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By-McHale, Thomas J.; Witzke, Paul T.

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The primary purposes of this developmental and demonstration project were to reduce the number of dropouts and failures and to increase the amount of learning in the technical mathematics core courses. In June 1965 a decision was made to pilot test locally developed programed units in technical mathematics. After the identification of the desired units, 700 pages of programed material and daily tests were written. Seventy-three students in the electrical, mechanical, and civil technologies were selected to participate in a pilot test of the material. Post-test means increased over the pre-test means for all 11 units given the pilot group. A test was administered to all students covering the units studied by all students. The pilot groups had a mean of 75 percent and a median of 80 percent while the conventional group of 295 students had a mean of 57 percent and a median of 61 percent. The material was later used with a large group of 395 students for 1 semester. For this group the final grade mean was 82 percent and the median was 85 percent. Eight hundred forty-three pages of second semester materials were tried with 303 students. The final exam mean for this group was 77 percent and the median was 78 percent. Information about the students' reactions and the unit contents are included. (EM)

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FINAL PROGRESS REPORT

TO

THE CARNEGIE CORPORATION

New York City

A Practical Demonstration Project In Teaching Technical Mathematics

by

THOMAS J. McHALE (Project Director)
and
PAUL T. WITZKE

Being Conducted by
The Milwaukee Institute of Technology
of
The Milwaukee Vocational Technical and Adult Schools

September, 1967

ABSTRACT

This report is a summary of a two-year project funded by the Carnegie Corporation for the development of a system to teach Technical Mathematics to industrial technicians. It includes a statement of the problem, a history of the development of the system, and a description of the results. Though the project is not yet completed, the following accomplishments have already been obtained:

(1) The dropout rate in the course has been substantially reduced.

(2) The level of achievement has been substantially increased to the point where students receive high grades even though grades are assigned on a strict percentage basis.

(3) The content of the course has been radically changed to make it more relevant for elementary science and technical work.

(4) A novel system of instruction has been developed in which daily personal attention is provided each student despite large numbers (about 500) of students.

(5) Over 2,000 pages of programmed materials have been written and revised.

(6) The students have responded enthusiastically to the system of instruction and the course.

TABLE OF CONTENTS

In December, 1964, the Milwaukee Institute of Technology received a $200,000 grant from the Carnegie Foundation in order to improve the instruction in the Technical Mathematics course. This course is a two semester core-course for all entering technical students in the industrial technology programs. The project was begun in June, 1965, and is currently still in progress. This report summarizes the problems faced and the accomplishments of the past two years.

Industrial Technicians: Nature and Need

Nature. Though the term "technician" is not well defined, technicians can be broadly subdivided into two categories: engineering technician and industrial technician. Both kinds of technicians are trained in two-year post-high school programs.

The training program for engineering technicians is academically oriented. Many of its courses, particularly in mathematics and science, are very similar to those in four year engineering programs. As a result, engineering technicians have considerable capability in theoretical work, plus some capability in practical, manipulative work. On the job, they usually work closely with engineers and scientists engaged in research and development. They hold job titles such as Engineering Assistant, Junior Engineer, Research Assistant, Engineering Aide.

Engineering technicians comprise only a small percent of the total technician group. Most technicians in the Milwaukee area are industrial technicians. Their training program and job requirements differ somewhat from that of engineering technicians. Though basic principles and skills are taught, their mathematics and science courses are not as academically oriented. They become more highly specialized in their technical fields and develop more manipulative skills with instruments and devices. Industrial technicians are more likely to work directly on the production aspects of industry. They hold job titles such as Instrument Technician, Production Control Technician,

Electronic Tester, Service Technician, Laboratory Technician, Quality Control Technician, Numerical Control Technician, Detail Draftsman.

Need. In the report Mathematical Expectations of Technicians in Michigan Industries, dated 1966, the following figures were cited concerning the need for technicians in the United States:

> To maintain the ratio of technicians to engineers (0.7 to 1), the 775,000 technicians employed in 1960 must increase (521,700 or 67.3 percent) to 1,296,700 by 1970. The increase must be 695,000 yearly by 1970, if losses due to deaths, retirement, and job changes are considered. Also, if the technician ratio is to be increased (2 to 1), the number of technical graduates yearly must increase to about 803,300. The 701 two-year colleges offering technical curriculums in 1963 represented a 300 percent increase from 1950, and it seems evident that in the State of Michigan the percent of increase will need to be even greater in the next decade.

What is true for the State of Michigan is also true for the State of Wisconsin. Approximately 500 more technicians are needed than are being graduated annually, and this deficit is bound to grow if the number of graduate technicians does not substantially increase.

Technician Training - Milwaukee Institute of Technology

The Milwaukee Institute of Technology is one of the largest public technical schools in the United States. It has been training industrial technicians since 1952. The following majors which require Technical Mathematics are offered:

Air Conditioning and Refrigeration
Architectural
Automotive
Chemical
Civil: Highway
Civil: Structural
Dental Laboratory
Diesel
Electrical: Communications
Electrical: Computer
Electrical: Electronics
Electrical: Instrumentation
Fire Fighting
Fluid Power
Mechanical: Design
Mechanical: Manufacturing
Metallurgical
Photo Instrumentation
Printing and Publishing

Although 14 technical majors are offered, 73% of the incoming students in 1966 enrolled in one of four programs: Electrical (30%), Civil (18%), Mechanical (16%) and Automotive (9%).

Curriculum. Graduates of the two year program receive an associate degree in applied science. Besides the technical courses themselves, the following core-courses are included: Technical Mathematics, Technical Science (Physics), Communication Skills, American Institutions, Psychology of Human Relations, Business and Industrial Relations, and Physical Education. Courses are not designed for transferability to a regular four year college program. This terminal nature of the courses fits the policy of the school. Because of the need for technicians in the Milwaukee area and the ability level of our incoming students, this policy is the only sensible one.

The somewhat nebulous role of technicians and the changing demands of industry prevent the curriculum from becoming fixed. Determining the optimum level of instruction for a course is difficult, and this difficulty is compounded by the fact that teachers are not trained specifically for the technical area. As a result, the level of instruction may deviate in either direction from the optimum blend of practical and theoretical. In a school of our size, the core-courses present a unique problem. Most of the teachers of these courses have no technical background, and consequently their courses frequently reflect too much of an academic flavor.

The Typical Student. The prerequisite for enrollment in a technology program is a high school diploma, including one year of algebra and one year of geometry. Since high school rank is ignored, an "open-door" policy is maintained. Naturally the ability level of the students is quite heterogeneous. Though a small percentage could qualify and succeed in a four year engineering program, the majority of the students could not. Many are slow learners with a history of low achievement, and many have poor study habits.

Dropout Rate. Only 35% or 40% of the students who enroll complete the two-year program. Since many do not really understand what a technician is when they enroll, some drop out simply because they do not want to become technicians. Others drop out to enter the Armed Forces, or because of job conflicts, or because they have developed sufficient skills to obtain a skilled job. However, a high proportion of the dropouts occur because of academic failure. Though some dropouts of the latter type must be expected because of the "open-door" policy, the percentage seems to be too high.

Technical Mathematics - A Problem Course

Technical Mathematics is a 2-semester, 8 credit-hour course. Because the level of mathematical proficiency of the students sets a lower limit on the level of all other technical courses, the quality of the whole program depends on the quality of the mathematical instruction. In our institution, the quality of this instruction has been seriously deficient. Because of an awareness of the nature and magnitude of the problems encountered by the students in the course, in 1960 the school selected Technical Mathematics as the first locally produced credit course to present over its educational television station. In spite of some improvement, the results were still not good. Coupled with a 40% dropout in the first semester and a 20% dropout of the remaining students in the second semester, the achievement level of students on final exams rarely exceeded 50% or 55%. Even when grades were curved, many students failed, and failure automatically disqualified them from continuation in the program. Furthermore, many students who passed did not possess the fundamental skills required for their technical courses. Because of the high dropout rate and low level of achievement, this course has been the source of many complaints from both students and technical teachers. In fact, it had become somewhat of a "whipping boy" for the lack of success of many students in their technical courses.

Although easy to criticize, a core-course of this type in a technical program is faced with many problems. The following list includes the more significant ones:

1) As a core-course, it must serve students from all technical programs. Unfortunately, these programs are diverse, and they do not require either the same level of mathematical sophistication or even the same mathematical topics.

2) Though designed to prepare students for their technical courses, the math course is taught parallel with technical courses. Given the restrictions of the necessary sequencing of mathematical topics and the speed with which the students can learn, it is almost impossible to treat all topics before they are encountered in the technical courses.

3) The entrance requirement of one year each of high school algebra and geometry is no guarantee that the students either learned or remember the fundamental skills from these courses. Therefore, their entry behavior is quite heterogeneous, and some decision must be made about the level at which the course is to begin. Of course, this decision has immediate effects on the dropout rate.

4) Though they must have received a passing grade for two years of high school math, many of the students have had the equivalent of a "failure" experience with mathematics, and they bring this defeatist attitude with them into the classroom.

5) The available textbooks are not satisfactory. Most seem to be watered-down versions of regular college textbooks, with not enough thought given to the relevancy of topics and the learning capabilities of the students.

6) The teachers assigned to this course are, for the most part, junior-college mathematics teachers. Since Technical Mathematics is not a status course, assignment to it is not entirely welcomed. The traditional lack of

Furthermore, many of the teachers are uninformed about the mathematical needs of students in a technology program. Instead of approaching mathematics from a technical-scientific point of view, they use traditional approaches. As a result, many irrelevant topics are covered, many relevant ones ignored, and the exposition of topics is too definitional and abstract for the ability level of the students.

Summary of the Grant Proposal

The grant proposal was formulated and written by staff members at the school. The grant was received from the Carnegie Foundation in December, 1964, before the project director or project staff was hired. In the proposal, a hypothetical solution to the problem was formulated. This hypothetical solution involved a systems-approach with the following major components:

1) Large-group instruction using participation television.
2) Individual instruction using programmed booklets.
3) Small-group instruction in tutorial sections.
4) Individual remedial and enrichment instruction in a special learning center.

As the project has developed, the list of components in the system has changed. The reasons for these changes will be discussed later.

Philosophy of the Project Staff

The whole development of the project has been guided by the following principles, excerpted and summarized from the original proposal:

I. There are three basic goals of the project:

   1) To reduce the number of dropouts.
   2) To increase the amount of learning.
   3) To reduce the number of failures.

II. The project is meant to be a practical demonstration and not basic research.

III. Since media are a means and not an end, they must be evaluated in terms of their administrative feasibility, flexibility and cost. Since hardware is expensive and the production of materials for hardware even more expensive, expenditure for equipment should be approached cautiously and only after considerable thought.

IV. The topics in the curriculum should be dictated solely by the needs of industrial technicians.

Pilot Groups - September, 1965

The intent of the proposal was to develop a systems-approach for the first semester of the course. A decision was made when the project began in June, 1965, to teach the whole first semester to some pilot groups in the fall semester, rather than to concentrate on one or two topics. Underlying this decision was the hope that published programmed materials could be found to piece together a one semester course. Unfortunately, such materials could not be found either because they did not exist or because the commercially available materials did not cover the topics from the desired approach. This lack of available materials meant, of course, that new materials had to be prepared by the project staff.

Survey of the Technology Teachers and Course Content.

A written survey of the technology teachers was conducted in order to decide on the topics to be included in the course. On the basis of this survey, the treatment of Higher-Degree Equations was dropped, and units on Powers of Ten and Estimation, Geometric Formula Evaluation, and Technical Measurement were inserted. The treatment of Formula Rearrangement was considerably expanded. Based on past experience with the course, a decision was made to review algebra completely, beginning with the number line and signed numbers. The staff felt that this review, though time-consuming, was necessary to reduce the dropout rate. Basic algebra, of course, is an absolute necessity for successful completion of a technician-training program.

Learning Materials.

Programmed materials were prepared for 11 topic-units. In addition to the programmed materials, television lectures were prepared for the slide-rule unit. These lectures included student participation in the form of paced exercises. Since the staff was hard-pressed to prepare the core materials within the available time, no attention was given to remedial or enrichment materials. 700 pages of programmed materials were written by the project

staff. Given the time limits and the staff's inexperience with such writing, these materials were somewhat crude.

Classroom Procedure.

After some experimentation during the first five topic-units, the following classroom procedure was adopted. No formal lectures were given. The bulk of the instruction was accomplished by means of the programmed materials. Daily criterion tests were administered to assess the daily assignments. These tests were corrected during class time by the teacher, and they served both as a check to see that the work had been done, and as a basis for tutoring. If the tutoring could not be handled immediately during the class time, students were assigned to the project center at some time before the next class meeting. Tutoring outside of class-time had to be kept at a minimum because the staff members who taught the pilot classes were also responsible for writing programmed materials. Considerable pressure was exerted on the students both to attend classes daily and to complete the daily assignments.

Selection of Students.

Three pilot classes were selected on the basis of a diagnostic test which had been specifically written for and used in the Technical Mathematics course during the previous five years. A representative cross-section of students was selected from the electrical, mechanical and civil technologies since these technologies include about 65% of the technology students. The three pilot classes included a total of 73 students.

Achievement Tests.

Parallel pre-tests and post-tests were given for each unit. The pre-test was used both to determine the level of entry skills and to determine the amount of behavioral change in each unit. Any behavioral change was the result of various factors: (1) the programmed materials, (2) tutoring, and (3) the recall of skills which had been forgotten because of disuse for

a number of years. The amount of this type of recall probably varied considerably among the different units. The names of the topic-units and the means and medians for the pre-tests and post-tests are given in the table below:

TABLE 1

ACHIEVEMENT TESTS - PILOT GROUPS

TOPIC-UNIT	PRE-TEST MEAN	PRE-TEST MEDIAN	POST-TEST MEAN	POST-TEST MEDIAN
1. Operations With Signed Numbers	81%	84%	96%	98%
2. Estimation and Powers of Ten	55%	58%	88%	93%
3. Slide Rule Operations	24%	20%	82%	83%
4. Basic Algebraic Operations, and Solving Simple Equations	69%	74%	87%	91%
5. Geometric Formula Evaluation	67%	65%	85%	88%
6. Technical Measurement	37%	35%	81%	83%
7. Graphing	47%	44%	86%	88%
8. Formula Rearrangement	61%	60%	90%	92%
9. Systems of Equations	32%	12%	84%	94%
10. Quadratic Equations	40%	38%	85%	38%
11. Exponentials and Logarithms	25%	19%	73%	76%

Final Exam - A Comparison.

To determine the comparative success of the pilot classes (59 students) and the conventional classes (295 students), a common final exam was administered to all students in all sections. The exam was written by one staff member and one conventional teacher. Since it included only items which had been covered by both groups, some topics taught uniquely to the pilot classes or to the conventional classes were not represented. The items were designed to test fundamental skills. The mean and median for the pilot classes were 75% and 80% respectively, the mean and median for the conventional classes were 57% and 61% respectively. This difference was obtained despite the fact that the dropout rate was only 19% in the pilot classes as opposed to 39% in the conventional classes. Therefore, in addition to cutting the dropout rate in half, the performance of the students in the pilot classes was superior.

Course Grades.

The final grades for the pilot students were based on both their post-test average (weighted 2/3) and their final exam score (weighted 1/3). Grades were given on a strict percentage basis (70% was required for a "D", etc.). Of the 59 students who finished the course, 16 received A's, 17 received B's, 17 received C's, 9 received D's, and none received an "F". The teachers expressed some reservations about the ability of the "D" students and some of the "C" students to complete a technology program.

Conclusions.

The following general conclusions were based on the experience with the pilot groups:

1) Since the dropout rate and number of failures was reduced and the amount of learning increased, the staff felt that it was moving in the right direction.

2) A decision was made to abandon the use of television. Production of

the video tapes was expensive, and they seemed to have no unique advantages. Any mathematical material presented on television can also be presented in booklets with the added advantage of the student's being able to progress at his own pace. The use of scheduled televised lectures also greatly increased the amount of time necessary to cover a topic, and it was difficult to reschedule these lectures for students who were absent.

3) Time prohibited the development and use of learning carrels at the project center for the pilot groups. Ideally, these carrels could contain remedial and enrichment materials, but in addition to the fact that the staff's time was absorbed in developing the core-materials, uncertainty existed about the nature of such materials.

4) The emphasis with the pilot groups was focused on the low-ability and/or unmotivated students. The teachers expressed some uneasiness about the probability of some of these students completing a technology program, even though they received a passing grade in the math course.

Technical Mathematics I - September, 1966

The original proposal stipulated that all students enrolled in the Technical Mathematics I course in September, 1966, be taught using the methods and materials developed with the pilot classes.

Preparation. Between the completion of the semester with the pilot students and the beginning of the next fall semester, the following were accomplished:

1) In an oral interview by a staff member, each technology teacher discussed the relevancy and sequencing of topics and the possible incorporation of new topics.
2) An error-analysis of each post-test and the final exam of the pilot students was completed. Errors on individual items were categorized so that more attention could be given to remedying the common errors.
3) The programmed booklets were revised on the basis of the error analysis. In the revised booklets, greater emphasis was given to a more expository style and better verbalization of the principles and strategies. The 11 revised booklets included a total of 1207 pages.
4) All tests were revised so that each item required a constructed response. The items on the 11 parallel pre-tests and post-tests tested terminal behavior related to the course objectives. The items on the 40 daily tests tested principles and some terminal behavior. In general, the items on the tests were more difficult than those on the tests prepared for the pilot groups.
5) Simple learning carrels equipped with rear-screen slide projectors were built. Exercises for improving skills in operations with signed numbers, estimating answers, and reading the slide-rule scales were developed.
6) A one-week in-service training program was conducted for the six teachers assigned to handle the Technical Mathematics sections.

During this program the teachers read and discussed the programmed materials, and they were introduced to the classroom procedure which had been developed with the pilot groups.

Classroom Procedure. The classroom procedure developed with the pilot groups was continued. For each of the 11 topic-units this procedure included:

1) A parallel pre-test and post-test.

2) Daily tests covering the daily assignments (plus tutoring in the classroom).

3) Tutoring outside the classroom for students needing further help.

All students were required to attend the regular classes. Pressure was exerted on the students to attend the classes and complete the daily assignments. It was each teacher's responsibility to see that his absent students completed the missed daily tests. If a student did not attain a satisfactory score (usually 80%) on a post-test, he was required to repeat the test until the satisfactory score was obtained. Each teacher was responsible for the tutoring preceding this retesting and the retesting itself.

Dropouts. 503 students enrolled in the course. (Enrollment was defined as "taking the diagnostic test," which meant that the student attended at least one class.) The original 24 sections were reduced to 21 before the end of the semester. Of the original 503 students, 395 completed the course. The 108 dropouts constituted a 21% dropout rate. Since only 11 of the 108 dropouts were actually failing the course at the time of withdrawal, the majority of the dropouts occurred for other reasons.

Achievement Tests. The mean and median for each of the 11 pre-tests and post-tests are given in the table on the next page. No partial credit was given for any item on any test. An item was scored "correct" only if the correct numerical or algebraic answer was given. The overall mean for the 11 post-tests was 91%. A description of the content of each topic unit is given in Appendix A.

ACHIEVEMENT TESTS - TECHNICAL MATHEMATICS I

TOPIC-UNIT	PRE-TEST MEAN	PRE-TEST MEDIAN	POST-TEST MEAN	POST-TEST MEDIAN
Number System, Number Line, Operations with Signed Numbers	80%	71%	97%	100%
Non-Fractional Equations	73%	85%	93%	95%
Fractions and Fractional Equations	48%	55%	90%	95%
Formula Rearrangement	47%	50%	91%	95%
Powers of Ten	59%	63%	97%	98%
Estimation	53%	50%	93%	95%
Slide Rule	20%	0%	80%	84%
Logarithms	21%	10%	88%	94%
Technical Measurement	69%	72%	93%	94%
Graphing	19%	14%	84%	88%
Triangles & Trigonometry	62%	64%	88%	93%

Final Exam and Course Grades. The mean and median for the final exam were 82% and 85%, respectively. Course grades were determined on a straight percentage basis with the overall percentage on the 11 post-tests weighted 2/3 and the final exam weighted 1/3. The distribution of the final grades was:

	Number of Students	Percent of Students
A (93% to 100%)	141	36%
B (85% to 95%)	136	35%
C (77% to 85%)	65	16%
D (70% to 77%)	40	10%
U (Below 70%)	11	3%

71% of the students received a grade of either "A" or "B"; only 3% failed. Since these grades were not curved, we feel that they represent a substantial level of achievement.

Conclusions. The following general conclusions are based on our experience with teaching a large group for one semester.

1) Given the reduced dropout rate and the reduced number of failures plus the increased achievement level, the staff felt that it had successfully demonstrated that its system worked with large groups.

2) Though the emphasis was again directed toward the lower-ability students, the higher-ability students attained a degree of mastery which they would never have attained in conventional classes.

3) The tutoring and retesting of lower-ability students absorbed much of the teacher's time. Given the low probability that many of these students would complete the two-year program, the staff felt that the efforts of the instructional team might have been misdirected. Concentration on the lower-ability students detracts from attention to the middle-ability students, and these latter students are much more likely to complete the program.

4) The learning carrels in the project center were used to develop skills with signed numbers, estimation, and slide rule scale-reading. Students with deficiencies were referred to the center by their teachers. The number of these students was comparatively small and typically from the lower-ability group. The staff felt that the time and expense involved in the preparation of materials for the learning carrels might have been better used elsewhere.

Technical Mathematics II - February, 1967

Though the original grant proposal did not include Technical Mathematics II, a decision was made during the first semester to develop materials for the second semester. The two reasons for this decision were: (1) the lack of a suitable textbook, and (2) a request by the teachers who felt that a change in the system of instruction at the end of the semester would present some serious difficulties. Ten new programmed booklets, including a total of 843 pages, were written. Ten post-tests and 46 daily tests were also written. Pre-tests were not given during the second semester since most of the topics were new to the students and the staff felt that pre-tests would take up too much time.

New Use of the Project Center. On the basis of the first semester's experience, the project center handled the following tasks during the second semester: (1) make-up work for the students who were absent, (2) retests for students who failed to attain a satisfactory level on post-tests, (3) the total instruction of a special group of students. The reason for each of these is discussed below.

1) Absences. Because the teachers were new to the system, the absentee rate during the first semester had been high for a few students. Administering the make-up work for such students was difficult during regular class time, and exerting pressure on these students to attend class made the student-teacher relationship unpleasant. Therefore, a new procedure was adopted in which a student, after being absent, reported to the project center instead of to his regular class. He was not permitted to return to his regular class until all missed work had been completed at the center. The explanation for his absence was discussed and recorded at the project center.

2) Re-tests. During the first semester, when a student attained an unsatisfactory score on a post-test, he was given remedial help and a re-test by his individual teacher. Since re-tests and their grading took a consi-

derable amount of time, the teachers had difficulty scheduling them. Therefore, during the second semester, these re-tests were administered at the project center. The individual teacher was still responsible, however, for the remedial tutoring before the re-test.

3) Special group. Many "A" and "B" students profit little from the daily tests since they understand the material. Yet if they remain in the regular classroom, the teacher must correct their daily tests and consequently he has less time for the students who really need his help. As an incentive for the good students, and as an attempt to maximize the efficient use of the teacher's time, a special group of good students were removed from their regular classes and taught completely from the project center. Students were assigned to this group upon recommendation by their regular teacher. Though this group was started with some caution, over 25% of the students were involved by the end of the semester.

In this special group, the burden of learning was placed on the student himself. When a new booklet was assigned, the date for the post-test was also assigned. All daily tests were taken at the center at the convenience of the student, with the stipulation that they had to be completed at least the day before the assigned date for the post-test. Ordinarily, the students corrected their own daily tests. If a student required much tutoring or did poorly on the post-test, he was returned to his regular class and the regular classroom procedure.

Dropouts. Of the 395 students who completed Technical Mathematics I, only 281 enrolled in Technical Mathematics II. Though 11 could not enroll because they had failed and 23 did not enroll because their technology did not require a second semester of math, 80 other students did not return. In Technical Mathematics I, 21 of these 80 received A's, 30 received B's, 20 received C's, and 9 received D's. Their reasons for not returning are unknown.

Since 22 students who had completed Technical Mathematics I in some previous year enrolled for the second semester, the total enrollment was 303. 37 of these students dropped out during the second semester for a dropout rate of 12%. Only 5 of the 37 dropouts were failing at the time of their withdrawal.

Achievement Tests. The mean and median for each post-test is given in the table below. The overall mean for the post-tests was 87%. This mean is somewhat lower than that obtained during the first semester, but the topics were more difficult and the requirement of correct calculation was more demanding. A description of the content of each topic unit is given in Appendix B.

ACHIEVEMENT TESTS - TECHNICAL MATHEMATICS II

TOPIC-UNITS	POST-TEST MEAN	POST-TEST MEDIAN
Oblique Triangles I	94%	94%
Vectors	85%	87%
Systems of Equations	89%	95%
Oblique Triangles II	85%	89%
Quadratic and Radical Equations	87%	92%
Geometry and Applied Trigonometry	84%	87%
Logarithms: Laws and Formulas	82%	88%
Exponentials: Base "e" and Natural Logarithms	83%	88%
Sine Waves	90%	92%
Further Topics in Trigonometry	90%	93%

Exams and Course Grades. The mean and median for the mid-semester exam were 80% and 83% respectively; the mean and median for the final exam were 77% and 78% respectively. The course grades were computed in the following way: the overall percentage on the 10 post-tests and the mid-semester exam (which was given the weight of 2 post-tests) was weighted 2/3; the final exam was weighted 1/3. The distribution of course grades was:

	Number of Students	Percent of Students
A (93% to 100%)	41	15%
B (85% to 93%)	97	37%
C (77% to 85%)	74	28%
D (70% to 77%)	41	15%
U (Below 70%)	13	5%

Conclusions. The following general conclusions are based on the results from Technical Mathematics II:

1) The level of achievement, dropout rate, and failure rate were very satisfactory, in spite of the fact that the materials covered were complex and, in general, required more computation.

2) The experiment with a special group who did not attend the daily classes was successful. This procedure did not decrease in any significant way their level of achievement, and it increased the amount of available teacher-time for the students who could benefit from tutoring. Furthermore, the students who participated liked it since they were given an opportunity to budget their own time.

Student Reaction to the Program

At the end of Technical Mathematics II, each student was asked to fill out a questionnaire concerning the method of teaching. Of the 266 students who completed the course, 244 filled out the questionnaire. Each question and the responses to it are given below.

(1) Compare the amount learned in this course and previous math courses.

	Number of Students	Percent of Students
Much More	127	52%
More	85	35%
Same Amount	14	6%
Less	11	4%
Much Less	4	2%
No Response	3	1%

(2) Compare how much you liked this course in relation to previous math courses.

	Number of Students	Percent of Students
Much More	114	47%
More	86	35%
Same Amount	25	10%
Less	15	6%
Much Less	4	2%

(3) Compare how hard you worked in this course in relation to previous math courses.

	Number of Students	Percent of Students
Much More	36	15%
More	75	31%
Same Amount	53	21%
Less	61	25%
Much Less	19	8%

(4) If you took another math course, would you prefer to study math with this same type of system?

	Number of Students	Percent of Students
Yes	218	89%
No	21	9%
No Response	5	2%

(5) Would you prefer to have this same type of system in other courses besides math?

	Number of Students	Percent of Students
Yes	192	79%
No	43	18%
No Response	9	3%

In general, the student reaction to this method of instruction was very favorable. Many of the students spontaneously commented that they had learned more math in this one year than they had learned in all of their math courses in high school. Others suggested quite emphatically that math should be taught in this manner in high school. The high percent of students who said they would prefer to take more courses using the same method dispelled some of our fears about the boredom of programmed instruction. Personal success is probably the main determinant of "liking" or "disliking" a method of instruction.

DISCUSSION

Success of the Project. Though the project is not yet completed (funds are available for at least one more year), we feel that it has already been successful. The dropout rate has been substantially reduced, and in spite of this reduction, the level of achievement has substantially increased. Even with grading on a strict percentage basis, the grades have been high and the number of failures low. Over 2,000 pages of programmed materials have been written, and these materials have altered the content of the course so that each topic is relevant to the needs of technicians. A new system of instruction has been developed. This system offered daily personal attention to each student despite the large number of students. Furthermore, the students liked this system even though it demanded consistent work and a fairly high level of achievement. Except for a very small group who needed frequent prodding, the motivation of the students was very satisfactory. A special class for high achievers was successfully begun. Even though the full responsibility for learning was placed on the students in this group, they responded enthusiastically.

Reasons for this Success. It is impossible to determine how much of the success was contributed by the programmed materials and how much by the system of instruction. If learning is to occur, the materials must communicate and the students must seriously interact with them. However, in our judgment, the materials played the basic role. Among average-ability students, no amount of motivation can overcome lectures or materials which do not communicate. And tutoring to a mastery level is not possible in a short period of time if the tutor must begin from scratch. Since the complexity of the materials and the quality of the booklets varied, the amount of tutoring also varied. In general, the amount of tutoring was never overwhelming, and it decreased as the year progressed. Aside from the tutoring required before the retests for students who performed below the acceptable level on a unit test, the amount of tutoring outside of class was minimal, and much of the tutoring during

class time was necessary because the students had not seriously done the assignment.

Learning Principles and the Programmed Materials. Programmed instruction was used because it offered the best opportunity of putting mathematics in learnable form. What is currently known about the psychology of human learning was taken into account in the writing of these materials. We feel that this emphasis has increased the amount of learning. The style of the writing was straight exposition. The principles of modern mathematics were used, with an emphasis on an intuitive understanding of them rather than on proof. Whenever possible, these principles were justified numerically. Attention was given to all details or learning sets, including a step by step strategy for complex procedures. Transfer of principles or strategies to new situations was never assumed; it was always explicitly taught. As much distributed practice as possible was included. Though more practice exercises were included than were needed by many students, the students were not forced to complete every problem unless they could not master the materials without doing so. Though some attention was given to the student's ability to verbalize the principles and strategies, we now feel that more verbalization should be required, especially in the algebra materials.

Possible Use of the Materials. Though prepared specifically for a Technical Mathematics course, the potential use of the materials is much broader than technician training, since their goal is a preparation for elementary science and technology. Some of the materials could easily be used in the training of apprentices and tradesmen; most of them could be incorporated in a high school mathematics program. During the coming school year, we hope to try some of them with apprentice groups and in a developmental program in the Junior College division of the Milwaukee Institute of Technology. This latter program is basically designed as a terminal program for students whose entrance exams for the Junior College are very low.

Emphasis on the Lower-Ability Students. Since the goal of the project was to reduce the number of dropouts and failures, the main effort was directed to the lower-ability or less motivated students. Though we found few students for whom mathematics seemed incomprehensible, some could not learn math at the pace required by the course. Furthermore, some lower-ability students manifested a serious memory problem even from day to day. If the successful training of technicians is taken as a criterion, too much time was devoted to this group of students. We found, for example, that no student whose average was below 80% at the midsemester of Technical Mathematics I successfully completed both semesters with passing grades. Also, only eight students who received "D's" as a final grade in Technical Mathematics I passed Technical Mathematics II, and only one of these eight did not receive some type of academic action (a warning or drop) for an overall grade-point average below "C". Without abandoning an "open door" policy, we feel that some decision should be made about students with a very low probability of success early in the year. Then they can be counseled into a program in which they can succeed, and the teachers' time can be devoted to the middle-ability students who are otherwise somewhat overlooked.

Personnel Problems. With the exception of the Project Director, the intent of the project was to use personnel from the existing mathematics staff of the Milwaukee Institute of Technology. This decision was made before the decision to write completely new programmed materials. Though the typical classroom teacher can handle the tutoring required in the course, he is not trained to write quality programmed materials. Time for such training is not available when production demands must be met within a delimited period of time. Of the 9 teachers who have attempted to write materials, only one has produced quality material with any degree of speed. Consequently, the two authors of this report have written the bulk of the materials. If other writers could be found, the speed of production would be considerably increased and other experiments with the system of instruction would be possible.

Plans for the Coming Academic Year (1967-1968)

During the coming academic year, the Technical Mathematics course will again be taught with the system devised by the project staff. The existing materials and tests will be revised, new materials will be added, and further experiments with the system of instruction will be conducted. If time permits, some preliminary materials will be developed for the Technical Science (Physics) course, which is also a core course for all technology students. These plans are discussed in more detail below.

Changes and Experiments with the Instructional System. The following changes or experiments with the instructional system will be made during the coming academic year:

(1) Since more content is being added to the course and more instructional time is consequently required, the following two changes are being made:

(a) Though remaining a 4 credit-hour course, the classes will meet 5 days per week.

(b) Aside from one algebra pre-test, the other unit pre-tests in Technical Mathematics I will be eliminated. We feel that the base rate is now known, and that the students will not change much from year to year.

(2) The Special Class will begin shortly after the beginning of the first semester. Students will be selected for it on the basis of their score on the algebra pre-test. Students in the Special Class will not report to daily classes, but will be handled from the Learning Center on a flexible and individual basis.

(3) During the second semester (Technical Mathematics II), the regular classes will be eliminated and all students will be handled directly from the Learning Center on as individual basis as possible. If this attempt is successful, both the amount of required physical space and the number of required teachers will be considerably reduced.

If time and money are available for future experimentation, the following features could be added to the instructional system:

(1) All testing and data analysis could be handled by a computer-based system, and evantually "tutoring" could be included along with the test items.

(2) Within the limits of certain deadlines, students could work at their own pace, even to the point of completing the course in a much shorter period of time.

Revision of Materials and Completion of the Course Content. On the basis of test results and error analyses of the unit tests, last year's materials will be revised. Approximately 10 new booklets will be included in the course. These booklets will introduce topics which, though not included in last year's course, are an essential part of a Technical Mathematics program. A detailed description of the content of each of the proposed booklets is given in Appendix C. The booklets are divided into six general content areas:

I. Algebra

II. Calculation

III. Graphing

IV. Trigonometry and Applied Geometry

V. Logarithms and Exponentials

VI. Measurement.

Certain unique aspects of each content area deserve separate mention. These unique aspects are listed in Appendix D.

Technical Science (Physics) and Problem Solving. The purpose of the Technical Science course is to introduce the fundamental principles of physics which are needed in elementary technical work. Student performance in this core course has been characterized by a low level of achievement and a high dropout rate. The traditional lecture-lab method does not seem to work with many of the students. Perhaps the lack of success can be attributed to too heavy an emphasis on "how a physicist thinks." The goals of the course should rather be an understanding of the principles of physics plus strategies for using these principles in elementary problem-solving. Furthermore, the content of the course needs serious reexamination to make sure that the topics correspond to the diverse needs of the various technology curricula.

The Technical Mathematics course, as we have designed it, has been criticized because of a lack of "problem-solving." We hope to meet this criticism by teaching formal strategies for problem-solving in elementary

science in the Technical Science course. Hopefully, these strategies will be generalizable to the problem-solving encountered in other technology courses. Aside from problem-solving in the area of geometry, we can think of no relevant problems to include in the Technical Mathematics course. Students cannot be asked to do problem-solving in science when they know no science. Before some basic concepts and principles are known, problem-solving which requires an understanding and use of them is impossible. Many of the traditional problems included in mathematics textbooks are trivial and artificial. They have to be trivial and artificial because the underlying principles and concepts must be common knowledge. Since such problems require strategies which are different than those used in elementary science, we see no reason to expect positive transfer from them to the scientific area.

General Mathematics Learning Center. Our institution includes trade programs for adults and apprentice programs. Common to most of these programs is a requirement of basic skills in arithmetic, algebra, calculation, and some applied geometry. Currently this requirement is being met by a proliferation of math courses for specific trades. The students in these courses represent a wide range of ability and mathematical skills. Though specific applications to various trades are not included in the materials we have prepared, many of the fundamental skills common to all of these math courses parallel those taught in the Technical Mathematics course. It would be possible to develop a Math Center where the students could be taught the core part of these courses. Such a Center could handle the students on as individual a basis as possible by means of an entry assessment of their skills. It could also handle adult students in the Milwaukee metropolitan community who desire to learn some specific math skills without enrolling in a formal mathematics course.

APPENDIX A

DESCRIPTION OF COURSE CONTENT
TECHNICAL MATHEMATICS I
1966-67 SCHOOL YEAR (FIRST SEMESTER)

Note: The topics were taught in the order in which they are listed.

(1) Algebra I: Decimal number system.
Number line and related concepts.
Addition and subtraction of signed numbers.
Multiplication of signed numbers.

(2) Algebra II: Addition axiom for equations.
Solution of non-fractional, first-degree equations in one variable.
Distributive principle; Factoring; Concept of opposites.

(3) Algebra III: Principles of fractions.
Multiplication of signed fractions.
Concept of reciprocals.
Division of signed numbers.
Multiplication axiom for equations.
Solution of non-fractional, first-degree equations whose roots are fractions.
Addition and subtraction of fractions.
Solution of fractional, first-degree equations in one variable whose roots are fractions.

(4) Algebra IV: Generalization of algebraic principles to literal expressions.
Rearrangement of non-fractional and fractional formulas for first-degree variables.

(5) Powers of Ten: Base 10 exponential expressions having integer exponents.
Laws of exponents for multiplication, division, and powers and roots.
Writing numbers in standard notation.

(6) Estimation: Estimating answers to multiplication problems by means of these techniques: gross inspection, decimal point shift, name-form, and powers of ten.
Estimating answers to division problems by means of these techniques: gross inspection, decimal point shift, name-form, and powers of ten.
Estimating answers to combined multiplication and division problems using powers of ten.
Estimating answers to squaring and cubing problems using powers of ten.
Estimating answers to square root and cube root problems using powers of ten.

(7) Slide Rule Operations: Slide rule operations as follow:
Multiplication.
Division.
Combined multiplication and division.
Squaring.
Square Root.
Cubing.
Cube Root.

(8) Logarithms: Review of the laws of exponents.
Meaning of fractional and decimal exponents.
Expressing positive numbers in power of ten form.
Calculations involving numbers in power-of-ten form.
Logarithmic notation and its meaning.
Calculations in logarithmic notation, with emphasis on powers and roots.

(9) Technical Measurement: Review of percent.
Measurement concepts and scale reading.
Precision in measurement.
Addition and subtraction of measurements.
Accuracy and significant digits in measurement.
Multiplication and division of measurements.
Error in measurement.

(10) Graphing: Properties of graphs.
Reading linear and non-linear graphs of equations and formulas.
Graphing linear and non-linear equations and formulas.
Intercepts of linear and non-linear graphs.
Concept of slope.
Slope-intercept form of linear equations and formulas.
Slope of non-linear graphs.

(11) Triangles and Trigonometry:
Properties and areas of rectangle, square, and parallelogram.
Volume and surface area of box and cube.
Properties and areas of general triangles and right triangles, including the angle-sum relationship and the Pythagorean Theorem.
Definitions of sine, cosine, and tangent of an acute angle.
Solving right triangles by means of the sine, cosine, and tangent ratios.

APPENDIX B

DESCRIPTION OF COURSE CONTENT
TECHNICAL MATHEMATICS II
1966-67 SCHOOL YEAR (SECOND SEMESTER)

Note: The topics were taught in the order in which they are listed.

(1) Oblique Triangles I:

Solving acute oblique triangles by right triangle methods.
Solving acute oblique triangles by law of sines.
Solving acute oblique triangles by law of cosines.

(2) Vectors:

Vectors on the rectangular coordinate system.
Components of a vector.
Definition of sine, cosine, and tangent of reference angles in all four quadrants.
Addition of two or more vectors by component method.
Concepts of equilibrium and vector opposite.

(3) Systems of Equations:

Graphical solution of systems of two linear equations.
Algebraic solution of systems of two linear equations.
Algebraic solution of systems of three linear equations.
Determinant solution of systems of two linear equations.
Formula derivation involving two equations.
Formula derivation involving three equations.

(4) Oblique Triangles II:

Standard position of angles between 0° and 360°.
Definition of sine, cosine, and tangent of angles between 0° and 360°.
Finding sine, cosine, and tangent ratios on the slide rule.
Solving obtuse oblique triangles by law of sines.
Solving obtuse oblique triangles by law of cosines.
Addition of two vectors by oblique triangle method.

(5) Quadratic and Radical Equations:

Meaning of quadratic equations.
Solving quadratic equations in one variable by factoring method.
Solving quadratic equations in one variable by quadratic formula method.
Operations with radicals.
Solving radical equations.
Rearranging formulas involving radicals.
Rearranging formulas involving second-degree variables.
Formula derivation involving squares and square roots.
Formula rearrangement using the quadratic formula.

(6) Geometry and Applied Trigonometry:

Problems involving chords and central angles of circles.
Problems involving circumferences and arcs of circles.
Problems involving areas of circles, sectors, and segments.
Areas of right triangles and oblique triangles.
Problems involving tangents and half-tangents of circles.
Volumes of prisms and non-prisms.
Problems involving density and weight.

(7) Logarithms: Laws and Formulas:
Review of logarithmic and exponential notation.
Laws of logarithms.
Negative logarithms.
Evaluating exponential and logarithmic formulas.
Rearranging exponential and logarithmic formulas.
Using tables of e^x and e^{-x}.
Table of natural logarithms.
Logarithmic scales.
Finding logarithms on the slide rule.

(8) Sine Waves:
Sine, cosine, and tangent of 0°, 90°, 180°, 270°, 360°.
Graphing the fundamental sine wave.
Sine, cosine, and tangent of angles greater than 360°, and of negative angles.
Sine wave amplitude and phase.
Fundamental sine wave equation.
Sine wave harmonics.

(9) Further Topics in Trigonometry:
Definitions of cosecant, secant, and cotangent of an angle.
Basic trigonometric identities.
Inverse trigonometric notation.
Degree and radian systems of angle measurement.
Angular velocity and circular velocity.

APPENDIX C

TECHNICAL MATHEMATICS I AND II
DESCRIPTION OF COURSE CONTENT
1967-68 SCHOOL YEAR

Note: Technical Mathematics I units are starred (*).
Technical Mathematics II units are not starred.
The order in which the units are listed is not necessarily the teaching order.

SECTION I: ALGEBRA

*ALGEBRA I: SIGNED NUMBERS

Concept of the number line.
Vectors on the number line.
Addition of two or more signed whole numbers.
Concept of opposites.
Subtraction of signed whole numbers.
Problems involving combined addition and subtraction.
Multiplication of two or more signed whole numbers.
Problems involving combined multiplication, addition and subtraction.

*ALGEBRA II: NON-FRACTIONAL EQUATIONS I

Note: All equations solved in this unit are non-fractional, first-degree, single-variable equations whose roots are integers.

Intuitive solution of equations.
Addition axiom for equations.
Solving equations by means of the addition axiom.
Distributive principle for multiplication over addition, and over subtraction.
Factoring involving the distributive principle.
Solving equations which involve the distributive principle.
Verbalizing the principles used in solving equations.

*ALGEBRA III: NON-FRACTIONAL EQUATIONS II

Note: All equations solved in this unit are non-fractional, first-degree, single-variable equations whose roots are integers.

Intuitive solution of more complicated equations.
Oppositing principle for equations.
Identity principle of multiplication.
Solving equations by means of the oppositing, identity, and distributive principles.
Solving equations which have one side equal to "0".
Solving equations which have the unknown variable on both sides.
The opposite of an addition or a subtraction.
Solving equations which contain expressions in these forms: $-(a + b)$, $-a(b + c)$, $-(a - b)$, and $-a(b - c)$.
Verbalizing the principles used in solving equations.

*ALGEBRA IV: NUMERICAL AND LITERAL FRACTIONS

Meaning of fractions.
Multiplication of fractions.
Factoring fractions.
Reducing fractions to lowest terms by means of the principles

$\frac{n}{n} = 1$ and $(n)(1) = n$.

Concept of reciprocals.
Definition of division in terms of multiplication.
Division of signed numbers.
Addition and subtraction of fractions.
Equivalent forms of fractional expressions.

*ALGEBRA V: FRACTIONAL ROOTS AND FRACTIONAL EQUATIONS

Note: All equations solved in this unit are non-fractional and fractional, first-degree, single-variable equations whose roots are fractions.

Multiplication axiom for equations.
Solving non-fractional equations using the multiplication axiom and other principles.
Solving fractional equations containing a single fraction.
Solving fractional equations containing two or more fractions having different denominators.

*ALGEBRA VI: FORMULA REARRANGEMENT

Note: In all formulas rearranged in this unit, the variable solved for is of first-degree.

Review of basic algebraic principles.
Rearranging non-fractional formulas using only the multiplication axiom.
Rearranging fractional formulas containing one fraction.
Rearranging fractional formulas containing two fractions.
Rearranging formulas containing an instance of the distributive principle.
Rearranging formulas which require factoring by the distributive principle.
Equivalent forms of formulas.

*ALGEBRA VII: SYSTEMS OF LINEAR EQUATIONS AND FORMULA DERIVATION

Meaning of a system of equations and its solution.
Graphical solution of a system of two equations.
Algebraic solution of a system of two equations.
Systems of equations containing decimals.
Systems of equations containing fractions.
Algebraic solution of a system of three equations.
Meaning of a system of two formulas.
Deriving a new formula from a system of two formulas by equating equal quantities.
Deriving a new formula from a system of two formulas by direct substitution.
Deriving a new formula from a system of three formulas.

ALGEBRA VIII: QUADRATIC AND RADICAL EQUATIONS

Meaning of quadratic equations.
Solving quadratic equations by the factoring method.
Standard form of quadratic equations.
Solving quadratic equations by the quadratic formula.
Operations with radicals.
Meaning of radical equations.
Solving radical equations.
Rearranging formulas involving radicals.
Rearranging formulas involving squared variables.
Rearranging formulas which are in quadratic form.
Deriving a new formula from a system of formulas containing radicals.
Deriving a new formula from a system of formulas containing squared variables.

ALGEBRA IX: FURTHER FORMULA DERIVATIONS

Further work with equivalent algebraic forms of complex fractional expressions.
Review of principles of formula derivation.
More complex derivations.

SECTION II: CALCULATION

*CALCULATION I: INTRODUCTION TO SLIDE RULE

Parts of the slide rule.
Reading the C and D scales.
Slide rule multiplication of two numbers lying between 1 and 100.
Slide rule multiplication of three and four numbers lying between 1 and 10.
Slide rule division of two numbers lying between 1 and 100, with dividend greater than divisor.
Placing the decimal point in slide rule products and quotients by rough estimation.

*CALCULATION II: ESTIMATION AND SLIDE RULE OPERATIONS

Estimating products by rough inspection.
Decimal number system, place-value, and decimal fractions.
Expressing numbers in "name-form."
Estimating products by "decimal point shift" method.
Estimating products by "name-form" method.
Estimating products involving both "decimal point shift" and "name-form" methods.
Working more complicated multiplication problems on slide rule.
Estimating quotients by rough inspection.
Estimating quotients by "decimal point shift" method.
Estimating quotients by "name-form" method.
Working more complicated division problems on slide rule.
Use of slide rule in solving simple first-degree equations in one variable.

*CALCULATION III: POWERS OF TEN AND SLIDE RULE OPERATIONS

Meaning of... 10^3, 10^2, 10^1, 10^0, 10^{-1}, 10^{-2}, 10^{-3} ...
Multiplication using powers of ten.
Law of exponents for multiplication.
Division using powers of ten.
Law of exponents for division.
Combined multiplication and division using powers of ten.
Writing general numbers in power of ten form, and vice versa.
Writing general numbers in standard notation using powers of ten, and vice versa.
Expressing a number as the product of a number multiplied by a particular power of ten.
Estimating multiplication problems by powers of ten.
Slide rule problems in multiplication.
Estimating division problems by powers of ten.
Slide rule problems in division.
Slide rule procedures for problems involving combined multiplication and division.
Estimating answers, using powers of ten, to problems involving combined multiplication and division.

*CALCULATION IV: SLIDE RULE POWERS AND ROOTS

Squaring numbers.
Square root of numbers.
Estimating answers to squaring and square root problems by means of powers of ten.
Short method for estimating answers to square root problems.
Cubing numbers.
Cube root of numbers.
Estimating answers to cubing and cube root problems by means of powers of ten.
Short method for estimating answers to cube root problems.
Squaring a product.
Squaring a quotient.
Use of simple table of squares and square roots.

*CALCULATION V: FURTHER SLIDE RULE WORK

Addition and subtraction of decimal numbers.
Percent concepts and calculations.
Review of combined numtiplication and division.
Working more complicated problems in combined multiplication and division.
Computations involving areas of rectangles, squares, parallelograms, and composite areas.
Computations involving volumes and surface areas of boxes and cubes.
Use of slide rule in evaluating the right-hand side of formulas which have a single variable on the left-hand side.
Finding reciprocals on the CI scale.
Multiplication using the CI scale.

SECTION III: GRAPHING

*GRAPHING I: READING AND CONSTRUCTING GRAPHS

Horizontal and vertical number lines.
Rectangular coordinate system.
Reading points on linear and curvilinear graphs (with uniform and non-uniform calibrations) of elementary formulas of science and technology.
Reading points on linear and curvilinear graphs (with uniform and non-uniform calibrations) of equations whose variables are x and y.
Reading the abscissa and ordinate of a point on a graph, and writing them as coordinates of the point.
Defining constants and variables.
Showing that only the coordinates of points on a graph satisfy the equation of the graph.
Constructing graphs of equations whose variables are x and y.
Basic requirements of technical graphs.
Constructing graphs of formulas in two variables.
Constructing graphs of formulas in three or more variables by holding constant all but two variables.
Constructing graphs of families of curves.
Constructing graphs from empirical data.

*GRAPHING II: STRAIGHT LINE AND SLOPE

Reading intercepts of linear and curvilinear graphs.

Concept of slope of a line, using vectors to represent the change in horizontal (Δh) and the change in vertical (Δv).

Computing slope of a line between two points.

Using a known slope value, to determine the change in one variable which accompanies a known change in the second variable.

Slope-intercept form of a line:

$$\boxed{\text{Variable}} = (\text{Constant})\boxed{\text{Variable}} + (\text{Constant})$$

Putting linear equations and formulas in slope-intercept form.

Equations of lines through the origin.

Equations of horizontal and vertical lines.

Slopes of curvilinear graphs: Meaning and signs.

Comparing the relative values of slope for several points on a curvilinear graph.

GRAPHING III: VARIATION IN FORMULAS AND GRAPHS

With all variables except two held constant in a technical formula, to show how the dependent variable increases or decreases as the independent variable increases or decreases, for fractional and non-fractional expressions.

Representation in formula form of these common relationships between two variables:

(a) Direct relation (including ratio and proportion).
(b) Direct square relation.
(c) Inverse relation.
(d) Inverse square relation.

Graphical representation of these four relationships.

Reducing formulas to one of these four relationships by considering all except two variables as constants.

Without graphing, to recognize the presence of these relationships in formulas.

Considering formulas that do not represent one of these four relationships.

Emphasizing that there is no formula associated with many graphs of empirical data.

SECTION IV: TRIGONOMETRY AND APPLIED GEOMETRY

*INTRODUCTION TO TRIGONOMETRY

Properties of a general triangle, emphasizing the angle-sum relation.

Properties of a right triangle, emphasizing the Pythagorean Theorem.

Computing side lengths of right triangles by means of the Pythagorean Theorem.

Definition of the sine ratio, cosine ratio, and tangent ratio of an acute angle.

Table of numerical values of the sine, cosine, and tangent ratios for acute angles.

Calculating sides and angles of right triangles by means of the sine, cosine, and tangent ratios.

Solving technical problems by means of the sine, cosine, and tangent ratios.

VECTORS AND TRIGONOMETRY

Horizontal, vertical, and slant vectors on the coordinate system.
Components of a vector.
Reference angle of a vector.
Designation of a vector by specifying its length and reference angle.
Definition of the sine, cosine, and tangent of reference angles in all four quadrants.
Calculating the length of a vector and the reference angle of a vector.
Finding the resultant of two vectors.
Finding the resultant of more than two vectors.
Condition of equilibrium in a system of vectors.
Finding the equilibrant of a system of vectors.
Setting up and solving a system of equations whose constants are trigonometric ratios in order to find the length and direction of vectors.

GENERAL ANGLES AND TRIGONOMETRIC GRAPHING

Standard position of angles on the coordinate system.
Relationship between reference angles and angles in standard position.
Definitions of the sine, cosine, and tangent ratios of the following angles:
(a) Positive angles in Quadrants I, II, III, IV.
(b) Negative angles in Quadrants I, II, III, IV.
(c) Positive and negative angles whose terminal sides are on the boundaries of quadrants.
Numerical values of the sine, cosine, and tangent ratios of angles of any size.
Graph of $y = \sin \theta$ between $0°$ and $360°$.
Graph of $y = \cos \theta$ between $0°$ and $360°$.
Graph of $y = \tan \theta$ between $0°$ and $360°$.
Finding the sine, cosine, and tangent ratios on the slide rule.

OBLIQUE TRIANGLES I

Definition of oblique triangles.
Solving an acute oblique triangle by resolving it into right triangles.
Law of sines, and its use in solving acute oblique triangles.
Law of cosines, and its use in solving acute oblique triangles.
Applied problems involving acute oblique triangles.

OBLIQUE TRIANGLES II

Solving an obtuse oblique triangle by resolving it into right triangles.
Solving obtuse oblique triangles by the law of sines.
Solving obtuse oblique triangles by the law of cosines.
Applied problems involving obtuse oblique triangles.
Finding the resultant of two vectors by oblique triangle methods.
Further work with two vectors involving oblique triangle methods.

APPLIED PROBLEMS IN GEOMETRY AND TRIGONOMETRY

Basic circle properties and related problems.
Circle central angles and related problems.
Circle circumference and related problems.
Circle area and related problems.
Triangle area and related problems.
Circle tangents and half-tangents and related problems.
Volumes of prisms and non-prisms.
Density and weight and related problems.
Additional applied problems of greater complexity.

COMPLEX NUMBERS AND VECTORS

Square root of negative numbers.
The general complex number $a + bj$, where $j = \sqrt{-1}$.
Addition and subtraction of complex numbers.
Multiplication and division of complex numbers.
Graphing complex numbers as vectors.
Expressing vectors as complex numbers.
Polar form of a vector.
Using complex numbers to add and subtract vectors.
Using complex numbers to multiply and divide vectors.
Simplifying complicated vector problems by complex number operations.

SINE WAVES I: FUNDAMENTALS

Review of the graph of $y = \sin \theta$.
Expanding the graph of $y = \sin \theta$ to include negative angles and angles greater than 360°.
Determining sine wave amplitude from a graph.
Determining sine wave amplitude from equations of the form $y = A \sin \theta$.
Shifting a sine wave horizontally to the right and to the left.
Fundamental sine wave equation: $y = A \sin(\theta \pm \gamma)$.
Sine wave harmonics: Equations and graphs.
Amplitude and phase of sine wave harmonics.

SINE WAVES II: RESULTANTS

Review of fundamental sine waves.
Graphing the resultant of two fundamental sine waves.
Equations of the resultant of two sine waves.
Showing that the resultant of two fundamental sine waves is also a sine wave.
Review of sine wave harmonics.
Graphing the resultant of a fundamental sine wave and a second harmonic.
Graphing the resultant of a fundamental sine wave and two or more harmonics.
Equations of the resultant of a fundamental sine wave and its harmonics.
Resultants which are square waves, sawtooth waves, triangle waves, pulse waves, and half waves.
Lissajous patterns as a special type of resultant.
Parametric equations and Lissajous patterns.

FURTHER TOPICS IN TRIGONOMETRY

Definitions of cosecant, secant, and cotangent of an angle.
Basic trigonometric identities.
Inverse trigonometric notation.
Degree and radian systems of angle measurement.
Angular velocity and circular velocity.

INTERPOLATION IN MATH TABLES

Concept of linear interpolation.
Linear interpolation in trigonometric tables.
Solving trigonometric problems requiring linear interpolation.
Linear interpolation in logarithmic tables.
Logarithmic calculation of powers and roots requiring interpolation.
Linear interpolation in exponential tables.

SECTION V: LOGARITHMS AND EXPONENTIALS

*LOGARITHMS I: INTRODUCTION TO LOGARITHMS

Review of the laws of exponents.
Meaning of fractional and decimal exponents.
Table of decimal exponents for base 10 (logarithm table).
Using the table to write any positive number N in the form 10^a.
Using the table to find the number represented by 10^a.
Using the table to perform the following operations on positive numbers: Multiplication, division, combined multiplication and division, raising to powers, and extracting roots.
Use of logarithmic notation.
Calculations by means of logarithmic notation and the logarithm table.

LOGARITHMS II: LAWS AND FORMULAS

Review of the concepts of logarithms and logarithmic notation.
Laws of logarithms.
Use of laws of logarithms in calculations involving multiplication and division.
Use of laws of logarithms in calculations involving powers and roots.
Further work with negative logarithms.
Rearranging and evaluating exponential formulas.
Rearranging and evaluating logarithmic formulas.

LOGARITHMS III: BASE "e" EXPONENTIALS AND LOGARITHMS

Plotting logarithmic and exponential graphs.
Solving exponential equations.
Base "e" exponential expressions.
Use of the table of e^x and e^{-x}.
Evaluation of formulas containing "e".
Rearrangement of technical formulas containing "e".
Natural logarithms (base "e").
Construction of logarithmic scales.
Finding base 10 logarithms on the slide rule.

SECTION VI: MEASUREMENT

BASIC MEASUREMENT CONCEPTS

Nature of measurement.
Upper and lower bounds of reported measurements.
Estimation in reading measurement scales.
Meaning of precision.
Addition and subtraction of measurements.
Meaning of accuracy.
Definition of relative error.
Accuracy reported exactly as relative error.
Defining significant digits in a measurement.
Accuracy reported crudely in terms of significant digits.
Multiplication and division of measurements.
Error and tolerance in measurement.

MEASUREMENT SYSTEMS AND CONVERSIONS

Note: All conversions are based on the algebraic principles
$\frac{n}{n} = 1$ and $(n)(1) = n$.

English system of measuring units.
Conversion of units within the English system.
Metric system of measuring units.
Conversion of units within the metric system.
English-metric conversion of basic units, and vice versa.
Introduction to derived units.
Conversions involving derived units.

FORMULA EVALUATION

Review of basic calculation operations.
Calculations involving more than one operation.
Evaluation of formulas by two techniques:
(a) Evaluation after unknown variable is solved for.
(b) Evaluation before unknown variable is solved for.
Discrimination as to when to use technique (a) or technique (b).
Evaluations involving conversions.

APPENDIX D

UNIQUE ASPECTS OF THE COURSE CONTENT FOR TECHNICAL MATHEMATICS I AND II

I. Algebra

(1) Besides basic skills in solving traditional equations, heavy emphasis is given to the manipulation and derivation of formulas. This emphasis requires a heavy emphasis on the meaning and manipulation of algebraic fractions.

(2) Evan the traditional equations in one variable are not limited to the usual letters "x" and "y".

(3) Except for factoring by the distributive principle, factoring is deemphasized.

(4) Multiplication and division of polynomials and higher-degree equations are not included.

II. Calculation

(1) Slide-rule exercises are designed to systematically cover the full range of possible numbers on each scale.

(2) The treatment of estimation techniques is very detailed.

(3) Initial emphasis is given to estimation techniques which do not involve powers of ten: Gross inspection, decimal-point shifts, and the use of name-form.

(4) A detailed treatment is given to the number system, powers of ten, and standard notation.

(5) Solutions of first-degree equations and formula evaluations which require use of the slide rule are briefly introduced.

III. Graphing

(1) All concepts are generalized beyond the traditional "xy" system.

(2) Linear and curvilinear relationships are introduced simultaneously.

(3) Heavy emphasis is given to the concept of slope and its use in determining changes in the values of variables.

(4) In defining slope, special emphasis is given to representing Δh and Δv graphically as vectors.

(5) Empirical graphs and their construction are introduced. These graphs of empirical data are related to no known equation or formula.

(6) The concept of variation, an analysis of the relationship between two variables, is considered both from an algebraic and graphical point of view.

(7) The formal properties of conics are not treated.

IV. Trigonometry and Applied Geometry

(1) Only the sine, cosine and tangent of angles are initially defined. The definitions of cosecant, secant, and cotangent are not only delayed but deemphasized.

(2) Instead of logarithms, the slide rule is consistently used in calculating unknown sides and angles of triangles.

(3) The initial treatment of vectors and their addition is based on reference angles only.

(4) A state of equilibrium for a system of vectors is defined, and related solutions involving systems of equations whose constants are trigonometric ratios are introduced.

(5) The components of standard position angles (and their accompanying reference angles) are defined as vectors.

(6) Obtuse oblique triangles are not introduced until the Law of Sines and the Law of Cosines are fully explored in the context of acute oblique triangles.

(7) The basic properties of circles and half-tangents are reviewed in the context of applied problems in trigonometry.

(8) For the electronics technicians, a more detailed and relevant treatment of sine-wave graphs and equations and sine-wave resultants is included.

(9) Complicated trigonometric identities are not treated.

(10) The treatment of interpolation in trigonometric tables is delayed until mastery of trigonometric principles and procedures has been attained.

V. Logarithms and Exponentials

(1) The meaning and laws of logarithms are based on a complete treatment of powers of ten and the laws of exponents.

(2) Except for the evaluation of decimal roots and powers, calculations by means of logarithms are not emphasized.

(3) Evaluating and manipulating logarithmic and exponential formulas is heavily emphasized.

(4) The treatment of interpolation in logarithmic tables is delayed until mastery of logarithmic principles and procedures has been attained.

(5) Considerable emphasis is given to exponential expressions involving base "e".

VI. Measurement

(1) Measurement concepts, measurement systems, conversions within and between systems, and formula evaluation are treated in detail. Ordinarily, these topics are covered quite superficially, if at all, in a mathematics course.

(2) Accuracy is defined as relative error, and the relationship of relative error (a continuous scale) to significant digits (a discrete scale and a cruder measure) is explicitly shown.

(3) All conversions within and between measurement systems are based on two algebraic principles:

$$\frac{n}{n} = 1 \quad \text{and} \quad n(1) = n$$

(4) Two strategies for formula evaluation are taught, and discrimination training is given concerning the situations in which one or the other is more efficient.