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THE DEVELOPMENT OF SINGLE CONCEPT AUDIO-VISUAL DEVICES FOR TEACHING ARITHMETIC CONCEPTS TO LOW ACHIEVING HIGH SCHOOL STUDENTS. FINAL REPORT.
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The main objective of this project was to develop effective multi-sensory aids for individual use by high school students who have not achieved a functional mastery of mathematics. Nineteen audiovisual devices directed specifically at the low achiever were developed from three specific devices--audio tape, audio tape with slides, and video tape. Two types of exercises were developed using audio tape. One type is an exercise designed to increase the low achiever's speed with basic facts and skills, and the other is a story problem exercise. Thirteen audio tape and slide programs were developed from materials utilizing fundamental concepts in mathematics. Finally, four video tape programs were developed for fractions, story problems, area, and volume. Responses to follow-up sheets administered to participating students have shown that these audiovisual materials represent an advance in the individualizing of instruction for low achievers among high school students. (RP)

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June 1967

U. S. DEPARTMENT OF
HEALTH, EDUCATION, AND WELFARE

Office of Education
Bureau of Research

The Development of Single Concept Audio-Visual Devices for Teaching Arithmetic Concepts to Low Achieving High School Students

Project No. 6-8510
Contract No. OEC-3-7-068510-0185

John D. Bingham

June 1967

Livonia Public Schools

Livonia, Michigan 48154

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INTRODUCTION

The main objective of this project was to develop effective multi-sensory aids for individual use by high school students who have not achieved a functional mastery of mathematics.

These students encounter a great variety of difficulties in their attempts to become competent in skills which are usually mastered by the average student by the end of the seventh grade. (U. S. Office of Education & NCTM, 1964.) They tend to be low ability, poorly motivated, or disadvantaged students. Many of them come from socially deprived homes from which they receive little motivation to study any academic subject (Schel, 1959). Their background and rates of growth are so varied that the setting of realistic goals must be done individually (Hilgard, 1956). This implies that at any given time in a class of fifteen, several students will be studying different topics (Mehl, 1960). Typically, short attention spans further complicate the problem of teaching these students (U. S. Office of Education & NCTM, 1964, p 2). The need for a large variety of single concept presentations which lend themselves to use by a few students at a time is critical. And, obviously, these presentations are useless unless they are clearly understood by the students.

Interest in individualizing instruction in mathematics has been high in recent years and good programmed materials and self-teaching tests are becoming available. However, it is generally necessary for publishers and authors to produce materials for mass consumption and since these students represent an often neglected minority, very little material has been created to deal with their learning problems. The text <u>Trouble Shooting Mathematics Skills</u> by Bernstein and Wells, published by Holt, Rinehart and Winston in 1963, is a notable exception. Most others simply amount to presenting sixth grade materials to tenth grade students who have been convinced that they are destined to remain fourth graders in their understanding of arithmetic. Even the Bernstein-Wells book does not deal with the motivational problems experienced by this student.

Teachers have not been successful in using existing materials with these students and the increase in occupational opportunities related to these skills is being highly publicized. Finding ways of solving the dilemma becomes increasingly critical. Shortages of technically skilled persons and efforts being made in anti-poverty programs are involved (U. S. Office of Education & NCTM, 1964, p 11). Among these students, those who are under-achievers must realize their capabilities, even though for some arithmetic reasoning will always be difficult. They must at least reach a skill level which will permit them to function in our modern society.

The basic problem is that of producing a variety of suitable materials which can be used by these students individually with the proper assistance of a skilled instructor, and then providing the proper atmosphere for their use (U. S. Office of Education & NCTM, 1964, p 9). Removing the reliance on reading as a prerequisite to success in arithmetic is also a part of this problem.

A survey of the literature shows that little is known about teaching arithmetic to low achievers. The following quotations from the "Preliminary Report of the Conference on Low Achievers in Mathematics,"[1] summarize the situation very well. When one notes the large number of leaders in mathematics and mathematics education attending that conference, the quotes take on added significance.

> The school administrator must encourage research and experimentation with new and different teaching techniques for low achievers in mathematics. Very little is known about methods which are most effective with low achievers. Very few, if any, follow-up studies have been made of pupils in this category. How do they learn best? What is an appropriate curriculum for them? What mathematical competencies do they need? What are the characteristics of teachers who work most effectively with them? The answers to these and similar questions may be found by a local mathematics staff.
>
> The learning materials should provide for many uses of objects, models, audio-visual aids, and manipulation devices, as well as the use of more complicated instruments and learning aids.
>
> Different ways of looking at the same mathematical concept may reinforce the idea or provide the insight needed. Every effort should be made to capitalize on the interest and motivation of the learner through the use of games, puzzles, short cuts, and discovery exercises that arouse curiosity and imagination.
>
> Competent teachers should be paid or given released time to write classroom materials and devise techniques that can be used to develop the learning potential of low achievers.

With the obvious need for materials directed specifically at the low achiever, this report portrays an attempt to supply three types. It includes the methods used in the development of the materials, the results recorded when the materials were tried in a classroom situation, and recommendations for the future use of the materials.

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[1] Conference on Low Achievers in Mathematics held in Washington, D.C., March, 1964.

METHOD

The project emphasized three specific devices:

1. Audio tape alone
2. Audio tape with slides
3. Video tape.

During the contract period, the director was released three hours per day to develop the materials accompanying this report. A teaching assignment of two refresher mathematics classes assured some immediate feedback regarding the success of the materials with the individual student.

The materials were developed and tried out at:

Franklin High School
31000 Joy Road
Livonia, Michigan 48150

RESULTS

In the audio tape area, two types of exercises were developed. One is concerned with speed drills of the four arithmetic operations with 100 basic facts. The other is a story problem exercise.

Thirteen programs were developed in the audio tape and slide area. The titles are self explanatory and are as follows:

0 Addition of Fractions
1 Subtraction of Fractions
2 Multiplication of Fractions
3 Division of Fractions
4 Improper Fractions and Mixed Numbers
5 Multiplication of Decimals
6 Division of Decimals
7 Ratio
8 Proportion
9 Ratio and Proportion
10 Story Problems and Proportion
11 Story Problems and Percent
12 Percent

Finally, four programs were developed on video tape. They dealt with the following topics:

1. $\frac{1}{2}$'s and $\frac{1}{4}$'s
2. Story problems
3. Area
4. Volume

DISCUSSION

The first type of audio tape exercise was simply to increase the listener's speed with the basic facts of addition, subtraction, multiplication, and division. A student first listens to 100 basic facts spaced five seconds apart on the tape, writing the answers on a prepared answer sheet. He then listens to the same basic facts spaced three seconds apart; again writing the answers on a prepared answer sheet. The basic objective in this exercise is to do as well on the three second tape as was done on the five second tape. This type of exercise is particularly good at the beginning of the school year in September because (1) the low achiever often has not done much thinking in terms of mathematics over the summer, and (2) his thoughts often return to his activities during the summer unless directed along the lines of some planned activity such as this.

The second type of audio tape exercise was developed to parallel the text used by refresher classes. The text used is Bernstein and Wells: Trouble Shooting Mathematics Skills. Permission was granted by Holt, Rinehart, and Winston to use the text in such a manner.

This exercise uses story problems from chapter eight of the aforementioned text. On the bulletin board just above the tape recorders in the classroom are located several lists showing the location of any particular story problem on a particular tape. (See Appendix "B".) For example, a student might be having trouble with problem eight of problem set two. That student can go to the bulletin board, look at the list of problems for set two, and find the location of problem eight on the tape for set two. He then would put the tape on a recorder and wind it to the correct position for problem eight. By listening to problem eight, the student hears a discussion of the problem and what is expected in order to solve it. He can then attempt to work the problem while listening to the tape. A student is encouraged to seek further help from the teacher if he does not understand what is said on the tape.

It is a known fact that inactivity often encourages a low achiever to "give up" on a problem if he does not understand it. Often a teacher finds it necessary to give help to a low achiever. But not all low achievers have trouble with the same problem. This story problem exercise is one example where individualized instruction can fill the void created when the teacher is the only source of help in the classroom. If the teacher is already helping another student, the tape recorder can be a source of help to others.

Each of the thirteen slide programs is followed by a sheet of five to ten problems which reiterate the concept found in it. Copies of the audio script, as well as the follow-up sheets, can be found at the end of this report and are therefore incorporated as part of it. (Appendix "C").

In all of the video programs, manipulative materials which the viewer could see were stressed. The program on $\frac{1}{2}$'s and $\frac{1}{4}$'s revolved around the 12 inch ruler and segments of 2 x 4 which displayed the concept of how the $\frac{1}{2}$ and $\frac{1}{4}$ receive their names.

The story problem program had cans of soup, gallon jugs of water, and dinner plates to represent the objects involved in each problem. Each problem was concluded by drawing small sketches of the physical objects the students had just seen. The idea here was to encourage the student to get some type of diagram on paper and not try to analyze the problem completely in his head.

The area program emphasized the meaning behind the formula: A = l x w. It included units of square inches and square feet made from poster board. It concluded with the idea that length tells one how many square units can be placed along a horizontal edge, in one row; width tells one how many rows there are.

The volume program was closely associated with the one on area. Again, the meaning behind the formula V = l x w x h was emphasized. Cubic inch units made from wood were used in the discussion of the volume of three cardboard boxes. Considering the "l x w" portion of the formula, it was shown how this tells one how many cubic units are in the first layer. The height is needed to tell one how many layers there are. Outlines and follow-up sheets of the four video tape programs can also be found at the end of this report. (Appendix "D".)

Audio tape was selected because it is one of the most popular as well as inexpensive audio aids found in most school systems. The tape recorders rented during the course of this project can be purchased for about $140 each. Tape, in quantity, runs about $2.00 for 1200 feet. When compared to the other two areas, audio tape is seen to be the most inexpensive.

Besides the audio tape expenses, the slide and audio tape devices have the additional cost of producing the slides. A 135 m.m. slide film of 36 exposures will run about $3.00. The cost of developing each slide is ten cents. A cartridge for storing the completed slide program costs $3.00. For additional information regarding the costs of this device, see Appendix "A". One advantage this method has over the audio tape is that it appeals to both the seeing and hearing senses of the viewer. Not only is the viewer hearing the concept, he also sees it on the screen. Another advantage it has over filmstrips is that the program can be varied to suit each teacher's ideas on how a concept should be presented. Certain slides may be omitted or others inserted to make a more meaningful program.

The video tape has the specific advantage of a more continuous audio and visual program than do the audio tape and slide programs. Again, it appeals to both sight and hearing. In a slide program, there are split seconds during the changing of a slide when nothing appears on the screen. On a video tape program, such split seconds are at a minimum. That is, once a video program has begun, there is usually always something on the screen in front of the viewer. Video tape is often prohibitive to most school systems because of its great expense. Besides the huge initial investment necessary to provide the studio and recording equipment, the video tape itself costs about $60.00 per 60 minute reel or approximately $1.00 a minute.

Livonia Public Schools had the video tape program available before this project received approval. It was for this reason that this director decided to include this device as one to be studied in relation to low achievers.

The ultimate goal of the director was to have four sources of help in the refresher class at the same time. These were: (1) the audio tape exercises; (2) the slide programs; (3) the video tapes; and (4) the teacher.

All four activities are performed in a classroom where individualized instruction is encouraged. In fact, the course has been structured around individualized instruction. The students have access to the four sources of help when they have need of one of them.

Anytime the teacher presents a general topic such as finding common denominators, he does this at the beginning of the period and usually for not longer than 15-20 minutes any given day.

The work sheets which follow the use of the video tape or slides have shown that these materials have accomplished their purpose--that is, the teaching of the various single concepts previously mentioned in this report. Nearly all of the scores on the worksheet were 100%'s. Two students received 90%'s and needed additional explanation of the particular problem missed before they could do it.

Below is a chart which shows the usage of materials during the week of May first to the fifth.

USE OF SINGLE CONCEPT MATERIALS

MONDAY	TUESDAY	WEDNESDAY	THURSDAY	FRIDAY	
1 student	3 students	0 students	1 student	2 students	Audio Tape
2 students	2 students	1 student	3 students	2 students	Slides-Tape
0 students	0 students	5 students	6 students	6 students	Video Tape

Friday best exemplifies the goal mentioned before. While these ten students were involved with the materials, the teacher was able to help others. Still others were working on assignments in the text used with refresher mathematics. Thus, five groups of students were functioning in a room where many teachers still feel that lecturing to the entire group on the same topic is the way to teach low achievers.

One hears talk about ability grouping in a class. And it is a lot easier to talk about it than to put it in practice--especially in a group whose average percentile rank is ten or lower on the Lee Clark Arithmetic Fundamentals Survey Test. As it turned out, there were several groups containing only one person. Yet, proceeding at his own rate, covering topics when he was ready for them, this person showed much growth in a year's time. (See Appendix "E").

The follow-up sheets for the single concept exercises, and the number of times each was used in the classroom, have shown that these materials represent an advance in the individualizing of instruction for low achievers assigned to these sections of refresher math. A more thorough evaluation, using various testing devices, is recommended.

CONCLUSIONS, IMPLICATIONS AND RECOMMENDATIONS

The Project was well accepted by the students. Very few realized that the director of The Project was actually their teacher. In fact, very few suspected there was anything different in refresher mathematics this year than there was last year or the year before that.

There was a definite change noticed by the director. This change was in the general morale of the students. A class such as this needs variety. Attention spans are short; variety helps maintain interest. If a sincere interest exists, attention does not wander. If attention does not wander, isn't the student more likely to absorb the subject being considered? This is one question the director has had answered during the course of this project. Comments like "This is my best class," and "Boy! This hour goes fast," were made in a class which usually lacks such statements.

Other facts were very emphatically driven home by this director's opportunity to work on this project. These facts will be presented here as the director saw them at Franklin High School. The regular assignment for a teacher is five classes with one period for consultation. Among these five classes are usually at least two classes of students having higher ability than those found in refresher mathematics. These two classes serve to balance the attitude so often found in refresher mathematics or similar courses. This director attempts to portray that attitude now, and later he will suggest some solutions which might remedy this attitude.

Unlike the higher ability courses where much of the feedback from the students is positive, the refresher mathematics type course often has prevalent an "I couldn't care less" atmosphere. The positive feedback might be exemplified by a student's show of interest for the subject material, by the kinds of questions that are asked, or by many other reactions known to those who have taught the higher ability student. There is a definite lack of such reactions in the lower ability classes.

This director was teaching only refresher level students. These are students achieving ten percentile or lower on the Lee Clark Fundamental Test. The director found that materials produced under this project helped create an interest on the part of the two classes of students in involved. Yet, having no higher level students to balance the lack of much positive reaction undoubtedly had an effect on this director's morale. He feels this may apply to anyone who has had an assignment of all lower level classes.

Teachers are so often loaded down with duties such as correcting papers, attending meetings, and other related activities that they have no time to prepare good lessons for their classes. It is one thing to know the subject matter thoroughly and quite another to present it to a class. To present it to a class in an interesting manner, time must be spent in thinking out a plan that will best suit the class involved. But, for lack of time, the amount of time spent along these lines is often a minimum. What classes often get the short end of this deal? Low ability classes do !!!!

Teachers of advanced placement classes in a few systems are sometimes given an hour of released time in addition to their consultation periods in which they prepare better lessons for the higher ability students. Might it not be just as important to give teachers of the lower ability students the same amount of time in which to prepare more interesting lessons for the refresher mathematics level student? A limit of two or three low ability level classes to an individual teacher should be enforced by a school system to ensure some positive feedback to that teacher from his other classes.

SUMMARY

There are not enough supplemental materials to be used in classes for the low achiever in mathematics. It has been the purpose of this project to investigate the development of such materials. Nineteen audio-visual devices directed specifically at the low achiever were developed under the auspices of this contract.

Two types of exercises were developed using audio tape. One type is simply an exercise designed to increase the low achiever's speed with basic facts skills. The other is a story problem exercise.

Thirteen audio tape and slide programs were developed. They are titled as follows:

0 Addition of Fractions
1 Subtraction of Fractions
2 Multiplication of Fractions
3 Division of Fractions
4 Improper Fractions and Mixed Numbers
5 Multiplication of Decimals
6 Division of Decimals
7 Ratio
8 Proportion
9 Ratio and Proportion
10 Story Problems and Proportion
11 Story Problems and Percent
12 Percent

Four video tape programs were developed. They dealt with:

1. $\frac{1}{2}$'s and $\frac{1}{4}$'s
2. Story problems
3. Area
4. Volume

As each exercise was completed, it was introduced into the classroom for use by low achieving mathematics students. This was made possible by the assignment of two such mathematics classes to the director during the course of the project. Followup sheets were administered to the participating students as they used the various exercises.

These follow-up sheets for the single concept exercises, and the number of times each was used in the classroom have shown that these materials represent an advance in the individualizing of instruction for low achievers assigned to these sections ot refresher math. A more thorough evaluation, using various testing devices, is recommended.

REFERENCES

Allendoerfer, Carl B. "The Second Revolution in Mathematics," Mathematics Teacher, December, 1965.

Hilgard, Ernest R. Theories of Learning, Appleton Century Crofts, Inc., N.Y. 1956, pp 273-88, quotes "Experiment by fears shows that realistic goal setting is characteristic of those who had experienced success."

Mehl, William S. "Providing for the Basic Student in the Junior High School," Mathematics Teacher, May, 1960.

"Preliminary Report of the Conference on the Low Achiever in Mathematics," U.S. Office of Education and National Council of Teachers of Mathematics, March, 1964. (Mimeographed)

Schel, Max. "Providing for the Slow Learner in the Junior High School," Mathematics Teacher, May, 1959, p 347.

APPENDIXES

REPRODUCTION COSTS OF SLIDE PROGRAM

		No. of Slides	Slides	Audio Tape	Total
Program 0	Addition of Fractions	23	4.60	.50	5.10
Program 1	Subtraction of Fractions	33	6.60	.50	7.10
Program 2	Multiplication of Fractions	36	7.20	.50	7.70
Program 3	Division of Fractions	33	6.60	.50	7.10
Program 4	Improper Fractions and Mixed Numbers	37	7.40	.50	7.90
Program 5	Multiplication of Decimals	22	4.40	.50	4.90
Program 6	Division of Decimals	28	5.60	.50	6.10
Program 7	Ratio	16	3.20	.50	3.70
Program 8	Proportion	21	4.20	.50	4.70
Program 9	Ratio and Proportion	17	3.40	.50	3.90
Program 10	Story Problems and Proportion	31	6.20	.50	6.70
Program 11	Percent in Ratio Form	28	5.60	.50	6.10
Program 12	Ratio and Percent	21	4.20	.50	4.70
					$75.70

The figures for the reproduction of the slides are based on 20¢ per slide in amounts of 100 or more. Otherwise, the cost for reproducing each slide is 25¢.

A cartridge of the type I have been using to store the slides while not in use costs $3.00. At least two of the programs will fit in one cartridge, possibly three, depending on the number of slides in the programs.

The actual cost of the audio tape will vary but the maximum amount should not be greater than the 50¢ shown for each program.

STORY PROBLEMS

Chapter 8

	Set 1, Pages 137-138 Position on Tape
Problem 1	000
Problem 2	016
Problem 3	036
Problem 4	082
Problem 5	121
Problem 6	145
Problem 7	170
Problem 8	204
Problem 9	223
Problem 10	246

	Set 2, Pages 140-141 Position on Tape
Problem 1	000
Problem 2	024
Problem 3	058
Problem 4	081
Problem 5	100
Problem 6	125
Problem 7	157
Problem 8	186
Problem 9	205
Problem 10	271

	Set 3, Pages 142-143 Position on Tape
Problem 1	000
Problem 2	073
Problem 3	092
Problem 4	127
Problem 5	192
Problem 6	225
Problem 7	248
Problem 8	272
Problem 9	298
Problem 10	350

	Set 4, Pages 144-145 Position on Tape
Problem 1	000
Problem 2	027
Problem 3	056
Problem 4	071
Problem 5	100
Problem 6	122
Problem 7	161
Problem 8	190
Problem 9	230
Problem 10	251

	Set 5, Pages 146-147 Position on Tape
Problem 1	000
Problem 2	034
Problem 3	053
Problem 4	084
Problem 5	099
Problem 6	137
Problem 7	165
Problem 8	197
Problem 9	218
Problem 10	240

ADDITION OF FRACTIONS

The purpose of this tape is to help you add numbers involving fractions. Every time you hear this sound ______ you should press the control button to see the next slide. You should now have slide 1 in front of you. If you do not, push the control button until it appears.

Notice that on slide 1, you are asked to add

$$\begin{array}{r} 3\ 2/5 \\ +\ \underline{1\ 4/5} \end{array}$$

Since you already have a common denominator of 5 in the fractions, all you have to do is add the numerators and put the answer over 5.

(2)
BEEP $\frac{2}{5} + \frac{4}{5} = \frac{2 + 4}{5} = \frac{6}{5}$

(3)
BEEP Same as number 1. Then add the whole numbers.

(4)
BEEP $3 + 1 = 4$ Then our original problem with its answer looks like this;

(5)
BEEP

$$\begin{array}{r} 3\ 2/5 \\ +\ \underline{1\ 4/5} \\ 4\ 6/5 \end{array}$$

We are not quite finished as $\frac{6}{5}$ is an improper fraction and must be reduced.

(6)
BEEP $\frac{6}{5} = \frac{5}{5} + \frac{1}{5} = 1\ 1/5$

(7)
BEEP So we now add 1 1/5 to the 4 we already had in our answer and we get 5 1/5 as the final answer to our problem.

(8)
BEEP Here is another problem.

$$\begin{array}{r} 5\ 1/8 \\ +\ \underline{2\ 3/16} \end{array}$$

(9)
(Beep) Notice it is quite simple to add the whole numbers.

$$\begin{array}{r} 5 \\ +\ \underline{2} \\ 7 \end{array}$$

(10)
BEEP Before we can add the 1/8 and 3/16, we must find a common denominator. Can we change 1/8 into 16ths?

(10) cont.

BEEP $\frac{1}{8} = \frac{?}{16}$

(11)

BEEP Ask yourself the question? $8 \times ? = 16$

(12)

BEEP $8 \times 2 = 16$

(13)

BEEP Here was our problem: $\frac{1}{8} = \frac{?}{16}$ (how many)

(14)

BEEP We found that $8 \times 2 = 16$. Since we must multiply the bottom by 2 to get 16, multiply the top by 2. What do you get?

(15)

BEEP 1×2 equals 2

so we get $\frac{1 \times 2}{8 \times 2} = \frac{2}{16}$ This means that $\frac{1}{8}$ equals $\frac{2}{16}$.

(16)

BEEP Our original problem was:

$$\begin{array}{r} 5\ \ 1/8 \\ +\ 2\ \ 3/16 \\ \hline \end{array}$$

Since we found 1/8 equals 2/16, let's put 2/16 in place of 1/8 and our problem looks like this.

(17)

BEEP

$$\begin{array}{r} 5\ \ 2/16 \\ +\ 2\ \ 3/16 \\ \hline \end{array}$$

We are now ready to finish

(18)

BEEP We already had

$$\begin{array}{r} 5\ \ 2/16 \\ +\ 2\ \ 3/16 \\ \hline 7\ \ 5/16 \end{array}$$

So the answer is 7 5/16.

(19)

BEEP Do this problem in the next 60 seconds. Write it on a sheet of paper. You have 60 seconds.

(20)

BEEP 60 seconds -- Your final answer should be 6 1/7.

(21) Try this one in 60 seconds.

BEEP

$$\begin{array}{r} 6\ 1/3 \\ +\ 3\ 1/12 \\ \hline \end{array}$$

Hint: Change both the bottom numbers of the fractions (denominators) to 12.

(22)

BEEP Your final answer should be 9 5/12.

Now ask your teacher for the problem sheet that goes along with this program.

ADDITION OF FRACTIONS

1. $5\frac{2}{7}$ + $3\frac{4}{7}$

$5 + 3 = 8$ Add the whole numbers.

$\frac{2}{7} + \frac{4}{7} = \frac{6}{7}$ Note that you add only the numerators.

Your answer should be $8\frac{6}{7}$

2. $3\frac{1}{4}$ + $6\frac{3}{16}$

Note that the denominators of the fractions are not the same. In this case, you must change $\frac{1}{4}$ to $\frac{4}{16}$ before you can solve this problem. This is how you do it:

Steps (1)

$4 \times \underline{?} = 16$

$4 \times \underline{4} = 16$

(2)

$\frac{1 \times 4}{4 \times 4} = \frac{4}{16}$

(3)

If you multiply the bottom number by 4 to get 16, you must also multiply the top number by 4.

So the original problem looks like this:

$3\frac{1}{4} = 3\frac{4}{16}$

$+ 6\frac{3}{16} \quad + 6\frac{3}{16}$

$3 + 6 = 9$

$\frac{4}{16} + \frac{3}{16} = \frac{4 + 3}{16} = \frac{7}{16}$

Your answer should be $9\frac{7}{16}$

Now try the following problems. Reduce all answers. Remember that before you can add factions, they must have the same denominator.

3. $4\frac{2}{3}$ + $1\frac{1}{6}$

4. $5\frac{3}{4}$ + $2\frac{1}{12}$

5. $14\frac{2}{5}$ + $6\frac{7}{10}$

6. $19\frac{1}{8}$ + $4\frac{5}{8}$

7. $3\frac{1}{4}$ + $5\frac{3}{4}$

8. $2\frac{3}{32}$ + $4\frac{1}{16}$

9. $9\frac{1}{2}$ + $3\frac{3}{16}$

10. $4\frac{2}{3}$ + $5\frac{7}{9}$

SUBTRACTION OF FRACTIONS

The purpose of this tape is to enable you to subtract numbers involving fractions. You should have slide #1 in front of you. After I am finished giving these directions and you hear this sound again, you should flip slide #1 over and look at slide #2. After I finish talking about slide #2, you will hear "this sound again" and should turn to slide #3. This will continue until the end of the program. Remember, every time you hear ______ you should flip a card.

Now, getting back to card #1.

(1) Notice the problem here is 12
 - 2-1/5

You are to subtract 2-1/5 from 12. (Beep*) To do this, we must borrow (1) from the 12, which leaves us with 11. Then, we must change the form of the 1 to 5/5 and you see the problem, 11-5/5 which you are to solve. - 2-1/5

(2) 1/5 from 5/5 is 4/5, and 2 from 11 gives us 9, and we see the completed problem of 11-5/5
 - 2-1/5
 9-4/5

(3) Reviewing now 1/5 from 5/5 is 4/5 and 2 from 11 is 9.

(4) Let's look at a similar problem: (Beep*) 12-2/5
 - 3-3/5

Notice in this problem, we're asked to subtract 3-3/5 from 12-2/5. You can't take 3/5 from 2/5, so this means you must borrow one from

(5) the 12, leaving 11. (Beep*) Change this 1 to 5/5 and since we already have 2/5 on top, we must add 2/5 to 5/5 and our problem now looks like this: 11-2/5 + 5/5 = 11-7/5
 - 3-3/5 - 3-3/5

(6) and after solving it, *3/5 from 7/5 is 4/5 and 3 from 11 is 8. We have the problem in its final form of: 11-7/5
 - 3-3/5

 8-4/5

(7) Now, can you do this example: 18-3/8
(Beep) - 5-5/8
Write the problem on a sheet of paper and solve it in 60 seconds.
Time ------------ 60 seconds. Your answer is 12-6/8. You should have

(8) a problem which looks like this: (Beep)
 17-3/8 + 8/8 = 17-11/8
 - 5-5/8 = - 5-5/8

Since 5/8 from 11/8 is 6/8, and 5 from 17 is 12, in final form, with its answer, the problem looks like this:

(9) (Beep)

$$\begin{array}{r} 17\text{-}11/8 \\ -\ \underline{5\text{-}5/8} \\ 12\text{-}6/8 \end{array}$$

Now it's not quite in formal form because the 6/8 can be reduced. What do I mean by reduced? Find a number that will divide both 6 and 8 evenly. Will 2 do this?

(10) 6 divided by 2 is 3. 8 divided by 2 is 4. (Beep) So it looks as though our answer of 12-6/8 == 12-3/4. Now consider only the fractional parts of the two answers on the card. That is 6/8 and 3/4.

(11) (Beep) I say that 6/8 = 3/4. These are called equivalent fractions even though each appears different from the other. You can always check to see if to fractions are equivalent.

(12) (Beep) Consider the form $\frac{X}{Y} = \frac{Z}{W}$

To see if 2 fractions are equivalent, one rule says that the number in the X position multiplied by the number in the W position must

(13) (Beep) give a product that is equal to the product of Y times Z. $\frac{X}{Y} = \frac{Z}{W}$ Let's write the 6/8 = 3/4 right next to the general form.

Is 6 x 4 = 8 x 3? Sure both answers are = 24. This tells us that 6/8 and 3/4 are equivalent fractions.

(14) (Beep) Let's practice on another couple of fractions. Is 2/3 = 4/6? Try this: Is 2 x 6 = to 3/4? Sure. Both products are = 12. This tells us that 2/3 and 4/6 are equivalent fractions. Equivalent

(15) (Beep) fractions lead us into a way of solving this type of problem:

$$\begin{array}{r} 21\text{-}2/3 \\ -\ \underline{11\text{-}5/12} \end{array}$$

Before we can even begin to solve this problem, we must change one or both of the fractions into <u>equivalent</u> fractions both having the <u>same</u> denominator. This denominator is called a common denominator because it must be the same or common to both fractions.

(16) How do we arrive at the common denominator in this example? We must have a number which both <u>3</u> and <u>12</u> will divide an even number of times. (Beep) 12 divides 12 once and 3 divides 12 four times. So, it looks as though 12 is our common denominator.

(17) (Beep) Now 5/12 already has 12 as a denominator so we leave that as it is. Our problem is to change the 2/3 into 12ths.

$$\begin{array}{rcl} 21\text{-}2/3 & = & 21\ \frac{\ }{12} \\ -\ \underline{11\text{-}5/12} & = & \underline{11\text{-}5/12} \end{array}$$

You can ask yourself: 3 times what number is 12. Your answer,

(18) 4. So, if you multiply the 3 by 4 to get 12, you must multiply
(Beep) the 2 x 4, which gives you 8. In other words:

$$\frac{2 \times 4}{3 \times 4} = \frac{8}{12}$$

Remember how to check equivalent fractions. Try it. Is

(19) 2/3 == 8/12?
(Beep) Remember 2 X 12 must = 3 X 8. Is it? Yes, both answers are 24.

Now we have come a long ways from our original problem.

(20) We started with
(Beep)

$$\begin{array}{r} 21\text{-}2/3 \\ -\ \underline{11\text{-}5/12} \end{array}$$

(21) and changed its form to
(Beep)

$$\begin{array}{r} 21\text{-}8/12 \\ -\ \underline{11\text{-}5/12} \end{array}$$

Can you finish the problem now?

(22) Your answer is 10-3/12, which can be reduced to 10-1/4.
(Beep)

(23) Is 3/12 = to 1/4?
(Beep) Is 3 X 4 = 12 X 1?
∴ Your final answer should be 10-1/4

(24) Can you do this problem in the next minute?
(Beep)

$$\begin{array}{r} 8\text{-}1/4 \\ -\ \underline{6\text{-}1/2} \end{array}$$

<u>Time: 60 seconds</u>

Remember, we must choose a common denominator. I say this time it's 4, because 4 divides 4 once and 2 divides 4 two times.
∴ Our problem takes form of:

(25)
(Beep)

$$\begin{array}{r} 8\text{-}1/4 \\ \underline{-\ 6\text{-}1/2} \end{array} = \begin{array}{r} 8\text{-}1/4 \\ -\ \underline{6\text{-}2/4} \end{array}$$

This is because I looked at the fraction $1/2 = \frac{\_\_\_}{4}$ and said that 2 X 2 = 4. Therefore, I must multiply the 1 X 2 also and I came up with 2/4.

(26) That is $\frac{1 \times 2}{2 \times 2} = \frac{2}{4}$
(Beep)

Now, consider the problem from the beginning again --

(27)
(Beep)

$$\begin{array}{r} 8\text{-}1/4 \\ -\ \underline{6\text{-}1/2} \end{array} = \begin{array}{r} 8\text{-}1/4 \\ \underline{-6\text{-}2/4} \end{array}$$

You can't take 2/4 from 1/4, so we must borrow 1 from the 8, leaving 7. Change the 1 to 4/4 and our problem takes the form

(28)

$$\begin{array}{r} 7 \\ \cancel{8}\text{-}1/4 + 4/4 \\ -\ 6\text{-}2/4 \end{array}$$

or

(28) (Beep)

$$\begin{array}{r} 7 \\ \cancel{8}\text{-}5/4 \\ -\ 6\text{-}2/4 \end{array}$$

(29) Then you're finally ready to arrive at the final answer of 1-3/4.
(Beep)

Now, rewind the tape and ask your instructor for the problem sheet which goes along with this tape.

(30) END

SUBTRACTION OF FRACTIONS

1. We can change the form of the number 1 many different ways. Try filling in the blanks of the chart below.

$1 = \frac{1}{1}$	$1 = \frac{4}{4}$	$1 = \frac{\_}{7}$
$1 = \frac{2}{2}$	$1 = \frac{\_}{5}$	$1 = \frac{8}{8}$
$1 = \frac{\_}{3}$	$1 = \frac{\_}{6}$	$1 = \frac{\_}{9}$

You often need to think about changing the form of the number 1 in order to solve certain subtraction problems.

2. Fill in the blanks using the first example below.

a. $14 = 13 + 1 = 13 + \frac{4}{4}$

b. $12 = 11 + \_\_ = \_\_ + \frac{5}{5}$

c. $\_\_ = 7 + 1 = 7 + \frac{\_}{8}$

3. Fill in the blanks using the first example below.

a. $12\frac{2}{7} = 12 + \frac{2}{7} = 11 + 1 + \frac{2}{7} = 11 + \frac{7}{7} + \frac{2}{7} = 11\frac{9}{7}$

b. $4\frac{1}{4} = \_\_ + \frac{1}{4} = 3 + \_\_ + \frac{1}{4} = 3 + \frac{4}{4} + \_\_ = 3\frac{5}{4}$

c. $2\frac{1}{5} = 2 + \_\_ = 1 + 1 + \_\_ = 1 + \_\_ + \frac{1}{5} = \_\_$

4. Now let's try to use some of the ideas above to solve this subtraction problem.

$$\begin{array}{r} 12\frac{2}{7} \\ -\ 3\frac{3}{7} \\ \hline \end{array} = \begin{array}{r} 11\frac{7}{7} + \frac{2}{7} \\ -\ 3\frac{3}{7} \\ \hline \end{array} = \begin{array}{r} 11\frac{9}{7} \\ -\ 3\frac{3}{7} \\ \hline \end{array}$$

If you have $8\frac{6}{7}$ for your answer, you should go on to problem 5.

5. Try this one.

$$\begin{array}{r} 5 \\ -\ 2\frac{1}{4} \\ \hline \end{array}$$

If you have $2\frac{3}{4}$ for your answer, do the next 3 problems. If you did not, get some help before going on.

6. $$\begin{array}{r} 5\frac{1}{8} \\ -\ 1\frac{3}{8} \\ \hline \end{array}$$

7. $$\begin{array}{r} 14\frac{1}{9} \\ -\ 5\frac{4}{9} \\ \hline \end{array}$$

8. $$\begin{array}{r} 3 \\ -\ 1\frac{1}{2} \\ \hline \end{array}$$

MULTIPLICATION OF FRACTIONS

The purpose of this tape is to assist you in multiplying numbers involving fractions. You should have slide #1 in front of you. Throughout this slide program you will hear a series of beeps, each one sounding like this: -- Each time you hear this sound -- that is the signal to press the button to see the next slide.

Let's consider the first problem:

(1)
Beep $5 \times 3/4 =$

Do you think we could write this problem graphically?

(2)
Beep I would like you to consider this rectangle as one whole unit. IMAGINE SEPARATING it into four equal parts. Like this:

(3)
Beep Now can you choose 3 of these parts? That is, 3 of the 4ths. We call them 4ths because we started with 1 whole rectangle and marked off four equal parts and each part is 1 of the 4. Thus, 1/4. Getting back to our original question, I want you to look at 3 of these 4ths. Especially these 3.

(4)
Beep Notice I have shaded 3 of these 4ths. 3 of the 4 equal parts. Now our original problem was

(5)
Beep We have represented the 3/4 graphically by shading 3 of the 4 parts. Now, how do we represent the $5 \times 3/4$?

(6)
Beep If I had 5 rectangles with 3/4 of each one shaded, wouldn't this represent $5 \times 3/4$? Sure it would. Each rectangle has 3 of the 4 equal parts shaded and we have 5 rectangles and therefore $5 \times 3/4$.

Now to solve the problem. That is, come up with an answer. Will you count the number of little shaded rectangles? You should have 15 of them. Each small rectangle is 1/4 of one of the large rectangles. So, actually, this 15 represents $\frac{15}{4}$.

(7)
Beep To solve this problem using arithmetic, you could end up with $\frac{15}{4}$ again by writing it as it is on this slide.

Notice we multiplied only the numerator 3 by the whole number 5. The denominator 4 stays just as it is.

(8)
Beep and $\frac{5 \times 3}{4} = \frac{15}{4}$

(9) Beep Now $\frac{15}{4}$ is an improper fraction because the numerator is larger than the denominator. So can you now change $\frac{15}{4}$ into a mixed number (a whole number and a fraction)? 4 divides 15, 3 times with 3/4 left, so our final answer is 3-3/4.

(10) Beep Now let's go back to our 5 shaded rectangles. Only this time, notice I have numbered the small shaded ones in the bottom rectangle.

(11) Beep Can you imagine moving the small No. 1 rectangle up to the empty space in the 1st rectangle; No. 2 rectangle up to the empty space in the second rectangle; and, finally, do the same with No. 3 rectangle up into the empty space in the third rectangle.

(12) Beep Once you have done this, we have a picture that looks like this. How many whole rectangles do we have? Three! How much of another 3/4? So, all together, we have 3-3/4 rectangles shaded.

(13) Beep Remember our original problem? $5 \times 3/4 = \frac{5}{4} \times 3$ and our answer of $\frac{15}{4}$ which was = 3-3/4.

(14) Beep We have just seen how from our original 5 rectangles we formed 3 whole shaded rectangles and 3/4 of another for a total shaded area of 3-3/4 rectangles.

And what you see in these 3-3/4 shaded rectangles is a graphic description of the answer to our problem.

(15) Beep In Algebra there is a rule that says $5 \times 3/4 = 3/4 \times 5$. Is it true in this example?

(16) Beep This means that $\frac{5 \times 3}{4} = \frac{3 \times 5}{4}$

Now 5 x 3 is 15 and 3 x 5 is 15, so we get

(17) Beep $\frac{15}{4} = \frac{15}{4}$ or

3-3/4 = 3-3/4 So you can see the rule holds or is true in this example we have just solved.

The problem we just solved was multiplication between a whole number and a fraction. Let's try one that multiplies a fraction by a fraction.

(18)
Beep try $\frac{2}{5} \times \frac{3}{4}$

(19)
Beep Rule for multiplication of 2 fractions says to multiply the 2 numerators together (top numbers) and put that answer over the one you get by multiplying the two denominators together. That is $\frac{2 \times 3}{5 \times 4} = \frac{6}{20}$

We must remember to reduce the answer to lowest terms if it is not already there.

(20)
Beep 1. If you look at $\frac{6}{20}$, both 6 and 20 can be factored. That is,

$$\frac{6}{20} = \frac{2 \times 3}{4 \times 5} = \frac{2 \times 3}{2 \times 2 \times 5}$$

Two shots, Since 2 is a common factor in the numerator and denominator, let's divide it out.

(21)
Beep Two shots, $\frac{2 \times 3}{2 \times 2 \times 5} = \frac{3}{10}$ Therefore, we can see that $\frac{6}{20}$ reduces to $\frac{3}{10}$.

(22)
Beep Let's double check to see if $\frac{6}{20} = \frac{3}{10}$

Remember the rule? 6×10 must $= 3 \times 20$

Does it? Sure - $6 \times 10 = 60$
and - $3 \times 20 = 60$

$$\therefore \frac{6}{20} = \frac{3}{10}$$

Now I'd like to see if we can represent this same problem graphically.

(23)
Beep Let's look at our original problem -

$\frac{2}{5} \times \frac{3}{4}$ You should see that it is 5ths and 4ths that we are multiplying by each other.

(24)
Beep Now let's take a rectangle and do two things to it. First, let's separate its top side into 4ths. Second, let's separate the left side into 5ths. Now extend the lines all the way through the rectangle and count the small rectangles we got by doing this. (Pause) Aren't there 20 of them? Our original problem was $\frac{2}{5} \times \frac{3}{4}$. We have marked off the 5ths and the 4ths. Now we must shade how many of each of these we want; that is, since we have $\frac{2}{5}$,

(25)
Beep We must shade 2 of the rows representing 5ths.

(26)
Beep Since we are also working with 3 of the 4ths, we must shade 3 of the columns which represent 4ths.

(27)
Beep Then to see what $\frac{2}{5} \times \frac{3}{4}$ looks like, let's put two pictures together. Now a question for you; how many of the small rectangles are in the two rows we shaded and are also in the three columns we shaded? Aren't there 6?

(28)
Beep Look at the small numbered rectangles. Don't these occur in the 2 rows representing 5ths and are also in the 3 columns representing 4ths?

(29)
Beep O.K., a few conclusions. How many small rectangles in all? 20.

(30)
Beep How many small rectangles occurred in the two rows and 3 columns at the same time? 6.

(31)
Beep So we ended up with 6 rectangles out of the 20 possible.

(32)
Beep Look at our original problem and its answer.

(33)
Beep Have we shown graphically the answer to this problem? Yes, we have.

(34)
Beep However, when done with arithmetic you should reduce the answer as we did before.

MULTIPLICATION OF FRACTIONS

1. Fractions are easy to multiply. Multiply the two top numbers together and put your answer on top. Multiply the two bottom numbers together and put that number on the bottom. Then reduce your fraction if possible.

Try this one:

$$\frac{2}{7} \times \frac{3}{4} = \frac{2 \times 3}{7 \times 4} =$$

Your answer should be $\frac{6}{28} = \frac{3}{14}$.

If you do not see how we reduced this answer, look at the example done in the next problem.

2. Now try this one:

$$\frac{3}{4} \times \frac{2}{6} = \frac{3 \times 2}{4 \times 6} =$$

Your answer should be $\frac{1}{4}$. Did you get $\frac{6}{24}$ and forget to reduce it? If you did, look at this: $\frac{6}{24} = \frac{{}^{1}\cancel{6}}{4 \times \cancel{6}_{1}} = \frac{1}{4 \times 1} = \frac{1}{4}$

Since we see that 6 is a number common to both top and bottom, we divide that number out.

3. Try factoring these numbers into prime factors. Break down the numbers as far as possible. Look at the first example.

$24 = 6 \times 4$

$= 2 \times 3 \times 2 \times 2$

$64 =$

$12 =$

$32 =$

$8 =$

$8 =$

4. Use the above factors to help you reduce these fractions:

$\frac{6}{12} =$ $\qquad \frac{6}{32} =$ $\qquad \frac{8}{32} =$

$\frac{8}{24} =$ $\qquad \frac{6}{8} =$ $\qquad \frac{12}{32} =$

5. No[illegible] multiply the fractions in problems 5, 6, 7, 8, 9, 10. Remember to try to reduce.

5. $\frac{2}{3} \times \frac{3}{4} =$ $\qquad$ 6. $\frac{2}{8} \times \frac{3}{4} =$ $\qquad$ 7. $\frac{3}{16} \times \frac{4}{2} =$

8. $\frac{2}{7} \times \frac{14}{16} =$ $\qquad$ 9. $\frac{5}{9} \times \frac{3}{5} =$ $\qquad$ 10. $\frac{2}{3} \times \frac{6}{10} =$

DIVISION OF FRACTIONS

The purpose of this tape is to help you understand the process of dividing one fraction by another. You should press the button until slide one appears. Throughout this slide program you will hear a series of beeps, each one sounding like this ___ . Each time you hear this sound ___ that is the signal to press the button to see the next slide.

We should look at several ideas before solving a division problem.

(1)
Beep

Getting back to slide one. Notice you are asked to divide 2/5 by 3/4. Before we even attempt to get an answer to this problem, there are several ideas we should think about.

(2)
Beep

Notice that on this slide we have four problems.

$4 \times 1 = 4$
$5 \times 1 = 5$
$8 \times 1 = 8$
$\frac{1}{2} \times 1 = \frac{1}{2}$

These are examples of our first principle. That is, you can multiply any number by one and get the same number you started with for an answer.

(3)
Beep

On this slide you can see four more problems.

$4/4 = 1$
$5/5 = 1$
$8/8 = 1$
$\frac{1}{2}/\frac{1}{2} = 1$

These problems are examples of our second principle. This principle says if you divide any number by itself, you get one for an answer. This is not true for 0/0 but you won't need to know this.

(4)
Beep

On this slide you see four more problems. These problems are examples of the principle of <u>reciprocals</u>.

$4 \times \frac{1}{4} = 1$
$5 \times 1/5 = 1$
$8 \times 1/8 = 1$
$\frac{1}{2} \times 2/1 = 1$

$\frac{1}{4}$ is the reciprocal of 4 because $4 \times \frac{1}{4} = 1$
1/5 is the reciprocal of 5 because $5 \times 1/5 = 1$
1/8 is the reciprocal of 8 because $8 \times 1/8 = 1$
2/1 is the reciprocal of $\frac{1}{2}$ because $\frac{1}{2} \times 2/1 = 1$

(5) Beep Let's look at another slide and see if we can find some reciprocals.

$3 \times \text{what} = 1$
$2/3 \times \text{what} = 1$
$3/4 \times \text{what} = 1$
$2/5 \times \text{what} = 1$

(6) Beep Well . . . . $3 \times 1/3 = \frac{3 \times 1}{3} = \frac{3}{3} = 1$ Therefore, 1/3 must be the reciprocal of 3.

(7) Beep $2/3 \times 3/2 = \frac{2 \times 3}{3 \times 2} = 6/6 = 1$ Therefore, 3/2 must be the reciprocal of 2/3.

(8) Beep $3/4 \times 4/3 = \frac{3 \times 4}{4 \times 3} = 12/12 = 1$ Therefore, 4/3 must be the reciprocal of 3/4.

(9) Beep $2/5 \times 5/2 = \frac{2 \times 5}{5 \times 2} = 10/10 = 1$ Therefore, 5/2 must be the reciprocal of 2/5.

Now let's look at all four of the examples we started with at first.

(10) Beep

$3 \times 1/3 = 1$ $3/4 \times 4/3 = 1$
$2/3 \times 3/2 = 1$ $2/5 \times 5/2 = 1$

In each of these four examples, the reciprocals are underlined. A rule to remember to get the reciprocal of a fraction; turn the fraction upside down. Notice the reciprocal of 2/3 is 3/2. The reciprocal of 3/4 is 4/3. The reciprocal of 2/5 is 5/2. Now 3 is the same as 3/1 (3 divided by 1 is 3) And the reciprocal of 3/1 is 1/3. So again we have turned the fraction upside down.

(11) Beep Here is our fourth principle.

$4/1 = 4$
$5/1 = 5$
$8/1 = 8$
$\frac{1}{2}/1 = \frac{1}{2}$

That is, any number divided by 1 <u>is</u> that number.

(12) Beep Let's review. Here is a slide that has all four of the principles applied to a number.

1. $4 \times 1 = 4$
2. $4/4 = 1$
3. $4 \times \frac{1}{4} = 1$
4. $4/1 = 1$

(12 continued)

1. Number one says we can multiply any number by one and get that number.

2. Number two says if we divide a number other than zero, by itself we get one.

3. Number three shows us that the reciprocal of a fraction is found by turning the fraction upside down (or inverting). In this example $\frac{1}{4}$ is the reciprocal of 4.

4. Number four tells us if we divide any number by one, we get that number for an answer.

(13)
Beep Now back to our original problem $\frac{2/5}{3/4}$. How do we solve this?

(14)
Beep Is this true by one of our principles? Sure. The principle that says if we multiply any number by one, we get that number.

(15)
Beep Now look at the 3/4. Does it have a reciprocal? Our reciprocal rule tells us that the reciprocal of 3/4 is 4/3.

(16)
Beep Can you multiply the 3/4 by 4/3? Now we aren't finished yet. Since we multiplied the denominator (at the bottom) or our original fraction by 4/3, we must multiply the numerator (at the top) by 4/3 also. And we get a problem that takes this form.

(17)
Beep $\frac{2/5 \times 4/3}{3/4 \times 4/3}$ Notice on this slide, I have not written the one. Do you know what happened to it?

(18)
Beep Answer this question? What is $\frac{4/3}{4/3}$ equal to? One! Do you see now that I have just changed the form of the one?

(19)
Beep Instead of saying $\frac{2/5}{3/4} = \frac{2/5}{3/4} \times 1$.

(20)
Beep I now have $\frac{2/5}{3/4} = \frac{2/5 \times 4/3}{3/4 \times 4/3}$ OK. We are almost ready for an answer.

(21)
Beep By our reciprocal rule, what is $3/4 \times 4/3$ equal to. If you have forgotten; What is 3/4? What is 4×3?

(22)
Beep Then what is 12/12? 12 divided by 12 is one.

(23)
Beep Now our original problem takes the form of $\frac{2/5}{3/4} = \frac{2/5 \times 4/3}{1}$

What did the rule say when we had a number divided by one? Didn't we get that same number?

(24)
Beep In other words, $\frac{2/5 \times 4/3}{1} = 2/5 \times 4/3$

Now finally we are ready to get our answer.

(25)
Beep $2/5 \times 4/3 = \frac{2 \times 4}{5 \times 3}$

(26)
Beep And: $\frac{2 \times 4}{5 \times 3} = \frac{8}{15}$ or $\frac{8}{15}$

(27)
Beep Let's review. We started with $\frac{2/5}{3/4}$

(28)
Beep We looked at the bottom number or denominator and asked the question. What is the reciprocal of 3/4? You said 4/3 and we multiplied both top and bottom by 4/3.

(29)
Beep Now 4/3 over 4/3 is equal to 1 so we haven't changed value of original problem. We know that: $3/4 \times 4/3 = \frac{12}{12} = 1$ So our problem took the form of

(30)
Beep $\frac{2/5 \times 4/3}{1}$ which was equal to $2/5 \times 4/3$ and working this multiplication problem out, we arrived at an answer of:

(31)
Beep $\frac{2 \times 4}{5 \times 3}$ Which is equal to $\frac{8}{15}$

(32)
Beep Can you do this problem using the four rules we have gone over today?

$\frac{1/3}{3/7}$

Good luck!

DIVISION OF FRACTIONS

1. Fill in the blanks using the first problem in each column as an example.

$4 \times 1 = 4$	$\frac{5}{5} = 1$	$\frac{2}{3} \times \frac{3}{2} = 1$
$8 \times \_\_ = 8$	$\frac{6}{} = 1$	$\frac{3}{5} \times \frac{5}{3} = \_\_$
$3 \times 1 = \_\_$	$\frac{}{7} = 1$	$\frac{2}{7} \times \frac{}{\_\_} = 1$

These problems are examples of the ideas you should have in mind when dividing fractions.

2. Here is one example of dividing one fraction by another.

$$\frac{\frac{3}{4}}{\frac{2}{7}} = \frac{\frac{3}{4}}{\frac{2}{7}} \times 1 = \frac{\frac{3}{4}}{\frac{2}{7}} \times \frac{\frac{7}{2}}{\frac{7}{2}} = \frac{\frac{3}{4} \times \frac{7}{2}}{1} = \frac{3}{4} \times \frac{7}{2} = \frac{3 \times 7}{4 \times 2} = \frac{21}{8} = 2\,\frac{5}{8}$$

Try this problem using the one above as an example. Fill in the blanks.

$$\frac{\frac{2}{3}}{\frac{4}{5}} = \frac{}{\frac{4}{5}} \times 1 = \frac{\frac{2}{3}}{\frac{4}{5}} \times \frac{}{\frac{5}{4}} = \frac{\frac{2}{3} \times \frac{5}{4}}{} = \frac{2}{3} \times \frac{5}{4} = \frac{2 \times }{ \times 4} = \frac{10}{} =$$

3. Try doing this problem writing in all the steps as in the above problems.

$$\frac{\frac{1}{5}}{\frac{3}{8}} =$$

4. The problem below shows the shortcut that is often used instead of the longer method in the above problems.

$$\frac{\frac{1}{8}}{\frac{7}{8}} = \frac{1}{8} \div \frac{7}{8} = \frac{1}{8} \times \frac{8}{7} = \frac{1 \times 8}{8 \times 7} = \frac{8}{56} = \frac{1}{7}$$

When you see $\frac{1}{8} \div \frac{7}{8}$, all you do is invert the $\frac{7}{8}$ (turn $\frac{7}{8}$ upside down) and multiply:

$$\begin{aligned} \frac{1}{8} \div \frac{7}{8} &= \frac{1}{8} \times \frac{8}{7} \\ &= \frac{1 \times 8}{8 \times 7} \\ &= \frac{8}{56} = \frac{1}{7} \end{aligned}$$

5. Use the method in problem 4 to solve this problem:

$$\frac{\frac{3}{5}}{\frac{4}{9}} = \frac{3}{5} \div \frac{4}{9} =$$

If your answer is $1\frac{7}{20}$, go on to the other problems. If you did not, try to find your mistake. Then do problems 6, 7, 8.

6. $\frac{1}{6} \div \frac{2}{3} =$

7. $\frac{3}{8} \div \frac{2}{5} =$

8. $\frac{5}{9} \div \frac{3}{5} =$

IMPROPER FRACTIONS AND MIXED NUMBERS

The purpose of this tape is to help you understand an improper fraction and how it is related to a mixed number. Often it is necessary to change an improper fraction to a mixed number or a mixed number to an improper fraction. These two operations will be seen on this program.

You should press the control button until slide one appears. Throughout this program you will hear a series of beeps, each one sounding like this ---. Each time you hear this sound ---, that is the signal to press the button to see the next slide.

(1)
Beep — Now on slide one you see two types of fractions. One is a proper fraction, the other is improper. Do you know the difference?

The first one, 5/12 is called a proper fraction. The top number (or numerator) is smaller than the bottom one (or denominator).

The second one, 12/5 is an improper fraction because the numerator is larger than the denominator. It is with this second type we will work first.

(2)
Beep — Here you see a series of 3 improper fractions. 12/5, 7/6, and 10/3. I want to change each of these to a mixed number. Let's work with the first one.

(3)
Beep — To change 12/5 to a mixed number, you should realize when you look at 12/5, you are saying 12 divided by 5.

(4)
Beep — Now it looks like a long division problem. How many times will 5 go into 12? Well, $5 \times 1 = 5$, $5 \times 2 = 10$, and $5 \times 3 = 15$, 15 is too large so it looks as though 5 will go into 12, two times with some left over. How many left?

(5)
Beep — 5 goes into 12, 2 times. 2×5 is 10. and 10 from 12 is 2. So we have a remainder of 2/5.

(6)
Beep — And our final answer looks like this: 12 divided by 5 is 2-2/5. 2-2/5 is called a mixed number because it has a whole number and a fractional part.

(7)
Beep — And our original problem of changing an improper fraction to a mixed number has been completed.

(8)
Beep Now our second improper fraction was 7/6. Can we change this to a mixed number?

(9)
Beep 7/6 equals 7 divided by 6.

(10)
Beep 6 goes into 7, once with 1/6 left. And our problem takes the form of:

(11)
Beep 1-1/6 is a mixed number because it has a whole number and a fractional part.

(12)
Beep And we have changed the improper fraction 7/6 into a mixed number of 1-1/6.

(13)
Beep Can you change the 3rd improper fraction we started out with to a mixed number? 10/3 is = to what mixed number?
You have 15 seconds to change 10/3 to a mixed number.

(14)
Beep You should have 3 and 1/3. 10/3 = 10 divided by 3.

(15)
Beep 3 divided by 10 = 3-1/3. 10 divided by 3 is equal to 3-1/3
Therefore, our original problem takes the form of:

(16)
Beep 10/3 = 3-1/3

(17)
Beep Let's look at the 3 mixed numbers we have come up with.

2-2/5 1-1/6 3-1/3

Let's see if we can start with each of these and change it back to a mixed number.

(18)
Beep First, 2-2/5 is equal to some improper fraction. Let's find it.

(19)
Beep 2-2/5 = 2 + 2/5

(20)
Beep 2 + 2/5 = (2 x 1) + 2/5 I would like to change the form of the 1 on the right side of this equation to 5/5. Do you know why I chose 5/5 rather than 3/3 or 6/6?

(21)
Beep Our problem now takes the form of (2 x 5/5) + 2/5

(22)
Beep And from this we get $(2 \times 5/5) + 2/5 = \frac{2 \times 5}{5} + \frac{2}{5} = \frac{10}{5} + \frac{2}{5}$

(23)
Beep And finally, $\frac{10}{5} + \frac{2}{5} = \frac{12}{5}$ And so the problem of changing 2-2/5 into an improper fraction has been completed.

(24)
Beep And we have $2\text{-}2/5 = 12/5$.

(25)
Beep Can we do the same with this mixed number? $1\text{-}1/6 = ?$

(26)
Beep $1\text{-}1/6 = 1 + 1/6$. Let's change the form of the 1 again. This time to 6/6.

(27)
Beep Now we have $1 + 1/6 = 6/6 + 1/6$

(28)
Beep And $6/6 + 1/6 = 7/6$

(29)
Beep Our problem is finished as we have changed 1-1/6 to 7/6.

(30)
Beep Now for our last mixed number. Can you change 3-1/3 into an improper fraction? Try it. You have 30 seconds.

Your answer is 10/3

(31)
Beep $3\text{-}1/3 = 3 + 1/3$

(32)
Beep $3 + 1/3 = (3 \times 1) + 1/3$.

(33)
Beep Change the 1 to 3/3 and you get

(34)
Beep $(3 \times 3/3) + 1/3 = \frac{3 \times 3}{3} + \frac{1}{3}$

(35)
Beep $\frac{3 \times 3}{3} + \frac{1}{3} = \frac{9}{3} + \frac{1}{3} = \frac{10}{3}$

(36)
Beep Then your problem is completed. You have changed 3-1/3 into 10/3. In other words: $3\text{-}1/3 = 10/3$

IMPROPER FRACTIONS AND MIXED NUMBERS

1. Change the forms of the given problems as is done in the example a)

a) $X \div Y = \frac{X}{Y} = Y\overline{)X}$

b) $11 \div 3 = \qquad =$

c) $\qquad = \frac{3}{5} =$

2. $\frac{12}{5} = 5\overline{)12}$ with quotient 2; $\frac{10}{2}$ below; $= 2\frac{2}{5}$

Change this improper fraction to a mixed number. Follow the example above.

$\frac{13}{4} =$

3. Here is an example of changing a mixed number to an improper fraction.

$5\frac{2}{3} = 5 + \frac{2}{3} = 5(1) + \frac{2}{3}$

$= 5(\frac{3}{3}) + \frac{2}{3}$

$= \frac{15}{3} + \frac{2}{3}$

$= \frac{17}{3}$

See if you can fill in the missing parts to this problem:

$4\frac{1}{4} = 4 + \_\_\_\_ = 4(1) + \frac{1}{4}$

$= 4(\ \ ) + \frac{1}{4}$

$= \frac{16}{4} + \_\_\_\_$

$= \_\_\_\_$

4. Change $3\frac{2}{5}$ to an improper fraction. Follow the pattern in problem 3 above.

In 5, 6, 7, change the mixed number to an improper fraction.

5. $7\frac{1}{2} =$

6. $5\frac{3}{4} =$

7. $4\frac{2}{3} =$

In 8, 9, 10, change the improper fraction to a mixed number.

8. $\frac{22}{7} =$

9. $\frac{41}{10} =$

10. $\frac{33}{6} =$

MULTIPLICATION OF DECIMALS

The purpose of this program is to help you multiply numbers involving decimals. You should have slide one in fron of you. If you do not, push the control button until it appears. Throughout this program, you will hear a series of beeps, each one sounding like this ---. Each time you hear this sound ---, that is the signal to press the button to see the next slide.

1. Now back to slide one.
Here you see one equation

(1)
Beep 2=2

Number one says that the whole number two might be said to have an invisible decimal point and all I have done is shown you where it is. This is true for any whole number, if the decimal point does not appear, it can always be placed to the right far behind the number as it is shown on this next slide.

(2)
Beep 3=3.
11=11.
101=101.

In each of these cases, you can see that the decimal point always is placed behind the number.

(3)
Beep Now, look at two more numbers.
2.1
3.25

These already have decimal points, so we do not have to be concerned with placing them any more.

(4)
Beep Now, will you look at this problem.

2.1
x 2.

We can go ahead and multiply the numbers and you come up with a problem that looks like this.

(5)
Beep 2.1
x 2.
42 And the numbers in our answer are: four-two

But you are not finished yet, because you must consider where to put the decimal point in your answer. I am going to tell you the rule and then show you why we need it.

(5 continued)

Look at the two numbers you multiplied together. How many digits or numbers are on the right side of either decimal point? The one is on the right side of the decimal point in 2.1 . There are no numbers on the right of the decimal point in 2. We have a total of one number on the right of either decimal point. Now start from the right in your answer and count one place and put your decimal in between the 2 and the 4.

(6)
Beep

And we have completed our problem and see its answer of 4.2

Now a question for you. Why must the decimal go there? Rather than in front of the 4 or behind the 2. To help you see why, let's look at the same problem again but in fractional form.

(7)
Beep

Since: 2.1=21/10= 21/10
and
$2. = \frac{2}{1}$ our problem takes the form of:

(8)
Beep

$$\begin{array}{r} 2.1 \\ \underline{x\ 2} \end{array} = \frac{21}{10} \times \frac{2}{1}$$

(9)
Beep

$\frac{21}{10} \times \frac{2}{1} = \frac{21 \times 2}{10} = \frac{42}{10}$ and $\frac{42}{10}$ can be reduced to $4\frac{2}{10}$

(10)
Beep

$\frac{42}{10}$ and the $4\frac{2}{10}$ written as a decimal takes the form 4.2
Isn't 4.2 the same answer as we had back on slide (6)?

(11)
Beep

Same as (6).

Sure it is. Now it is a lot easier to get the answer directly through decimals but the work on fractions often helps you to understand why the decimal goes wher it does.

(12)
Beep

Let's try another problem. 3.25
x 2. multiply the numbers. We get a problem that looks like:

(13)
Beep

$$\begin{array}{r} 3.25 \\ \underline{x\ \ 2.} \\ 650 \end{array}$$

and the numbers in our answer are six, five, zero.

(13 continued)

But don't forget to put the decimal point in your answer. Otherwise, it is wrong. How many numbers are on the right of either decimal point? Aren't there two? The 2 and the 5 are on the right of the decimal point in 3.25 so you must start on the right side of your answer and count over two numbers. The 0 would be one and the 5 would be 2 so your decimal goes between the 5 and the 6.

(14)
Beep   And our problem takes the final form of

$$\begin{array}{r} 3.25 \\ \underline{x\ \ 2.} \\ 6.50 \end{array}$$

and the problem is completed.

Again let me show you why the decimal must go there.

(15)
Beep   Since: $3.25 = 3\frac{25}{100} = \frac{325}{100}$ and $2. = \frac{2}{1}$

(16)
Beep

We can change the form of our problem from decimals to fractions like we did before.

$$\begin{array}{r} 3.25 \\ \underline{x\ \ 2.} \end{array} = \frac{325}{100} \times \frac{2}{1}$$

Now multiply the two fractions together, We see:

(17)
Beep $\frac{325}{100} \times \frac{2}{1} = \frac{325 \times 2}{100 \times 1} = \frac{650}{100}$ Reducing $\frac{650}{100}$ we get:

(18)
Beep $6\frac{50}{100}$ which when changed to a decimal is 6.50. Isn't 6.50 the same answer that we came up with by multiplying the two decimals together?

(19)
Beep   Same as 14. Sure! You can see that it is. Try this problem now.

(20)
Beep

$$\begin{array}{r} 3.25 \\ \underline{x\ 2.1} \end{array}$$

Write it on a sheet of paper and work it in the next 60 seconds. Remember, you have 60 seconds. Begin!

60 seconds lapse.

(21)
Beep You should have a problem that looks like this:

$$\begin{array}{r} 3.25 \\ \underline{2.1} \\ 325 \\ \underline{650} \\ 6.825 \end{array}$$

Remember how your decimal is placed? There is a total of 3 numbers on the right side of either decimal point so we must start at the right of our answer and count 3 numbers:

. the 5 is one
the 2 is two
and the 8 is our third number.
• the decimal point goes between the 8 and the 6.

(22)
Beep (End)

Now you should rewind the tape and ask your instructor for the problem sheet which accompanies this program.

MULTIPLICATION OF DECIMALS

FILL IN THE BLANKS IN THE FIRST FOUR PROBLEMS

1. $1.5 = 1\frac{\ }{10} = \frac{\ }{10}$

2. $2.1 = 2\frac{\ }{10} = \frac{\ }{10}$

3. $\frac{15}{10} \times \frac{21}{10} = \frac{\ }{100}$

4. $\frac{315}{100} = 3\frac{\ }{100} = 3.$

5.
```
  1.5
 X2.1
```

Your answer to problem 5 should be the same as the answer to problem 4. If it is not, try to find your mistake. Then continue with problems 6, 7, 8, and 9.

6.
```
  4.5
 X  6.
```

7.
```
  8.5
 X1.4
```

8.
```
  .041
 X .81
```

9.
```
  3.25
 X   .4
```

Your decimal point must be in the correct position in the answers of 6, 7, 8 and 9. You should have counted as many places from the right in your answer as the places to the right of either decimal point up in the problem. If you did not, try problem 10.

10.
```
  3.451
 X   .11
```

You should have your decimal point located 5 places from the right of your answer. If you do not, ask your teacher for some help.

DIVISION OF DECIMALS

The purpose of this program is to help you divide numbers involving decimals. You should have slide one in front of you. If you do not, push the control button until it appears. Throughout this program, you will hear a series of beeps, each one sounding like this __--__. Each time you hear this sound __--__, that is the signal to press the button to see the next slide.

Now, back to slide 1.

(1)
Beep $x \div y = \frac{x}{y} \quad y\overline{)x}$ The reason for this slide is to remind you that all three ways say the same thing. x divided by y. Now let's take a look at a problem.

(2)
Beep $15 \div .05 = \frac{15.}{.05} = .05\overline{)15.}$

(3)
Beep The first thing we must do is to move the decimal point outside the division sign to the right side of the 5. $.05.\overline{)15.}$

Now how many places have we moved it? Two. Then we must also move the decimal point inside two places to the right, since there are no numbers on the right of the decimal point, we make some by writing some zeros on the right of the decimal point. Like this: $.05\overline{)15.00}$

(4)
Beep Notice I have added 2 zeros to our picture. Now remember, we moved the decimal outside two places to the right so we must also move the decimal point inside two places to the right. Like this.

(5)
Beep $.05.\overline{)15.00.}$

(6)
Beep Now divide the problem. 15 divided by 5 is 3. $3 \times 5 = 15$ 0 divided by 5 = 0. Then placing our new decimal place in the answer, our answer has the final form of:

(7)
Beep __300.__ Now to help you understand why the decimal must be moved, let's change our original problem to fractions and work out an answer.

(8)
Beep Now 15 is the same as 15/1 (read 15 over 1) and .05 = 5/100 so we can change the form of this problem to:

(9)
Beep $\dfrac{\frac{15}{1}}{\frac{5}{100}}$ Looking at this problem, it is dividing one fraction by another. From our program on division of fractions, you should see that this problem takes the form of:

(10)
Beep $\dfrac{\frac{15}{1} \times \frac{100}{5}}{\frac{5}{100} \times \frac{100}{5}} = \dfrac{\frac{1500}{5}}{1}$

Notice we multiply both numerator and denominator by $\frac{100}{5}$
100
over
5

Any number divided by 1 is that number.

(11)
Beep So $\dfrac{\frac{1500}{5}}{1} = \dfrac{1500}{5} = 300$

(12)
Beep Isn't this the same answer we got by dividing the decimals by long division? Yes it is.

(13)
Beep Now let's look at another type: $18.6 \div .3 = \dfrac{18.6}{.3} = .3\overline{)18.6}$

Here we again have to move the decimal outside the division sign one place to the right so our problem takes the form of:

(14)
Beep $.3.\overline{)18.6}$ But don't <u>forget</u>. If we moved the decimal on the outside one place to the right, we must also move the decimal inside one place to the right. So our problem has the final form of:

(15)
Beep $.3.\overline{)18.6.}$ Now we are ready to use regular long division to come up with our answer.

(16)
Beep

$$\begin{array}{r} 62. \\ .3.\overline{)18.6.} \\ \underline{18} \\ 6 \\ \underline{6} \\ 0 \end{array}$$

Why did we move the decimal again? This could be shown using a plan like we did for the first problem on this program. But we won't do it here.

(17)
Beep One more type. $16.8 \div 8 = \dfrac{16.8}{8} = 8\overline{)16.8}$

Notice in this problem we are dividing a decimal by a whole number. See that form on the right side of the screen. I could also write it this way.

(18) Beep

$8.\overline{)16.8}$

This comes from the idea we discussed before on the placing of decimals in whole numbers. We have not moved the decimal at all. We have just shown where it is in a whole number.

(19) Beep

If we don't move the decimal outside, we don't move the one inside either and just put it up in our answer. Now work out the problem in long division.

(20) Beep

$$\begin{array}{r} 2.1 \\ 8.\overline{)16.8} \\ \underline{16} \\ 8 \\ \underline{8} \\ 0 \end{array}$$

and we have an answer of 2.1

(21) Beep

We have had 3 types of problems. Type I: whole numbers divided by a decimal. Read problem $.05\overline{)15.}$

(22) Beep

Where we had to add some zeros before moving both decimal points over 2 places.

(23) Beep

Type II: read problem $.3\overline{)18.6}$ decimal divided by a decimal.

(24) Beep

Where we just moved both the decimal points over 1 place.

(25) Beep

Type III: read problem $8\overline{)16.8}$ decimal divided by a whole number.

(26) Beep

Where we were dividing by a whole number and just left our decimal where it was except for moving it up to become part of our answer.

(27) Beep

Using the ideas discussed on this program, copy these problems on a sheet of paper. Copy all of them down. Then after shutting the projector off, work them out and hand them to your teacher.

DIVISION OF DECIMALS

1. Write the given problem in two other ways as the first example.

   a) $\frac{X}{Y} = X \div Y = Y\overline{)X}$

   b) $\frac{3}{4} =$ ______ = ______

   c) ______ $= 5 \div 2 =$ ______

   d) ______ = ______ $= 3\overline{)2}$

2. Write this problem in two other ways as you did those in problem 1.

   $1.7\overline{)3.4}$ = ______ = ______

3. Change 3.4 to a fraction: 3.4 =

4. Change 1.7 to a fraction: 1.7 =

5. Now divide the fraction in problem 3 by the fraction in problem 4.

   $$\frac{\frac{34}{10}}{\frac{17}{10}} =$$

   Your answer should be 2.

6. Look at the same problem using decimals.

   $1.7\overline{)3.4}$

   You must move the decimal outside on place to the right. So you must also move the decimal inside one place to the right.

   Now divide. You should get 2 for an answer. If you did, do 7, 8, 9, and 10. If you did not, check your work.

7. $.6\overline{)3.612}$

8. $.10\overline{)1.20}$

9. $.12\overline{)24}$

10. $1.1\overline{)22.33}$

RATIO

Ratio is a way to compare two items. The next three slides show examples of the ratio between two items.

(1)
Beep If you were asked to set up a ratio of games won to games played for a ball team who had won 10 games out of 12, your ratio would look like this:

Remember - $\frac{\text{games won - that is 10}}{\text{games played that is 12}} = \frac{10}{12}$

(2)
Beep If you were asked to set up a ratio of girls to boys for a class that had 8 girls and 5 boys, your ratio would look like this: girls to boys - 8 girls over 5 boys. $\frac{8}{5}$ would be the ratio of girls to boys.

(3)
Beep Let's say you took a test. The test had 10 problems on it and you had 9 of them right. Here is the ratio of problems right to total problems.

$\frac{\text{9 right}}{\text{out of 10}}$ $\frac{9}{10}$

(4)
Beep You see on this slide 3 forms which say the same thing. The first form is the ratio of x to y. The middle form is read x to y. And the third form says x to y.

It is the third form that is the one that tells you how to set up the 1st form.

(5)
Beep Take a closer look at the third form. x is the first letter when you read the form from left to right. So we call x the 1st term. y is the 2nd letter in the sentence. So we call the y the 2nd term.

(6)
Beep This is always the case. When you set up a ratio the 1st term is on top and the 2nd term is on the bottom.

(7)
Beep Remember x is the first term, y is the second term. So when we set up the ratio of x to y, we have $\frac{x}{y}$ over

(8)
Beep Remember, the first ratio I asked you to set up. A team had won 10 games out of 12 played. And I asked you to set up the ratio of games won (that's 10) to games played (that's 12) so we wrote:

(9)
Beep $\frac{10}{12}$ over — This ratio can be reduced, very much the same way we do fractions.

(10)
Beep 10 can be factored into 5 x 2.
12 can be factored into 2 x 2 x 3.
Since 2 is a factor common to both numbers, let's divide this factor out.

(11)
Beep We have left $\frac{5}{2x3}$ over or $\frac{5}{6}$ This is the final form of the reduced ratio.

(12)
Beep The second ratio was girls to boys for a class that had $\frac{8 \text{ girls}}{5 \text{ boys}}$
so we wrote:

(13)
Beep $\frac{8}{5}$ over as to the ratio of girls to boys.

The third ratio was about a test you took and had 9 out of 10 problems. I asked you for the ratio of problems right to the total number of problems.

(14)
Beep $\frac{9 \text{ right}}{10 \text{ total problems}}$

(15)
Beep So you wrote $\frac{9}{10}$ over as to the ratio of problems right to total problems.

Ask your teacher for the work sheet that accompanies this program. Try setting up the ratios on it.

RATIO

1. A ratio has a first term and a second term. Here is a ratio $\frac{X}{Y}$. It is read X to Y.

Which letter comes first in that last sentence? Isn't it the X? The X is called the first term. Which letter comes second? It is the Y. So we call Y the second term of the ratio.

$\frac{5}{6}$ is a ratio. The first term is the ______. The second term is the ______.

2. You took a test with 10 problems on it. You had 9 of them correct. What is the ratio of correct problems to total problems?

You should have written $\frac{9}{10}$.

Read this sentence: What is the ratio of correct problems (1st) to total problems (2nd)?

The phrase "correct problems" comes first in the sentence so you should have that as the first term in the proportion.

The phrase "total problems" comes second so that is the second term of the proportion.

3. Try this one.

Our class has 14 boys and 9 girls. Set up the following ratios.

a. Ratio of boys to girls.
b. Ratio of girls to boys.
c. Ratio of girls to total in class.
d. Ratio of boys to total in class.

4. A car traveled 150 miles on 10 gallons of gas. Set up a ratio of miles to gallons.

If you had $\frac{150}{10} = \frac{15}{1}$ as the ratio of miles to gallons, go on to problem 5. Notice in this problem, we had to reduce our original ratio of $\frac{150}{10}$ to $\frac{15}{1}$.

5. A football team lost two games and won 8 games. What is the ratio of games won to total games played?

PROPORTION

A ratio compares two items - A proportion is a way to compare four items.

(1)
Beep Here is an example of a proportion. Notice the fou items x,y,z, and w.

(2)
Beep Look at this ratio.

(3)
Beep Look at this ratio and see they must be equal.

(4)
Beep In a proportion these two items must be here in order to have a proportion.

(5)
Beep (1) There must be two ratios.
(2) They must be equal.

Let's take a look at two different problems.

(6)
Beep Does 3/8 = 6/16 represent a proportion?
First of all, it does have 2 ratios so it fills the first rule that we had for a proportion. Next, we must check to see if the ratios are equal. There is a quick way to do this.

(7)
Beep Look at this picture. See the two numbers that are connected by the red line. Multiply them together.

(8)
Beep 16 x 3 = 48.

(9)
Beep See the two numbers connected by the line. Multiply these two numbers together.

(10)
Beep

$$\begin{array}{r} 8 \\ \times 6 \\ \hline 48 \end{array}$$

(11)
Beep Since 16 x 3 = 48 and 8 x 6 = 48, we have equal ratios.

(12)
Beep Here are the two rules in order to have a proportion.

(1) Must have two ratios
(2) Ratios must be equal

We have two ratios. We have just shown that they are equal.

(13)
Beep Our original expression is a proportion and we don't need the question mark because we have shown the two ratios equal.

Let's look at another problem now.

(14)
Beep 3/4 = 2/3 a proportion. 1st of all, we do have two ratios. So our first rule holds. Now let's see if the two ratios are equal.

(15)
Beep Do it just like we did the other. Multiply the two numbers connected by the line.

(16)
Beep $3 \times 3 = 9$

(17)
Beep Multiply the two numbers connected by the blue line.

(18)
Beep $4 \times 2 = 8$

(19)
Beep Since $3 \times 3 = 9$
$4 \times 2 = 8$, we do not have equal ratios.

(20)
Beep Then 3/4 = 2/3 is not a proportion. Now ask your teacher for the practice sheet for proportions.

PROPORTION

1. Here is an example of a proportion:

$$\frac{X}{Y} = \frac{Z}{W}$$

$\frac{X}{Y}$ is a ratio. $\frac{Z}{W}$ is a ratio. The " = " sign tells you the ratios must be equal.

Here are two ratios: $\frac{1}{2}$ and $\frac{6}{12}$

Since they are equal, we can set up the proportion: $\frac{1}{2} = \frac{6}{12}$.

2. Here are two more ratios: $\frac{3}{8} \stackrel{?}{=} \frac{6}{16}$. Notice the question mark over the equal sign. This indicates that you must check to see if $\frac{3}{8} = \frac{6}{16}$. If the ratios are equal, then you have a proportion.

Cross multiply: $3 \times 16 = 48$

$8 \times 6 = 48$

Since you get 48 for both answers, the ratios are equal. Then you can say that $\frac{3}{8} = \frac{6}{16}$ is a proportion.

3. Are $\frac{2}{3}$ and $\frac{8}{12}$ proportional? To answer this question, set up this problem:

$$\frac{2}{3} \stackrel{?}{=} \frac{8}{12}$$

Cross multiply: $2 \times 12 = 24$

$3 \times 8 = 24$

Since you get 24 for both answers, you can say yes, $\frac{2}{3}$ and $\frac{8}{12}$ are proportional.

4. Try this one: Are $\frac{3}{4}$ and $\frac{5}{6}$ proportional? Set up this problem:

$$\frac{3}{4} \stackrel{?}{=} \frac{5}{6}$$

Cross multiply: $3 \times 6 = 18$

$4 \times 5 = 20$

Since 18 and 20 are not the same, $\frac{3}{4}$ and $\frac{5}{6}$ are not proportional.

In the problems below, check to see if the ratios are proportional. If they are, write yes. If they are not, write no.

5. $\frac{2}{5} \stackrel{?}{=} \frac{4}{10}$

6. $\frac{3}{5} \stackrel{?}{=} \frac{5}{10}$

7. $\frac{3}{8} \stackrel{?}{=} \frac{6}{9}$

8. $\frac{5}{9} \stackrel{?}{=} \frac{10}{18}$

9. $\frac{4}{9} \stackrel{?}{=} \frac{5}{10}$

10. $\frac{4}{7} \stackrel{?}{=} \frac{16}{28}$

RATIO AND PROPORTION

Quite often we run into the problem of telling whether two items are proportional to two other items. The problem is quite simple if you know how to set it up. This program has three problems we will consider.

(1)
Beep A family traveled 400 miles in ten hours.

(2)
Beep The next day, they traveled 320 miles in 8 hours.
The problem---are these items proportional?

(3)
Beep There are two ways we can set this up. This way 400 over 10 ? = 320 over 8

400 - the miles 320-miles
10 - the hours 8-the hours

or this way:

(4)
Beep 10/400 = 8/320   10 the hours to go 400 miles   8 the hours to go 320 miles

Either way will allow you to check the proportion.
Let's work with this one here. Remember how to check a proportion?
Try cross multiplying.

(5)
Beep 10 x 320 = 3200   8 x 400 = 3200

(6)
Beep An easy rule to remember is to multiply 1st term (the top number) of one ratio with the 2nd term (the bottom number) of the other ratio.

(7) Since both answers are 3200, the times are proportional.
Beep

(8)
Beep Try this problem: A salesman earned a commission of $75.00 for making sales of $1,500.00. The next month he made $5,000.00 worth of sales for a commission of $250.00. Let's set up a proportion.

(9)
Beep 75/1500 = 250/5000   Notice: 75 commission over 1500 sales = 250 commission 5000 sales

Both commissions on the top of the ratios.
Both sales on the bottom of the ratios. Now check it out. Cross multiply.

(10)
Beep 75 times 5000
1500 times 250

(11)
Beep $75 \times 5000 = 375{,}000$ Since both answers are equal to 375,000,
$1500 \times 250 = 375{,}000$ the items are proportional.

(12)
Beep Try this one. Sam studied 6 hours and got 82 on an examination. The next week he studied 4 hours and received 69 on his exam.

(13)
Beep Let's set it up.

6 hours over 82 score	4 hours over 69 score	It took 6 hours of study for a score of 82. So we put 6 over 82. It took 4 hours of study for a score of 69. So we put 4 over 69.

Try to check this problem in the next 30 seconds.

30 seconds - - - -

(14)
Beep You should have multiplied 6 times 69 and 4 times 82.

(15)
Beep $6 \times 69 = 414$
$4 \times 82 = 328$

(16)
Beep Since the answers are not equal, the items are <u>not</u> proportional. Now ask your teacher for the practice sheet which goes along with this program.

RATIO AND PROPORTION

1. A family traveled 80 miles and used 4 gallons of gas. The next day they traveled 60 miles and used 3 gallons of gas. Are these items proportional?

First, you must set up the two ratios: $\frac{80}{4}$ and $\frac{60}{3}$.

Next, set up the problem: $\frac{80}{4} = \frac{60}{3}$.

Cross-multiply: $80 \times 3 = 240$
$60 \times 4 = 240$

Since both answers equal 240, the items are proportional.

Note that we could have set up the ratios: $\frac{4}{80}$ and $\frac{3}{60}$ and still come up with the same answer.

2. Remember that what you put in the first term of the second ratio must be the same type of thing as is in the first term of the first ratio.

EXAMPLE: $\frac{\text{80 miles}}{\text{4 hours}} = \frac{\text{60 miles}}{\text{3 hours}}$ We have miles in the first term of the first ratio and must also have miles in the first term of the second ratio.

Try this one: A salesman made a commission of $40. for making sales of $800. The next week he made $1,600. worth of sales for a commission of $80. Are these items proportional?

HINT: Your proportion could be $\frac{40}{800} = \frac{80}{1{,}600}$ or $\frac{800}{40} = \frac{1{,}600}{80}$

Now finish the problem.

If you answered yes, go on to 3, 4, 5. If you answered no, check your cross-multiplication before going on.

3. A family traveled 120 miles in 2 hours. The next day they traveled 300 miles in 5 hours. Are these items proportional:

4. JoAnne studied 1 hour for her history exam and received a 40. The next week she studied 2 hours and received an eighty. Are these items proportional?

5. Rick bragged that he got 14 miles to one gallon of gas. Bill bragged that he went 21 miles on 1 and one-half gallons of gas. Show that the items are proportional.

STORY PROBLEMS AND PROPORTIONS

Proportions can make some types of story problems very easy if you know how to use them. This slide program has only three story problems on it. But if you watch and listen carefully, you will find the problems on the work sheet easy.

(1)
Beep A family on a trip drove 400 miles in 10 hours. The next day they drove at the same rate and went 320 miles. How long did it take them?

Notice here: 400 miles in ten hours
320 miles in how long?
let x be the number of hours it took them.

(2)
Beep We set up a proportion. In this problem since we are talking about miles and hours, this is one way we could set it up:

Look at the left side. Miles are on top. How about the right side? Are the miles on top here too? Yes. If you start with miles on top on the left, the miles must be on top on the right also.

(3)
Beep We could put it this way too. If you start with hours on top on the left, hours must be on top on the right also.

(4)
Beep Let's go back to our problem: 400 miles in 10 hours
320 miles in x hours.

Remember that we want to know how long it took the family to go 320 miles so x is what we want to find.

(5)
Beep Let's start by setting up the left side of our proportion. Put the 400 over the 10 to show the family went 400 miles in 10 hours. Notice we have miles over hours.

(6)
Beep So on the right side of our proportion, we must also have miles over hours. OK, 320 over x. Again, miles over hours where x is the number of hours we are trying to find.

(7)
Beep Since this is a proportion, we can cross-multiply. 400 times x must equal 320 x 10.

(8)
Beep So we get 400 x = 3200 Now we have to find x. Divide both sides by 400, the number in front of the x.

(9)
Beep $\frac{400\ x}{400} = \frac{3200}{400}$

(10)
Beep $\frac{400}{400} = 1$ 400 divided by 400 = 1

(11)
Beep So we have 1 x or $x = \frac{3200}{400}$ (Read 3200 divided by 400.)

(12)
Beep $400\overline{)3200}$ with quotient 8; subtract 3200 — 3200 divided by 400 = 8

(13)
Beep So we have x = 8. Since x was the number of hours it took the family to drive 320 miles, this means the family took 8 hours to drive the 320 miles.

(14)
Beep Let's try another common problem. Joe used 15.4 gallons of gasoline to go 320 miles. How many miles per gallon was that?

When they say per gallon, this means 1 gallon. The question, How many miles for 1 gallon means that we let x be the number of miles. That is what we are asked to find.

(15)
Beep Set up the proportion $\frac{320}{15.} = \frac{x}{1}$ We put the 320 over the 15.4 because for 320 miles, we used 15.4 gallons. We put the x over 1 because we have the 320 miles on top on the left so we must put x on top on the right

(16)
Beep This goes back to the idea shown here. If you put miles on top on the left, you must put miles on top on the right.

(17)
Beep Back to our problem. Now cross-multiply. That is, 320 times 1 = 15.4 times x

(18)
Beep 320 = 15.4 x Now divide both sides by 15.4, the number in front of the x.

(19)
Beep $\frac{320}{15.4} = \frac{15.4\ x}{15.4}$

(20)
Beep $\frac{15.4}{15.4} = 1$ So we have

(21)
Beep $\frac{320}{15.4} = 1 \text{ x or x}$ So to find x we must divide 320 by 15.4

(22)
Beep

$$\begin{array}{r} 20.7 \\ 15.4\overline{)320.0.0} \\ \underline{308} \\ 1200 \\ \underline{1078} \\ 122 \end{array}$$

You see an answer of 20.7. This could have been carried out further. But this gives you an idea that x is about 20.7

(23)
Beep What was x again? The number of miles per gallon. This means that the car went 20.7 miles per gallon of gas.

(24)
Beep Now try solving this problem. Set up a proportion and solve as we have done to the others. You have one minute.

Paul drove 75-miles in 2 hours. How far did he drive in 7 hours?

ONE-MINUTE TIME

(25)
Beep One proportion is this $\frac{75}{2} = \frac{x}{7}$ Cross-multiply and you have:

(26)
Beep $75 \times 7 = 2 \times x$ or:

(27)
Beep $525 = 2x$ Divide both sides by 2, the number in front of the x.

(28)
Beep You get $\frac{525}{2} = x$

(29)
Beep 525 divided by 2 is 262.5 Since we let x be the number of miles Paul could drive in 7 hours, our answer is

(30)
Beep 262.5 miles

If you want to review these three problems, go back to the beginning and run through the whole program again. Then ask your teacher for the problem sheet that goes with this program.

STORY PROBLEMS AND PROPORTIONS

1. A car traveled 200 miles in 4 hours. At the same rate, how long will it take the car to go 300 miles?

Set up a proportion: $\frac{200 \text{ miles}}{4 \text{ hours}} = \frac{300 \text{ miles}}{X \text{ hours}}$

Notice that I put the miles in the first term of the first ratio so I must also put miles in the first term of the second ratio.

Cross multiply: $200 \times X = 300 \times 4$

$$200X = 1200$$

Now, divide both sides by 200: $X = \frac{1200}{200} = 6$ The number of hours it took the car to go 300 miles.

2. Solve this problem using a proportion:
John drove 360 miles on 18 gallons of gas. How many miles per gallon was this?

HINT: You should use either one of these proportions:

$$\frac{360}{18} = \frac{X}{1} \quad \text{or} \quad \frac{18}{360} = \frac{1}{X}$$

You should get 20 miles per gallon as your answer. If you do not, check your work before going on with problems 3, 4, and 5.

3. A car can go 100 miles in 2 hours. At the same rate how far will it travel in 7 hours?

4. Golf balls cost $ 1.95 for 3. At this same cost, how much will 27 cost?

5. A certain type of wire is bought at $ 1.25 for 100 feet. How much wire can be purchased for $ 5.00?

PERCENT IN RATIO FORM

This program has two of the problems involving percent. Watch the steps which use the ratio form of percent to solve the problems. Every time you hear this sound **, you should press the button to see the next slide. You should now have slide 1 in front of you. If you do not, press the button until it appears.

On slide one you see the ratio $\frac{x}{\text{over}}\Big/100$. You will use this ratio to solve the first problem of this program.

(2)
Beep I want you to look for these two words in the first problem; what percent.

(3)
Beep In fact every time you see these two words in a story problem, (pause), you should realize that a good way to write this in ratio form by writing $\frac{x}{\text{over}}\Big/100$. Now let's look at our first problem

(4)
Beep Karen had a score of 54 on a test with 60 points. What percent of the total points did she have?

(5)
Beep Notice the words WHAT PERCENT. So let's start out by putting $\frac{x}{\text{over}}\Big/100 =$ on our papers.

(6)
Beep Now back to the problem. Same as number 4. What other numbers do you see in this problem. There is a 54 and a 60. Think now! How many would she have to get right in order to have 100% as her score?

(7)
Beep Karen would have to get all 60 points for 100%. This tells you that 60 must go right across from the 100 in the ratio we had before.

(8)
Beep Notice, 60 is on the same level as the 100. Now the 54 goes in the only other empty spot in the proportion.

(9)
Beep We have a proportion: $\frac{X}{100} = \frac{54}{60}$ Now, cross-multiply.

(10)
Beep $60\,X = 5400$ Divide both sides by 60.

(11)
Beep $\frac{60\,X}{60} = \frac{5400}{60}$ and $\frac{60}{60} = 1$ $\frac{5400}{60} = 90$, so, we have

(12)
Beep 1 X or X = 90

(13)
Beep Our original ratio was $\frac{X}{100}$ so we found X = 90

(14)
Beep $\frac{90}{100}$ which is = to 90% $\frac{90}{\text{over } 100}$ is the ratio form of 90%.

This final 90% tells us that Karen has a score of 90% on her test.

(15)
Beep We have come quite a ways from our original problem so let's try another one of the same type. This is one:

Out of 20 students, 16 took a test. What percent took the test? Can you find the two words, WHAT PERCENT?

(16)
Beep So we again start by writing $\frac{X}{100}$.

(17)
Beep Now back to our problem. What other numbers do you see in this problem? There is a 20 and a 16. Think now!! If all 20 of the students had taken the test, 100% would have taken the test.

(18)
Beep So, 20 students represent 100%. This tells you that the 20 must go right across from the 100 in the ratio we had before.

(19)
Beep Notice 20 is on the same level as the 100. Now, the 16 goes in the only other empty spot in the proportion.

(20)
Beep And we have this proportion. $\frac{X}{\text{over } 100} = \frac{16}{\text{over } 20}$ now, cross-multiply.

(21)
Beep $20\,X = 1600$ Divide both sides by 20.

(22)
Beep $\frac{20\,X}{20} = \frac{1600}{20}$ And $\frac{20}{20} = 1.$ $\frac{1600}{20} = \underline{80}$ so we have

(23)
Beep 1 X or X = 80.

(24)
Beep Our original ratio was $\frac{X}{\text{over}}$ / 100. We found X=80, so let's put 80 in place of X.

(25)
Beep $\frac{80}{100}$ which is = to 80%. $\frac{80}{100}$ is the ratio form of 80%.

This final 80% tells us that 80% of the students took the test.

(26)
Beep Here is another problem. See if you can set up the proportion to solve it.

9 of the 12 cars in the race crossed the finish line. What percent of the cars crossed the finish line?

You have 60 seconds to set up the proportion.

(27)
Beep Does your proportion look like this?
(pause) Now ask your teacher for the problem sheet that goes along with this program.

STORY PROBLEMS AND PERCENT
(Percent in Ratio Form)

1. Jim threw the bowling ball down the alley and knocked over 8 of the 10 pins. What percent of the pins did he knock over?

See the two words <u>what percent</u>? This tells you to write the ratio $\frac{X}{100}$.

Now we have to decide what to do with the 8 and the 10. If Jim were to knock over all 10 pins, what percent of the pins would this be? Wouldn't 10 pins be 100%? Then the 10 must go opposite the 100 on the same level. The 8 goes in the other position.

We now have a proportion that looks like this: $\frac{X}{100} = \frac{8}{10}$

Now cross-multiply: $10\ x = 800$

Divide by 10: $X = 80$ This tells us that Jim knocked over 80% of the pins.

2. The math club has 15 members. At the last meeting only 12 members were there. What percent of the members were there? Set up the ratio: $\frac{X}{100}$
Now can you finish the problem?

Your proportion should be $\frac{X}{100} = \frac{12}{15}$ because if all 15 members were there, they would have 100% of their members present. This is why the 15 goes on the same level as the 100. Your answer should be 80%.

3. The typing class had 20 students in it. Fifteen of the students were girls. What percent of the students were girls?

4. Forty people took the test. Only 26 passed it. What percent of the people passed the test?

5. Roy had a score of 42 on a test with 70 points. What percent of the total points did he have?

RATIO AND PERCENT

This program has two problems involving percent. Watch the steps which use the ratio form of percent to solve the problems. Every time you hear this sound ---, you should press the button to see the next slide. You should now have slide one in front of you. If you do not, press the button until it appears.

On slide one, you see an equation: $9\% = \frac{9}{100}$, $\frac{9}{\text{over } 100}$ is the way you write 9% in ratio form.

(2)
Beep On slide 2, you see the equation $5\% = \frac{5}{100}$. Notice that again you have 100 as the second term (that is, the bottom number) of the ratio. This is always true when you are asked to change a percent to ration form.

(3)
Beep Here is the first problem we want to solve.

Laura was told that she had to answer 75% of the questions on the test correctly in order to pass it. If the test had 80 questions on it, how many correct answers did she need to pass the test?

Can you find the only % that is mentioned in the problem? Isn't it the 75%? Let's write this % in ratio form.

(4)
Beep $75\% = \frac{75}{100}$ Let's put this ratio on a side by itself. This will help us solve the problem.

(5)
Beep $\frac{75}{100} =$ Now we have to set up the other side of the proportion. Before we do this, let's look at our original problem.

(6)
Beep Same as number 3. What other number do you see in this problem? The 80? Remember, the 80 is the total number of questions on Laura's test. What percent would she get if she got all of these right? Wouldn't she have 100% if she got all 80 questions right?

(7)
Beep So this 80 must go right across from the 100 on the same level. We have one empty spot left. Let x be the number of problems Laura must do correctly to pass the test. The proportion should now look like this:

(8)
Beep $\frac{75}{\text{over } 100} = \frac{X}{\text{over } 80}$

(9)
Beep Now, cross-multiply. 100 X = 6000

(10)
Beep Divide both sides by 100

X = 60 And X was the number of questions Laura had to answer correctly to pass the test.

(11)
Beep Here is the second problem.

There are 20 students in the class. One day, 25% of them were absent. How many students were absent that day?

Look for the % in this problem. Can you write it in ratio form?

(12)
Beep $25\% = \frac{25}{100}$ Let's put this ratio on a slide by itself.

(13)
Beep 25 over 100 = Let's look at our original problem again.

(14)
Beep Same as number 11.
If all 20 of the students had been absent, there would have been 100% absent, so the 20 must go opposite the 100.

(15)
Beep See that the 20 is opposite the 100 on the same level? Again this is because if all 20 had been absent, there sould have been 100% absent. In this case, we let X be the number of students absent and put this over the 20. 25 over 100 = X over 20

(16)
Beep Now, cross-multiply. 100 X = 500

(17)
Beep Divide both sides by 100
X = 5 and X is the number of students absent.

(18)
Beep Try setting this problem up in the form of a proportion in the next 60 seconds.

There are 25 students in our mathematics class. 40% of them are girls. How many students are girls?

You now have 60 seconds time.

(19)
Beep Your proportion should look like this:

$$\frac{40}{100} = \frac{X}{25}$$

Now, ask your teacher for the problem sheet that goes along with this program.

RATIO AND PERCENT

1. Jack answered 65% of the 80 questions correctly. How many of the questions did he answer correctly?

   Write the 65% in ratio form: $\frac{65}{100}$

   If Jack had answered all 80 questions correctly, he would have had 100%. So the 80 must go opposite the 100 on the same level:

   $$\frac{65}{100} = \frac{X}{80}$$

   Cross-multiply now and solve for X. X is the number of questions Jack answered correctly.

2. 75% of the students in the class had their homework done. There were 28 students in the class. How many of them had their homework done?

   Your ratio for this problem should look like this: $\frac{75}{100} = \frac{x}{28}$

3. 15% of the 2000 students in the school were absent. How many students were absent?

4. 87% of the 500 people in the parade were Irish. How many of the people were Irish?

5. 10% of the 2,350 students had all A's on their last report card. How many of them had all A's?

VIDEO TAPE I

I. TITLE

A. Fractions and the Ruler

$\frac{1}{2}$'s and $\frac{1}{4}$'s

II. INTRODUCTION

A. Relate how a fraction gets its name

1. paper

2. wire

3. piece of 2 x 4

a. Show a piece of 2 x 4 and 2 smaller and equal pieces whose combined lengths are equal to the length of the original piece.

b. Talk about the length of 1 of the two pieces.

c. Transfer idea to a line segment.

(1) Name points of line segment

$$0, \frac{1}{2}, \frac{2}{2}, \frac{3}{2}, \frac{4}{2}$$

d. Use same development as in a., b., and c. above but with 4 smaller and equal pieces whose combined lengths are equal to the length of the original piece.

(1) Name points of line segment: $0, \frac{1}{4}, \frac{2}{4}, \frac{3}{4}, \frac{4}{4}$

4. Enlarge ruler to point out fractional parts on it.

a. Marks of ruler have been drawn to coincide with points on line segments mentioned in (1) under c. and d. above.

5. Closeup of marks on regular 12" ruler.

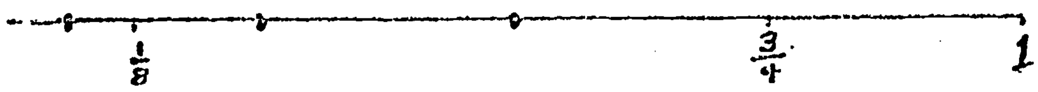
III. CONCLUSION

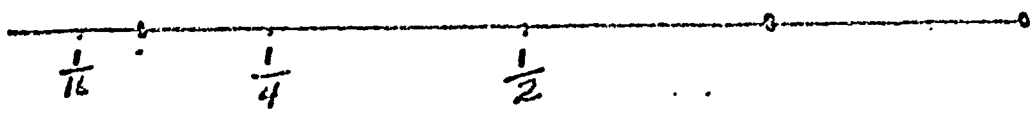
A. Point out that each mark on the ruler represents its distance from the end of the ruler.

VIDEO TAPE I

$\frac{1}{2}$'s & $\frac{1}{4}$'s

1. Label the three points on these lines as I have done the other three.

a) 

b) 

2. Measure these lines:

a) ________________________________

b) ______________________________________

c) _____________________

d) ____________________________

3. Draw lines having the lengths indicated.

a) 2-3/16

b) 1-1/2

c) 3-3/4

d) 1-1/8

VIDEO TAPE II

1. Mrs. Jaskin bought 12 cans of soup which were on sale at 3 cans for .25¢.

   What was the sale price for the goods?

2. Mr. and Mrs. Banta bought 2 tickets to see the play "Oklahoma" at Franklin High School. The tickets cost $2.25 each. Then they went to the " Skipper's Table" and bought 2 dinners costing $2.00 each. What was the total cost of the evening?

3. On the way home the other day, Joe stopped in at a gas station and had 8 gallons of gas put in his car, each costing $.32. He also needed 2 quarts of oil at $.55 for each one. What was his total bill?

VIDEO TAPE II

Story Problems

1. Soup is priced at 2 cans for $ .37 Find how much you will have to pay for a dozen cans.

   a) Step 1 Draw a picture. Let little rectangles be the cans of soup. Since the problem says a dozen, you will need 12 rectangles. Since each group of 2 cans will cost $ .37, you should draw the little rectangles in groups of 2.

      Step 2 Beside each group of 2 cans, write $ .37

      Step 3 Add the $ .37's

      Step 4 Label your answer "Cost of the dozen cans of soup"

2. John gets $ 2.50 allowance per week. He must spend $ .10 per day for milk in school and $ .20 per day for bus fare. Draw a picture as you did in problem 1. Then tell me how much he has left for other items.

3. A car traveled 125 miles and used 5 gallons of gas. How far did it travel on 1 gallon of gas? Draw a picture as you did in problem 1.

4. You take home $ 138.50 a week for the 12 weeks during the summer. How much money did you take home in all? Again, draw a picture as in problem 1.

5. Oranges are priced at 3 for $ .41 Jane wanted 18 oranges. How much did she have to pay for them? Draw a picture.

VIDEO TAPE III

AREA PROBLEMS

OUTLINE

1. Line segments have length. Draw several segments

2. Area is the name applied to surface enclosed by segments.

Show 3 segments

4 segments

Put the four segments in a form called a rectangle.

Area of a rectangle is the amount of surface enclosed by four segments.

Now to find this area. Before we start,

Area is measured in square units Square inch

Square foot

Square yard

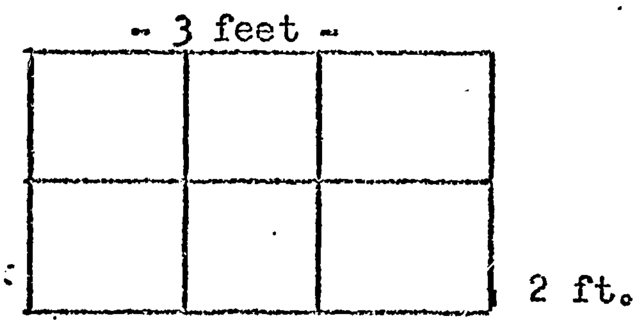
Square mile

So, 6 square inches means 6 little square areas 1 inch on a side

8 square feet means 8 square areas, 1 foot on a side

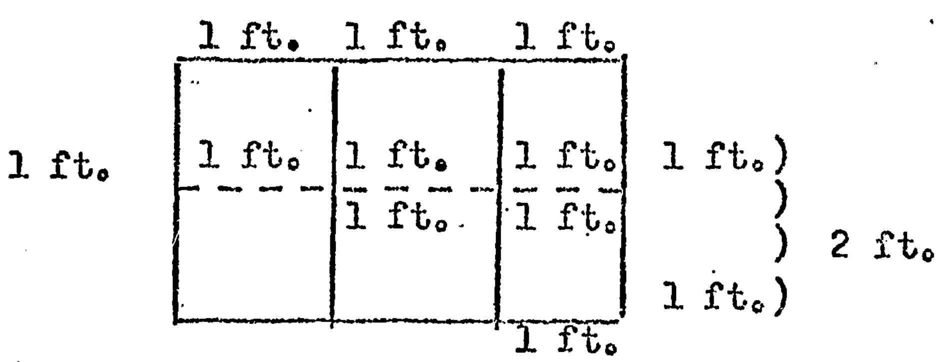
4 square yards means 4 square areas, 1 yard on a side

2 square miles means 2 square areas, 1 mile on a side



Now to find area of our rectange 3 feet long, 2 feet wide

3 feet long separate into feet



6 square feet is area of our rectangle

Video Tape III
Area Problems (Continued)

There is a shortcut to area problems. How many square feet along top of rectangle? 3 --- same as length.

How many square feet in each of the 3 columns? 2 -- same as width.

2 in each of 3

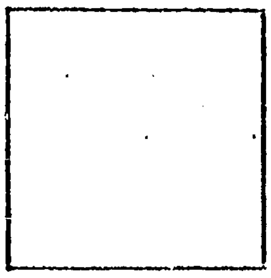
2 + 2 + 2

2 x 3

Width x length = number of square units, or area.

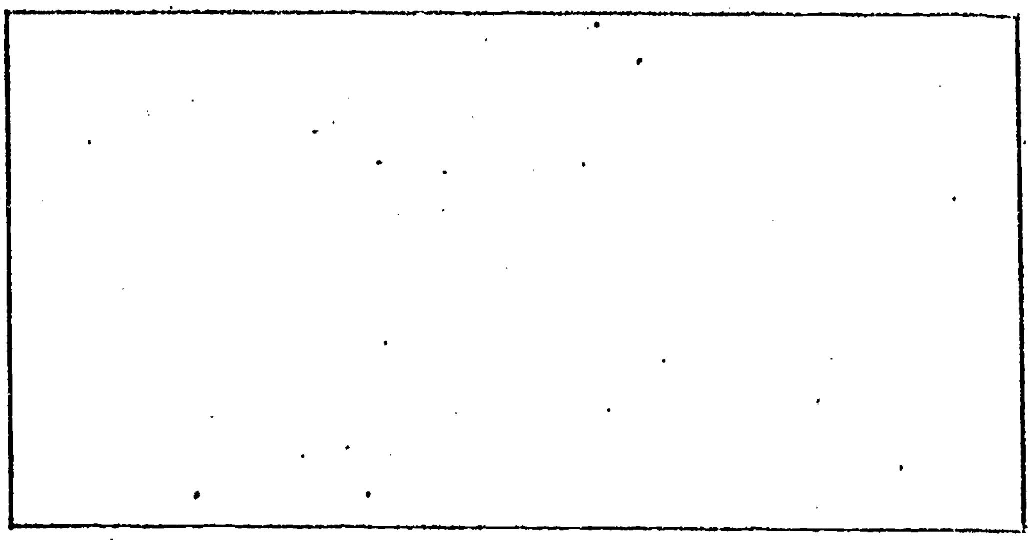
VIDEO TAPE III

Area



1. FILL IN THE BLANKS:

   This in 1 _________ inch.
   It is on unit that is used
   to measure ____________.



2. This rectangle is 4 inches long and 2 inches wide.

   How many squares like the one in problem 1 can fit into this rectangle? Draw in the lines that will show how many square inches are in this rectangle.

   You should have 8 small squares.

   Its area is 8 square inches.

3. The formula for area is:

   A = L X W

   A stands for area
   L stands for length
   W stands for width

   The length of the rectangle is 4 inches. The length also tells one how many square inches are in one row.

   The width of the rectangle is 2 inches. The width also tells one how many square inches are in each column.

   Then the area of the rectangle in problem 2 is found like this:

   A = 4 x 2 = 8 square inches.

   Remember your answer must always be labeled square units when indicating area. The unit will be the inch, the foot, the yard, or whatever you are using as the unit of area. For example, your answer will be labeled square inches, square feet, square yards, etc.

4. Find the area of a rectangle five feet long and three feet wide. Show all work.

5. Find the area of a square foot in square inches. Show all work.

VIDEO TAPE IV

VOLUME TO SHOW

I. Have a box to fill with sand. To measure amount of sand in box, we use a unit of measure called a cube.

Pick up grains of sand and ask how easy it would be to measure it in feet; in yards?

Measure it by how much sand can be contained in a cube of some size -- cubic inch, cubic foot, cubic yard.

Take the box full of sand

3 x 2 x 1 How many cubic inches does it contain? To start with, each cubic inch will have 1 square inch as the size of one of its faces. Let's consider only the bottom of this box -- 3 x 2; 6 square inches. If we put a cubic inch on each of these square inches, how many will it take to cover all 6? Six cubic inches.

That means that this box of 3 x 2 x 1 has a volume of 6 cubic inches. Or, as far as sand goes, this box will contain 6 cubic inches of sand.

Let's take another box that is 3 inches long, 2 inches wide and 3 inches high. Can we figure out what it's volume is? The area of its base is 6 square inches. How many cubic inches will fit on it? Six. If we take a look at this box 3 inches high, we must add more layers to this first one. Take 6 more. Still don't have right size. So take 6 more. Now we have shown the number of cubic inches in this box.

6 +	6 +	6	=	18
1st	2nd	3rd		

How many layers? 3. What was the height of our box? 3.

So we have a formula for volume of a solid; length x width x height =Volume

The length x width gives us the area of the bottom. Then when we take the area of the bottom and multiply it by 1, we get the cubic inches in the first layer. Finally, when we look at the height that tells us how many layers of cubes we have. In this example, we have 3 layers with 6 cubes in each, for a volume of 18 cubic inches.

Using the formula 3 x 2 x 3 = 18 cubic inches.

Now, using this same formula, can you tell me what the volume of a box . . . . . 6 inches long
4 inches wide
and 3 inches high is?
Using the formula 5 x 3 x 2 = 30 cubic inches

Let's use our cubic inches to build a picture of it. First of all, area of our base is 5 x 3 = 15 square inches.

15 x 1 = number of cubic inches in 1st layer.

We have 2 of these layers (Build on the 2 layers)

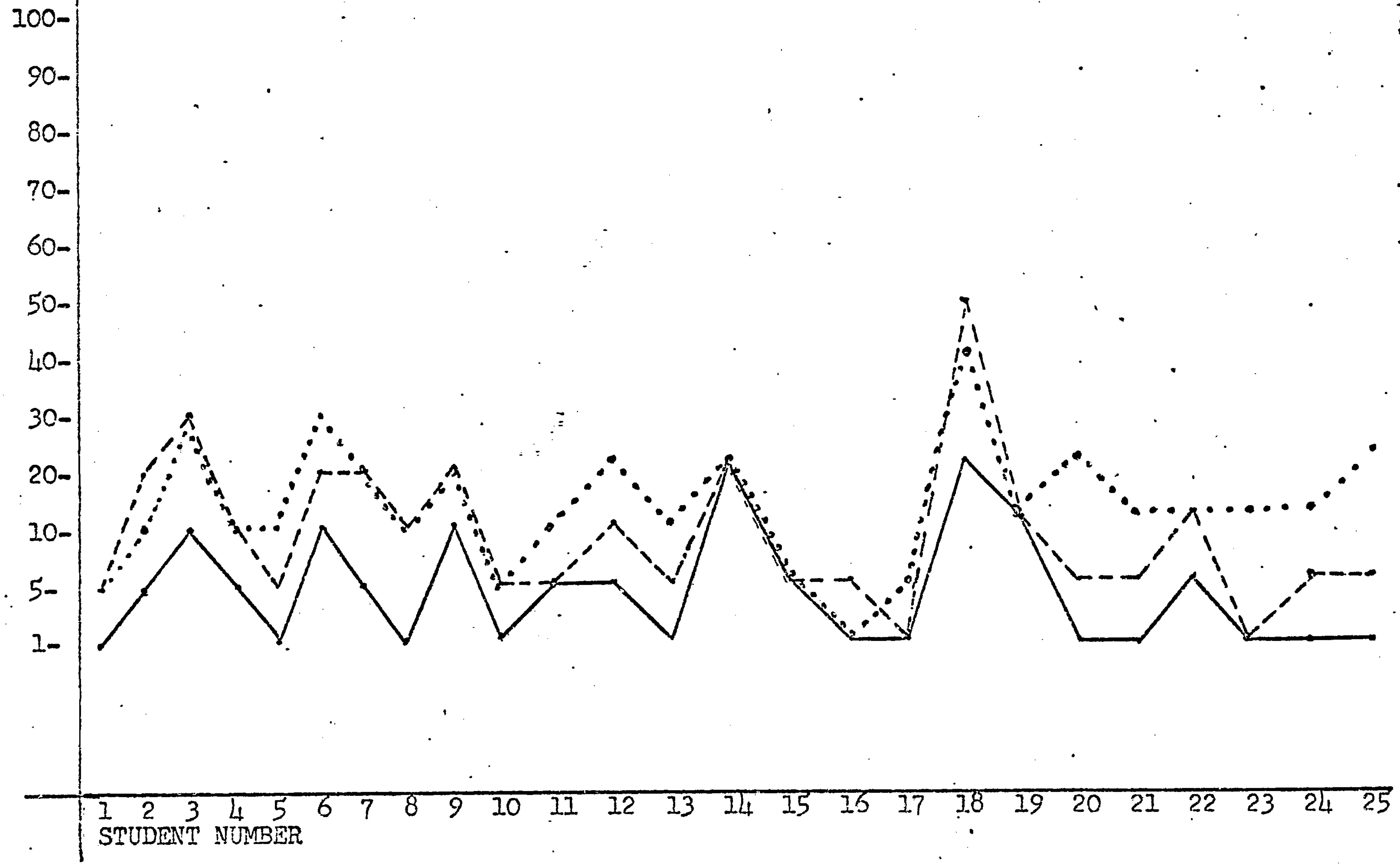
Then go over dimensions of box, then count cubes.

Today, we have had a very short look at volume. Try the five volume problems which go along with this program.

Exhibit "E"

LEE CLARK ARITHMETIC FUNDAMENTALS SURVEY TEST

PERCENTILE RANK



The 25 students listed here are those who had project materials available to them both semesters and were tested three times with Lee Clark Arithmetic Fundamentals Survey Test.

Testing Period

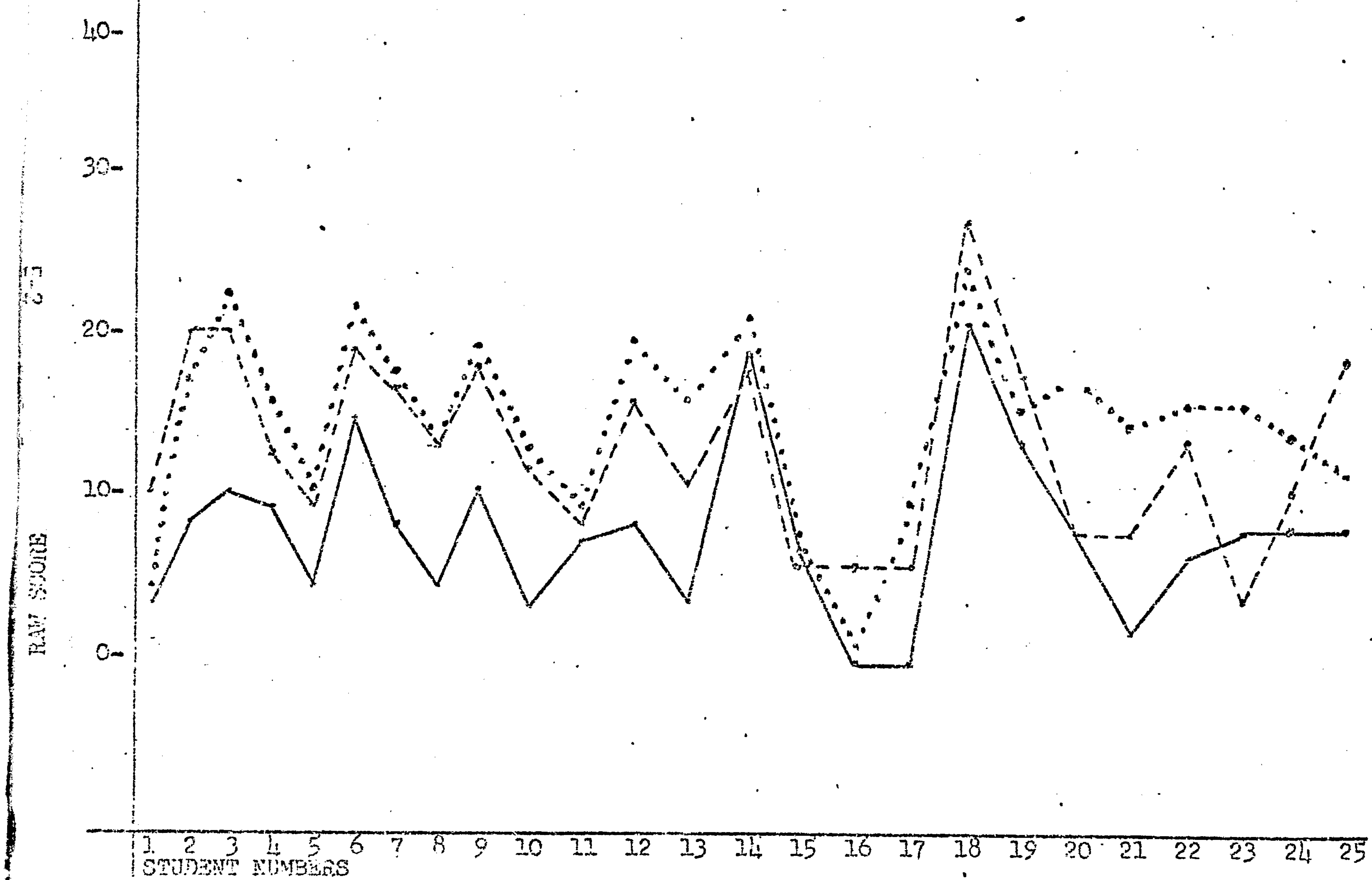
——— September, Form A

– – – January, Form B

······· June, Form A

LEE CLARK ARITHMETIC FUNDAMENTALS SURVEY TEST

RAW SCORE



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Testing Period

September, Form A

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June, Form A