This programed mathematics textbook (Volume I) is for student use in vocational education courses. It was developed as part of a programed series covering 21 mathematical competencies which were identified by university researchers through task analysis of several occupational clusters. The development of a sequential content structure was also based on these mathematics competencies. After completion of this program the student should know that a number \( X \) having an exponent \( n \) means that \( X \) is multiplied by itself \( n \) times and be able to perform addition, subtraction, multiplication, and division with numbers containing exponents, convert any number into standard scientific notation, convert a number from standard notation into standard decimal notation, and perform addition, subtraction, multiplication, and division using scientific notation. The material is to be used by individual students under teacher supervision. Twenty-six other programed texts and an introductory volume are available as VT 006 882-VT 006 909, and VT 006 975 (EM).
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Occupational Mathematics

SCIENTIFIC NOTATION

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U.S. DEPARTMENT OF
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State Coordinating Council for Occupational Education, Olympia, Washington
OBJECTIVES

1. The student should know that a number $x$ having an exponent $n$ means that $x$ is multiplied by itself $n$ times.

2. The student should be able to perform the basic operations of addition, subtraction, multiplication, and division with numbers containing exponents.

3. The student should be able to convert any number into standard scientific notation.

4. The student should be able to convert a number from scientific notation into standard decimal notation.

5. The student should be able to perform the basic operations of addition, subtraction, multiplication, and division using scientific notation.
Greetings! You are about to begin improving your knowledge of basic mathematics. There are many important uses for the mathematics you are learning.

This booklet is not like your ordinary books. It is designed to help you learn as an individual. On the following pages you will find some information about mathematics. After the information is presented, you will be asked a question. Your answers to these questions will determine how you proceed through this booklet. When you have selected your answer to the question, turn to the page you are told to.

Do not write in this booklet. You may wish to have a pencil and some paper handy so you can write when you want to.

Remember this is not an ordinary book.

1. Study the material on the page.
2. Read the question on the page (you may want to restudy the material on the page).
3. Select the answer you believe is correct.
4. Turn to the page indicated by your answer.

Are you ready to begin?

(a) Yes Turn to page 1
(b) No Turn to page C
(c) HELP Go see your teacher
Your answer was (b) No.

Well, this booklet is a little different.

Go back and read page B again. After you have read it, you will probably be ready to begin.
In this unit you will learn how to simplify some of your work through the use of Scientific Notation. If we consider, for instance, the size of an atom, we are dealing with a very small number. When talking about the distances from earth to the stars we are talking about very large numbers. By studying Scientific Notation, you will learn a convenient way to deal with very small and very large numbers. You should find it both useful and time-saving.
Before we get into the actual methods of how to use scientific notation, we must first have a thorough understanding of how to work with exponents. Once this is accomplished, the actual procedures for using scientific notation should be quite easy. So work hard on the first part of this unit, and you shouldn't have much trouble later.

Are you ready to start? Good.

Turn to page 8.
Very good!

Try one more.

Another way to write \( R \times R \times R \times R \times R \) is:

(a) \( R^5 \) Turn to page 18
(b) 5R Turn to page 11
No!

Remember that $2^3$ means "2 multiplied by itself 3 times."
That is, $2 \times 2 \times 2$, or 8. $2^3$ does not mean $2 \times 3$. The
3 written above has a very special meaning.

Now, see if you can do this problem.

$3^4 = ?$

(a) 12 Turn to page 11
(b) 81 Turn to page 3
(c) 64 Turn to page 10
Incorrect! Let me explain it to you.

The small 2 in \( a^2 \) is called an exponent. Notice it is written slightly above the line. It is a way of saying, "Multiply a by itself 2 times." If the exponent had been 5, it would mean, "Multiply a by itself 5 times."

In other words,
\[
\begin{align*}
    a^2 &= a \cdot a & \text{(Read "a squared")}
    \\
    a^5 &= a \cdot a \cdot a \cdot a \cdot a & \text{(Read "a to the fifth power")}
    \\
    3^4 &= 3 \cdot 3 \cdot 3 \cdot 3 & \text{(Read "3 to the fourth power")}
    \\
    (4x)^3 &= (4x) \cdot (4x) \cdot (4x) & \text{(Read "4x to the third power")}
\end{align*}
\]

Do you see how the exponent works? Study the above examples very carefully.

When you are sure you understand the examples, turn to page 12.
Correct!

Try this one.

\[ 2^3 = ? \]

(a) 6   Turn to page 4
(b) 8   Turn to page 18
(c) 23  Turn to page 9
Wrong answer!

I think you need to study exponents in more detail before continuing this unit.

Go work the first section of Unit 20 that deals with exponents. Then return to page 1 of this unit.
Here is your first question.

Does $a \cdot a = a^2$?

(a) Yes  Turn to page 6
(b) No  Turn to page 5
2^3 does not mean 23. Notice that in 2^3, the 3 is raised above the line.

Go back to page 6 now and choose another answer.

Turn to page 6.
Whoops!

Your answer would have been correct for $4^3$. But the problem was $3^4$. They are not the same.

See if you can find the right answer this time.

Turn to page 4.
No!

You had better go study the examples on page 5. Then continue from there.

Turn to page 5.
Now, see if you can do this one.

\[ x \cdot x \cdot x = ? \]

(a) \( 3x \)  Turn to page 7
(b) \( x^3 \)  Turn to page 6
Question:

The quantity $x^6$ means:

(a) $\cdot\cdot\cdot\cdot\cdot\cdot\cdot$ Turn to page 17
(b) $6x$ Turn to page 22
(c) Same as $2^6$ Turn to page 21
Incorrect!

I think you multiplied 4 \times 3, which is wrong. Remember, the exponent does not indicate simple multiplication.
See if you can get back on the track with this one.

For \( x = 5 \), \( x^2 = ? \)

(a) 10    Turn to page 20
(b) 25    Turn to page 19
Okay! Let's see what the problem means.

The problem was to find the value of $6^y$ when $y = 2$.

Well, if $y = 2$, then you simply put 2 where $y$ was.

The problem then becomes $6^2$. Can you finish it?

So back to page 19 and try.

Turn to page 19.
No:

If $m = 3$, then $m^3 = 27$. But that wasn't what the problem said.

Better go back to page 17 and try again.

Turn to page 17.
Correct! Let's continue.

If $m = 4$, then $m^3 = ?$

(a) 64    Turn to page 23
(b) 27    Turn to page 16
(c) 12    Turn to page 14
Excellent! Keep up the good work.

As you proceed, always keep in mind what an exponent means. That is, "multiply the number (or letter) by itself the number of times that the exponent indicates."

Turn to page 13 and continue.
Very good! 25 was correct.

Here's one a little different.

When $y = 2$, $6^y = ?$

(a) 12  
(b) 36  
(c) I'm not sure what to do

Turn to page 20  
Turn to page 23  
Turn to page 15
No! I think you have forgotten the idea we have been stressing.

If a number has an exponent of $k$, then you should multiply that number by itself $k$ times.

Here are a few more examples:

$m^3 = m \cdot m \cdot m$

$6^2 = 6 \cdot 6 = 36$

$y^5 = y \cdot y \cdot y \cdot y \cdot y$

Now, go to page 14 and work the problem there.
No! $x^6$ is not the same as $2^6$.

$x^6$ is a general case, and could represent any value of $x$. If $x = 2$, then $x^6 = 2^6$, but that is a special case. Be careful not to confuse the general case, which is represented by letters, with specific numeric cases.

Now, go back to page 13 and try again.

Turn to page 13.
Incorrect!

I don't think you read page 18 very carefully.

Turn to page 18.
Good! Let's proceed.

So far we have been dealing with the meaning of an exponent. We now wish to learn how to add, subtract, multiply, and divide numbers containing exponents.

Let's discuss addition and subtraction first. In order to add or subtract numbers containing exponents there are two rules you must follow:

Rule 1: The exponents on all numbers must be the same.

Rule 2: The bases must be the same.

Let's look at an example.

In a number like $4y^5$,

- 5 is the exponent,
- $y$ is the base
- 4 is called the coefficient

So, in order to add or subtract this from another number, that number must also contain a $y^5$ in order to follow both rules.

As an example, $4y^5 + 2y^5 = 6y^5$.

Turn to page 28.
No! You subtracted the exponents.

You never change an exponent on an addition or subtraction problem. You only subtract coefficients, leaving the base and the exponent the same.

So, $3x^5 - (1)x^5 = 2x^5$. It's that easy!

Now, what is $9y^3 - y^3$?

(a) $9$   Turn to page 33
(b) $8y^3$   Turn to page 36
Incorrect!

Let's review what we just said. In order to add or subtract numbers with exponents they must have both the same base and the same exponent. $m^2$ and $m^3$ are not the same, so we can't add them.

Are you ready to try another problem?

Turn to page 34.
Good! You were aware that \( x \) and \( x^2 \) could not be added.

Now, what is \( 2z^2 + 3z^2 \)?

(a) \( 5z^2 \)  
(b) \( 5z^4 \)  
(c) You can't add them
Oops! You slipped up on that one.

You should view $3x^5$ as a single quantity. Do not take the $x^5$ away from the 3. The correct solution was simply $3x^5 - x^5 = 2x^5$. It's that easy!

Try this one.

What is $9y^3 - y^3$?

(a) 9  
(b) $8y^3$
Let's go on.

See if you can work this problem.

What is $m^2 + m^3$?

(a) $5m$  
(b) $m^5$  
(c) You can't add $m^2$ and $m^3$
No! You just haven't grasped the idea.

Go to page 23 and study the material very carefully. Then continue from there.

Turn to page 23.
Excellent! You've got the right idea.

Keep up the good work.

What is $3x^5 - x^5$?

- (a) $2x^0$ Turn to page 24
- (b) $3$ Turn to page 27
- (c) $2x^5$ Turn to page 36
No! You just haven't grasped the idea.

Go to page 23 and study the material very carefully. Then continue from there.

Turn to page 23.
Incorrect!

Let's review what we just said. In order to add or subtract numbers with exponents they must have both the same base and the same exponent. $m^2$ and $m^3$ are not the same, so we can't add them.

Are you ready to try another problem?

Turn to page 34.
No!

Let's look at $9y^3$ in a different way. Suppose $y^3$ represents apples. Then the last problem is 9 apples - 1 apple. Do you see that you must look at the whole quantity $9y^3$. You cannot separate the 9 and the $y$.

See if you can get it this time.

Turn to page 24.
Question:

What is the sum of $x$ and $x^2$?

(a) $3x^2$  
(b) $x^3$  
(c) You can't add them
Hold on there! The base was $z$ in both cases and the exponent was 2. Both rules for addition are satisfied, so you should be able to add them.

Better go back to page 26 and re-work it.

Turn to page 26.
Very good!

Now, what is the sum of $5y^8 + 6y^8$?

(a) $11y^8$  
(b) $11y^{16}$  
(c) You can't add them
Oops! You forgot our two rules.

Both exponent and base must be the same before you can add or subtract.

See if you can do better with this one.

What is $5k^2 - k$?

(a) 4k  
(b) You can't subtract them
Oh, Oh! You made a bad mistake.

When adding or subtracting, you must never change the exponent. Only the coefficients are added or subtracted. The exponent and base remain the same.

Let's see if you can get back on the track with the next problem.

Turn to page 44.
Fine! You recognized that \( k \) could not be subtracted from \( 5k^2 \).

Try this one now.

What is \( 3n^4 - n^4 \)?

(a) \( 2n^4 \)   
(b) You can't subtract them

Turn to page 47

Turn to page 43
Page 40

Good! Your answer was correct.

You're doing well.

Subtract: \(7y^6 - 3y^3 = ?\)

(a) \(4y^3\)  
(b) \(4y^6\)  
(c) You can't subtract them

Turn to page 37  
Turn to page 45  
Turn to page 47
Correct!

Here's another.

\[ m^3 + 2m^3 = ? \]

(a) \( 2m^6 \) \hspace{1cm} (b) \( 3m^3 \)  
Turn to page 38 \hspace{1cm} Turn to page 40
You said you can't add $5y^8$ and $6y^8$.

Let's take a closer look:

The exponents are 8 in both quantities.

The base is $y$ in both quantities.

These are precisely the two conditions we need to add or subtract. So, you merely add the coefficients, leaving the base and exponent the same.

Now, see if you can work the previous problem.

Turn to page 36.
No! You don't understand the main ideas of this section.

Go back to page 23 and study it carefully. Then continue from there.

Turn to page 23.
What is $2x^2 + 3x^2$?

(a) $5x^2$  
(b) $5x^4$  
(c) You can't add them

Turn to page 41  
Turn to page 38  
Turn to page 43
Oops! You forgot our two rules.

Both exponent and base must be the same before you can add or subtract.

See if you can do better with this one.

What is $5k^2 - k$?

(a) $4k$  
(b) You can't subtract them

Turn to page 43

Turn to page 39
Incorrect! I don't think you have been paying attention.

We said that in a multiplication problem you always keep the same base, multiply coefficients and add exponents.

What is $9^3 \cdot 9^5$?

(a) $9^8$ Turn to page 51
(b) $9^{15}$ Turn to page 56
(c) You can't multiply them Turn to page 57
Excellent! You seem to grasp how to do addition and subtraction problems.

Now, let's try some multiplication problems with numbers containing exponents.

There is only one restriction necessary in order to multiply two quantities. The bases must be the same.

Then here are the rules to follow:

Rule 1: Multiply the coefficients together.

Rule 2: Add the exponents

Here are some examples for you to study.

Example 1: \(2^5 \cdot 2^3 = 2^8\)  (Add the exponents)

Example 2: \(x^2 \cdot x^5 = x^7\)

Example 3: \((4y)^3 \cdot (4y)^9 = (4y)^{12}\)

Example 4: \(3x^2 \cdot 2x^5 = 6x^7\) (Multiply coefficients and add the exponents)

Example 5: \(4m^2 \cdot 5m = 20m^3\)

Example 6: \(7z^5 \cdot z^5 = 7z^{10}\)

Now, turn to page 50 for a problem to work.
No! I think you forgot our rules for multiplication.

First of all, the bases must be the same. Then:

1. Multiply coefficients
2. Add exponents

So, \(3x^2 \cdot 2x^4 = (3 \cdot 2)x^{2+4} = 6x^6\)

Okay, try another one.

What is \(5y^3 \cdot 4y^4\) ?

(a) \(9y^{12}\) Turn to page 57
(b) \(20y^7\) Turn to page 49
Good! You got that one.

Now, $5^5 \cdot 5^{10} = ?$

(a) $5^{15}$ Turn to page 61
(b) $5^{50}$ Turn to page 46
(c) $25^{50}$ Turn to page 53
See if you can do this one.

\[ 2^4 \cdot 2^2 = ? \]

(a) \( 2^6 \)  
(b) \( 2^8 \)  
(c) \( 4^8 \)  

Turn to page 54
Turn to page 58
Turn to page 52
Good!

See if you can do this one.

What is $m^4 \cdot m^5$?

(a) $m^9$  
(b) $m^{20}$

Turn to page 54

Turn to page 46
Hold on there! You made two mistakes.

First, you do not change the base when multiplying. Second, add exponents when multiplying.

So, $2^4 \cdot 2^2 = 2^6$ is correct.

Let's see why. You know that $2^4 = 2 \cdot 2 \cdot 2 \cdot 2$

Also, $2^2 = 2 \cdot 2$

Then, $2^4 \cdot 2^2 = (2 \cdot 2 \cdot 2 \cdot 2) \cdot (2 \cdot 2) = 2^6$

How, do you see why you add exponents?

Turn to page 55.
Incorrect!

You treated 5 as if it were a coefficient. In the last problem, the 5 is the base. Also, you multiplied exponents when you should have added.

Go back and see if you can do better this time.

Turn to page 49.
Excellent!

Here's a little tougher one.

$3x^2 \cdot 2x^4 = ?$

(a) $6x^8$  
(b) $6x^6$  
(c) You can't multiply them
What is $3^2 \times 3^3$?

(a) $9^5$   Turn to page 46
(b) $3^6$   Turn to page 59
(c) $3^5$   Turn to page 51
You are having difficulty, aren't you?

I think it would help you to review the rules needed for multiplication. Study carefully before continuing.

Turn to page 47.
You are having difficulty, aren't you?

I think it would help you to review the rules needed for multiplication. Study carefully before continuing.

Turn to page 47.
No! In multiplication problems you are supposed to **add exponents**.

So, $2^4 \cdot 2^2 = 2^6$ is the correct answer.

Let's see why this is true.

You know that $2^4 = 2 \cdot 2 \cdot 2 \cdot 2$.
Also, $2^2 = 2 \cdot 2$
Then $2^4 \cdot 2^2 = (2 \cdot 2 \cdot 2 \cdot 2) \cdot (2 \cdot 2) = 2^6$

Now, do you see why you add exponents?

Turn to page 55 for your next problem.
No, no, no: You multiplied exponents.

He just said that in multiplication you add exponents.

Now, go back to page 55 and see if you can do better.

Turn to page 55.
No! I think you forgot our rules for multiplication.

First of all, the bases must be the same. Then:

1. Multiply coefficients
2. Add exponents

\[ 3x^2 \cdot 2x^4 = (3 \cdot 2)x^{2+4} = 6x^6 \]

Okay, try another one.

What is \( 5y^3 \cdot 4y^4 \)?

(a) \( 9y^{12} \)  Turn to page 57
(b) \( 20y^7 \)  Turn to page 49
Very good: You are moving along well. Let us now discuss division with quantities containing exponents.

Division is just opposite from multiplication. However, you must still have the same base in problems of division.

Follow these rules to divide:

1. Divide the coefficients
2. Subtract the exponent in the denominator from the exponent in the numerator.

Here are some examples for you to study:

Example 1: \( \frac{2^8}{2^2} = 2^6 \) (Subtract exponents)

Example 2: \( \frac{x^7}{x^5} = x^2 \)

Example 3: \( \frac{(4k)^5}{(4k)^2} = (4k)^3 \)

Example 4: \( \frac{6x^9}{3x^6} = 2x^3 \) (Divide coefficients and subtract exponents)

Example 5: \( \frac{15m^2}{5m} = 3m \)

Example 6: \( \frac{21y^3}{3y} = 7y^2 \)

Now, turn to page 67 for a problem to work.
Come now, you didn't read the last page very carefully.

Go back to page 70 and try again.
Correct!

Try this one

What is the quotient of $6x^5/2x^3$?

(a) $3x^2$  Turn to page 76
(b) $4x^2$  Turn to page 66
(c) $3x^{5/3}$  Turn to page 74
All right! Let's go on.

What is $6x^{18} + 3x^6$?

(a) $3x$  Turn to page 71  
(b) $2x^3$  Turn to page 70  
(c) $2x^{12}$  Turn to page 76
See if you can apply what you just read to this problem.

What is $4^6 \div 4^2$?

(a) $4^3$  Turn to page 66
(b) $4^3$  Turn to page 74
(c) $4^4$  Turn to page 72
No! You don't see the correct way of working division problems.

You'd better go re-study the explanation. Then continue your work from there.

Turn to page 61.
Here is your problem.

What is $3^9/3^5$?

(a) $3^{9/5}$ Turn to page 73
(b) $3^4$ Turn to page 72
(c) $3^4$ Turn to page 68
No! You divided 3 by 3. That is incorrect.

Do not change the base. In the last problem of $3^9/3^5$, the correct answer was $3^4$. You keep 3 as the base and simply subtract the exponents.

Turn to page 65.
Oh, oh! You slipped up on that one. You subtracted exponents all right. But did you notice anything about the bases? The base in the numerator was 6, while the base in the denominator was 3. So, you could not even divide $6^5$ by $3^2$.

Be sure you check to make sure the bases are the same before you attempt any operation with exponents.

Now, turn to page 64.
No! I think you need to review the rules for division again.

First, you must have the same base. Then, you merely divide the coefficients and subtract the exponent in the denominator from the exponent in the numerator.

Okay, then what is $\frac{10y^9}{5y^3}$?

(a) $2y^3$ Turn to page 62
(b) $2y^6$ Turn to page 63
No! I think you need to review the rules for division again.

First, you must have the same base. Then, you merely divide the coefficients and subtract the exponent in the denominator from the exponent in the numerator.

Okay, then what is \( \frac{10y^9}{5y^3} \) ?

(a) \( 2y^3 \) Turn to page 62
(b) \( 2y^6 \) Turn to page 63.
Excellent! Your answer was correct.

Try your hand at this one.

The quotient of $\frac{6^5}{3^2}$ is?

(a) $2^3$  
(b) You can't divide them.
No! You divided exponents. That is incorrect. You should have subtracted exponents. Let's see why this is true.

The problem was \( \frac{3^9}{3^5} = \frac{3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3}{3 \cdot 3 \cdot 3 \cdot 3 \cdot 3} = 3 \cdot 3 \cdot 3 = 3^4 \)

Do you see why you subtract now. What you are really saying is that \( \frac{3^9}{3^5} = \frac{3^4 \cdot 3^5}{3^5} = 3^4 \)

Okay, let's see if you can get the next one.

Turn to page 65.
No! You don't see the correct way of working division problems.

You'd better go restudy the explanation. Then continue your work from there.

Turn to page 61.
No! The correct answer was that \((5y)^0 = 1\). Remember, we just said that any quantity to the zero power is one.

Try this one.

\[(mx)^0 = ?\]

(a) \(m\)  
(b) \(1\)

Turn to page 78

Turn to page 84
Fine! That completes our brief discussion of how to work with positive integral exponents. But what about 0 as an exponent? Can zero be used as an exponent? The answer to that is "yes." We will define any quantity (except zero) to the zero power as being equal to 1.

So, $2^0 = 1$
$x^0 = 1$
$185^0 = 1$
$(4y)^0 = 1$
$(7x+3)^0 = 1$

Turn to page 79.
Right!

Now, try this one.

\[(5y)^0 = ?\]

(a) 1  \hspace{1em} \text{Turn to page 85}
(b) 5  \hspace{1em} \text{Turn to page 83}
(c) 5y \hspace{1em} \text{Turn to page 75}
No! In a problem like \((mx)^0\), it is not just the \(x\) that has the exponent of zero. The whole quantity is to the zero power. That is what the parenthesis indicate. So, \((mx)^0 = 1\) was the correct answer.

Now, \((62)^0 = ?\)

(a) 1  Turn to page 84
(b) 62  Turn to page 80
Here is a problem for you.

What is the value of $3^0$?

(a) 0 Turn to page 82
(b) 1 Turn to page 77
(c) 3 Turn to page 81
You're off the track. The zero exponent is really quite simple.

You need to re-read the explanation about it. Then continue from there.

Turn to page 76.
Hold on there. You weren't paying attention.

Go back to page 76 and read the explanation again. Then continue from there.

Turn to page 76.
Hold on there. You weren't paying attention.

Go back to page 76 and read the explanation again. Then continue from there.

Turn to page 76.
No! In $(5y)^0$, it is the whole quantity that is to the zero power, not just the $y$. So, $(5y)^0 = 1$.

Try this one.

\[(mx)^0 = ?\]

(a) $m$ Turn to page 78
(b) $1$ Turn to page 84
Right!

Let's see if you can do one more.

If $x = 2$, then $x^0 =$ ?

(a) 1   Turn to page 85
(b) 2   Turn to page 80
Very good: One was the correct answer.

You may wonder if it is possible to have a negative exponent. For example, can we talk about $5^{-2}$? The answer is yes.

$5^{-2}$ is the reciprocal of $5^2 = \frac{1}{5^2} = \frac{1}{25}$

Other examples are:

$2^{-4} = \text{the reciprocal of } 2^4 = \frac{1}{2^4} = \frac{1}{16}$

$4^{-3} = \text{the reciprocal of } 4^3 = \frac{1}{4^3} = \frac{1}{64}$

In general, for any value of $n$ we can say that $x^{-n} = \frac{1}{x^n}$

Using this, let's see if you can work a few problems.

Turn to page 93.
Correct! \( \frac{1}{2^2} \) was the answer.

See if you can do as well on this one.

Another way to write \( 6^{-3} \) is:

(a) 216  Turn to page 89
(b) 3  Turn to page 92
(c) \( \frac{1}{6^3} \)  Turn to page 90
Good! $1/8^4$ was correct.

Try this one.

$10^{-6}$ can also be written as:

(a) $1/10^6$  Turn to page 109
(b) 60      Turn to page 89
(c) 4       Turn to page 96
The correct answer was that $4^{-5} = \frac{1}{4^5}$.

Here are a few more examples showing how we eliminate the use of negative exponents.

1. $3^{-7} = \text{the reciprocal of } 3^7 = \frac{1}{3^7}$
2. $8^{-4} = \text{the reciprocal of } 8^4 = \frac{1}{8^4}$
3. $(15)^{-2} = \text{the reciprocal of } (15)^2 = \frac{1}{(15)^2}$
4. $x^{-n} = \text{the reciprocal of } x^n = \frac{1}{x^n}$

Turn to page 94.
You missed the main idea.

Go back and re-study the material on page 85. Pay close attention to the examples.

Turn to page 85.
Excellent! Your answer was correct.

See if you can do this one.

Write $10^{-3}$ without using a negative exponent.

(a) $1/10^3$  Turn to page 109
(b) 30  Turn to page 91
(c) 7  Turn to page 97
The correct answer was that \(10^{-3} = 1/10^3\).

That's all there is to it. Remember, in general we can say that \(x^{-n} = 1/x^n\).

Now, what is an alternate form of \(8^{-4}\)?

(a) 32 Turn to page 89
(b) 4 Turn to page 98
(c) \(1/8^4\) Turn to page 87
Careful, now! It looks like you made a careless error.
$6^{-3}$ is not the same as $6 - 3$. Remember, the $(-3)$ is
the exponent.

Go back to page 86 and make a better selection.

Turn to page 86.
Remember the rule: \( x^{-n} = \frac{1}{x^n} \)

Question:

What is another way to write \( 4^{-5} \)?

(a) \( 5 \cdot 4 \) or 20
(b) \( \frac{1}{4^5} \)
(c) There isn't any other way

Turn to page 88
Turn to page 90
Turn to page 95
Try this one.

The expression $2^{-2}$ can also be written as:

(a) $\frac{1}{2^2}$ Turn to page 86
(b) 4 Turn to page 89
(c) 0 Turn to page 99
Your answer was incorrect! $4^{-5}$ can be written in another way.

You'd better re-study the material on page 85.

Turn to page 85.
It looks like you're going a bit too fast. $10^{-6}$ is not the same as $10 - 6$!

Go back to page 87 and try again. This time be more careful.

Turn to page 87.
Come now! You didn't read the last question very carefully. The (-3) was an exponent.

Go back to page 90 and be a little more careful.

Turn to page 90.
You made a careless error. The quantity $8^{-4}$ does not mean $8 - 4$.

Go back to page 91 and see if you can make a better selection.

Turn to page 91.
Sorry! 2 - 2 is 0. However, you didn't read the problem correctly. The problem asked for $2^{-2}$.

Go back to page 94 and make another selection.

Turn to page 94.
Good! Your answer was correct.

See if you can do this one as well.

The decimal value of $10^{-6}$ is:

(a) $\frac{1}{10^6}$ Turn to page 112
(b) .000001 Turn to page 103
(c) .001 Turn to page 108
Good!

Try this one.

The decimal .01 can also be written as:

(a) $10^{-2}$  Turn to page 118
(b) 100       Turn to page 102
(c) $\frac{1}{10000}$ Turn to page 110
I think you're off the track.

See if you can get this one.

Another way of expressing .1 is:

(a) \( \frac{1}{100} \) \hspace{1cm} \text{Turn to page 104}
(b) \( 10^{-1} \) \hspace{1cm} \text{Turn to page 101}
(c) I'm not sure \hspace{1cm} \text{Turn to page 117}
Excellent! Now, lets see if you can go from a decimal
to a number with a negative exponent.

The decimal .00001 can be written as:

(a) $10^{-5}$
(b) $10^{-4}$
(c) I'm not sure
You are having trouble with decimals. This is an area you must understand before completing this unit.

Take Unit 8 before you go on with this one.

Proceed to Unit 8.
Let's look at the correct solution to the last problem.

\[ .00001 = \frac{1}{100000} = \frac{1}{10^5} = 10^{-5} \]

This process is exactly the reverse of the last one you worked. Now, you try this one.

The decimal .001 is equal to:

(a) \( \frac{1}{1000} \)  
(b) \( 10^{-3} \)  
(c) \( 10^2 \)

Turn to page 116  
Turn to page 101  
Turn to page 102
Let's look at the correct solution to the last problem.

\[
0.00001 = \frac{1}{100000} = \frac{1}{10^5} = 10^{-5}
\]

This process is exactly the reverse of the last one you worked.

Now, you try this one.

The decimal .001 is equal to:

(a) \( \frac{1}{1000} \)  \( \text{Turn to page 116} \)
(b) \( 10^{-3} \)  \( \text{Turn to page 101} \)
(c) \( 10^2 \)  \( \text{Turn to page 102} \)
Very good! You said you are ready to try some problems yourself.

What is the decimal value of $10^{-2}$?

(a) 20  Turn to page 111
(b) $\frac{1}{100}$  Turn to page 115
(c) .01  Turn to page 103
You don't quite have it. Remember, a negative exponent indicates you put the number in fractional form. Then, you convert it to a decimal if necessary.

$10^{-3}$ is equal to what decimal number?

(a) $.1$  
(b) $.001$  
(c) $.000001$
Excellent!

Now, let's look at some negative powers of 10. These are very important in scientific notation.

For example, \(10^{-3} = \frac{1}{10^3} = \frac{1}{1000} = .001\)

or, \(10^{-6} = \frac{1}{10^6} = \frac{1}{1000000} = .000001\)

If you do not know how to express such fractions as decimals, go work Unit 8 before continuing this unit.

Otherwise, turn to page 107.
I think you're off the track.

See if you can get this one.

Another way of expressing .1 is:

(a) \( \frac{1}{100} \)  
(b) \( 10^{-1} \)  
(c) I'm not sure
Your answer was incorrect!

Let's use the problem for an example to see how it should be worked.

\[ 10^{-2} = \frac{1}{100} = .01 \]

See how easy it is.

Just go through every step and you should be able to do it correctly. Here's another example.

\[ 10^{-4} = \frac{1}{10000} = .0001 \]

Turn to page 114
Come now! You're not being very careful. \( \frac{1}{10^6} \) is not a decimal.

Go back to page 100 and see if you can choose the correct decimal answer.

Turn to page 100.
Oops! You didn't choose a decimal.

Go back to page 114 and make another selection.

Turn to page 114.
All right! Let's see if you can get back on the right track.

What is the decimal value of $10^{-1}$?

(a) .1  Turn to page 100
(b) 1/10  Turn to page 113
(c) 100.0  Turn to page 108
Hold on there! Did you read the problem correctly?
It asked for the **decimal** value.

Go back to page 107 and be more careful this time.

Turn to page 107.
Yes! \( .001 = \frac{1}{1000} \)

But, what other value is this equal to?

(a) \( 10^2 \) Turn to page 102
(b) \( 10^{-3} \) Turn to page 101
You are having trouble with decimals. This is an area you must understand before completing this unit.

Take Unit 8 before you go on with this one.

Proceed to Unit 8.
Correct! You have now completed the section dealing strictly with the use of exponents. You will need to remember what you have learned as you continue into the actual use of Scientific Notation.

Now, turn to the first page of Booklet II of this Unit. It is page 119.