DOCUMENT RESUME

ED 022 930 08 VT 006 884

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OCCUPATIONAL MATHEMATICS; REPRESENTING NUMBERS BY LETTERS. REPORT NO. 16-B. FINAL REPORT.

Washington State Coordinating Council for Occupational Education, Olympia.; Washington State Univ., Pullman. Dept. of Education.
Spons Agency-Office of Education (DHEW), Washington, D.C.
Bureau No-BR-7-0031
Pub Date Jun 68
Grant-OEG-4-7-070031-1626
Note-52p.
EDRS Price MF-$0.25 HC-$2.16
Descriptors-*ARITHMETIC, *FUNDAMENTAL CONCEPTS, *MATHEMATICAL CONCEPTS, *PROGRAMED TEXTS, *VOCATIONAL EDUCATION

This programed mathematics textbook is for student use in vocational education courses. It was developed as part of a programed series covering 21 mathematical competencies which were identified by university researchers through task analysis of several occupational clusters. The development of a sequential content structure was also based on these mathematics competencies. After completion of this program the student should know that a letter can represent a number and that algebraic and arithmetic rules of operations apply to letters and numbers. He should be able to make correct numeric substitutions for general literal expressions and construct general formulas that represent simple relationships. The material is to be used by individual students under teacher supervision. Twenty-six other programed texts and an introductory volume are available as VT 006 882-VT 006 909, and VT 006 975. (EM)

FINAL REPORT
Project No. OE7-0031
Contract No. OEG-4-7-070031-1626
Report No. 16-B

Occupational Mathematics

REPRESENTING NUMBERS BY LETTERS

June 1968

U.S. DEPARTMENT OF
HEALTH, EDUCATION AND WELFARE

Office of Education
Bureau of Research

Occupational Mathematics

REPRESENTING NUMBERS BY LETTERS

Project No. OE7-0031
Contract No. OEG-4-7-070031-1626
Report No. 16-B

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June 1968

The research reported herein was performed pursuant to a contract with the Office of Education, U.S. Department of Health, Education, and Welfare. Contractors undertaking such projects under Government sponsorship are encouraged to express freely their professional judgment in the conduct of the project. Points of view or opinions stated do not, therefore, necessarily represent official Office of Education position or policy.

Washington State ~~University,~~ Department of Education, Pullman, Washington
State Coordinating Council for Occupational Education, Olympia, Washington

OBJECTIVES

1. The student should know that a letter may be used to represent a number.

2. The student should know that algebraic and arithmetic rules of operation apply to letters as well as numbers.

3. The student should be able to make correct numeric substitutions for general literal expressions.

4. The student should be able to construct general formulas that represent simple relationships.

Greetings! You are about to begin improving your knowledge of basic mathematics. There are many important uses for the mathematics you are learning.

This booklet is not like your ordinary books. It is designed to help you learn as an individual. On the following pages you will find some information about mathematics. After the information is presented, you will be asked a question. Your answers to these questions will determine how you proceed through this booklet. When you have selected your answer to the question, turn to the page you are told to.

Do not write in this booklet. You may wish to have a pencil and some paper handy so you can write when you want to.

Remember, this is not an ordinary book.

1. Study the material on the page.
2. Read the question on the page (you may want to restudy the material on the page).
3. Select the answer you believe is correct.
4. Turn to the page indicated by your answer.

Are you ready to begin?

(a) Yes Turn to page 1

(b) No Turn to page C

(c) HELP Go see your teacher

Your answer was (b) No.

Well, this booklet is a little different.

Go back and read page B again. After you have read it, you will probably be ready to begin.

In this unit you will learn how to use LETTERS to REPRESENT NUMBERS.

If I had 7 apples yesterday and ate 2 of them today, how many are left? We know that the number of apples left is some number. Let us call it A.

In this example A = 7 - 2, the number of apples left. Or, A = 5. We could have used the letter B, X, Y, or any other letter. The important point is that the letter represents the number of apples.

Question:

If I am A years old, which expression represents my age 10 years from now?

(a)	10 + A	Turn to page 4
(b)	A - 10	Turn to page 5
(c)	A + 10	Turn to page 4

Fine! Your answer was correct.

Turn to page 10.

No. The correct choice was 2L. If your boat is L feet long and mine is twice as long, then 2 x L or 2L represents the length of my boat. Try using some numbers for L and see how it works.

Now, turn to page 1 and go over the material again.

Very good! Both 10 + A and A + 10 are correct.

The order you add letters or numbers does not matter.

You will get the same answer.

Turn to page 10.

No! A - 10 would represent your age 10 years ago.

Let A equal your own age and see what happens?

Which of the following expressions would correctly give your age 5 years ago?

(a) E - 5 Turn to page 2

(b) 5 + W Turn to page 6

No! The correct answer was E - 5.

If you are 15 now, then E = 15; and your age 5 years ago, E - 5 = 15 - 5 = 10. Got it? Notice that it is just as good to use E or A or any other letter.

Question:

If my sailboat is twice as long as yours, what expression represents the length of my boat?

(a) L - 2 Turn to page 3

(b) 2L Turn to page 2

Very good! 24 is correct.

Suppose a and c represent numbers which are related in such a way that

$$c = a/3.$$

If a = 12, then c has what value?

(a) 4 Turn to page 15

(b) 36 Turn to page 11

That's not it. Remember that $w = 2$ means that we replace the letter w by 2, and then $Z = w \cdot 12$ becomes $Z = 2 \cdot 12$. Since $2 \cdot 12 = 24$, we see that Z must have the value 24 when w has the value 2.

Try one more.

Suppose S and d are letters which represent numbers and suppose $S = 3 \cdot d$.

If $d = 7$, then what value does S have?

(a) 21 Turn to page 14

(b) 3 Turn to page 9

Sorry! Your answer was not correct. You seem to be having some difficulty with this type of problem. Why don't you turn to page 10 again and reread this explanation.

Suppose we have an equation like $A = 3B$. If $B = 5$, then we have $A = 3 \cdot 5$, or $A = 15$. This is found simply by substituting 5 for B, since <u>B merely represented the number 5</u>.

Question:

Suppose $X = 7Y$. If $Y = 3$, what value must X have?

(a) 7 Turn to page 13

(b) 21 Turn to page 14

Your answer was not correct. If a = 12, we replace a by 12 and c = a/3 becomes c = 12/3. Therefore, c must have the value 4.

Here is another problem.

Suppose that B and F represent numbers that are related in such a way that

$$F = 6/B.$$

If B = 2, then F has what value?

(a) 12 Turn to page 9

(b) 3 Turn to page 12

That's it! 3 is correct.

Let's do one more to make sure that you have it for keeps.

If $d = 9/t$, then what value must d have if $t = 3$?

(a) 9 Turn to page 9

(b) 3 Turn to page 15

No! That is not correct. The correct answer is 21. When we say $Y = 3$, we mean that we are replacing Y by 3. In this case, $X = 7Y$ becomes $X = 7 \cdot 3$, and we see that X must have the value 21.

Try another one.

Suppose Z and w represent numbers which are related in such a way that

$$Z = w \cdot 12.$$

If $w = 2$, then what value must Z have?

(a) 24 Turn to page 7

(b) 6 Turn to page 8

Very good! 21 is correct.

Suppose a and c represent numbers which are related in such a way that

$$c = a/3.$$

If a = 12, then c has what value?

(a) 4 Turn to page 15

(b) 36 Turn to page 11

Fine! Your answer was correct.

Suppose R, E, and I represent numbers which are related in such a way that

$$E = IR.$$

If $R = 6$ and $I = 3$, then E has what value?

(a) 2 Turn to page 18

(b) 18 Turn to page 19

That is not correct. When we replace R by 2 and E by 12, then $I = E/R$ becomes $I = 12/2$. Therefore, I must have the value 6.

Suppose A, P, and T represent numbers related in such a way that

$$P = A/T.$$

If $A = 9$ and $T = 3$, then what value does P have?

(a) 9 Turn to page 17

(b) 3 Turn to page 20

No, that is not correct. Perhaps you should review the material beginning at page 10.

No, that's not correct. If $R = 6$ and $I = 3$, we replace R by 6 and I by 3 and then $E = IR$ becomes $E = 3 \cdot 6$. Therefore, E must be 18.

Try this one.

If $I = US$ and $U = 2$ and $S = 9$, then what value does I have?

(a) 18 Turn to page 19

(b) 2 Turn to page 17

Very good! 18 is correct.

Suppose R, E, and I represent numbers which are related in such a way that

$$I = E/R.$$

If R = 2 and E = 12, then I has what value?

(a) 6 Turn to page 21

(b) 24 Turn to page 16

Very good! 3 is the correct answer. I think you have it, but let's do cne more problem just to make sure.

Suppose A, P, and T represent numbers related in such a way that

$$T = A/P.$$

If P = 5 and A = 30, then what value does T have?

(a) 6 Turn to page 21

(b) 150 Turn to page 17

You have now had some practice in substituting numbers for letters. Let's see how to write rules like the ones you have been using.

Example 1: You know that $3 \times 2 = 2 \times 3$. In fact, any two numbers can be multiplied in reverse order, and the answer remains the same.

In general, we can then say that $ab = ba$, where a and b represent any two numbers.

Example 2: Suppose Hi Fi records cost \$4 each. If you wish to buy 3, the total price is 3×4 or \$12. As a general formula, we can write:

Total price = (Quantity) x (\$4)

Using only letters, we could say that $P = Q \cdot 4$, where P is price and Q is the number of records we wish to buy. Notice that this formula will give the correct price for any number of records we wish to purchase, i.e., if $Q = 10$, then $P = 10 \cdot 4 = \$40$.

(Continued)

Question:

If we travel at 40 miles per hour and drive for 5 hours, we travel a distance of 200 miles (which is $5 \cdot 40$). If we drive for 7 hours, the total distance traveled is 280 miles. $(7 \cdot 40)$

Which of the following equations shows the rule for finding distance if D stands for distance and T is the time we travel?

(a) $D = 40/T$ Turn to page 25

(b) $D = T \cdot 40$ Turn to page 26

(c) $D = T/40$ Turn to page 30

Incorrect! Any letters could be used, as long as the relationship is the same.

We could have said $S = C \cdot 40$

$D = T \cdot 40$

or $X = Y \cdot 40$

These all express the same relationship.

Question:

How many of the following equations would also work?

$d = t \cdot 40$ $\quad S = w \cdot 40$ $\quad B = 5 \cdot 40$

(a) One Turn to page 27

(b) Two Turn to page 24

(c) Three Turn to page 32

You are so right! $S = C \cdot 40$ could be used to find the distance traveled. In this case, C would represent the number of hours traveled; and then S would represent the number of miles traveled. Other letters could be used also.

Turn to page 31.

That's fine! You noticed that the first two equations would work. The last equation $B = 5 \cdot 40$ only works for 5 hours, and we wanted a formula which would work for any number of hours.

Turn to page 31.

No, that's not correct. Remember that the formula must work when numbers are used in place of the letters.

Go back and restudy page 21 and try the problem again.

Your answer $D = T \cdot 40$ is correct! T represents the time traveled, and D would be the distance traveled.

If $T = 5$, we travel 200 miles; or if $T = 2$, we would travel only 80 miles.

Could the equation $S = C \cdot 40$ be used to show the same rule for finding distance?

(a) Yes Turn to page 23

(b) No Turn to page 22

No, this is not correct. Go back to page 22 and look at the equations again. Perhaps it will help if you see how many equations give you the correct distance traveled when you let one of the letters represent 4 hours.

No, this is not quite the answer. Remember that the general rule must work when the letters are replaced by any numbers. Let's see why the answer you chose is not correct. You chose $a \cdot 1/b = 1$. Since the letter a can represent any number, let's let $a = 10$; and, since b can represent any number, let's let $b = 2$. Then $a \cdot 1/b = 1$ becomes $10 \cdot 1/2 = 1$ which is the same as $5 = 1$. This, of course, is nonsense. Therefore, the general rule does not work.

Turn back to page 31 and see if you can choose a better answer now.

No, this is not the correct answer. Remember that we want a general rule which will work when the letters represent any numbers. Let's see why your answer $c \cdot d/r = d$ is not a general rule.

Since the letters are supposed to represent any numbers, let's choose some values for these letters. Let's let $c = 4$, $r = 2$, and $d = 1$. Then $c \cdot d/r = d$ becomes $4 \cdot 1/2 = 1$ which is the same as $2 = 1$. This, of course, is nonsense. Therefore, the general rule is not even a rule.

Turn back to page 31 and see if you can choose a better answer.

No, that's not correct. Remember that the formula must work when numbers are used in place of the letters.

Go back and restudy page 21 and try the problem again.

Let's do another problem which is just a little different. I'll give you some examples using numbers, and you try to figure out the general rule which the examples illustrate. The examples are:

$3 \cdot 1/3 = 1$ $\quad$ $7 \cdot 1/7 = 1$ $\quad$ $11 \cdot 1/11 = 1$

What is the general rule? (The general rule should work when the letters are replaced by any numbers.)

(a) $a \cdot 1/b = 1$ Turn to page 28

(b) $n \cdot 1/n = 1$ Turn to page 33

(c) $c \cdot d/r = d$ Turn to page 29

That's not quite right. The last equation $B = 5 \cdot 40$ only works if you drive for 5 hours. We wanted a formula for the distance when we drive for 4 hours, or 7 hours, or for any number of hours.

Turn back to page 22 and try again.

That was good figuring. $N \cdot 1/n = 1$ was correct. $1/n$ is called the reciprocal of n, and this rule shows that a number times its reciprocal always equals one.

See if you can figure out this one. The examples are:

$$5/17 + 3/17 = \frac{5 + 3}{17}$$

$$2/17 + 7/17 = \frac{2 + 7}{17}$$

$$1/17 + 9/17 = \frac{1 + 9}{17}$$

What general rule is illustrated by these examples?

(a) $a/17 + b/17 = \frac{a + b}{17}$ Turn to page 35

(b) $a/c + b/d = \frac{a + b}{c}$ Turn to page 36

Fine! Your answer ab = ba is correct. This is the <u>commutative rule</u>, which tells us that multiplying the first number by the second is the same as multiplying the second by the first.

Turn to page 40.

Excellent! $a/17 + b/17 = \frac{a + b}{17}$ shows that when adding two fractions with a common denominator of 17, you obtain a fraction with denominator 17 and new numerator the sum of the old numerators.

Look at these examples:

$2 \cdot 5 = 5 \cdot 2$

$4 \cdot 9 = 9 \cdot 4$

$6 \cdot 3 = 3 \cdot 6$

What general rule do these examples illustrate?

(a) $ab = cd$ Turn to page 37

(b) $ab = ba$ Turn to page 34

No. Your answer was incorrect. The correct general rule is $a/17 + b/17 = \frac{a + b}{17}$. If we let a = 5 and b = 3, we obtain $5/17 + 3/17 = \frac{5 + 3}{17}$. Notice that this is the first example on page 33. Convince yourself that this rule is valid for any a and b.

Look at these statements:

$$\frac{4 - 2}{7} = \frac{4}{7} - \frac{2}{7} \qquad \frac{9 - 7}{13} = \frac{9}{13} - \frac{7}{13}$$

What general rule is illustrated by both statements?

(a) $\frac{x - y}{7} = x/7 - x/y$ Turn to page 39

(b) $\frac{4 - 2}{x} = 4/x - 2/x$ Turn to page 39

(c) $\frac{m - n}{r} = m/r - n/r$ Turn to page 38

No, that's not correct. This problem should have looked familiar to you. Go back to page 21 and continue from there.

Right! $\frac{m - n}{r}$ = m/r - n/r is correct.

Let's do another one.

We want to write the general rule which the following examples illustrate. Remember that the rule should work when the letters are replaced by <u>any</u> numbers. The examples are:

$2 \cdot 5 = 5 \cdot 2$

$4 \cdot 9 = 9 \cdot 4$

$6 \cdot 3 = 3 \cdot 6$

What is the general rule?

(a) ab = cd Turn to page 37

(b) ab = ba Turn to page 34

No. You just don't seem to get it!

Go see if your teacher can give you a hand. Then return to page 21 and begin there.

Congratulations! You have completed the unit. Let's review what we have done.

1. You have seen several ways in which letters may represent numbers.

2. You have practiced the correct methods for substituting particular numbers for letters.

3. You have seen and have practiced writing general rules with letters and numbers.

Now go see your teacher for a test over this unit.

TEST QUESTIONS

UNIT 2 - REPRESENTING NUMBERS BY LETTERS

1. Suppose that K = 6/3. Then you know that K is equal to

   (a) X

   (b) 2

   (c) You can't tell from t[illegible] information

2. If we have an expression like 15 = 5B, the letter B

   (a) Does not mean anything without further information

   (b) Represents the number 15

   (c) Represents the number 3

3. What do we mean by A·B?

   (a) Multiply A and B

   (b) Add A and B

   (c) Divide A by B

   (d) It doesn't mean anything unless there are definite numbers.

4. How would you express this in symbols: "Add X and Y and divide the sum by Z"?

   (a) $\frac{X + Y}{Z}$

   (b) $\frac{4 + 5}{6}$

   (c) $X \cdot (Y + Z)$

5. If X = 4·K and K = 5, then X =

   (a) 9

   (b) 20

   (c) 45

6. If $39 = 13 + X$, what is X?

   (a) 13

   (b) 3

   (c) 26

7. If you are X feet tall, how would you express your height plus 5 feet?

   (a) 5X

   (b) $X + 5$

   (c) $X \div 5$

   (d) You can't do it without knowing the value of X.

8. My house is K feet long. What is the expression for a house 4 times as long as mine?

   (a) 4K

   (b) $4 + K$

   (c) $\frac{4}{K}$

9. If $X = Y + M$ and $M = 5$, the value of X is

   (a) 8

   (b) 14

   (c) You can't do it without knowing the value of Y.

10. For $A = 7$ and $B = 3$, then $A + B =$

    (a) 10

    (b) 21

    (c) 4

11. Given that $X = 4$, the value of $13 - X$ is

    (a) 9

    (b) 52

    (c) 17

12. Given 10X = Y, this might express the fact that

    (a) I am 10 years older than you are.

    (b) It is 10 minutes past noon.

    (c) There are 10 pennies in a dime.

13. If you knew the values of M and N and that Y = M-N how would you find Y?

    (a) Subtract N from M.

    (b) Subtract M from N.

    (c) Divide M by N

    (d) You couldn't do it.

14. The relationship X = 2Y means that

    (a) Y = 12

    (b) X = Y + 2

    (c) X is twice as large as Y

15. If this year is 1968, what was the date X years ago?

    (a) 1968·X

    (b) 1968 - X

    (c) 1968 + X

16. Suppose I weigh X pounds and you weigh twice as much as I do. How could you express your weight?

    (a) 2X

    (b) 2 + X

    (c) X - 2

17. For K = 12 what is K - 4?

    (a) 16

    (b) 8

    (c) Not enough information is given.

18. The expression 3Y + 7 means to

(a) Add 3, Y, and 7

(b) Subtract 7 from 3 + Y

(c) Multiply 3 by Y and add 7

19. Suppose R = S/T. If S = 12 and T = 4 then

(a) R = 3

(b) R = 48

(c) R = 16

20. If 3·1/3 and 7·1/7 express a general rule, this rule is

(a) a·1/b = 1

(b) n·1/n = 1

(c) c·d/r = d

21. Suppose X = 4 + 8. Then

(a) you know X = 12

(b) X can stand for any number

(c) you don't know anything about X

22. If your house is X miles from here and my house is 10 miles farther, how would you express the distance from here to my house?

(a) 10X

(b) 10 - X

(c) X + 10

23. Look at these statements 3·4 = 4·3, 9·8 = 8·9 The rule they illustrate is

(a) 3·X = X·4

(b) X·Y = Y·X

(c) 9·R = 9R

24. The quantity M/17 means simply

(a) Add M and 17

(b) Divide 17 by M

(c) Divide M by 17

25. Assume that $X = \frac{3K + 17}{2}$ and that K = 1. Then X =

(a) 10

(b) 7

(c) 30

Answer Sheet - Unit 2

Objective	Question Number	Answer
1	1	b
1	2	c
2	3	a
2	4	a
3	5	b
3	6	c
4	7	b
4	8	a
3	9	c
2, 3	10	a
2, 3	11	a
4	12	c
2	13	a
2	14	c
4	15	b
4	16	a
3	17	b
2	18	c
3	19	a
4	20	b
1	21	a
4	22	c
4	23	b
1	24	c
3	25	a

Objective	Questions
1	1, 2, 21, 24 (In reality all questions are testing objective 1
2	3, 4, 10, 11, 13, 14, 18
3	5, 6, 9, 10, 11, 17, 19
4	7, 8, 12, 15, 16, 20, 22, 23

0 (REV. 9-66) DEPARTMENT OF HEALTH, EDUCATION, AND WELFARE
OFFICE OF EDUCATION

ERIC REPORT RESUME

ERIC ACCESSION NO.

CLEARINGHOUSE ACCESSION NUMBER | RESUME DATE June 1968 | P.A. | T.A.

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TITLE

Occupational Mathematics - REPRESENTING NUMBERS BY LETTERS

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INSTITUTION (SOURCE)
Washington State University, Pullman, Washington, Dept. of Education
SOURCE CODE

REPORT/SERIES NO. Final Report No. 16-B

OTHER SOURCE
SOURCE CODE

OTHER REPORT NO.

OTHER SOURCE
SOURCE CODE

OTHER REPORT NO.

PUB'L. DATE June 1968 | CONTRACT/GRANT NUMBER OEG-4-7-070031-1626

PAGINATION, ETC

44 pages

RETRIEVAL TERMS
Occupational mathematics
Programmed instruction
Mathematical numbers and letters

IDENTIFIERS
Vo-Tech. Education Research and Development Project (no. 7-0031)

ABSTRACT

One book of a 21-book series of programmed instruction materials designed to help pupils acquire mathematics capabilities most useful in sub-professional level occupations. Other programmed books in the series are:

- Symbols
- Equivalent Forms
- Ratios and Fractions
- Addition of Fractions
- Subtraction of Fractions
- Multiplication of Fractions
- Division of Fractions
- Concepts of Decimals and Fractions
- Addition and Subtraction of Decimals
- Multiplication of Decimals
- Division of Decimals
- Conversion of Fractions into Decimals
- Equivalent Forms of A = BC
- Solutions of A = BC
- Percentage
- Commutative Law
- Reciprocals
- Scientific Notation
- Proportions
- Concepts of Number Bases