta
$$

If you fill in the box with the number of discs you are working with and then compute the expression it will give you the number of moves, in the

How many moves does it take with 3 discs?
How many moves does it take with 4 discs?
How many moves does it take with 5 discs?
How many moves does it take with 6 discs?

## E S P-(B) TEN MEN IN A BOAT

## What is it?

## How to make it?

A puzzle in which you try to interchange the position of the pieces.
It can be made out of a block of wood (1" $\times 2^{\prime \prime} \times 6^{\prime \prime}$ ) having eleven evenly spaced holes drilled in it. The players are two different colors of plastic golf tees, 5 red and 5 white.


ES P
TEN MEN IN A BOAT
B 1
Object:
Rules: 1. Red men can move only to the right. White men can move only to the left
2. You may either jump a man (as in checkers) or you may just move to an adjacent empty hole.
3. You may not jump a man of the same color.

Can you do it? What is the fewest number of moves you can do it in?

## ES P

TEN MEN IN A BOAT
B 2
Put just four men in the boat, two on each side of the center hole. How many moves are required to interchange the men?
Try the same idea with six men. How many moves are required?
How many moves do you need with eight men?
Could you predict how many moves it would take for twelve men?

## E S P-(C) CENTIGRID

What is it?

How to make it?
Materials:

A set of transparent grids and overlays which can be used to teach many concepts, among them fractions, per cent, ratio and proportion.

See page 8-10.
6 Grids
8 Overlays

## ESP <br> CENTIGRID <br> C 1

1. Locate grid A. How many squares are there?
2. Place overlay number 1 on grid A. How many squares are shaded?
3. Can you express the shaded region as a fraction? What would the fraction be?
4. How would you name the shaded region as a per cent, $(1 \%=1$ per 100)?

## ESP

CENTIGRID
C 2
5. Place overlay number 2 on grid $A$. How many squares are shaded now?
6-7. Express the shaded region as a fraction, as a per cent.
8-10. Repeat the process above for overlay number 3.
11-13. Repeat the process above for overlay number 4.
14-16. Repeat the process above for overlay number 5.
17-19. Repeat the process above for overlay number 6.
20-22. Repeat the process above for overlay number 7.
23-25. Repeat the process above for overlay number 8.
E S P CENTIGRID

1. Locate grid E. How many squares are there?
2. Can you tell what per cent overlay number 1 is of grid E ? Is there
an easy multiplication that will convert 40 to 100 ?
A new way of doing this problem is called a proportion. Try setting
up the problem like this:
$\frac{\text { Shaded Area }}{\text { Total Area }}=\frac{\mathrm{N}}{100}$ or $\frac{8}{40}=\frac{\mathrm{N}}{100}$
You solve a proportion by cross-multiplying and simplifying.
$40 \mathrm{~N}=800, \mathrm{~N}=20$
CENTIGRID C 6
1 - Use the proportion method for finding per cent as explained on C 5 to find out what per çents overlays 2-8 are of E. Also give the fractions in lowest terms.
15 - Find what per cents the overlays are of grid $\mathbf{F}$, using the proportion method.
C 4
3. Locate grids $\mathbf{C}$ and $\mathbf{D}$. How many squares are there on each?
4. Place overlay number 3 on grid C. How many squares are shaded?
5. Place overlay number 3 on grid D. How many squares are shaded? What per cent is this?
6. Repeat the process above for overlay number 4.

## ES P-(D) DISECTED SQUARE

What is it?
A set of five geometric shapes which when properly positioned form a perfeet square.

How to make it? Tagboard will do the job. However a wood model from $1 / 4$ " plywood is best. Have a student make one as a project. Many cuttings other than the one shown will work just as well. The idea is easily extended to disected rectangles, circles, and triangles.


## ES P <br> DISECTED SQUARES <br> D 1

## Contents: 5 geometric shapes

Object: See if you can make a perfect square using all 5 pieces. Every piece must lie flat on the desk and none may be put on top of any other.

Sketch your solution on a response card.
Is it possible to make a square with just 4 pieces? Which one did you leave out?

## E S P-(E) TOPOLOGY 1

What is it?

How to make it?

Solution:

A puzzle in which the topological nature of an apparently closed curve is utilized to separate two interlocked parts.

The main piece can be made from any flexible material such as plastic. Artificial leatherette upholstery works real well. Cut two slits and a hole in the material. String and two large buttons complete the puzzle. Make sure the buttons will not pass through the hole and that the width of the loop is less than the diameter of the hole.

Stick the loop through the hole and remove the string with buttons attached.


## $\mathbf{E} \mathbf{S} \mathbf{P}$

TOPOLOGY 1
E 1
It is possible to separate the string from the loop and hole. You may not cut the string, tear the loop, pass the buttons through the hole, or untie the buttons. Can you do it? Briefly describe your solution on a response card.

## ES P-(F) TOPOLOGY 2

## How to make it?



A more complicated version 'f the apparently closed curve. The string can be removed from the center hole by the proper manipulation.

The puzzle could be made out of heavy card board, but $1 / 4$ " plywood works best. Drill three $1 / 2$ " holes. Use a piece of string with white buttons in the middle. Make sure the buttons are large enough that they cannot be forced through the holes. When you put it together place the two black buttons on the string, pass the ends of the string through the two outside holes and tie on the white buttons at each end. Spread the two black buttons apart and make a loop in the middle of the string. Pass this loop through the center hole from the bottom. Then pass the loop through each of the outside holes from the bottom, passing the white button through the loop.


Solution: Use the instructions above in reverse.

ESP
TOPOLOGY 2
It is possible to remove the string from the center hole completely so that the two black buttons come together. You may not pass any of the buttons through any of the holes, cut the string, or untie the white buttons. Can you do it? Describe your solution on a response card.

## E S P-(G) TOPOLOGY 3

What is it?

How to make it?

A wire heart interlocked with a U-bolt and pin. It appears to be two interlocked closed curves but is not.

Fairly heavy wire which will not bend easily. Use the diagram below as a pattern. Special attention must be paid to the size of the loops on the yoke, since the indentation of the heart has to fit into it.

Solution: Thread 1 through the ring labeled 2 as below.



Push 3 through the ring formed by 1 and 2.

Remove 2 from one moving towards the center of the heart.

To remove the yoke, reverse the pro-


The yoke and pin can be removed from the heart without bending the wire or undoing any of the joints. Try and separate them. If you succeed describe your method of solution on a response card. Please be careful not to force the wire. It will bend.

## E S P-(H) MOEBIUS STRIPS

What is it? A set of paper belts having different numbers of twists, which yield rather fasinating results when they are cut.

How to make it?
Acquire the following materials:
-a roll of adding machine tape (or calculator)

- a pair of scissors
-a roll of scotch tape


## ESP

MOEBIUS STRIPS
H 1
Cut from roll of paper a strip about two feet long. Bring the ends together and tape them to make a flat belt. If you were to cut down the center of the belt lengthwise, what would you end up with? Do it. Were you right? Record your results on a response card.

ESP
MOEBIUS STRIPS
H 2
Take another strip of paper about two feet long, and bring the ends together, but before taping turn one end over (bottom's up). If you were again to cut down the center of the strip, what would you end up with? Do it. Were you right? Record your results on a response card.

Cut another two foot strip from the roll. Join the ends together, turn one end over and tape as you did on H 2. This time instead of cutting down the center, cut a third of the way over. What do you suppose the result will be? Do. it. Were you right? Record your results on a response card.

ES P
MOEBIUS STRIPS
H 4
Take a st-ip of paper about two feet long, join the ends together, turning one of the ends over twice (two flips). What will you get if you cut this strip down the center? Do it. Were you right? On a second strip made the same way, cut a third of the way over. Record your results on a response card.

## ES P

MOEBIUS STRIPS
H 5
Take a two foot strip of paper, bring the ends together, give one end three turn-overs and then tape. What do you suppose will happen when you cut this strip down the center? Do it. Repeat the experiment with strips having $4 \& 5$ twists. Summarize the results of the entire experiment on a response card.

## ESP-(I) TRIANGLES

What is it?

How to make it?

An exercise in measuring angles and in discovering the sum of the angles of a triangle.

Acquire the following:

- a protractor
— a pair of scissors
- a stack of triangles (made out of tagboard on the paper cutter, varying in size and shape)

Select a triangle from the bcx. Measure each of its angles and record their sum. Repeat this process with several other triangles. Now add the totals together and find an average by dividing by the number of totals. Can you tell what the sum of the angles of every triangle should be?

## ES $\mathbf{P}$

TRIANGLES
Take a triangle from the box. Label the angles A, B, and C. Now take the scissors and cut the triangle into three pieces, taking care not to cut any of the three angles. Can you fit the angles together to form a straight line? How many degrees are there in a line? What is the sum of the angles of a triangle?

## E S P-(J) MAKE-A-CUBE

What is it?

How to make it?

A puzzle which can be taken apart and hopefully put back together. It teaches the necessity of a logical sequence in problem solving.

This type of puzzle can be bought rather inexpensively at most any novelty shop. Besides the cube, there are many others, a pig, a mouse, a sphere, a barrel, etc. Most of the puzzles work on the same pattern and cost about 75 cents apiece.


E S P
MAKE-A-CUBE
Depending on its last user's ability, this box contains either a cube or a number of pieces which when properly combined make a cube. If it is in one piece, see if you can take it apart and then put it back together. If it is in pieces, see if you can put it together without having taken it apart. This is much more difficult to do.

## ESP-(K) MYSTERIOUS RECTANGLE

What is in?

## How to make in?

A $6 \times 10$ rectangle which has been divided into 12 pieces each having an area of 5 square units.

It can be made out of tagboard using the pattern below. If you do $\mathbf{s c}$, make your unit an inch so that the rectangle measures 6 " $\times 10^{\prime \prime}$, and each piece has an are $\lrcorner$ of five square inches. A model made of $1 / 4$ " plywood would be better. This puzzle can be obtained commerically under the name of "Hexed" from most toy or novelty stores. It is made by the Kohner company and costs one dollar.


## E S P MYSTERIOUS RECTANGLES

This box contains 12 pieces which will make a perfect rectangle. Each piece has an area of five square units and the rectangle is 6 units by 10 units. There are supposedly over 200 different ways of putting the rectangle together. Can you find just one of them?

A 3-dimensional form of the mysterious rectangle is called Penta-cubes, and can be made by gluing childrens' building blocks together in pieces like the ones above. Each piece now has a volume of five cubic units. Using the pentacubes it is possible to make a $1 \times 6 \times 10$, a $1 \times 5 \times 12$, a $2 \times 5 \times 6$, and a $3 \times 4 \times 5$ rectangular prism.

## ESP-(L) Soma Cubes

What is it?

How to make it?
A set of seven pieces composed of cubes glued together in various combinations. Soma cubes are used to make a variety of configurations, but by far the simplest one is a $3 \times 3 \times 3$ cube. There are supposedly over 500,000 solutions to this puzzle, but it takes a while to get just one. Can you do it?

Use childrens' building blocks or special 1" cubes and glue thern together following the pattern below. Making a set is a good student project which can be repeated frequently by just using rubber cement and taking the blocks apart when finished.


Ref: Martin Gardner, Scientific American Puzzle Book II

## ES P-(M) Puzzling Pyramid

What is it?

How to make it?
Two identical pieces which when positioned together correctly form a tetrahedron or pyramid having a triangular base. Each piece is composed of 5 faces: 2 equilateral triangles, 2 isosceles trapezoids, and 1 square.
The puzzle is difficult to make out of wood, but quite easy using oak tag or other light cardboard. Using the pattern below, make two pieces. Any dimensions can be used, however, take care in assigning lengths so that the resulting pyramid is equilaterol - that is, the base of the trapizoids must be in a $1: 2$ ratio, and the sides of the trapezoids must be equal to the sides of the square, and to the sides of the triangles.


## E S P-(N) Quadrix

What is it?

How to make it?
A puzzle similar to 10 -men-in-a-boat, involving 8 pegs, four of one color and four of another, and a set of 10 evenly spaced holes in a straight line.


Object: End up with the empty holes at the opposite end with the teams separated and to do this in just four moves.

Rules: 1. You may move any two adjacent pegs at a time, and you must move 2 each time.
2. You may never change the order of the two pegs being moved.


Starting position:

Two empty holes at one end, the players alternating.

Drill 10 holes in a 1 " $\times 1$ " $\times 12$ ".piece of wood and use piastic golf tees for players, or if you have the 10 -men-in-a-boat game just cover one of the holes with a piece of tape and use 8 of the pegs.

## ES P-(O) CURVESTITCHING

What is it?
The construction of beautiful and graceful curves using just plain straight lines.
How to make it? Begin with a simple example such as an angle. Mark off the same number of units ( $1 / 4$ " is fine) on each side of the angle. Label the points as in Fig. 1 below and then connect the 1's, 2's 3 's etc with straight lines. Once the pattern is established the numbers are unnecessary.


## (FIG I)

Encourage your students to be creative and make up designs of their own. By using the angle above we can make a famous curve known as the Astroid Curve Fig. 2.


Rearranging the same four right angles into the form of a square yields a different effect.


Sometimes it is disireable to curve-stitch two lines of different lengths. In doing so, be careful to mark off the same number of units on each line, allowing the size of the units on the longer line to be larger. (NOTE: All units on a given line must be the same size). An interesting application of this is the curve-stitching of the diagonals and sides of a square. The result is another fiamous curve, the Lemniscate.


The circle is one of the most interesting figures to curve-stitch. Choose a unit ( $1 / 4$ " or $1 / 2^{\prime \prime}$ ) and mark it off around the circumference of the circle. If it doesn't come out exact, simply split the difference between the last few points. Now, choose two points on the circumference and draw a straight line between them. (Avoid the lines which pass through the center of the circle.) Now move one unit in a counter-clock-wise direction and connect the next two points. Continue on around the circle until each point has two lines drawn from it. The effect can be emphasized by repeating the process several times on the same circle, each time choosing a different set of starting points.


I CYCLE


2 CYCLES

## E S P-(P) PAPER CUBE

What is it?
A simple exercise in measurement and paper foiding.

## How to make it?

Why use it?

## P. S.

(Fig 2)

(Fig 1)



On a piece of regular notebook paper, make six identical squares with $1 / 2$ inch flaps on one pair of opposite sides (Fig 1).
Cut the pieces out and fold the flaps back at right angles to each square. (Fig. 2) Now, see if you can build a cube like the one illustrated above. By alternating the flaps and keeping them on the outside, it is possible to make a rather rigid cube. No tape or glue, please.

It is a good way to show the necessity of accuracy in measurement and also au interesting way of leading into a discussion on volume and surface area.
It is possible to make the same paper cube using other patterns and the same principle of alternating exterior flaps. Students have made models using 3 pieces and 2 pieces. See if your students can come up with working models.
Also, the same idea can be applied to a rectangular solid. Be careful about which sides you put the flaps on.

A variation of the curve-stitched circle can be created by connecting each point on the circumference to all the other points.


Once the idea has been introduced and explored, encourage your students to make up their own designs using one or more of the types studied. Emphasize creativity and composition.
Students have adapted curve-stitching to many mediums other than pencil and paper. One used golf-tees in a peg-board as points, and string stretched between the tees as lines. Another stretched felt cloth over cardboard and used needle and thread to do the stitching. While still others have used plastic coffee can lids and thread.

Why use it?
First of all it creates an appreciation of the structural principle of building a curve with straight lines. Then to it serves as an introduction to geometric art. Some students profit from just wanting to learn to draw a straight line. The finished products serve as an excellent way of brightening up any mathematics room.

Ref:

## ESP-(Q) TANAGRAMS

What is it?

How to make it?

Why use it?

A 4000 year old Chinese puzzle, made up of 7 pieces used to make various figures whose outline is given.

Cut out of oak tag using the patterns, above, or assign as


A a class project using the instruction sheet which follcws.

It is an excellent way of building form perception. Also, a good in measurement exercise.

## CLASS PROJECT

You can have a class make the puzzle using the instruction sheet which follows. While the students are working, the teacher would be free to give individual help to those needing it.

To complete the assignment successfully the student must be able to:

1. Construct a perpendicular line
2. Duplicate a given line segment
3. Construct a line twice as long as a given line
4. Bisect a line segment
5. Bisect an angle (the 135 degree angle is formed by bisecting a right angle, $90+45$.)

The puzzle construction serves as an excellent conclusion to a unit on geometry and constructions.

## ESP-(Q) TANAGRAMS

1. Draw a large square on your paper and label as illustrated:
2. Draw $\overline{\mathrm{AC}}$
3. Find the midpoint (center) of line $\overline{\mathrm{AB}}$, label it E


Find the midpoint (center) of line $\overline{\mathrm{BC}}$, label it F Find the midpoint (center) of line $\overline{\mathrm{AC}}$, label it $G$ Find the midpoint (center) of line $\overline{\mathrm{AG}}$, label it $\mathbf{X}$ Find the midpoint (center) of line $\overline{\mathbf{G C}}$, label it $\mathbf{Y}$
4. Draw: $\overline{\mathrm{EF}}, \overline{\mathrm{EX}}, \overline{\mathrm{DG}}$
5. Extend $\overline{\mathrm{DG}}$ to intersect $\overline{\mathrm{EF}}$, and label the point of intersection Z .
6. Draw $\overline{Y Z}$
7. Cut out the 7 pieces: 5 triangles, 1 square, 1 paralleìogram.

Can you make the figures below:


## C)



## E S P <br> PYTHAGORAS, EUCLID, TORMENTOR, \& VOODOO

$\therefore$ of puzzles which are commercially made by the Kohner company. Each game is * e up of plastic geometric forms. Included with each game is a booklet of silhouettes which can be made using all the pieces of each puzzle. As a student succeeds in constucting a silhouette he would sketch the relationship of the pieces on a response card. The puzzles are not limited to any particular age group.

The preceeding pages represent only a small part of the tremendous number of possibilities for E S P boxes. This treatment is intended only to spark the interest of the teacher in this idea. E S P has unlimited potential. It is hoped that you will use these ideas as well as create many of your own. Once the idea has been started it seems to grow naturally. Again, remember the purpose of ES P is to arcuse the interest of students in mathematics through the use of the problem solving technique.

贲



## REAL - LIFE - PROBLEMS

What is it?

Why use it?

## How to use them?

A set of problems which have been taken from businesses around the Des Moines area. The idea of using real business forms and actual letterheads helps the student see the practical side of mathematics and thus provides motivation. The problems use realistic figures in everyday situations.

General Mathematics students are always asking why they should study mathematics. The use of practical problems in their natural settings helps answer this question. It is also a good source of variety.

Duplicate the problems as they are or adapt them to new situations by adding your own ideas to $i=\mathrm{i}$. A problem is easily altered by blocking out a section with a piece of paper cut to the correct size and shape. Additions can be made in pencil.
Use the problems two or three times a week for ten to fifteen minutes. Have the students do their work on the sheets. Other class work can be done on the back. Encourage class discussion of the problem and the business it comes from.
A special assignment which the students will take an interest in is to have them bring real-life-problems of their own. Have them ask their parents or friends in business for letterheads, business forms, and prohlems. Not only does this involve the student in the work, but it also gives the teacher a rich source of new real-lifeproblems.
Added interest can be obtained by field trips to some of the businesses studied or by having a speaker come into the classroom.
A potential drop-out will listen much more intently to an auto-machanic telling about how he needs mathematics than he will just hearing the teacher say it.

## NOTES:

C) CITY DF DES MOINES, iowa

DEPARTMENT OF AVIATION
TERMINAL BUILDING
MUNICIPAL AIRPORT
DIS MOISES, IOWA 50321
STATISTICAL SUMMARY
Department of Aviation
Month of June 1966

| Activity | This Month <br> June | This Month <br> Last Year | This Year <br> To Date | Last Year <br> To Date |
| :--- | ---: | ---: | ---: | ---: |
| Regular Scheduled Flights | 2,246 | 2,094 | 13,104 | 11,852 |
| Air Carrier Traffic | 2,266 | 2,255 | 13,590 | 12,604 |
| Military Traffic | 900 | 875 | 4,926 | 4,960 |
| Civil Itinerant Traffic | 8,251 | 7,136 | 39,914 | 34,886 |
| Local Flights (all types) | 6,836 | 6,548 | 33,868 | 30,491 |
| Total all Flights (In \& Out) | 18,253 | 16,814 | 92,298 | 82,941 |
| Passengers (In \& Out) | 50,288 | 41,152 | 282,563 | 235,721 |
| Express Handled Total Lbs. | 144,881 | 123,163 | 801,671 | 690,039 |
| Freight Handled Total Lbs. | 576,109 | 487,210 | $3,355,780$ | $2,522,779$ |
| Mail Handled Total Lbs. | 203,065 | 155,427 | $1,188,834$ | 886,324 |

Ire above statistics show a sigsiffeant increase in air activity at the Der maine airport. Figure the incerate the per cant of increase of your first fire problems

UNDERWRITING AND ISSUE DIVISION EDMUND J. SIMMONS Manager Polly issue Section
miss Jeannette ahrens MISS HELEN HARMON Polly loewe Aeviotants


# Lynn's Motorayde Shop 1700 East University Avenue 

CYLINDER REARING. COMPLETE MOTOR REBUILDING IS OUR SPECIALTY.

The Yamaha YGI-K is made in Japan where the metric system is used. If you work with one there are two conversions you must be able to make 1 kilometer $(\mathrm{km}$. $)=0.62137$ miles and $1 \mathrm{~mm}=.03937$ inches. The speedometer of the Yamaha cycle measures speed in terms of $\mathrm{km} / \mathrm{h}$ (kilometers per hour).

What would be the $\mathrm{km} / \mathrm{h}$ equivalents for: $20 \mathrm{mp} / \mathrm{h}$ in terms of millimeters. What would be the equivalent measures in terms of inches?

Acceleration $1 / 8$ mile in 12.5 sec Bore \& Stroke $47 \times 42 \mathrm{~mm}$ Compression Ratio 7.5:1 Speed Range $50-60 \mathrm{mp} / \mathrm{h}$ Gearbox 4 speed Overall Length 1,815 mm Overall Width 625 mm Overall Height 960 mm Wheelbase $1,145 \mathrm{~mm}$ Ground Clearance 150 mm

Fuel Capacity 6.2 liters Oil Capacity 1.1 liters Weight 70 kg

```
                                2.5 sec
```

$\qquad$1
sales, inc.
444 East Fourth 288-2271
DIS MOINES, IOWA 50309

Dear Mr. Thomas:
In response to your letter of the 24 th, I am happy to inform you that we can deliver the ' $662+2$ Fastback Mustang with the 289 engine to you by the list of the month at the quoted price of \$ 2787.50 . In case you desire to finance through us I'm including a monthly payment chart for your convenience.

| Balance | 12 | 18 | 24 | 30 | 36 |
| :--- | :---: | :---: | :---: | :---: | :---: |
| 2000 | 180.00 | 124.44 | 96.67 | 80.00 | 68.89 |
| 2100 | 189.00 | 130.67 | 101.50 | 84.00 | 72.33 |
| 2200 | 198.00 | 136.89 | 106.33 | 88.00 | 75.78 |
| 2300 | 207.00 | 143.11 | 111.17 | 92.00 | 79.22 |
| 2400 | 216.00 | 149.33 | 116.00 | 96.00 | 82.67 |
| 2500 | 225.00 | 155.56 | 120.83 | 100.00 | 86.11 |
| 2600 | 234.00 | 161.78 | 125.67 | 104.00 | 89.56 |
| 2700 | 243.00 | 168.00 | 130.50 | 108.00 | 93.00 |
| 2800 |  |  |  |  |  |
| 2900 |  |  |  |  |  |

Mrithomas pays $\$ 287.50$ drum and decides to pay the balance off in 2 years. What will be the total cast of the car? What rate of interest is he paying?
$\qquad$
$\qquad$


RETURN THIS RHET TO OFFICE FOR DELIVERY TICKET

## INSTRUCTIONS: <br> Find the running balance and place it in the proper indicated column.




INCLUDE US IN YOUR
REMODELINS
PLANS

USE REVERSE SIDE FO? RECONCILING YOUR ACCOUNT.

SUMMARY OF ACCOUNT

| EPORTED IN TEN DAYS. SUMMARY OF ACCOUNT |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| UMBER OF closures | BEGINNING BALANCE | WE HAVE SUBTRACTED DEBITS |  | AND ADDED CREDITS |  | AT A SERVICE CHARGE OF |  | RESULTING IN BALANCE OF |
|  |  | NUMBER | AMOUNT | NUMBER | AMOUNT | NUMBER | AMOUNT |  |
| 31 | 19479 | 28 | 681137 | 3 | 93154 | 28 | 163 | 443 |



WESTERN AUTO SUPPLY COMPANY


SHIP TO Western Auto Supply Co. $\neq 3$



ERIC

In the Des Moines area there are about 85,000 radio-listeninghomes. A recent survey showed that more people listen to KIOA than any other radio station. On the chart below are listed the various per cents for each station in the Des Moines area. Figure out how many homes listen to each station in the morning and in the afternoon. What per cent of the homes were tested? (hint: compare Sample size to the number of radio-homes)

| TIME | $\begin{aligned} & \text { HOMES } \\ & \text { USING } \\ & \text { RADIO } \end{aligned}$ | A | KİA | B | C | D | E | F | OTHER <br> $A M \& F M$ | SAMPLE SIZE |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| OODAY TERU FRIDAY $\therefore: 00$ A.M $-12: 00$ NOON | 16.4 | 5.9 | 38.5 | 28.8 | 3.2 | 4.9 | 15.5 | $1.8 \dagger$ | 2.1 | 6,157 |
| $\because O D A Y$ THRU FRIDAY $\because: 00: \because O O N-6: 00 ~ P . M$, | 9.6 | 4.4 | 40.9 | 23.0 | 4.1 | 6.2 | 15.0 | 2.1 | 4.2 | 7,541 |



SERVICE ORDER


PHILCHECK MERCHANDISE SERVICE


ON THIS ORDER THE FOLLOWING WERE CHECKED:

Differential level \& vent
Conv. trans. level
Exhaust system
Shock absorbers
these services were PERFORMED FOR SAFETY AND/APPEARANCE:

All glass cleaned 5 Wiper blades checked Floors vacuumed Q Ash trays emptied All lights checked Ares inspected Tire pressure checked Front 28 Rear 30 Spare $\qquad$
Mileage Sticker Completed \& Attached remarks and suggested automotive needs.
$\qquad$


[^0]

Find the total amount due on the servicing of this auto.

Don't forget to include State Tax of 2\% on all items except gas and labor.

## INSTRUCTIONS :

No. 5
name


1. Calculate your own salary at the rate of $\$ 2.17$ per hour for the first 40 hours.
2. Any time over 40 hours will be counted as overtime and calculated at the rate of time and a half.

## IN ACCOUNT WITH

member of credit reference and reporting company


The above statement is the monthly billing that is given to each family receiving milk delivery at their door.

Find the amount of each item ordered, total amount and sales tax.

Find the carrying charge for each patron if the rate of interest is $7 \frac{1}{2} \%$ on the unpaid balance $:$

| Patron | Unpaid Bal. Carrying Charge | Total Pay back |
| :---: | :---: | :---: |
| A | $\$ 31.60$ |  |
| B | 125.60 |  |
| C | 98.95 |  |
| D | 1145.69 |  |
| E | 10.75 |  |

Find the carrying charge for each patron if the rate of interest is $7 \frac{1}{2} \%$ on the unpaid balance.

| Patron | Unpaid BaI. Carrying Charge | Total Pay back |  |
| :---: | :---: | :---: | :---: |
| A | $\$ 31.60$ |  |  |
| B | 125.60 |  |  |
| C | 98.95 |  |  |
| D | 1145.69 |  |  |
| E | 10.75 |  |  |

# Bell \& Howell Company 

7100 MCCORMICK ROAD, CHICAGO 45, ILL.
offire of

## PETER G. PETERSON <br> PRESIDENT

We are planning to manufacture a camera that we will sell for $\$ 180.00$.

The camera will cost $\$ 135.00$ to make.
How much will we make on each camera?
It will cost us $\$ 125,000$ for tools and dies to make the camera.

How many cameras must we sell to get back our tooling cost?

How many must we sell to make a profit equal to $20 \%$ of our investment?

## Massey-Ferguson Inc.

I90I Bell Avenue Des Koines, Iowa 50315

WAGE RATES IN OCCUPATIONS IN METALWORKING ESTABLISHMENTS
DE MOINES, IOWA
September 1965


What is the average starting arose An each occupation? COMPANY

DES MOINES 9, IOWA
July 2, 1965

Woodrow Wilson Junior High
East 24th \& University Ave.
Dis Moines, Iowa 50317
Dear Sir:
We are pleased to quote on the list of school supplies used in your school. All of the items are available from our regular stock and could be delivered to your school within a few hours after your order is placed. We have Dis Koines' largest stock of office supplies and equipment, art and engineering supplies. Also, we invite you to visit our Gift Department next time you are downtown. Remember, we offer fast free delivery and charge accounts are welcome.

Quantity No.
5000 \#1 15,000 \#400 6 lbs. \# 16 8 rms 20 lb . 600 ea 17H 4 qu. \#314 6 ea \#310

Item
Unit
Gem paper clips $\quad 1.00 / \mathrm{M}$
Pilot staples Janus rubber bands $2.65 / 1 \mathrm{~b}$. Triad mime lir. size 2.20 rm . $3 \times 5$ ruled cards .20/C Mime stencils Shaffer pencils
3.50 qu.
1.95 ea.

Total

Hoping we can be of help to you in the future, we remain, Sincerely,
DE MOINES STATIONERY COMPANY
$\bigcup_{\text {W. A. Wilson/pm }}$


INSTRUCTIONS:

1. A man comes in to the McDonalds Drive-In and orders the following:

6 hamburgers
3 cheeseburgers
9 french fries
2 coffee
1 milk
4 large Root Beer
3 Choc. milk shakes
4 Strawberry milk shakes.
2. Can you find his total bill? How much will it cost him including $2 \%$ tax?
3. How much change should he receive if he gives you a $\$ 10$ bill?

## Des Moines Waterworks

A meter book is used to record the meter readings from which the water bill is figured. The price of water is $80 \phi$ per thousand gallons used.

The average semer rental $\$ 1.50$. Find the water bills including sewer rental and $2 \%$ state tax.

| Name | present reading | last reading |
| :--- | :---: | :---: |
| Appling | 31596 | 30414 |
| Anson | 21430 | $20: 11$ |
| Bray | 71205 | 60114 |
| Cawson | 2057 | 40 |
| Downing | 94721 | 24756 |
| Jarnagan | 39724 | 31277 |

1

With reference to your inquiry as to prices, discounts, etc. on our Divisumma 24, Arithmetic Typing Computer, I am pleased to quote below our price to the Commercial User, and for your comparison, our special prices to Schools and Educational Institutions.


Schools \& Educational Institutions- - - - \$500.00 (Tax exempt)
In quantities of ten (10) $-\ldots-\ldots-\ldots$. $\$ 475.00$

I trust this will give you the desired information.

Yours very truly
Yurtahner
R.W. Holmes

Regional Supervisor

- By how much does the school cost differ from the total cost for the commercial user? - What per cent discount do the schools receive? $\qquad$ for burning quantities of 10 or more?


Pete: Figure the total amain due
including 290 state tar on all ittons

ITS TIME FOR DX SAFETY LANE SERVICE



NAME
ADDRESS
CITY
state

$$
\text { car at } 9: 30 \text {. }
$$

Ed

lICK UP
ME Wanted $9!30$
SLIVERED BY
HONE

$\qquad$


# (S)SAFEMAY stores, incorporated 

115 South 46 th Street, P.O. Box 931, Omaha 1, Nebraska
Store at East 14th and University

One of our cash registers last Saturday calculated the following items in one hours time.

To give us an idea of the inventory check we find the amount sold of each item: meat (Mt), produce ( Pr ), and Groceries (Gr).

What was the amount of each item sold in the one hours time?


Ind the amount after a $10 \%$ discount.
invoice
Northwestern Candy Co.
Manufacturers and Jobbers
FINE CONFECTIONS
SHOOL SUPPLIES, WORK GLOVES, PIPES, CIGARS, FOUNTAIN SUPPLIES AND SPECIALTIES

OFFICE: 100 East Locust St.
PHONE 4-3147

$$
\text { DES MOINES 3, IOWA,_ } 19 \sim \text { So rs }
$$

Sold To $\qquad$
Address $\qquad$
TERMS: NET



Sack J. Calhoun. Gen. Mgr.

HOTEL
Kirthumod
Des Moines, Lowa

Administrative Expense

Office Salaries
Officiers Salaries and Bonus Insurance
Employees Insurance
Depreciation Office Equipment
Attornye's Fees
Telephone and Telegraph
Office Supplies
Permits
Dues
Custodian Service
Rayroll Taxes
Bank Service Charges
Entertainment and Gifts Utilities

$$
\$ 18,260.02
$$

$\$ 18,260.02$
$25,650.00$
450.00
$1,823.23$
275.34
15.00
$1,221.24$
646.95
157.00
224.25
11.87
420.48
12.00
382.30
120.00

$$
\begin{array}{r}
25,650.00 \\
450.00
\end{array}
$$

$$
1,823.23
$$

$$
\begin{array}{r}
7.34 \\
15.00
\end{array}
$$

$$
\begin{array}{r}
1,221.24 \\
646.95
\end{array}
$$

$$
157.00
$$

$$
224.25
$$

$$
11.87
$$

$$
+2.00
$$

$$
382.30
$$

$$
120.00
$$

The above expenses are for the past month. We need to know the per cent for each item. (Naturally, we first find the total.)

Parkway Inns, Inc.

## LESSON - PLANS

What is it?
One of the most difficult and yet one of the most important steps in making an interesting class. There are two key words in lesson planning: variety and creativity. It is essential to the success of the program that the teacher plan each class in greater detail than is neressary for other classes. It is recommended that each class be divided into four sections and that a different activity be planned for each. The emphasis here is not only on variety but also on change of pace. We suggest that these plans be made on a weekly basis. In this way both variety and continuity can be achieved. An effort should be made to make each day as independent as possible with its own special goal or objective. Friday could be reserved for summary and test day. One way of setting up a week's lesson plan is to divide each class into four activities: review, instruction, application, and practice. Each of these activities can be done in a variety of ways.

Review: through class discussion, teacher questioning, students doing problems on the board, written quiz, question games, etc.
Instruction: teacher-lecture, student-report, film-strip or movie, studerts reading from the text, teacher-student dialogue, guest-speaker, experimentation, etc.
Application; discussion, use of real-life-problems, oral drill, board work, questioning, games, puzzles, etc.
Practice: individual study, small group study, calculator work, class drill, tests, etc.

Every class should be different as well as interesting. This is where creativity comes into play. It is only through the use of creative imagination that interesting classes are made. Good teaching is not an accident. It comes only as the result of thorough, pre-meditated, and imaginative planning.
The following pages are examples of this type of planning.


|  | Monday | Tuesday | Wednesday | Thursday | Friday |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | rilinstrip Introducing Inew matival Discuscim | Encle chant quiz <br> Discues ansurens | fundiustion: 身terdent reppart | Inacose ocnut- <br> 2abseci... on <br> set. ofof sumew probleme. | Revieur urth stidente at <br> s. boud |
| 2 | Shot assignment on new work from lert | Reading assignment <br> from tept + <br> witten <br> axignment <br> Cluck on calulator | Practice: <br> Seniew <br> from supple fractem-gram <br> (student choice) | $\downarrow$ | Test |
| 3 | Reviem past learninge using Baseball quyz (prepare quations) | whem finished <br> sive undividual | $\downarrow$ | Instruction: from tept | V |
| 4 | $\checkmark$ | relp. <br> E\& $P$ for bettu studente who finier eanly | Submarine Sraphing | $\begin{gathered} \text { Oppluaturn: } \\ \text { Wead-life } \\ \text { problem- } \end{gathered}$ | Discussum of tand $t$ $+$ ansurere |




[^0]:    PRINTED IN U.8.h FORM 22184.64

