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The objective of this study was to explore methods of using commonplace mechanical devices at the fourth grade level to expedite and extend the treatment of two important segments of the lower elementary arithmetic curriculum. The experimental testing of classroom materials took place at the Middle School of the Laboratory Schools of the University of Chicago. The results of this investigation show (1) that common mechanical devices can be effective teaching instruments for up-grading the level of achievement in elementary arithmetic, and (2) that within the pre high school program students can attain a working mastery of the decimal model of the real number system. (RP) (RP)

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HEALTH, EDUCATION, AND WELFARE

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FEASIBILITY OF A NEW BASIS FOR SCHOOL ARITHMETIC

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Chicago, Illinois

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Four teachers of the Laboratory Schools contributed to this project: Mrs. Fay Abrams, Fourth Grade; Mr. Joseph Dispensa, Shop; Miss Daphne Harwood, Second Grade; and Mrs. Ann C. Howe, General Science. Their participation was essential in obtaining the results of this study: the extra efforts and counsel, and help in securing and adopting materials in large measure were responsible for its success as a group project and as an experiment in curriculum development. Mrs. Abrams' fourth grade class made important points during the course of the study which were not anticipated in planning sessions. We also wish to thank the staff of the Laboratory Schools who aided this project in many ways.

Mr. David Flight, Principal of the Lower School at the time of this project, was very helpful in arranging to include this project in a very busy schedule.

Finally, we wish to thank Mr. William Klupar of the Peoples Gas Light and Coke Company of Chicago for securing some household gas meter dials, and Mr. Marshall Julian, Manager of the Chicago Plant of the Equipment Rebuilding Department, Standard Oil Company of Indiana for adapting and donating a gasoline pump computer for classroom use. These materials were very important in attaining the goals of this study.

SUMMARY

The real number system is a principal tool in the improvement of the secondary mathematics curriculum ([2], [5], [10], [12])[1]. The postulational treatment favored by most influential reform groups is not suited to the skills or the sophistication of pre-high school pupils. A working mastery of a more concrete model of the form of infinite decimals and their arithmetic was shown to be attainable as an eighth grade project in a preceding study [8].

Much of the concern of the "New Math" curricula in the lower grades is given to a preparation for the postulational treatment of the reals which involves level of sophistication. The infinite decimal system as a model of the reals has concreteness, power to resolve critical issues, and an arithmetic naturally developing from the basic skills possessed by elementary school pupils. The approach via decimals does not require, as does the New Math, the lengthy development of concepts and terminology which use valuable time that should be devoted to genuine mathematical experience and development.

The present study explored methods of using commonplace mechanical devices at the fourth grade level to expedite and extend the treatment of two important segments of the lower elementary arithmetic curriculum. Efficient design of this part of the curriculum can provide time and a suitable context for the introduction of the arithmetic of infinite decimals in the fifth and sixth grades.

The first part of this study required the cooperation of the shop, science, and arithmetic teachers of a fourth grade class. Its objective was to design and build a lever balance; to learn how to operate it for balance under loads and discover the mathematic formulation of the law that governs it; and to apply this knowledge to the study of fractions and their applications.

The second part of the study, exploited mechanical gear connections for similar purposes, but it was done within the arithmetic class. Clocks, meter dials, and other such gadgets were used to introduce, expedite, and extend the treatment of finite decimal, independently of fraction facts. The mechanical gadgets served as "analog computers" both for demonstration of methods and checking of work.

[1]Numbers in square brackets denote sources listed in the References on pages

Team teaching permitted pupils to work in small groups, and the teachers to plan, prepare, observe, and evaluate the experimental classes. Work sheets and anecdotal accounts were kept for later examination.

The results of this study are clear indications of the value of commonplace accounting and metering instruments for expediting the mathematical maturation of elementary school pupils. These resources are used presently only for incidental purposes, but this study shows that they have a more central and effective role to play.

INTRODUCTION

The need for a working model of the real number system as a tool for the improved design of the secondary mathematics curriculum has been widely recognized ([2], [3], [5], [10])[1]. In the report of the Cambridge Conference [5] for example, the major part of the curriculum for grades 3 to 6 is committed to the development of such a system. An effective model accessible to beginning high school students is essential to the significant advancement planned for the secondary mathematics curriculum in the Cambridge Report. It is needed, not only to clarify the theoretical foundations of algebra and geometry, but to make possible a viable high school course in calculus.

At present, a mathematically sound and complete form of the reals occurs very late in the secondary curriculum -- if at all -- and its late appearance precludes any opportunity of exploiting its potential for improvement. One reason for its late introduction is that the usual treatment presupposes a high level of mathematical sophistication by the very nature of its postulational presentation.

The organization outlined for the grades 3 to 6 in the Cambridge Report has been criticized [12] as being unrealistic in not including means of attaining the goals it sets for these grades. Nevertheless the goals themselves are realistic in the sense that there can be no significant secondary mathematics curriculum change unless the obstacles posed by these goals are resolved.

The traditional route (not suggested or recommended in the Cambridge Report) to a model of the reals is via Dedekind cuts in the rationals. This is not only an exhausting program to carry out, but it is decidedly unappealing and unsuitable to school mathematics due to its remoteness from the actual arithmetical skills developed in the lower curriculum.

This study is an outgrowth of a previous project designed to evaluate the feasibility of attaining a working mastery of the complete decimal model of the reals within the limitations of the existing elementary curriculum [8]. The program tested in that study was in turn adapted from a program designed and developed for use at much higher levels ([6], [7], [9]). But one important

[1]Numbers in square brackets denote sources listed in the References on pages

conclusion of that study was a clear indication that there are no inherent obstacles to attaining the arithmetical mastery described above.

A significant feature of the program described in [8] is its decidedly non-postulational character. This means that methods exploiting mechanical characteristics of the (decimal) representation are of particular interest. The value of such resources has been noted in connection with other objectives ([4], [11]). By and large, however, the elementary curriculum uses these resources only for incidental purposes. It is however very important that considerable familiarity and manipulative skill with dial, metering, and registration devices be available early in the lower curriculum.

The purpose of the present study was to explore opportunities and methods of improving the treatment of two important segments of the elementary arithmetic curriculum with the help of such resources.

The two parts of this study were conducted with the same class of fourth grade pupils of the Laboratory Schools at the University of Chicago, partly because of the interest and enterprise of their teacher, Mrs. Fay Abrams.

The first of these sub-projects was a cooperative effort involving the teacher of the shop class, Mr. Joseph Dispensa; the science teacher, Mrs. Ann C. Howe; and the arithmetic teacher, Mrs. Abrams. Miss Daphne Harwood, a second grade teacher, also participated in the arithmetic section. The objective here was to design and build a balance lever in the shop class, discover and formulate the law of the lever in the science class, and to apply both the mechanical skill and scientific knowledge acquired to develop a considerably extended treatment of fractions capable of dealing with sophisticated applications.

The second sub-project was confined to the arithmetic class. In it, experience with dialing and metering devices was used to support extensive arithmetical work with the finite decimals independently of fraction facts and to develop suggestions of the infinite decimal notation from these experiences.

METHOD

The character of this study was necessarily heuristic. The project had the benefit of advance planning by the cooperating teachers, review of each class experiment, evaluation of the individual pupil's progress through examination of the paper work that was systematically retained, anecdoted accounts of class proceedings, and comparison with the content of the regular curriculum familiar to the teachers. The low level of financial support did not permit use of control groups and instruments, or statistical evaluations, and was further limited by the fact that the work of the project had to be scheduled into an already ambitious academic program of the Laboratory Schools. While this method of investigation cannot support conclusive results, it served very well the purposes and scope of this project.

The first portion of the study dealt with the treatment of fractions that evolved from the work on the lever. At the outset, the group of teachers and the investigator, Mr. Klein, discussed the amount of time available in the various classes and set objectives for each. Mrs. Abrams outlined to her class what we hoped to accomplish with their help. A class discussion was devoted to the significance of the lever from an engineering viewpoint and the stability and sensitivity characteristics of the different types in common use. We were able to secure, very inexpensively, scrap end pieces of acrylic material cut from transparent quarter inch sheets, and extrusions, about $1\frac{1}{2}$ inches in diameter in the cross-section shape of a five pointed star. The former was used as a rigid balance bar (2" x 15") and the latter, cut to $2\frac{1}{2}$ inch lengths with one of the edges sharpened, was used for a fulcrum. This work was done in the shop where each pupil, using only a file, sharpened his own fulcrum, cut a V-groove in the balance bar, removed material from either end to obtain a balanced system. Mr. Dispensa and Mr. Klein supervised this work and helped check out each system for balance. The pupils kept these levers in or on their home-room desks and were generally quite proud to take possession of them.

In the science class the pupils worked in small groups of three or four. The science department had available an assortment of washers, discs, nuts, nails, and other small objects convenient to place and shift on the balance boards. The laboratory experiments were designed to lead to understanding of the principles of the lever balance in two stages:

The first was to recognize that through the lever balance, weight and distance, though different physical entities, were interchangeable in certain engineering respects. This was learned in the development of methods of attaining balance under various restrictions, such as allowing position but not weight to be changed, etc.

The second objective was to discover and formulate the mathematical law governing the lever balance. This was accomplished with the help of laboratory record sheets designed and prepared by Mrs. Howe and Mr. Klein on which data were recorded by the pupils as collected. These laboratory records also contained schematic diagrams representing experimental problems. These diagrams proved useful later in the mathematics classes. Numerous exercises and problems were presented using these diagrams.

As a result the pupils completed the science portion of the cooperative experiment with newly acquired manipulative skills, engineering insight, and an important scientific discovery of their own expressed in mathematical form.

The third part of this coordinated project was the arithmetic section devoted to fractions. It was conducted by Mrs. Abrams with the cooperation of Miss Harwood and Mr. Klein. This arrangement permitted close observation of the progress of the lessons and anecdotal recording and review of them. Also this team effort made it convenient to set up small groups of pupils working on different projects and keep track of the progress of individual pupils.

The purpose of arithmetic section was to use the pupils skills with and knowledge of the lever in treating a wide range of fraction notions, skills, procedures, and applications. The intention was to extend the application of fractions notions and skills to a broader range than usual, and to accomplish this earlier in the curriculum.

A typical application of the lever principle was to divide a small amount of rice into two parts, (say) one twice the weight of the other. This was done by placing two containers on the lever bar so that one was twice as far from the fulcrum as the other. The rice was then shifted between the containers to obtain a balance and the required portioning.

Ideas developed through use of the lever were also used independently of it. This was done partly through class discussion and through the use of other portioning devices, such as paper cups and rulers for measuring such corresponding features of the objects as volume and length. Much of the work was done on problem sheets using first diagrams from the science section and later story problems. "Each pupil wrote his own story entitled "$\frac{1}{2} = \frac{2}{4}$"." These were prepared lesson by lesson and were retained for later examination. Miss Harwood later found some of this material useful and accessible to her second grade class.

After completion of the fraction project there was an interruption of several weeks before beginning the part of the study on decimals. This was considered to be a continuation of the same project by the pupils, since our intial session concerned a mechanical topic: the working relationship of the hands of a clock. Work with the clock led to introduction and use of time notation (eg. $12{:}30^{\underline{15}}$). A typical problem was introduced by setting different times on a pair of clock faces representing the beginning and end of a period. The clocks were used as "analog computers" to determine the intermediate times which separated the given period into prescribed portions--another link with the first part. Calendar notation entered the lessons naturally and were used in solving problems of considerable complexity with sophisticated methods.

In addition to clocks, meter dials taken from household gas meters were also used. These were donated by the Peoples Gas Company of Chicago and proved to be extremely valuable. The relationship between the individual hands, though each had its own face, was recognized easily as akin to the clock hands. The Children could activate changes in setting by turning the gear discs, and by observing this relationship. Most importantly, each hand was regarded as giving information about the position of the adjoining hands. There was a clear suggestion that extra faces could be attached on either end if the purpose at hand required it. Choice of one dial to represent units gave the faster moving ones meaning as tenth', hundredth's etc., and the slower ones their usual place value significance.

Again problems of finding various intermediate readings were done using a pair of such meters an "analogue computers", just as the clocks were. Worksheets were prepared, progressing from facsimile sketches of the meter faces to problems using finite decimal notation. The experience represented by this work was capable of supporting a "proof" that the midreading of two meter readings was the arithmetic average.

Class and individual students' progress was discussed at regular meetings, and problems and exercises were designed to help the slower pupils and occupy the faster ones. Class worksheets were retained for later examination.

A gasoline price computer was obtained from the Standard Oil Company's Chicago Equipment Rebuilding plant through the courtesy of its manager, Mr. Marshall Julian. It had been fitted with a small electric motor, but its mechanism could be still studied by the pupils. One class project with it was to adjust the price setting so that the price register kept time with and counted the seconds on the wall clock. After noting the price setting, the class could use the wall clock to determine periods in which the computer showed a given amount of gasoline pumped by volume or by price. This experiment showed that even fourth graders could help adapt machines to do jobs of another, through arithmetical technology.

Finally a tour was made of the basement of Blaine Hall, the building of the Laboratory Schools occupied by the Lower School. It was replete with dials and recording instruments of all sorts: steam and compressed air recording instruments, time clocks, electric meters, etc. The pupils were able to follow the explanations of their use and purpose made by the building engineer. Both he and the children enjoyed the occasion very much.

RESULTS

This study was heuristic for a variety of reasons, and the results are not supported by statistical analysis. But the character of the work done by this class of fourth graders is nevertheless significant for design of the elementary mathematics arithmetic curriculum.

It shows very clearly that common mechanical gadgets can be powerful teaching instruments and machinery for upgrading the level of achievement in elementary arithmetic. It shows that within the pre-high school program a working mastery of the decimal model of the real number system can be provided.

The evaluations of the participating teachers are reflected in Mrs. Abrams' remarks which are given below:

Fractions Project

> In the Spring Quarter of 1966, Mr. George Klein, with the help of Miss Daphne Harwood, explored an experimental program on fractions in my fourth grade class at the Laboratory School. I participated in the planning, teaching and record keeping with special attention to the understandings developed by individual children. The 25 children ranged widely in ability from a couple of math buffs to several math casualties.
>
> The children had previously constructed simple balance boards in the school shop under the supervision of Mr. Dispensa. They had also acquired skill in using the lever and had discovered in law of the lever in Mrs. Howe's science class.
>
> Following the science class sequence, the arithmetic lessons were given over to reviewing this previous work and extending it. We planned to use these skills to build our units on fractions. We collected all sorts of material: rice, beans, peanuts; and borrowed washers, nuts, etc. from the science classroom. Originally the balance bars were used to portion out these materials into fractional parts but later the ideas and methods were applied independently of the lever; using cups etc. to portion by volume, for example. At a later stage the pupils were able to work out fairly complex problems with the ideas they acquired, without need to resort to actual doing.

Examination of problems in "which share would you rather have" again led to building sets of equivalent fractions and went further into the concept of greater and smaller fractions. The work culminated with a rate problem-- (Danny can mow Mrs. J's lawn in 3 hours, Nina in 5 hours. If they both set to work at the same time, how long will it take them to do the job?) which was satisfactorily resolved by the class.

The results of the work was indeed positive. All the children were completely absorbed and highly motivated; they learned a great deal of translating many of the problems into physical activity with objects. They could all work at their own pace and receive individual help as needed. Moreover, the problems no longer seemed an imposed abstraction. Many of them could build generalizations from their own experience.

This experiment proved valuable for my own teaching. I find I still use ideas worked out at that time to clarify concepts normally presented more formally.

Decimal Project

Mr. Klein had collected an assortment of discarded clocks, counters, meters, and a gasoline computer. The children were intrigued by them and delighted to be able to get their hands on them.

In the classes, these gadgets proved to have great powers of suggestion, for several pupils who were marginal students in arithmetic before showed sharp improvement in interest and performance. The idea that commonplace instruments such as clocks can be used for solving complicated arithmetic problems is a lession I am sure these children will find useful for many years.

The decimal notation was particularly natural when linked to the gas meter dials. For example, addition and even division problems involving decimals could be done on work sheets and could be checked against the dials.

Even allowing for the excitement generated by an unusual project, it was clear that with the use of the gadgets, the children made solid and significant progress in topics not normally covered in the fourth grade.

The day that we went down to the basement where the maintenance area was, was a red letter day. The children were amazed to find how many of the technical gadgets they were able to read and understand.

CONCLUSIONS

It is not unreasonable to attribute the first place standing in the recent International Study of Mathematical Achievement (11) to the common use of the abacus in that country. It is consistent with the implications of this study to expect a considerable upgrading of the levels of achievement and sophistication in the elementary mathematics curriculum from a more systematic use of mechanical resources that are readily available but are used only incidentally at present.

The tendency of the "New Math" curriculum has been criticized for its wasteful allocation of time and resources that does very little to enhance the goals of the elementary mathematics program (14). This study supports that criticism . Abstractions introduced in the elementary program need to be built on experiences and skills that illustrate the concepts in several interpretations. Well chosen implements which have the capacity and adaptability to act as teaching instruments are more suitable to the goals of the elementary curriculum than abstract ideas which do not carry their own weight within the context of elementary mathematics.

The indication of the present study is that goals described in the report of the Cambridge Conference (15) are not beyond reach if we use the best resources available without regard to doctrinaire prejudgements on the purposes and character of the elementary mathematics curriculum.

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