This manual outlines the nature of some of the major curriculum projects in mathematics and lists materials which are available from these projects. An introductory statement concerning the history and philosophy of each program is indicated. The curriculum projects reported on are: (1) Boston College Mathematics Institute, (2) Cambridge Conference on School Mathematics, (3) Commission on Mathematics of the College Entrance Examination Board, (4) Computer-Based Mathematics Instruction, (5) Greater Cleveland Mathematics Program, (6) The Madison Project of Syracuse University and Webster College, (7) Mathematical Association of America, (8) Minnesota School Mathematics and Science Teaching Project, (9) National Council of Teachers of Mathematics, (10) New York State Mathematics Curriculum Activities, (11) Ontario Mathematics Commission, (12) Patterns in Arithmetic, (13) School Mathematics Study Group, (14) University of Illinois Arithmetic Project at Educational Services Incorporated, (15) University of Illinois Committee on School Mathematics, and (16) University of Maryland Mathematics Project. (RP)
MATHEMATICS CURRICULUM MATERIALS

Wisconsin Department of Public Instruction
William C. Kahl, State Superintendent
FOREWORD

In recent years, mathematics courses and mathematics curriculum materials for kindergarten through high school have been developed at a rate that makes it very difficult for classroom teachers to stay up-to-date. Finding and reading current materials, and discovering which developments offer the most promise for implementation in local schools are even more difficult.

However, in spite of the difficulty, it remains the professional responsibility of teachers and school administrators to take advantage of all resources which promise to improve their educational programs. To help with this responsibility in mathematics education, the Wisconsin Department of Public Instruction, with support of Title III of the National Defense Education Act, has attempted to make information about such developments easily available through this publication.

It is intended that those who make use of this booklet treat it as an introduction to further investigation. The listing of materials for individual projects is not exhaustive. The philosophies and learning theories upon which the mathematics teaching programs are based are only briefly described. There has been no attempt at critical evaluation. For these reasons an extensive bibliography is included as well as the addresses of project directors from whom much additional information can be obtained. It is hoped that, when a particular project or some of its materials have special pertinence to a local program, the reader will make the extra effort necessary to obtain greater information.

William C. Kahl
State Superintendent
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INTRODUCTION TO MAJOR NEW MATHEMATICS CURRICULUM PROJECTS

The term "new mathematics" is applied indiscriminately nowadays to just about anything which is being taught in mathematics. In particular, the term is sometimes used to refer to simply a new emphasis upon mathematics and a use of modern methods of instruction; in such cases, the content of a mathematics program remains substantially what it has been in the past. In connection with new curriculum programs, sponsored for the most part by agencies of the federal government or major private foundations, the new mathematics is something more than this; it contains new content as well as a new approach.

A lag of over four hundred years exists between the discovery of new knowledge in mathematics and the appearance of this knowledge in the school mathematics curriculum. The development of mathematics did not end with Newton. On the contrary, as has often been pointed out, more mathematics has been created since the year 1900 than in all of previous history. As an example of this lag, over eighty years ago Cantor showed that sets and the idea of a one-to-one correspondence are the basis for an understanding of arithmetic. Until recently these ideas were taught only to graduate mathematics students; now they are frequently introduced in kindergarten.

In addition, applications of mathematics are playing an ever-increasing role in modern life, particularly in scientific and economic areas, but also in many other fields once considered completely non-mathematical. Automation, high-speed computers, and space exploration are obvious examples of areas which are using profound mathematics—and are in turn contributing to the fund of mathematical knowledge.

An understanding of the role of mathematics in our society has, thus, become a prerequisite for intelligent citizenship. The student today cannot know what mathematical skills will be necessary in his future career; perhaps he will be using mathematics that has not yet been developed. It is important that mathematics be taught in such a way that a student is able in later life to acquire new mathematical skills when it is necessary for him to do so. The student must be given the tools needed in the continuing process of self-education. Consequently, the new programs emphasize methods of acquiring knowledge, rather than rote memorization of facts.

Recently much research has been done on the processes involved in learning mathematics and children's capabilities for learning mathematics. The new programs have attempted to utilize the results of such research. In particular, an effort has been made to build a coherent structure in mathematics, rather than to teach an accumulation of isolated facts. Research has also shown that children can easily absorb many topics in mathematics at a much earlier age than that at which such topics were traditionally taught. A characteristic of nearly all of the new programs is the introduction of some topics at an earlier grade level.

In new mathematics the emphasis is on concepts. The various programs differ in many ways, but they all have certain elements in common:

1. Explanations are given of the why as well as the how. The student learns that there is a valid reason for every bit of manipulation—"symbol pushing."

2. Extensive use is made of deductive reasoning and proof. The basic laws of logic are applied to algebra as well as to geometry.

3. The structure of mathematics is emphasized. Mathematics is developed as an organized body of knowledge, founded on a surprisingly small number of basic assumptions.

4. The discovery method of teaching is utilized. Questions and illustrative examples often lead the student to make and test conjectures of his own. As a result, both the teaching and the learning of mathematics are more interesting and rewarding than before.
5. Great emphasis is placed on the precise use of language. Definitions are stated carefully. The ability to read intensively for meaning is essential for success.

6. The new courses are built on unifying ideas (structure, operations and their inverses, logical deductions, valid generalizations, etc.) that are essential for the understanding of advanced mathematics. There is, therefore, less need for unlearning and relearning at higher stages.

The new mathematics curricula have not been in operation long enough for any kind of long-range evaluation studies to have been made. Most students seem to enjoy the new mathematics better than the old. Evidence indicates that students in the new programs do at least as well as other students on traditional tests and that they also learn more of the basic principles underlying mathematics.

In the pages which follow, an attempt has been made to outline the nature of some major curriculum projects in mathematics and to list materials which are available from these projects. The introductory statements concerning the history and philosophy of each program are generally extracted or summarized from materials published by the project under discussion.

Other publications which give information about curriculum projects in mathematics are An Analysis of New Mathematics Programs, National Council of Teachers of Mathematics, 1963, and the reports of the Information Clearinghouse on New Science and Mathematics Curricula, compiled under the direction of J. David Lockard, Science Teaching Center, University of Maryland, College Park, Maryland 20742. The Fourth Report is scheduled for publication during the summer of 1966 and will be available free upon request. Another pamphlet, now somewhat out-of-date, Studies in Mathematics Education: A Brief Survey of Improvement Programs for School Mathematics, published in 1960 by Scott, Foresman and Company, contains information about the beginnings of the various programs, as well as descriptions of projects no longer in operation.
II SOME OF THE MAJOR NEW MATHEMATICS CURRICULUM PROJECTS

BCMI

BOSTON COLLEGE MATHEMATICS INSTITUTE (BCMI)

DIRECTOR AND ADDRESS: Rev. Stanley J. Bezuska, S. J.
Boston College
Chestnut Hill, Massachusetts 02167

GRADE LEVEL: 7-12; teacher training

The Boston College Mathematics Institute has grown out of a modern mathematics program for mathematics majors at Boston College. This program was started in 1953. In 1957, the program took on the formal organization of BCMI after it had become apparent that incoming college freshmen had little or no background in modern mathematical terminology and basic mathematical concepts or in thinking of mathematics as a structure. The chief goal of the institute has been the re-education of secondary school teachers in the elements of contemporary mathematics. With this goal in mind, the institute has prepared modern mathematics courses for secondary schools and courses for increasing the professional competence of teachers, with support from the National Science Foundation. Throughout the program there has been an attempt to relate modern mathematics with the past history and development of mathematics. Teacher preparation has been carried out during the academic year and summer by means of inservice institutes and correspondence courses.

PUBLICATIONS

Junior High School Texts:

A two-year course, Contemporary Progress in Mathematics, has been prepared for students in the upper 50 percent of the ability range and is accompanied by a manual for the teacher. In addition, a seventh-grade course, General Contemporary Mathematics, has been developed for students in the lower 50 percent of the ability range. Another course, Sets, Operations and Patterns, was originally developed for the ninth grade, but is now recommended for very bright seventh and eighth grade students who are beginning their study of mathematics. This course has a teacher's supplement.

High School Texts:

Two courses are available for high school students: Computer Oriented Mathematics and Calculus.

Reference Materials:

A reference book, Heritage Builders in the Arts and Sciences, describes important people in the arts and sciences and emphasizes those in mathematics and the physical sciences.

Materials for Teachers:

Three correspondence courses are available for teachers, all entitled, Cooperative Unit Study Program. Course IE is for elementary school teachers, and Courses 1 and 2 for junior and senior high school teachers.

FUTURE PLANS: An elementary text series is contemplated.
CAMBRIDGE CONFERENCE ON SCHOOL MATHEMATICS

GRADE LEVEL: K-12

The Cambridge Conference was held June 18 to July 12, 1963, in Cambridge, Massachusetts under the sponsorship of the National Science Foundation and the administration of Educational Services, Inc., to discuss the future of mathematics curricula. The main purpose was to reconsider the structure of mathematics education and to sketch a rough outline of a possible new framework for the primary and secondary school. Participants included twenty-five mathematicians and users of mathematics from universities and industry. The report of this conference, which has become known as "The Cambridge Report," details the recommendation of the conference. It has as its stated purpose to present "tentative views upon the shape and content of a pre-college mathematics curriculum that might be brought into being over the next few decades." This highly-controversial report suggests a revised mathematics curriculum for grades K-12, which essentially entails compressing the mathematical program currently taught during the period of twelve years of school and three years of college into just twelve years. That is, the report suggests that this program be completed by the end of high school.

PUBLICATIONS


COMMISSION ON MATHEMATICS OF THE COLLEGE ENTRANCE EXAMINATION BOARD

ADDRESS: Commission on Mathematics
College Entrance Examination Board
425 West 117th Street
New York 27, New York

GRADE LEVEL: 9-12

The College Entrance Examination Board prepares entrance examinations for universities and colleges throughout the country. The Commission on Mathematics, headed by Professor A. W. Tucker of Princeton University, prepared its report from 1954 to 1959 through a special grant from the Carnegie Corporation of New York. Its purpose was to consider broadly the college preparatory mathematics curriculum in the secondary school and make recommendations with regard to its modernization, modification, and improvement. The overriding objective of the Commission was to produce a curriculum suitable for students and oriented to the needs of mathematics, natural science, social science, business, technology, and industry in the second half of the twentieth century.

The report of the Commission was in a sense the "grandfather" of the current revolution in mathematics instruction. Many of its recommendations have now been adopted by the major commercial publishing companies as well as by many of the curriculum studies outlined in this publication. It is still considered a standard reference on the "new mathematics," although its recommendations are less radical than some of the proposals which have since appeared.

PUBLICATIONS

The Commission's report came out in May 1959 in two parts. The first part, Program for College Preparatory Mathematics, includes detailed outlines of its recommended
courses. It also states the Commission's premises, its recommendations for teacher training, and its proposals for articulating high school and college programs in mathematics. The second part, Appendices, provides information, instruction, and enrichment to assist teachers in carrying out the Commission's recommendations. The appendices amplify and clarify certain of the Commission's recommendations. Some of the appendices introduce ideas and facts that may be new to some teachers; others set forth new methods of approaching old topics.

CBMI

COMPUTER-BASED MATHEMATICS INSTRUCTION (CBMI)

DIRECTOR AND ADDRESS: Professor Patrick Suppes
Ventura Hall
Stanford University
Stanford, California 94305

GRADE LEVEL: 1-5

The Computer-Based Mathematics Instruction program is an outgrowth of two earlier projects directed by Professor Suppes: Set Theory in First Grade and Geometry for Primary Grades. During these earlier projects, textbooks were produced for each of these courses and a course in mathematical logic was also developed for able fifth-grade students.

The current project is an attempt to develop a computer-based curriculum for the elementary school. Because the project has been conducted on a small scale at Stanford with facilities for only six students to work independently at one time, it has been possible in the computer-based environment to use much deeper behavioral analysis than could be used in ordinary curriculum work. Continuous study using the tools of modern learning theory has been carried out.

PUBLICATIONS

Elementary School Texts:

The Sets and Numbers books, grades K-6, which were developed by the set theory project are available commercially from L. W. Singer Co. *First Course in Mathematical Logic* has been published commercially by Blaisdell Publishing Co. A text, *Geometry for Primary Grades*, was produced under the geometry project.

Materials for Teachers:

A number of descriptive brochures and reprints of articles published in professional journals are available free upon request.

FUTURE PLANS: Present courses are being continued and revised. During the summer of 1966, a computer-based system with 16 terminals will be installed in a local school so that the students will not have to go to the Stanford University Laboratory.

GCMP

GREATER CLEVELAND MATHEMATICS PROGRAM (GCMP)

DIRECTOR AND ADDRESS: George Baird
Educational Research Council of Greater Cleveland
75 Public Square
Cleveland 13, Ohio

GRADE LEVEL: K-8; teacher training
The Educational Research Council of Greater Cleveland was created in 1959 as an independent, non-profit organization to make a continuous effort to improve the quality of elementary and secondary education in twenty-one school districts in the Greater Cleveland area (not including the Cleveland City Public Schools). Its first major project was a research and implementation study in mathematics education, the aim of which was "to develop a comprehensive, sequential mathematics program for all children in grades kindergarten through twelve, a program that is both mathematically correct and pedagogically sound."

The GCMP is the outgrowth of this project. It is a concept-oriented mathematics program with primary emphasis on thinking, reasoning, and understanding, rather than on purely mechanical responses to standard situations. Much emphasis is put on the discovery method. Only after the students have discovered fundamental concepts and the logical structure of mathematics is the established symbolism introduced and are computational skills developed, often through story-problem situations. After extensive testing, the materials developed by the project were revised in 1962-63. Most of the materials are now available commercially.

**PUBLICATIONS**

**Elementary School Texts:**
The "Elementary Mathematics Series" for grades K-6 is available commercially from Science Research Associates, 259 East Erie Street, Chicago, Illinois 60611. Teachers' guides and many audiovisual aids are also available from SRA.

**Junior High School Texts:**
Texts for grades 7-8 are available directly from the Educational Research Council of Greater Cleveland.

**Materials for Teachers:**
In addition to the teachers' guides a separate booklet containing a more extensive treatment of the mathematical concepts developed in the GCMP was designed for in-service programs.

**Films:**
A series of teacher-training films, stressing both content and teaching methods, is available from SRA.

**FUTURE PLANS:** In line with the original guidelines of GCMP, the program will eventually include grades 9-12 as well.

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**MADISON PROJECT**

**THE MADISON PROJECT**
**OF SYRACUSE UNIVERSITY AND WEBSTER COLLEGE**

**DIRECTOR AND ADDRESS:** Robert B. Davis
Webster College
8756 Big Bend Boulevard
Webster Groves, Missouri 63119

**GRADE LEVEL:** K-9; teacher training

The Madison Project is so named because it was first operated by Syracuse University in the Madison Junior High School of Syracuse, New York in 1957-58 with low-I.Q., culturally-deprived seventh-graders. The full title of the project is "A Modern Mathematics Program as It Pertains to the Interrelationship of Mathematical Content, Teaching Methods and Classroom Atmosphere" (The Madison Project). In 1962 a second
office was started in connection with the Weston, Connecticut Public Schools, and in 1961 a third office was opened at Webster College. The activities of the project have included studies of student learning and growth and of teacher growth, evaluation of the curriculum, testing of general curriculum theory, and the operation of the project itself.

The materials developed by the Madison Project have been primarily of a supplemental nature, intending to broaden the curriculum. Schools using the project materials generally use them once a week along with their own arithmetic programs. In this way, concepts of algebra, coordinate geometry, and logic as well as relations of mathematics to science are introduced into the elementary grades.

In an attempt to instill a more creative flavor into the school mathematics curriculum, the project has emphasized the discovery approach, "experience-without-formal-instruction," which always precedes formal instruction. In fact, the project has found that in many instances the actual formal instruction can be eliminated completely. Another curriculum objective has been to achieve greater variety in the child's experience through "seminars," games, physical apparatus, etc.

The principal objectives for the student in this program are to develop his ability to discover patterns in abstract situations; to develop a habit of exploratory behavior; to learn really basic mathematical ideas, such as variable, function, graph, matrix, isomorphism, and implication; to acquire a reasonable mastery of important techniques; and to know certain basic mathematical facts.

From this combination of objectives comes what the project calls an "informal formal" approach. The teaching methods are informal; the mathematics taught is formal.

Because of the nature of the program, much effort has been devoted to the development of the teachers. Consequently, much of the project has been focused on the production of courses and films for teacher training.

**PUBLICATIONS**

**Elementary School Texts:**

Two texts, *Discovery in Mathematics* and *Matrices, Logic and Other Topics*, were published commercially by Addison-Wesley Publishing Co., Inc., Palo Alto, California. Each consists of a student discussion guide and a text for teachers.

**Materials for Teachers:**

Two inservice courses, which come in a package including films, written materials, and laboratory equipment, are available for teachers. A volume on *Axioms for Arithmetic and Algebra* is intended to introduce elementary teachers to the axiomatic approach. A large number of films of actual classroom lessons are available at various grade levels. Also available are some taped classroom lessons. Some of the films have pamphlets accompanying them.

**Reports:**

A large number of reports of the project's activities as well as reprints of articles from professional journals are available from the project. They describe its history, philosophy, and accomplishments. In particular, *Newsletter No. 1*, issued in July 1965 listed materials and prices. Pamphlets describing the work of the project, available for a nominal fee, include "The Madison Project--A Brief Introduction to Materials and Activities," and "Goals for School Mathematics: The Madison Project View." A much longer and more complete description of the activities of the project to date was issued in 1965 in a report to the U. S. Office of Education.

The project has also issued two experimental course reports, one on kindergarten and one on ninth grade courses.
Laboratory Equipment:

To enable an individual student to learn independently, the Madison Project has developed a number of "shoeboxes," each containing written and physical materials designed to lead the student to a discovery of some specific mathematical ideas. These materials may be used in a library or resource center, in a mathematics laboratory or by a specialist teacher.

FUTURE PLANS: Materials relating science and mathematics are planned as is the development of ninth-grade and kindergarten courses and individualized study materials for grades 6-12.

MATHEMATICAL ASSOCIATION OF AMERICA (MAA)

DIRECTOR AND ADDRESS: H. M. Gehman
Mathematical Association of America
SUNY at Buffalo
Buffalo, New York 14214

The Mathematical Association of America is a professional organization devoted to undergraduate collegiate mathematics. A number of its activities are of interest to the school mathematics teacher, however. Its official journal, The American Mathematical Monthly, contains reports of curriculum projects and of research into educational problems in its section "Mathematical Education Notes."

A series of films, "Mathematics Today," has been produced by the association's Individual Lectures Project, Committee on Educational Media. These films are distributed by Modern Learning Aids, 160 East Grand Avenue, Chicago, Illinois 60611. These basic films require little or no mathematical knowledge and could be used in high school mathematics classes. They cover such topics as mathematical induction, limits, topology, and intelligent guessing. Some of the collegiate films might also be used by advanced high school classes or by teachers.

The Committee on the Undergraduate Program in Mathematics (CUPM) has published a number of recommendations for undergraduate mathematics curricula. These are available free from the CUPM Central Office, P. O. Box 1024, Berkeley, California 94701. Of particular interest to school teachers are "Recommendations for the Training of Teachers of Mathematics," "Mathematics Text Materials for the Undergraduate Preparation of Elementary School Teachers," and "Course Guides for the Training of Teachers of Junior High and High School Mathematics." They have also printed a Basic Library List, which lists some 300 books in an attempt to define a minimal college mathematics library. Although aimed primarily at colleges, this list could be used by the high school teacher for suggestions of books to add to his school or personal library.

MINNEMAST

MINNESOTA SCHOOL MATHEMATICS AND SCIENCE TEACHING PROJECT (MINNEMAST)

DIRECTOR AND ADDRESS: James H. Wertsch, Jr.
Minnesota School Mathematics and Science Center
720 Washington Avenue, S. E.
University of Minnesota
Minneapolis, Minnesota 55455

GRADE LEVEL: K-9; teacher training

MINNEMAST is a long-range curriculum project established for the purposes of determining what children can learn, producing appropriate instructional materials, and preparing teachers to use these new materials effectively. The project began in 1961.
under the direction of Dr. Paul C. Rosenbloom, aided by a grant from the National Science Foundation. Course materials in mathematics and science are being developed, and experimental classes are being conducted concurrently to evaluate the appropriateness of the materials being developed.

Although MINNEMAST is working in both mathematics and science, the two projects are apparently independent of one another. The discussion presented here pertains only to mathematics.

In contrast to many of the other curricular programs in mathematics, MINNEMAST has made a very marked departure from the traditional curriculum and, consequently, cannot be taught by present teachers without consultant help. Several college courses for prospective mathematics teachers have been developed to help overcome this difficulty.

In grades K-9, MINNEMAST stresses three main mathematical structures: the real number system, Euclidean space, and a space with a measure. The emphasis in the early grades is on those areas where arithmetic, algebra, and geometry overlap.

**PUBLICATIONS**

**Elementary School Texts:**

Materials are available at cost for grades K-3. These are divided into twenty units, which may be purchased separately and which cover such topics as geometry, sets, measurement, numeration, and "Squareville."

**Materials for Teachers:**

A college mathematics course outline with sample chapters has been produced as has a mathematics methods course. There are also mathematics tests for grades K-3. A quarterly publication, Minnemath Center Reports, will be sent free to any teacher who asks that his name be put on the mailing list. A small brochure describing the project, "Questions and Answers about MINNEMAST," is also available.

**FUTURE PLANS:** Curriculum materials for grade 4 are currently being developed, and materials for grades 5-9 will follow. A college geometry film project is underway. Further information can be secured from Dr. Seymour Schuster at the address listed above.

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**NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS (NCTM)**

**DIRECTOR AND ADDRESS:** James D. Gates, Executive Secretary
1201 Sixteenth Street, N.W.
Washington, D. C. 20036

**GRADE LEVEL:** K-12; teacher training

The National Council of Teachers of Mathematics is the professional organization for mathematics teachers. It is a department of the National Education Association and is affiliated with the American Association for the Advancement of Science. Through its many committees and publications, it has long been a leader in curricular changes in mathematics.

**PUBLICATIONS**

(Note: It would be impossible to try to list all of NCTM's publications here. A brochure entitled "Current Publications" is available upon request. Only those publications which are pertinent to the various curriculum projects are listed here.)
Journals:

The two official journals of the NCTM are the Mathematics Teacher and the Arithmetic Teacher. These are probably the best periodic sources on modern trends in mathematics education. The March 1964 issue of the Mathematics Teacher carried an annotated list of articles and books about the changing mathematics curriculum at the secondary level.

A similar list for the elementary level appeared in the April 1964 issue of the Arithmetic Teacher. A bibliography of materials for parents was printed in the January 1966 issue of the Arithmetic Teacher.

Yearbooks:

Since 1926 the NCTM has published yearbooks, which have made significant contributions to the literature in this field. The yearbooks deal with timely problems in the teaching of mathematics. Most of the recent yearbooks, from the 23rd (1957) to the 29th (1964), have dealt with the problems of curriculum and of teaching modern mathematics.

High School Texts:

A series of booklets under the general title "Experiences in Mathematical Discovery" has been published for the non-college bound, ninth-grade student. Each of the five booklets costs $.50. They provide independent units for a general mathematics class.

Reference Materials for Teachers:

Most of the materials produced by NCTM fall into this category, but of particular interest are several pamphlets concerning the new curricula: An Analysis of the New Mathematics Programs, The Revolution in School Mathematics, The Secondary Mathematics Curriculum, Program Provisions for the Mathematically Gifted Student in the Secondary School, and Aids for Evaluators of Mathematics Textbooks. A book on computer oriented mathematics was written for secondary school teachers, but it is also of interest to students. A series of eight booklets entitled "Topics in Mathematics for Elementary School Teachers" may be purchased separately, as a set, or bound together as the 29th Yearbook. Each booklet gives the key principles for understanding a particular topic, such as sets, whole numbers, rational numbers, and numeration systems.

NEW YORK STATE MATHEMATICS CURRICULUM ACTIVITIES

DIRECTOR AND ADDRESS: Frank Hawthorne, Chief
Bureau of Mathematics Education
State Education Department
Albany, New York 12224

GRADE LEVEL: K-12

In New York State all students are tested by examinations developed by the Board of Regents. Consequently, curriculum is one of the basic duties of the State Education Department as it is so closely related to the examinations. Over a number of years the State Education Department has developed syllabuses and other publications concerning mathematics. These are free to teachers in New York and may be purchased at a nominal fee by those outside New York State. In general the types of materials which are available include syllabuses for grades K-9 and several reference pamphlets for teachers. Future plans call for writing units for grades 10-12. Note that no text materials have been produced by this group.
The initial encouragement, both moral and financial, for a thoroughgoing review of the provincial curriculum in mathematics was given by the Ontario Teachers' Federation. In August 1960, the Ontario Mathematics Commission was set up as a representative, independent body supported jointly by the Ontario Teachers' Federation and the Ontario Department of Education.

Among the objectives of the Commission as set forth in its constitution are the following:

1. Encouraging the production of experimental teaching material in mathematics and seeking the cooperation of the Department of Education in testing such material in the schools of the Province.

2. Cooperating with the universities, the Ontario Teachers' Federation, and other appropriate bodies in providing courses which will enable teachers to improve their qualifications and keep abreast of current experimentation in curriculum changes and in teaching techniques.

3. Undertaking curriculum research aimed at keeping the Province abreast of the best contemporary practice, and circulating such information to interested persons.

The writing of textbooks by the Commission followed very much the same pattern as that used by SMSG in this country. University, college, and high school teachers cooperated in the writing of the materials. Extensive testing of written materials was made by a large number of experienced high school teachers, after which revisions were made by the authors based upon the criticisms of the teachers participating.

The materials developed are an attempt to present "modern" mathematics, with emphasis on structure and concepts. No attempt has been made, however, to use the discovery method, and much greater emphasis is placed upon manipulative skills than generally appears in the curriculum materials produced in the United States.

**PUBLICATIONS**

**High School Texts:**

Three textbooks have been produced: Mathematics 9, Mathematics 10, and Mathematics 11. The material in Mathematics 9 deals with arithmetic and algebra; Mathematics 10 deals with algebra and geometry as does Mathematics 11.

**FUTURE PLANS:** Extensive testing of the materials produced is planned, and the suitable sequence of topics for grades 12 and 13 is being studied. (In Ontario, grade 13 is generally taken in high school.)
The Patterns in Arithmetic program is a television course in arithmetic for the elementary school. It is an attempt to meet the problem, faced by many school systems, of trying out new arithmetic materials with a staff that may not be adequately trained to teach some of the new ideas which are coming into the curriculum. The program is based upon the premise that the only way elementary school teachers can learn the "new mathematics" is to teach the "new mathematics." In order to establish necessary guidelines for such teachers, Patterns in Arithmetic has prepared a series of fifteen minute television programs to be shown to the students once or twice a week for thirty-two weeks of the school year. Each lesson is supported by pupil exercises and suggestions for the teacher. Each lesson develops the new ideas that will be used in class until the next lesson. The Patterns in Arithmetic program is self-contained, and no supplementary texts are needed when the television series is used.

The key ideas used in the program are sets, number systems, numeration systems, operations, the mathematical sentence, measurement, and geometry. Applications of these ideas are included, and certain enrichment topics are provided for more capable students.

Since the program is associated with the Research and Development Center for Learning and Re-Education, it has been able to incorporate recent research studies made at the center in recent years. In particular, the first grade program utilizes work which has been done in pre-number experiences. The teaching is based upon an informal approach to basic mathematical ideas. Although computational techniques are taught, the main emphasis is upon mathematical ideas.

The television programs were developed at the University of Wisconsin in cooperation with the Wisconsin School of the Air, WHA-TV. The first- and third-grade programs were shown during 1965-66, and the programs for grades 1-4 will be shown during 1966-67. In addition, the programs for grades 1 and 3 will be shown during 1966-67 on WMVS-TV, Milwaukee and over the stations of the Alabama State-Wide Educational Network.

PUBLICATIONS

Elementary School Texts:

Although there is no text for the television course, pupil exercise books are available.

Materials for Teachers:

There is a teacher's commentary to accompany each of the courses. In addition, the program has a manual which describes the program in detail.

FUTURE PLANS: The Patterns in Arithmetic programs will be submitted to field tests in the WHA-TV, WMVS-TV, and Alabama areas. Extensive data will be gathered relative to the success of the program. Future plans call for extending the program to higher grades.
SCHOOL MATHEMATICS STUDY GROUP (SMSG)

DIRECTOR AND ADDRESS: E. G. Begle
SMSG-Cedar Hall
Stanford University
Stanford, California 94305

GRADE LEVEL: K-12; teacher training materials

The School Mathematics Study Group was formed in the spring of 1958 by the president of the American Mathematical Society in consultation with the presidents of the National Council of Teachers of Mathematics and the Mathematical Association of America. Professor E. G. Begle of Yale University was appointed director of the study group, and an organizing committee was appointed, consisting of college and university mathematicians, high school teachers of mathematics, experts in education, and representatives of science and technology. Headquarters for SMSG were originally at Yale University, but they were later moved to Stanford University. Financial support for SMSG has come largely from the National Science Foundation.

The study group, realizing that the present world demands more mathematical knowledge on the part of more people than ever before and that future needs for mathematical skills will be even greater, decided that mathematics must be taught in such a way that students will be able in later life to learn the new mathematical skills which will be required of many of them. In order to meet this objective, SMSG decided to try to meet three requirements: 1) to provide an improved curriculum offering students not only mathematical skills, but also a deeper understanding of basic concepts and structures; 2) to attract and train more of those students who are capable of profitably studying mathematics; 3) to provide as much help as possible to teachers who are preparing themselves to teach challenging and interesting courses. Each of SMSG's projects has been concerned with one or more of these three objectives.

"The primary purpose of the SMSG is to foster research and development in the teaching of school mathematics. The work of SMSG consists primarily in the development of courses, teaching materials, and teaching methods. It is part of SMSG's task, in cooperation with other mathematical organizations, to encourage exploration of the hypothesis underlying mathematics education."

PUBLICATIONS

NOTE: Most of the texts produced by SMSG are available from Yale University Press, SMSG, 92A Yale Station, New Haven, Connecticut 06520. Most of the other materials are available from A. C. Vroman, Inc., 367 South Pasadena Avenue, Pasadena, California.

Elementary School Texts:

Texts for grades K-6 together with teachers' commentaries are available from Yale. The title of the series is "Mathematics for Elementary School." For kindergarten there is a teacher's book only. The texts for grades 4-6 presuppose a conventional program through grade 3. A considerable amount of informal geometry is included. Special editions of texts for grades K and 1 have been prepared for the culturally disadvantaged, and these are available from Vroman.

Junior High School Texts:

"Mathematics for Junior High School," Volumes I and II are available for grades 7 and 8 together with teachers' commentaries. In addition, "Introduction to Secondary School Mathematics," Volumes I and II have been produced for slower learners. These texts cover essentially the same material as the regular texts for grades 7-8, but the reading difficulty has been reduced to make them suitable for average or slightly below average students. All of these texts are available from Yale.
High School Texts:

First Course in Algebra, Geometry, and Intermediate Functions are the texts for grades 9, 10, and 11, respectively. For grade 12 and fifth-year programs, there are five one-semester texts available: Elementary Functions, Introduction to Matrix Algebra, Analytic Geometry, Calculus, and Algorithms, Computation and Mathematics (computer mathematics). Each text has a teachers' commentary. These texts are designed for average and above average college preparatory students.

In addition, there is an algebra course for slower learners, an alternate geometry text using coordinates, and a programmed algebra text. All of these texts are available from Yale, except for the text on computer mathematics which is available from Vroman.

Reference Materials:

"New Mathematical Library" is a series which consists of short expository monographs on various mathematical subjects, designed as enrichment material. They are written by mathematicians interested and well versed in the topics they treat. Seventeen titles are now available, covering such topics as contest problems, calculus, inequalities, graphs, and large numbers. A trade edition in paperback is available from Random House, Inc., 457 Madison Avenue, New York, New York 10022. A hardbound library edition is also available from Random House School and Library Services, Inc. at the same address. In addition, there is a special edition for high school students and teachers only, available at a reduced rate from L. W. Singer, Co., Inc., 249 West Erie Boulevard, Syracuse, New York 13201. A reference guide to the NML is available from Singer, a complete description of which is contained in Newsletter No. 21.

Supplementary Materials:

A number of booklets, each covering less than a full academic year, has been prepared. They cover such topics as mathematics in science, geometry, number theory, etc. They are available from Vroman. In addition, there is a "Supplementary and Enrichment Series," consisting of small pamphlets designed to allow teachers to try short modern treatments of particular topics in class. Some of these materials have teachers' commentaries.

Materials for Teachers:

Information concerning SMSG is disseminated at irregular intervals through its newsletters. A postcard request is sufficient for one to be placed on the mailing list. To date, twenty-two newsletters have been distributed. Limited numbers of back issues are available upon request. Complete information concerning SMSG publications is given in the newsletters. (See especially Newsletter No. 22, April 1966.)

"Studies in Mathematics" is a series of fourteen volumes available from Vroman and intended for teachers. Some volumes provide background for a specific student course, whereas others are more general in nature.

Study Guides in Mathematics are annotated bibliographies on various aspects of mathematics, all bound in one pamphlet. They are intended for teachers wishing to study by themselves and also for those planning courses for teachers. Available from Vroman.

A filmed course intended primarily for inservice training of elementary teachers has been developed which consists of thirty half-hour color films. Volume 9 of "Studies in Mathematics" is designed to accompany the filmed course. The films are distributed by Modern Learning Aids, 160 East Grand Avenue, Chicago, Illinois 60611.

Reports of various conferences sponsored by SMSG are available from Vroman. These cover the contents and objectives of SMSG texts, as well as discussions of problems in mathematics education and the role of SMSG in attacking these problems.
Other Publications:
The texts for grades 6-12 are available in Spanish translation, as is Volume 9 of "Studies in Mathematics."

"A Very Short Course in Mathematics for Parents" is a booklet designed to give parents a chance to work through a small sample of "modern" mathematics to see how it differs from a traditional treatment.

Philosophies and Procedures of SMSG Writing Teams and SMSG: The Making of a Curriculum are books providing background information about SMSG.

All of these publications are available from Vroman except the last one which is distributed by Yale.

FUTURE PLANS:
Additional texts will be translated into Spanish. An inservice training text for seventh-grade teachers is being written as an example of an inservice course.

SMSG has largely finished its task of providing sample curricula for grades K-12. In the future its major efforts will lie in three areas. The first is that of providing closer connections between mathematics and the various areas in which mathematics is used. The second is that of research on the learning of mathematics by students in school. The third is that of providing mathematics curriculum materials suitable for students whose achievement in mathematics is below average.

UNIVERSITY OF ILLINOIS ARITHMETIC PROJECT
AT EDUCATIONAL SERVICES, INCORPORATED

DIRECTOR AND ADDRESS: David A. Page
Educational Services, Inc.
372 Main Street
Watertown, Massachusetts 02172

GRADE LEVEL: K-6

The University of Illinois Arithmetic Project is engaged in developing materials and techniques for instructing elementary school teachers in mathematics and its teaching. The central theme of the project is that the study of mathematics should be an adventure, requiring and deserving hard work. Children who grasp some of the inherent fascination of real mathematics while they are in elementary school are well on the way to success in further study of mathematics and science. Students who are not to continue a formal study of mathematics need at least as appealing a diet.

Beginning in 1958 at the University of Illinois, the Arithmetic Project uncovered new ways of interesting children in genuine mathematics and began to explore the possibilities of communicating these ideas to teachers through direct instruction by project members, summer institutes, written materials, and other means. The project now addresses itself to the task of creating courses in mathematics for a significant number of elementary school teachers in this country. In 1964 the project moved to Watertown, Massachusetts and is currently operating in association with Educational Services, Inc.

The course materials, frequently misnamed "new" mathematics, comprise not new mathematics, but novel ways of doing old mathematics—new structures or schemes within which there are large numbers of interrelated problems that children can solve and that contain significant mathematical ideas. Such innovations can be developed by creative mathematicians and teachers working with children, and they can be successfully learned by other teachers. In order to invent and try out these new ideas,
project staff members currently teach regularly in Watertown and inter-city Boston schools in grades K-6. Throughout its work the project has found that improved computational skills on the part of children are an automatic adjunct of their working successfully with project materials. Children will do impressive amounts of computation in order to solve problems that interest them.

Due to the experimental nature of the project materials, it is necessary for teachers to be familiar with them if they are to be used effectively. In addition, it is helpful to have teachers become acquainted with ideas and methods similar to those that have been found effective with children by project staff members. The courses for teachers, therefore, make extensive use of problem sequences intended to transmit ideas, and motion pictures of classroom teaching of the materials, showing classes of various grades and a variety of teaching techniques. In addition to continuous teaching by staff members, the project conducts a limited number of sixteen-week invitational institutes for teachers from local school systems.

PUBLICATIONS

Elementary School Texts:
The only text which has been produced is a book by Mr. Page, *Number Lines, Functions, and Fundamental Topics*. It is published commercially by the MacMillan Company.

Materials for Teachers:

To help communicate its ideas to other teachers, the Arithmetic Project has a variety of reprints of articles, pamphlets, and monographs available. In most cases single copies are complimentary. You may have your name put on the project mailing list for information about available material and other developments that might be of interest.

The course is now being put on films, but general distribution of the films is not contemplated until the entire course is ready for release. However, some motion pictures are available for showing in certain circumstances to groups especially interested in the current work of the project.

FUTURE PLANS: Work on the course continues. When it is ready, the project plans to begin work on three further courses, to adapt these courses for preservice university courses, and to conduct institutes for university teachers concerned with the training of future elementary school teachers.

UICSM

UNIVERSITY OF ILLINOIS COMMITTEE ON SCHOOL MATHEMATICS (UICSM)

DIRECTOR AND ADDRESS: Max Beberman
Professor of Education
University of Illinois
Urbana, Illinois

GRADE LEVEL: 7-12

The UICSM was established in 1952 and is the oldest of the current curriculum projects in mathematics. Its major concern has been the development of instructional materials and their experimental trial in schools throughout the country. The Committee has "introduced some new content, rearranged some of the traditional content, and ... developed many promising pedagogical techniques and approaches." It has conducted a large number of summer institutes for teachers using its materials, emphasizing both content and pedagogy.

Until 1958 the UICSM textbooks were not available except to trained teachers who were willing to help evaluate the texts. They are now generally available, but the committee feels that they should still be used only by teachers trained in their use or
by teachers who have made an intensive study of their content and methods of instruction.

The content of the texts is structure-oriented. The methods stress discovery through the presentation of "a sequence of activities from which a student may come to independently recognize the desired knowledge." The texts indicate a fundamental belief in the importance of sophisticated and precise language, although the teacher's commentaries indicate that the teacher is not expected to require such precision from the students in the classroom.

**Publications**

**High School Texts:**

UICSM High School Mathematics, Units 1-11 (including teacher's commentaries) is sequential. Consequently, the problem of grade placement is resolved in a variety of ways in schools using this material. Most schools begin with Unit 1 in grade 9 and may or may not complete the course with Units 9-11 in grade 12. Others start with Unit 1 in grade 8 or even grade 7. The appropriateness of a particular topic in any unit must be judged in terms of the mathematical experiences of the students, rather than in terms of grade level. Courses 1 and 2 are also published commercially by D. C. Heath and Company in a hard-cover text.

Besides the basic course, UICSM has available self-instructional texts on solid geometry, logic, and an introduction to algebra.

**Materials for Teachers:**

Examinations for Units 1-6 have been prepared. There is a series of teacher-training films showing demonstrations of a class studying the 9th grade course. Reprints of articles concerning UICSM programs and research are also available.

**Future Plans:**

Presently being developed are courses in mathematics for grades 7-8 for under-achievers, a grades 10-11 vector geometry sequence, and teacher-training films for ninth-grade algebra teachers. Also planned are materials presenting elementary topics in geometry for seventh and eighth grades.

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**University of Maryland Mathematics Project (UMMaP)**

**Director and Address:**

John R. Mayor

University of Maryland Mathematics Project

University of Maryland

College Park, Maryland 20742

**Grade Level:** 7-8; teacher training

UMMaP was organized in 1957 with support from the Carnegie Corporation of New York. Its work has been carried out in three rather distinct phases.

The first phase (1957-60) was devoted to providing sample courses in mathematics for grades 7-8. It was, thus, the first of the major curriculum projects to devote its attention to junior high school mathematics. It was recognized that in 1957 junior high school mathematics was probably the most unsatisfactory of all the school mathematics courses, in that it was largely a repetition of earlier arithmetic with emphasis on skills and so-called "social applications." Better students found nothing new and were bored; poorer students found no new inspiration in studying material they had already failed or rejected.

The Maryland courses attempt to introduce mathematical sophistication in a modern manner. Language and mathematical structure are stressed. Unlike some of the other
seventh- and eighth-grade courses which were developed later, UMaP covers a complete first year in algebra in the two courses. In addition, there is considerable work with logic and geometry as well as with some topics in trigonometry, statistics, and probability.

After sample texts were tried for several years in a number of schools, chiefly in the Washington area, the books were published in hardcover editions in 1961.

The second phase (1962–65) was concerned with preparing experimental courses in mathematics for elementary teachers, with support from the National Science Foundation. The discovery approach is used in these algebra and geometry courses. The third phase (1960 to the present) has been devoted to learning research. It has overlapped the second phase and has resulted in a number of publications in scientific and teaching journals.

**PUBLICATIONS**

**Junior High School Texts:**

The junior high school courses are called Mathematics for the Junior High School: First Book, and Second Book. A teacher's guide is available for each, and there is an answer book for the first course. Some programed units for elementary and junior high school students have been produced.

**Materials for Teachers:**

The courses for elementary teachers prepared in the second phase of the program are Mathematics for Elementary School Teachers: Book I and Book II. The first course is concerned with algebra and the second with geometry. In addition, there are a number of monographs and reprints of articles on learning theory which are free upon request.
Grade levels. Mathematics curricula have been developed at all levels from kindergarten through twelfth grade. Some of the projects (Cambridge Report, NCTM, New York State, SMSG) have devoted themselves to the entire range of school mathematics. Those concerned with the elementary grades only are CBMI, Illinois Arithmetic Project, MINNEMAST, and Patterns in Arithmetic, whereas the Madison Project and GCMP have covered the elementary and junior high school grades. The Commission on Mathematics (CEEB) and OMC have been concerned with only senior high school, and BCMI and UICSM have worked with both junior and senior high school courses. UMMaP is the only program which has produced texts for only the junior high school level.

Ability levels. Most of the programs have developed curricular materials for only the better students, variously described as "college-bound," "average and above-average," and the like. In some cases, notably UMMaP, it was deliberately decided not to develop special materials for low achievers, as it was felt that the best approach for such students was to use the regular materials at a greatly reduced pace. On the other hand, BCMI, the Madison Project, NCTM, and SMSG have produced textual materials for the average or slightly below average student, and UICSM is planning some work along these lines. None of the projects has concerned itself very extensively with the lowest quartile of students.

Student texts. Most of the projects have produced or are producing their own student texts. This is true of BCMI, CBMI, GCMP, the Madison Project, MINNEMAST, OMC, SMSG, UICSM, and UMMaP. The NCTM has produced a few textual materials, notably their course units for ninth-grade general mathematics. The other projects mentioned are primarily interested in teacher training or in making recommendations about curricula. Patterns in Arithmetic has a pupil exercise book to accompany its television series.

Enrichment materials for students. Reference materials to be used by students for enrichment purposes have been produced by BCMI and SMSG as well as by NCTM. "Laboratory equipment" with the same general aim is available from GCMP and the Madison Project.

Teachers' guides. BCMI, GCMP, the Madison Project, SMSG, UICSM, and UMMaP have published teachers' guides (supplements, commentaries) to accompany their student texts. The report of the Commission on Mathematics also contains a teacher's guide or syllabus, which is not connected with any specific text. The same is true of materials produced by New York State and the Illinois Arithmetic Project. Patterns in Arithmetic has a teacher's commentary to supplement its television series.

Teachers' courses. A number of the programs have developed courses for teachers as one of their primary tasks. These include BCMI (which also has a correspondence course available), the Illinois Arithmetic Project, and UMMaP. MINNEMAST is working on pre-service courses for prospective teachers. In addition, GCMP, the Madison Project, SMSG, and UICSM have produced courses or special textbooks for teachers in conjunction with courses for their students. Many of the projects offer inservice and summer institutes.

Reference materials for teachers. All of the projects listed have materials available for teachers' reference. These include curricular recommendations (Cambridge Report, Commission on Mathematics, the Madison Project, MAA, NCTM, New York State, and SMSG), reports on the activities of the programs, monographs and articles concerning research into curriculum and learning theory as well as books and pamphlets dealing with mathematical topics. (See the listings of individual projects for details.)

Films. Films, either for teacher training or for use in the classroom, have been produced by GCMP, the Madison Project, MAA, MINNEMAST, Patterns in Arithmetic (television), SMSG, the Illinois Arithmetic Project, and UICSM.

Special materials. Computer-oriented high school courses have been developed by BCMI, NCTM, and SMSG. BCMI and SMSG have written special calculus courses for high school students. Programed material is available from SMSG and UMMaP. Most of the SMSG texts are now produced in Spanish-language editions.
Science and mathematics. Several of the curriculum projects have developed some materials on the uses of mathematics in science, although not too much work has been done in this area. BKN has produced a reference book on famous scientists. The Madison Project has attempted to introduce some units on mathematics in science. Some of the MAA films deal with problems in physics and other areas of physical science. The MINIBEST program is devoted to both science and mathematics teaching, but so far there seems to be little attempt at integrating them. Some of the units on measurement are used in common. SMSG has written four supplemental books using simple experiments to stimulate mathematical ideas. Mathematics through Science, Parts 1-3, is devoted to the physical sciences and designed for grades 7-9. Mathematics through Living Things is a similar book for grade 8, based on biological science.
IV SELECTED BIBLIOGRAPHY


- 22 -


Shryock, Jerry. "A Mathematics Course for Prospective Elementary School Teachers," The Arithmetic Teacher, X (April, 1963), 208-211.


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