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VOLUME AND SURFACE AREA.
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THIS BOOKLET, ONE OF A SERIES, HAS BEEN DEVELOPED FOR THE PROJECT, A PROGRAM FOR MATHEMATICALLY UNDERDEVELOPED PUPILS. A PROJECT TEAM, INCLUDING INSERVICE TEACHERS, IS BEING USED TO WRITE AND DEVELOP THE MATERIALS FOR THIS PROGRAM. THE MATERIALS DEVELOPED IN THIS BOOKLET INCLUDE (1) MEASURING VOLUMES OF RECTANGULAR SOLIDS, RIGHT RECTANGULAR PYRAMIDS, CYLINDERS, CONES AND SPHERES, AND (2) FINDING THE SURFACE AREA OF ELEMENTARY GEOMETRICAL CONFIGURATIONS. ACCOMPANYING THESE BOOKLETS WILL BE A "TEACHING STRATEGY BOOKLET" WHICH WILL INCLUDE A DESCRIPTION OF TEACHER TECHNIQUES, METHODS, SUGGESTED SEQUENCES, ACADEMIC GAMES, AND SUGGESTED VISUAL MATERIALS. (RP)

ESEA Title III

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VOLUME AND SURFACE AREA

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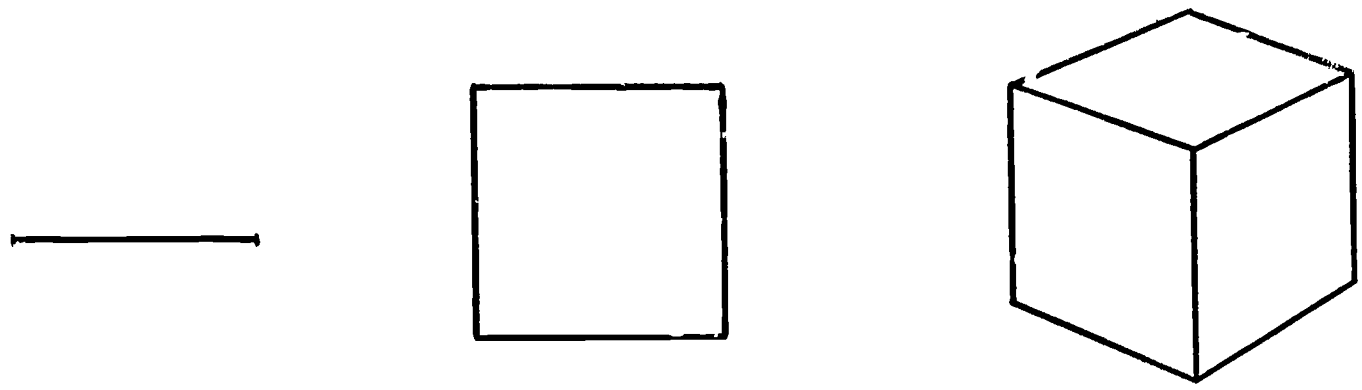
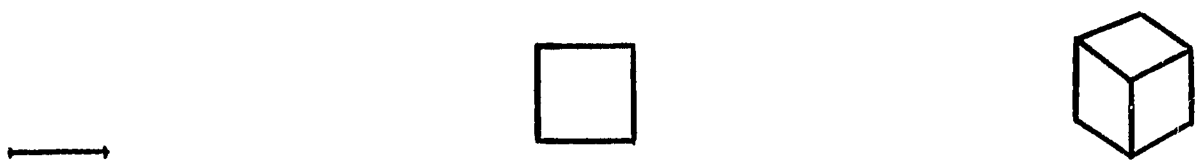
MEASURING VOLUME

Three different units are pictured below:



1 inch | 1 inch square | 1 inch cube

One unit is used to measure length (inch), another to measure area (square), and the third to measure volume (cube). Other linear, area, and volume units could be either "smaller" or "larger"--depending on what is to be measured. Some examples of "smaller" units are:



1 centimeter | 1 centimeter square | 1 centimeter cube

Linear units are units such as the inch, centimeter, yard, meter, mile, and kilometer. They are used to measure length (height, width, depth, etc.). A linear unit has only one dimension--length.

Area units are generally squares--of different sizes--depending on what is to be measured, such as: a centimeter square, an inch square, a foot square, a yard square, or a mile square. An area unit has two dimensions--length and width.

Volume units are generally cubes, or can be related to cubes, such as: a centimeter cube, an inch cube, a foot cube, a yard cube. A volume unit has three dimensions--length, width, and height.

Volume is often given in terms of a liquid measure, such as the tablespoon, cup, pint, quart, gallon, liter, etc. However, these measures can be related to a cubic unit.

Examples: $7\frac{1}{2}$ gallons = 1 cubic foot

1000 cubic centimeters = 1 liter

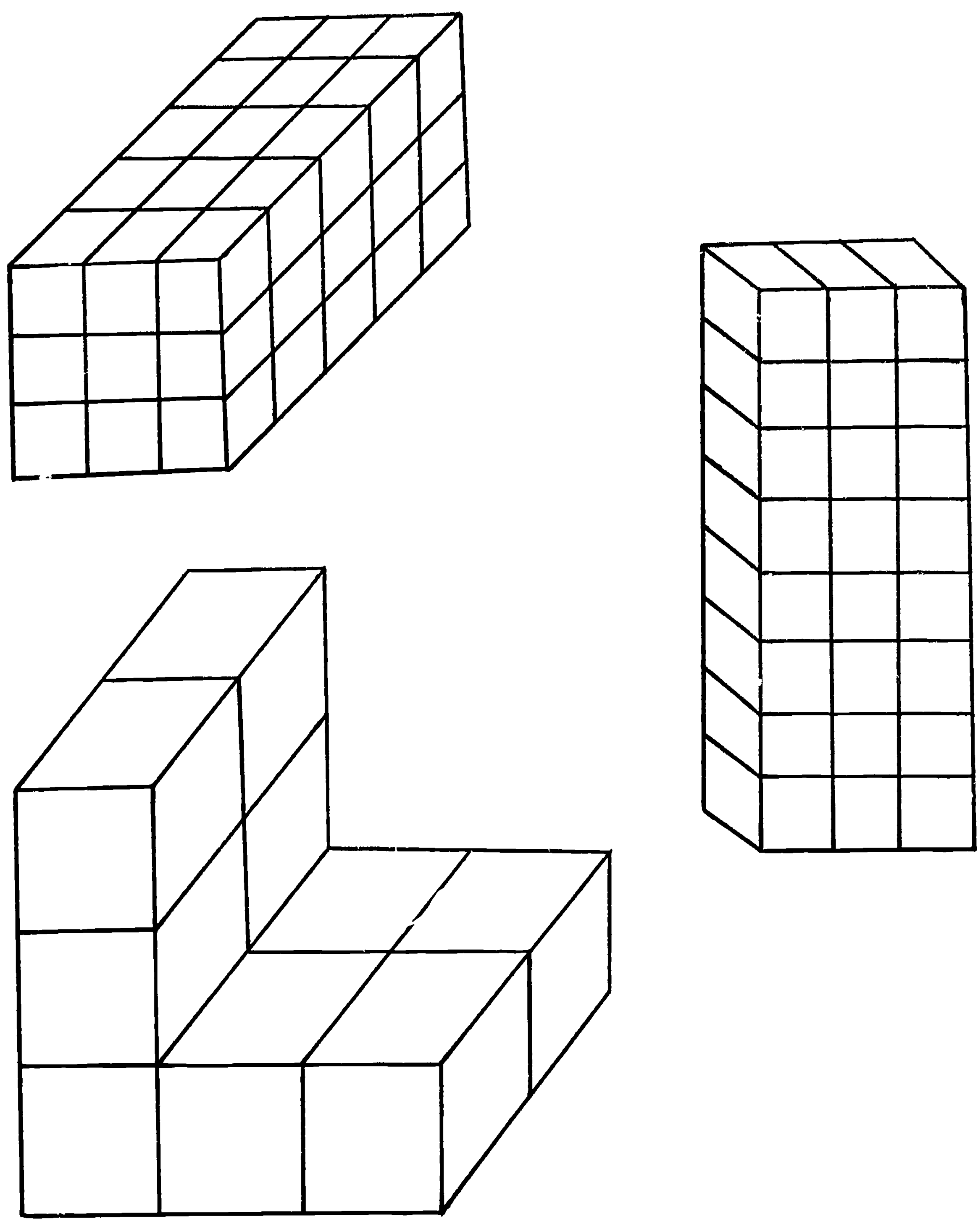
Activities

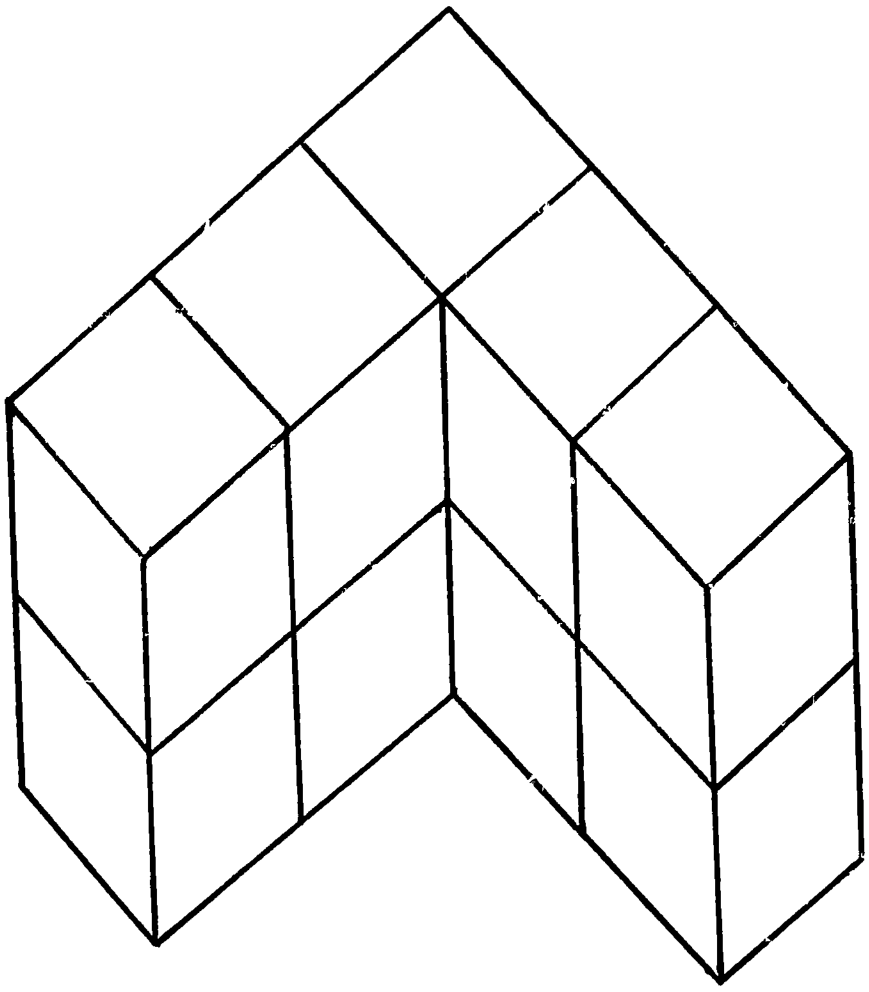
Which unit would be used to measure each of the following--a linear unit, an area unit, or a cubic unit?

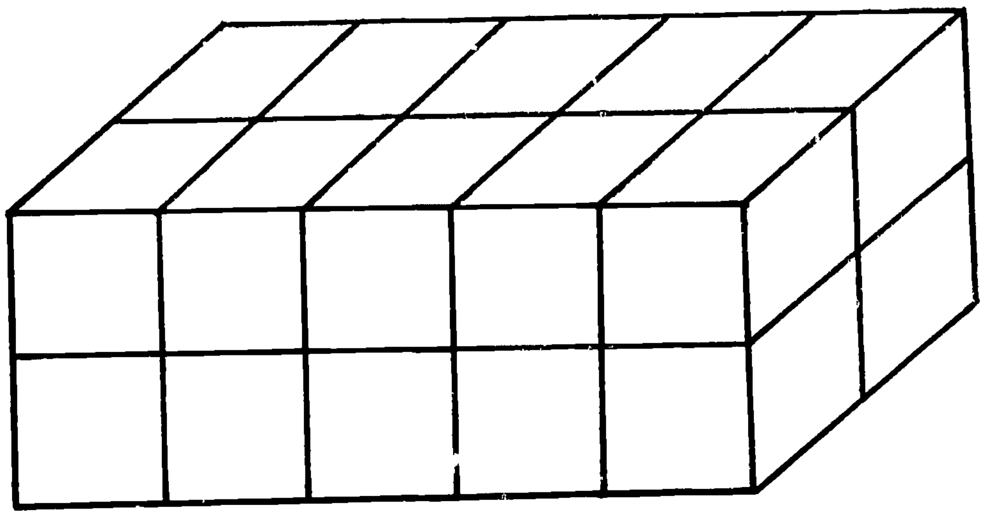
1. The height of a flag pole. __________ unit
2. The amount of water needed to fill an aquarium. __________ unit
3. The amount of floor surface in a room. __________ unit
4. The distance from Palm Beach to East Lake. __________ unit
5. Circumference of a circle. __________ unit
6. Perimeter of a rectangle. __________ unit
7. The amount of concrete needed to build a sidewalk. __________ unit
8. The amount of gas to fill the tank of a car. __________ unit
9. The size of shoe a person wears. __________ unit
10. Viewing surface of a T.V. screen. __________ unit
11. Surface of a dance floor. __________ unit
12. Storage space in a freezer. __________ unit

Rectangular Solids

In measuring, we count the number of units contained in what is being measured. There are different units for measuring length, area, and volume. A volume unit is the cube. What is the volume of each of the following solids? Are all of these rectangular solids?

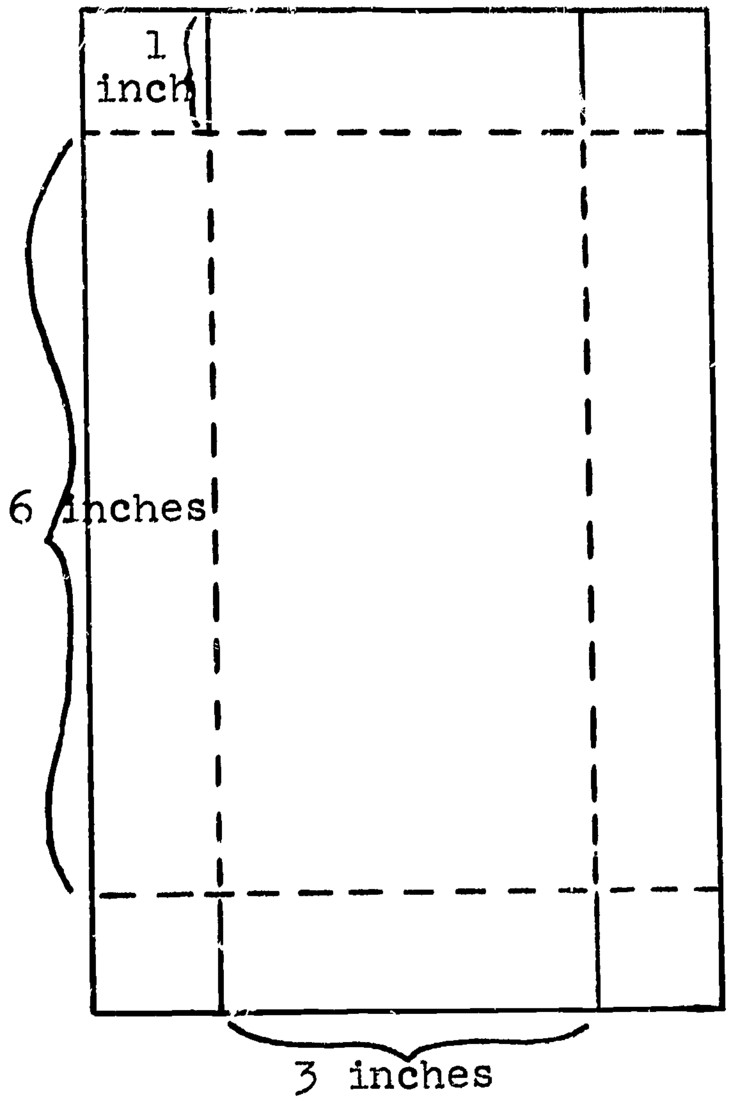


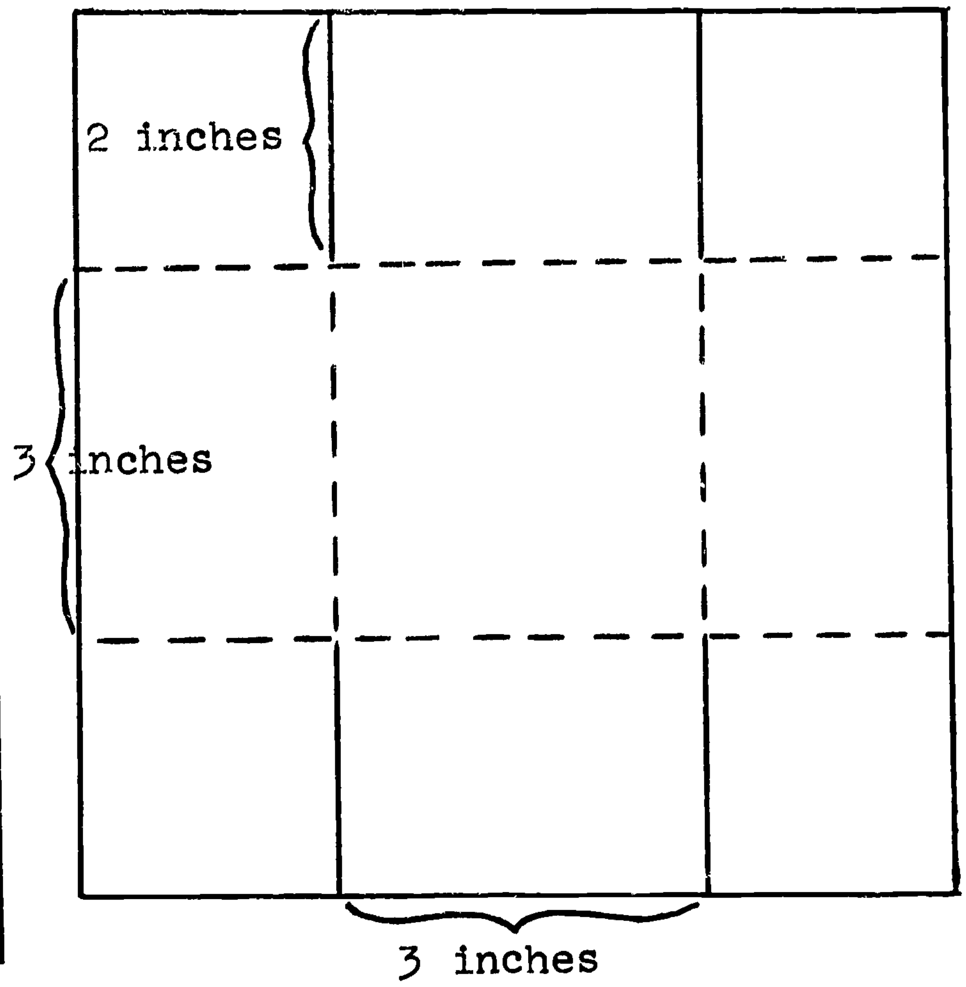


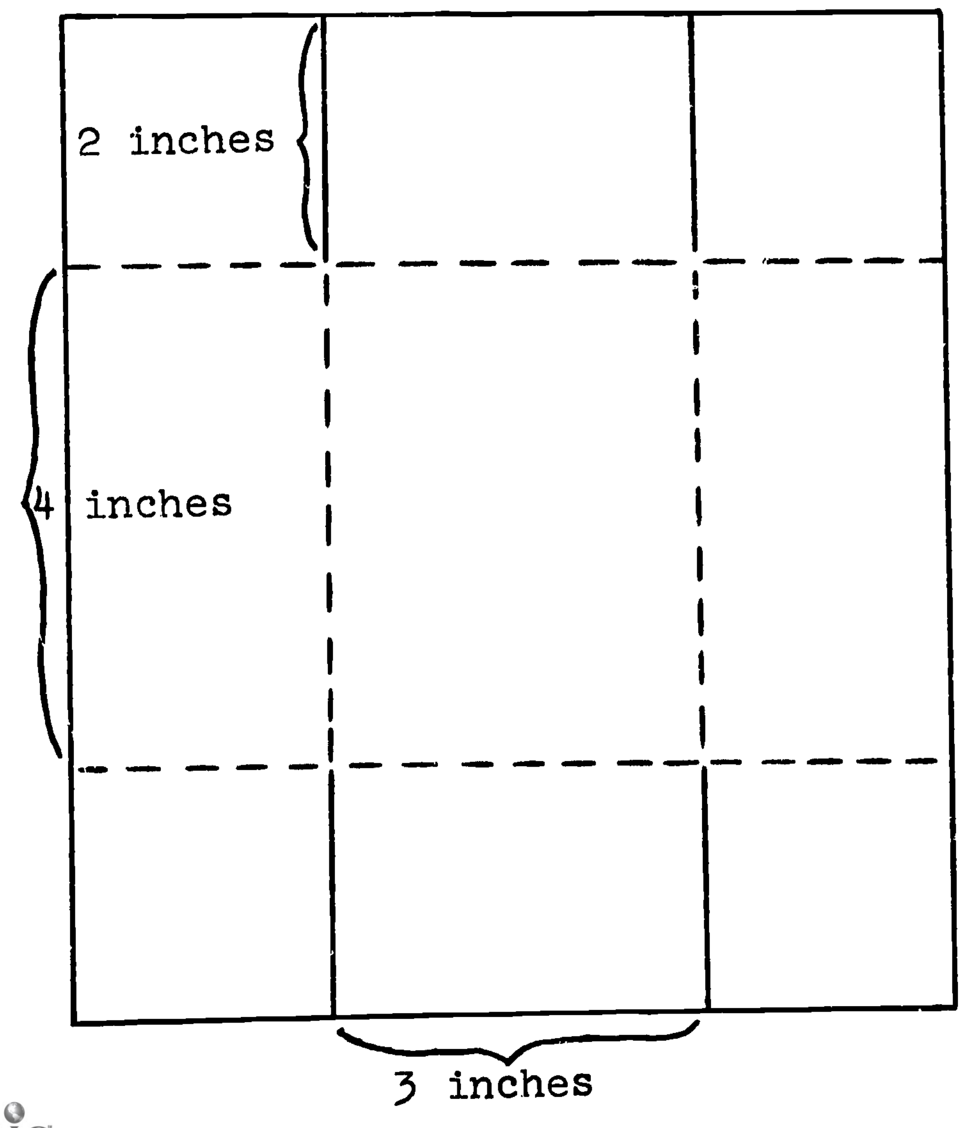


In each case, the number of cubes were counted in order to determine the measure. Suppose the size of each cube counted was 1 inch X 1 inch X 1 inch. Then the volume is given in cubic inches. If each cube is a 1 centimeter cube, the volume would be in cubic centimeters.

The volume of a rectangular box can be determined by actually counting the number of cubes that can be stacked in the box.



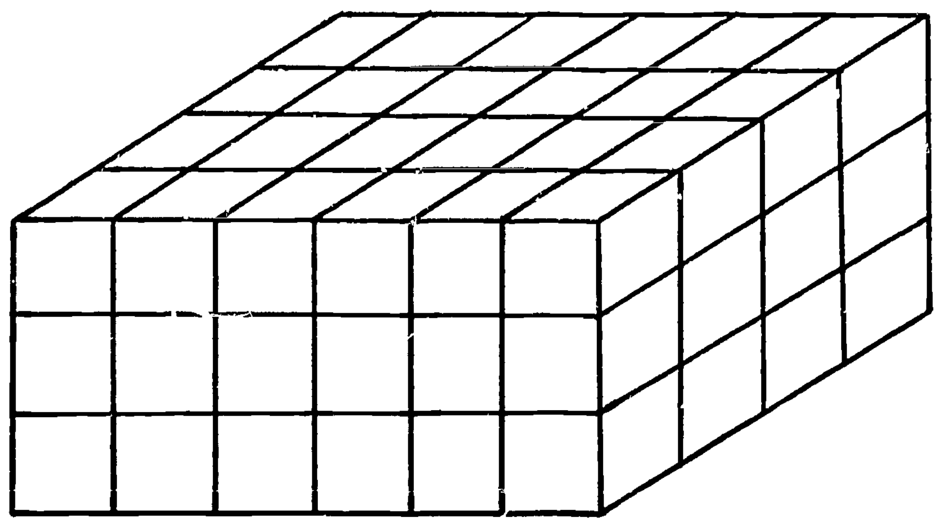




On three separate sheets of paper draw the above figures to scale. Then cut, or tear with your ruler along all solid lines. Fold up the sides along the dotted lines to form boxes. Stack the cubes in the boxes until each box is filled. The number of cubes in a box is the volume within that box.

It would not be very practical to determine volumes by filling various containers with cubic inches. What we do is to find the dimensions of the container and by the process of multiplication determine the volume. We have already seen that the volume inside a box or a rectangular-shaped solid can be found by determining the number of cubes in one layer and then multiplying by the number of the layers. The number of cubes in one layer can be determined by multiplying the number of cubes along the length by the number of rows. The number of rows is given by the width. All of this measuring can be done using linear units, without counting any cubes.

The volume of a rectangular box 6 feet long, 3 feet high, and 4 feet wide is 6 x 4 x 3, which equals 72 cubic feet. The product 6 x 4 gives the number of cubic feet in one layer. The number 3 gives the number of layers. The drawing below illustrates the idea--even though these are not one foot cubes.



1st layer -	6 rows of cubes with 4 in each row (or 4 rows with 6 in each row)
then	6 x 4 = 24 cubes in one layer
then	3 layers with 24 cubes in each
gives	3 X 24 = 72 cubic units

The volume can also be pictured as the area of the base multiplied by the height. For the area of the base, we count squares (not cubes). The area of the base is:

(6 X 4)
↓
(length X width)

multiplied by the height:

(6 X 4) X 3 = 72 cubic units

(length X width) X height

Then the volume of a rectangular solid is

Volume = length X width X height

or Volume = area of base X height

Use this idea to find the volume of the following rectangular solids. Be careful that the length, width, and height are measured in exactly the same unit.

Example 1. Find the volume of a rectangular solid with these dimensions:

length = 1 foot

width = 5 inches

height = 4 inches

Notice these are not in the same units. If the foot (length) is changed to inches, the volume can be determined: 1 foot = 12 inches

Therefore:

V = l X w X h

V = 12 X 5 X 4

V = 240 cubic inches

Activities

1. Find the volume of each rectangular solid with the following dimensions:

   a. l = 4 inches
      w = 2 inches
      h = 1 inch

   b. l = $\frac{1}{2}$ inch
      w = 6 inches
      h = 2 inches

   c. l = 2 feet
      w = 1 yard
      h = 12 inches

   d. l = 3 feet
      w = 2 yards
      h = 6 feet

2. Give the length, width, and height--in feet--of a cubic yard. How many cubic feet in a cubic yard?

3. Give the length, width, and height--in inches--of a cubic foot. How many cubic inches in a cubic foot? How many cubic inches in a cubic yard?

4. A concrete drive is to be constructed. The dimensions are

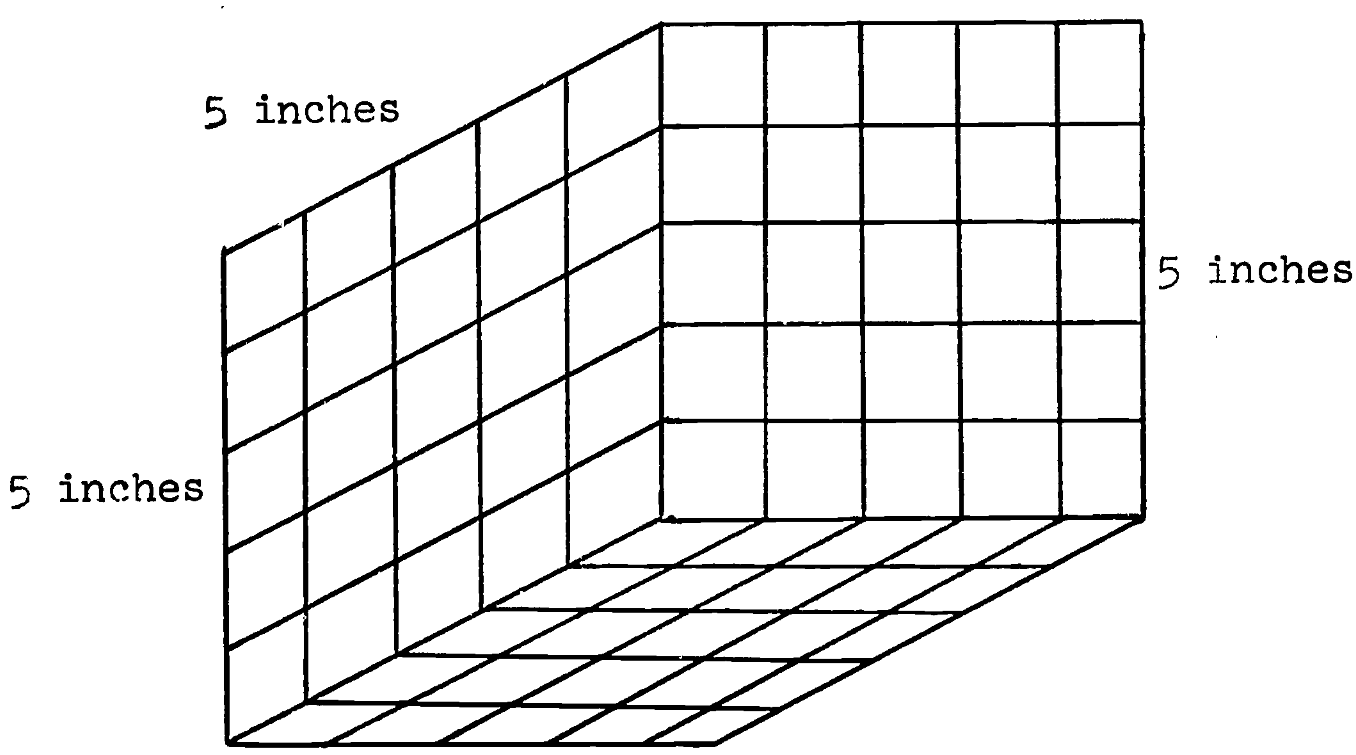
   length = 40 ft., width = 27 ft., depth = 6 in.

   If concrete cost $18.00 per cubic yard, what is the volume of concrete needed and what is the cost?

5. Estimate the volume of your classroom--in cubic feet. Check your estimate by measuring the length, width, and height and multiplying.

6. Suppose a large cube was constructed using <u>one</u> inch cubes. The dimensions are:

$$l = 5 \text{ in.}, w = 5 \text{ in.}, h = 5 \text{ in.}$$



If the outside of the cube is painted blue, how many <u>one</u> inch cubes would have:

a. Exactly one side blue?

b. Exactly two sides blue?

c. Exactly three sides blue?

d. No blue paint on them?

e. Exactly four sides blue?

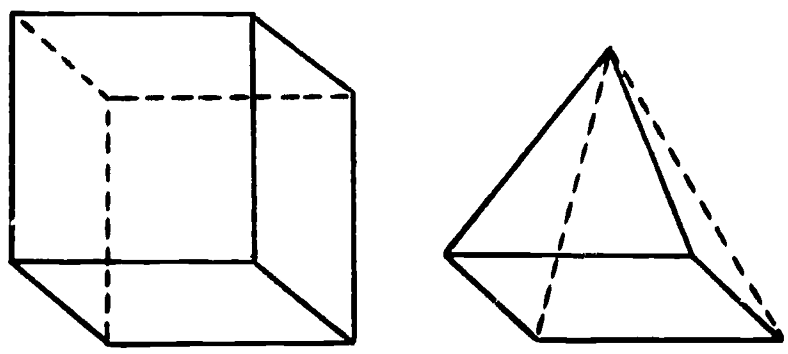
f. Exactly five sides blue?

g. All sides blue?

7. Find the number of gallons of water in Lake Okeechobee. (Hint: The surface area of this lake is 700 square miles. Its average depth is 6 feet. The volume of the water in Lake Okeechobee can be found by treating the lake as a rectangular solid with dimensions 35 miles by 20 miles by 6 feet. Then use the fact that there are 7.5 gallons in 1 cubic foot.)

Right Rectangular Pyramids

Pictured below are two solids, a rectangular solid and a right rectangular pyramid. The base of the pyramid is the same as the base of the rectangular solid and also, the heights of the two solids are equal.



Compare the volumes of the two solids. How many pyramids full of sand or water would it take to fill the rectangular solid? Demonstration models can be constructed or they may already be available. Does it check if you say it takes 3 of the pyramids to fill the rectangular solid?

The volume of the rectangular solid was:

$$V = l \times w \times h$$

Then what would be the volume of a right rectangular pyramid?

One-third of the rectangular solid is correct.

$$V \text{ (pyramid)} = \frac{1}{3} \times l \times w \times h$$

Remember that the base of the pyramid is a rectangle, otherwise this would not hold true.

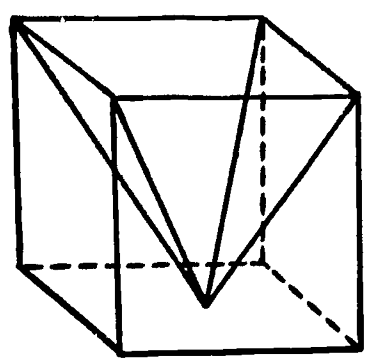
Activities

Find the volume of the following:

1. A right rectangular pyramid with these dimensions.
   a) a square base: the length of one side of 2 inches
   b) a height of 6 inches

2. A right rectangular pyramid with:
   a) the area of the base = 20 square inches
   b) height = 8 inches

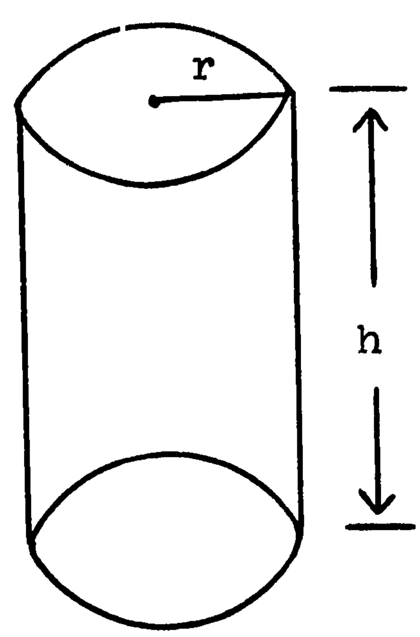
3. A right rectangular pyramid is cut from the following cube. Find the volume of the cube and also the pyramid.



Cylinders

The drawing below represents a right circular cylinder. What is the area of the base of the cylinder? Since the base is a circle, the area is found by: (As an approximate value for π, use $\frac{22}{7}$ or 3.14.)

$$A\ (\text{base}) = \pi r^2$$

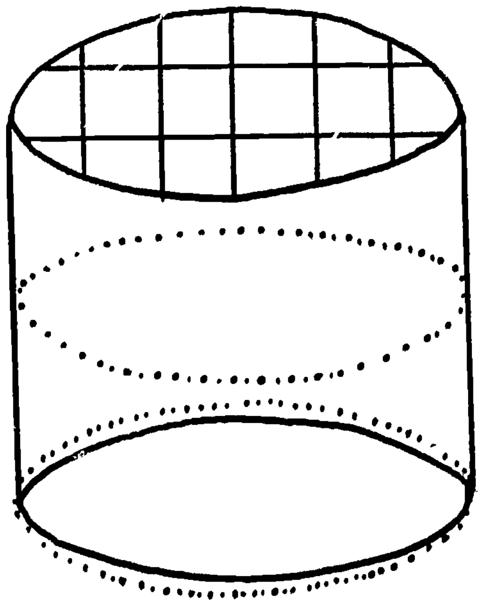


The volume of the cylinder is found by:

V (cylinder) = (area of the base) X (height) or

V (cylinder) = $\pi \times r^2 \times h$

This may also be pictured by cutting the cylinder into one inch or one centimeter layers. Now find how many layers and how many cubic inches there are in each layer. The drawing will help to illustrate this idea.



The expression $1 \times \pi r^2$ gives the number of cubes in each layer. Notice that one is used as a factor. It gives the height of a layer.

Activities

1. Bring in some cans (cylinders) of various sizes. Measure and find the volume of each can by using the relationship $V = \pi r^2 h$. (Use $\frac{22}{7}$ or 3.14 as an approximate value for π.)

   Construct an inch cube and check the results by actually counting the number of inch cubes of sand or water needed to fill each can.

2. You may also determine the volume within a tin can (cylinder) the following way:

   a. Fill the tin can with sand.
   b. Pour the sand into a box.
   c. Determine the volume of the sand in the box.
   d. Measure the tin can (cylinder) and use the cylinder formula $V = \pi r^2 h$, to check the volume again.

3. Pour a can of water into a rectangular container. Measure the height of water in the rectangular solid and find the volume of water by using the relationship $V = l \times w \times h$.

   How does this compare to the volume of the can when the relationship used is $V = \pi r^2 h$?

4. Find the volume of some cylinder tanks found in the community (gas tanks, water tanks, etc.).

5. Find the volume for each cylinder:

   a) h = 6 inches
      r = 3 inches

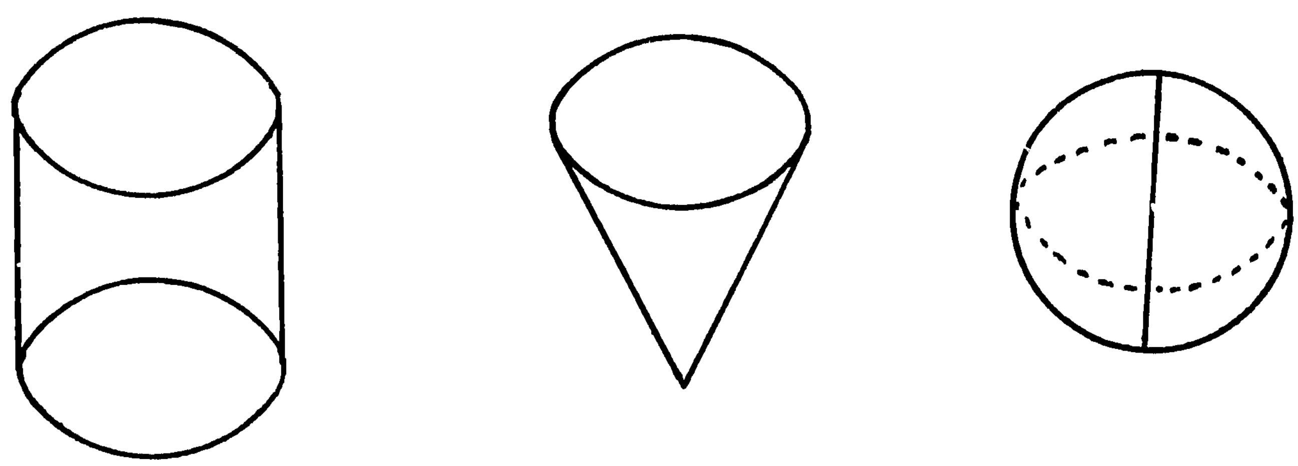
   b) h = 10 inches
      r = 7 inches

   c) h = 1 foot
      r = 1 foot

   d) h = 5 inches
      r = 2 inches

Cones and Spheres

Shown below are a cylinder, cone, and a sphere. The height of the cone and cylinder is equal to the diameter of the sphere. The base of the cylinder is equal to the circular top of the cone.



cylinder   cone   sphere

As an experiment, determine how many conefuls of sand or water it would take to fill the cylinder. (If the cone, cylinder, and sphere aren't available, they can be constructed.) Look back at the relationship between the pyramid and the rectangular solid. Did you find that the cylinder would hold 3 conefuls of sand? Since the volume of a cylinder is V (cylinder) = $\pi r^2 h$, what would be the volume of a cone? According to the experiment it is one-third of the cylinder, or V (cone) = $\frac{1}{3} \pi r^2 h$.

How many conefuls do you think it would take to fill the sphere? Guess and then measure. Did you find it would take 2 conefuls? Then the volume of the sphere is 2 times the volume of the cone or:

$$V \text{ (sphere)} = \frac{2}{3} \pi r^2 (h)$$

However, the height of a sphere (h) is called the diameter, which is two times the radius (2r). Replace h with 2r.

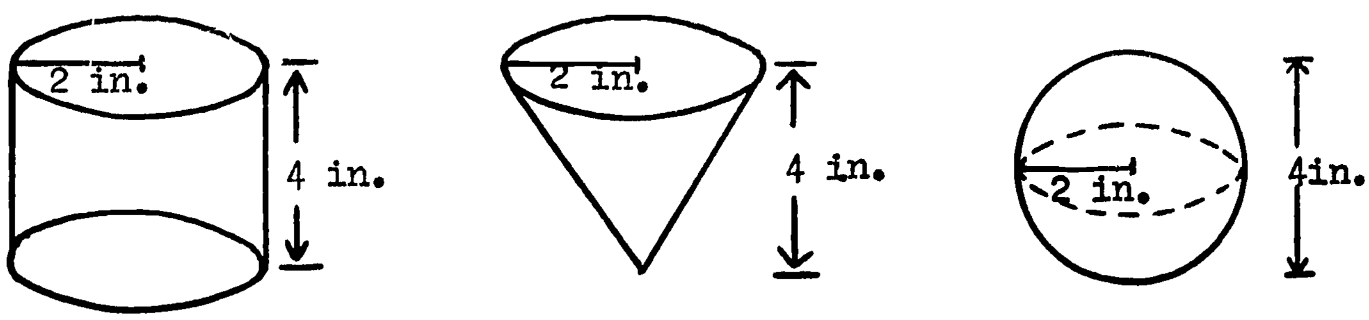
$$V \text{ (sphere)} = \frac{2}{3} \pi r^2 (2r)$$

Multiplying $\frac{2}{3}$ X 2 and r^2 X r gives us: V (sphere) = $\frac{4}{3} \pi r^3$

$$\begin{aligned} V \text{ (sphere)} &= \frac{2}{3} \pi r^2 h \\ &= \frac{2}{3} \pi r^2 (2r) \\ &= 2 \left(\frac{2}{3}\right) \pi r^2 r \\ V \text{ (sphere)} &= \frac{4}{3} \pi r^3 \end{aligned}$$

Some examples of finding the volume of a cylinder, cone, and sphere are: (Use as an approximate value for π, $\frac{22}{7}$ or 3.14.)

Examples:



$V = \pi \times (2\text{ in})^2 \times 4\text{ in.}$

$V = \pi \times 4\text{ sq.in.} \times 4\text{ in.}$

$V = \frac{22}{7} \times 16\text{ cu.in.}$

$V = 50\frac{2}{7}$ <u>cubic inches</u>

$V = \frac{1}{3} \times \pi \times (2\text{ in})^2 \times 4\text{ in.}$

$V = \frac{1}{3} \times \pi \times 4\text{ sq.in.} \times 4\text{ in.}$

$V = \frac{1}{3} \times \frac{22}{7} \times 16\text{ cu.in.}$

$V = 16\frac{16}{21}$ <u>cubic inches</u>

$V = \frac{2}{3}\pi \times (2\text{ in})^2 \times 4\text{ in.}$

$V = \frac{2}{3}\pi \times 4\text{ sq.in.} \times 4\text{ in.}$

$V = \frac{2}{3} \times \frac{22}{7} \times 16\text{ cu.in.}$

$V = 33\frac{11}{21}$ <u>cubic inches</u>

Activities

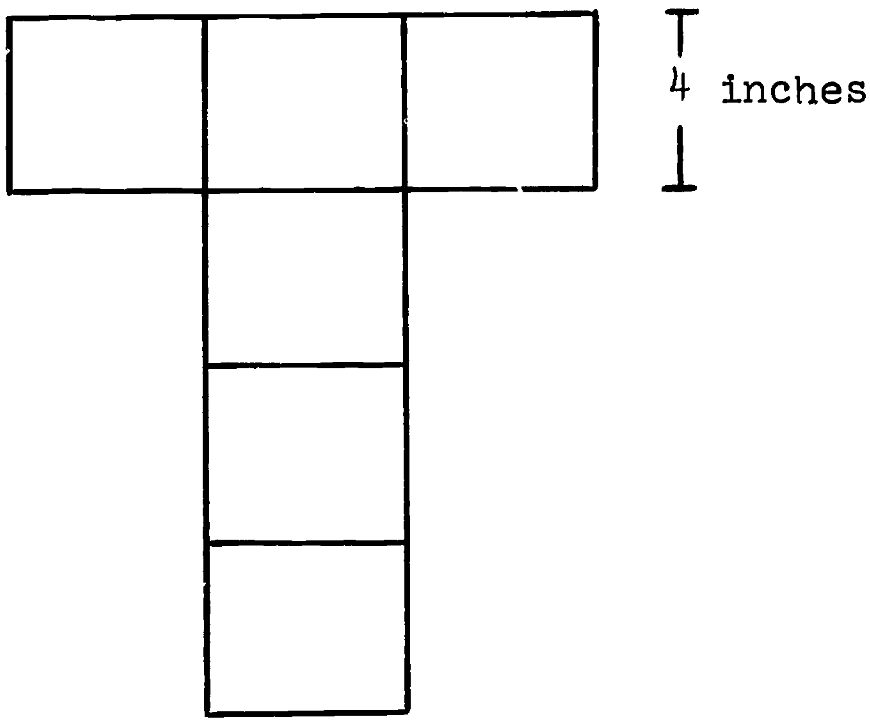
1. Find the volume within these right circular cones.
   a) h = 6 inches
      r = 3 inches
   b) h = 5 inches
      r = 2 inches
   c) h = 5 centimeters
      r = 2 centimeters
   d) h = 1 foot
      r = 1 foot

2. Find the volume within these spheres.
   a) r = 4 inches
   b) diameter = 8 inches
   c) h = 6 inches
   d) r = 4 centimeters

3. As a classroom activity, bring to class and find the volume of such objects as:
   a) a softball
   b) an ice cream cone (empty please, melted ice cream is messy)
   c) a baseball
   d) a basketball
   e) any other objects that are cones, cylinders or spheres

SURFACE AREA

Recall that squares are used in measuring area. Many times it is necessary to find both the volume and surface area of solids. If just the surface area of a solid is considered, the solid could be pictured in this way. Suppose a rectangular solid (box) is cut along the edges and folded out. The drawing below shows a cube folded out. To find the surface area, find the area of one face and multiply by the number of faces. Examine the example below.



The area of one face is: 4 X 4 = 16 square inches

For a total of six faces: 6 X 16 square inches = 96 sq. inches

If a cube four inches on a side has 96 square inches of surface area, what would be the volume?

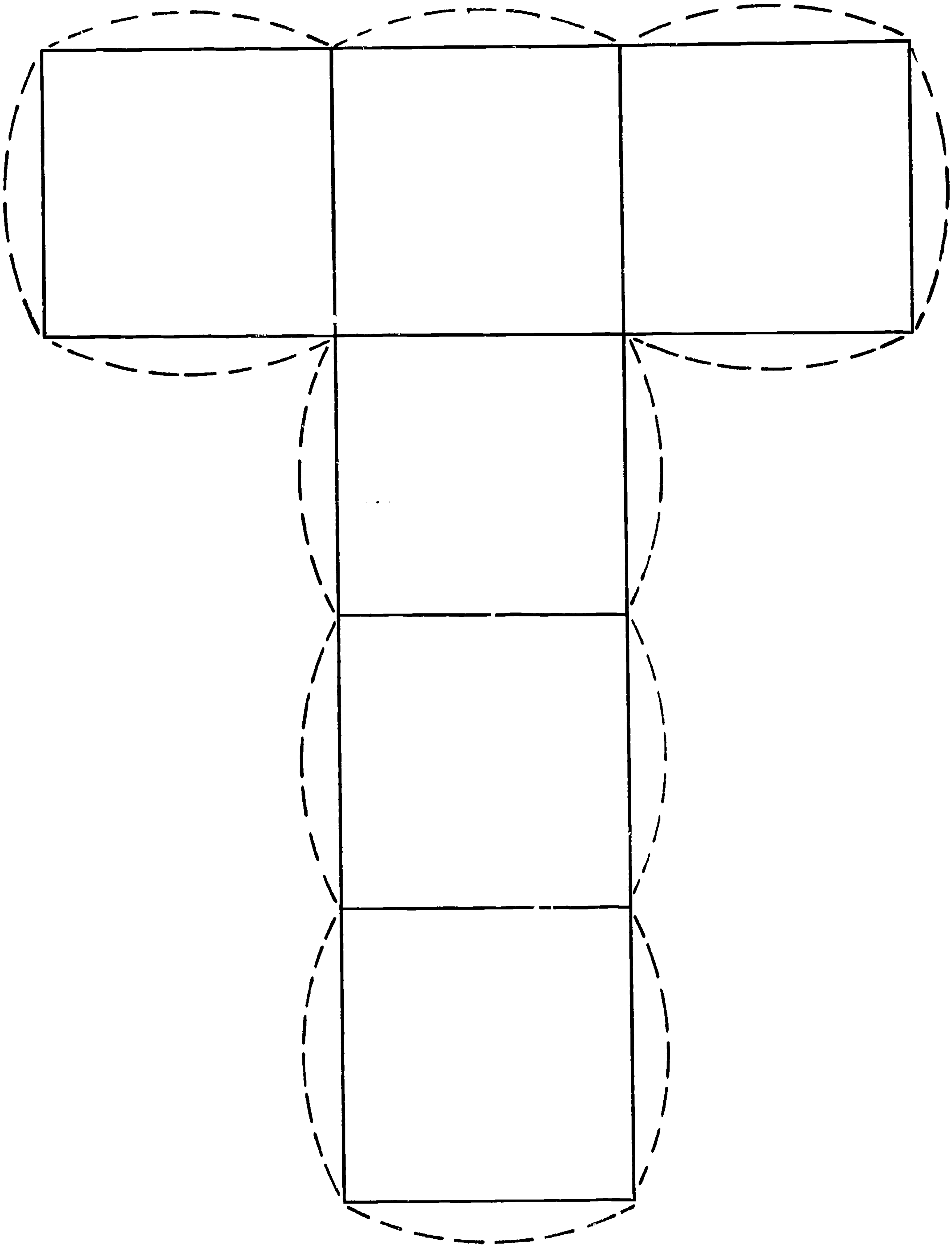
V = 4 X 4 X 4

V = 64 cubic inches

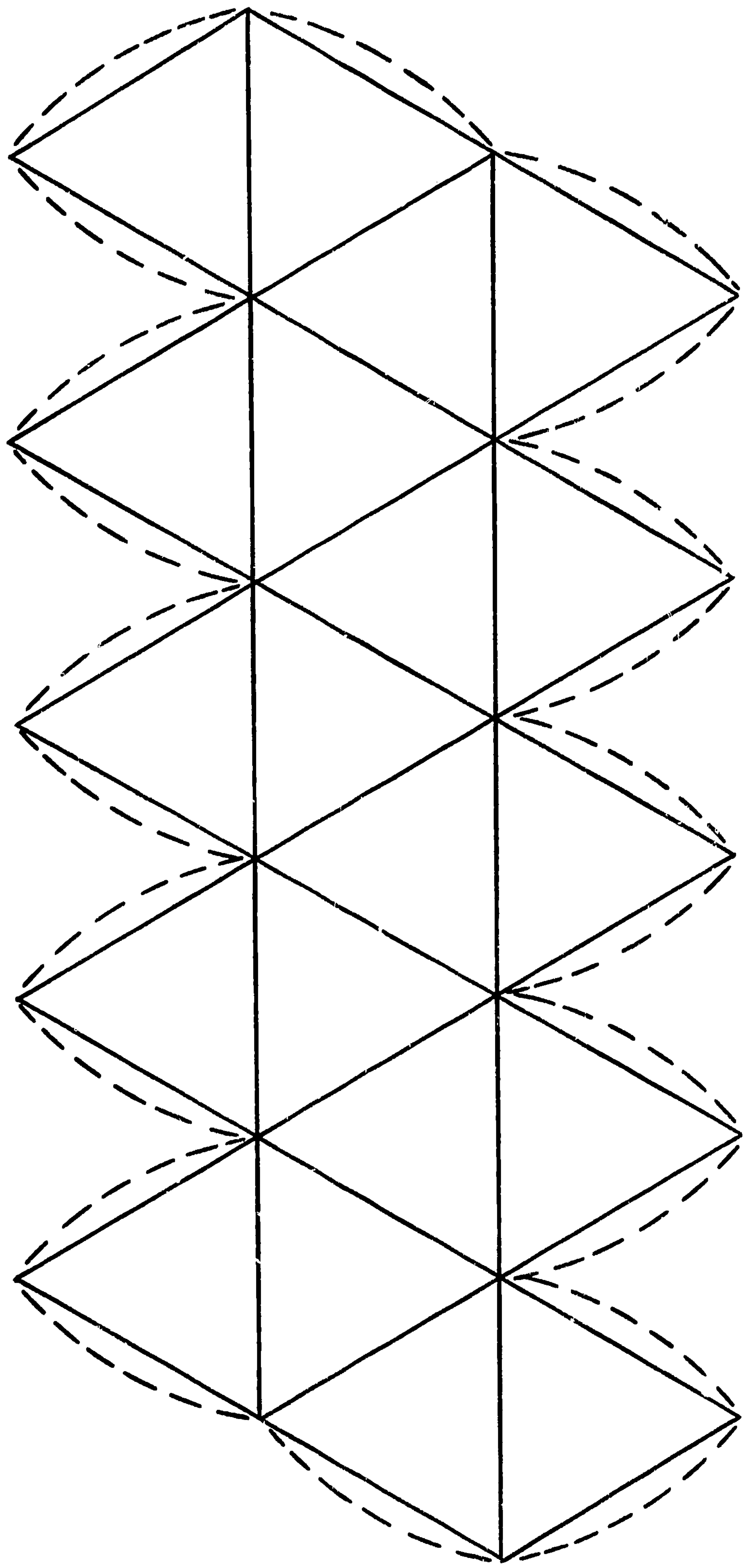
Activities

Using your ruler and protractor when necessary, find the surface area of the hexahedron, icosahedron, octahedron (in square inches) and the hexahedron, tetrahedron, and rectangular right pyramid (in square centimeters).

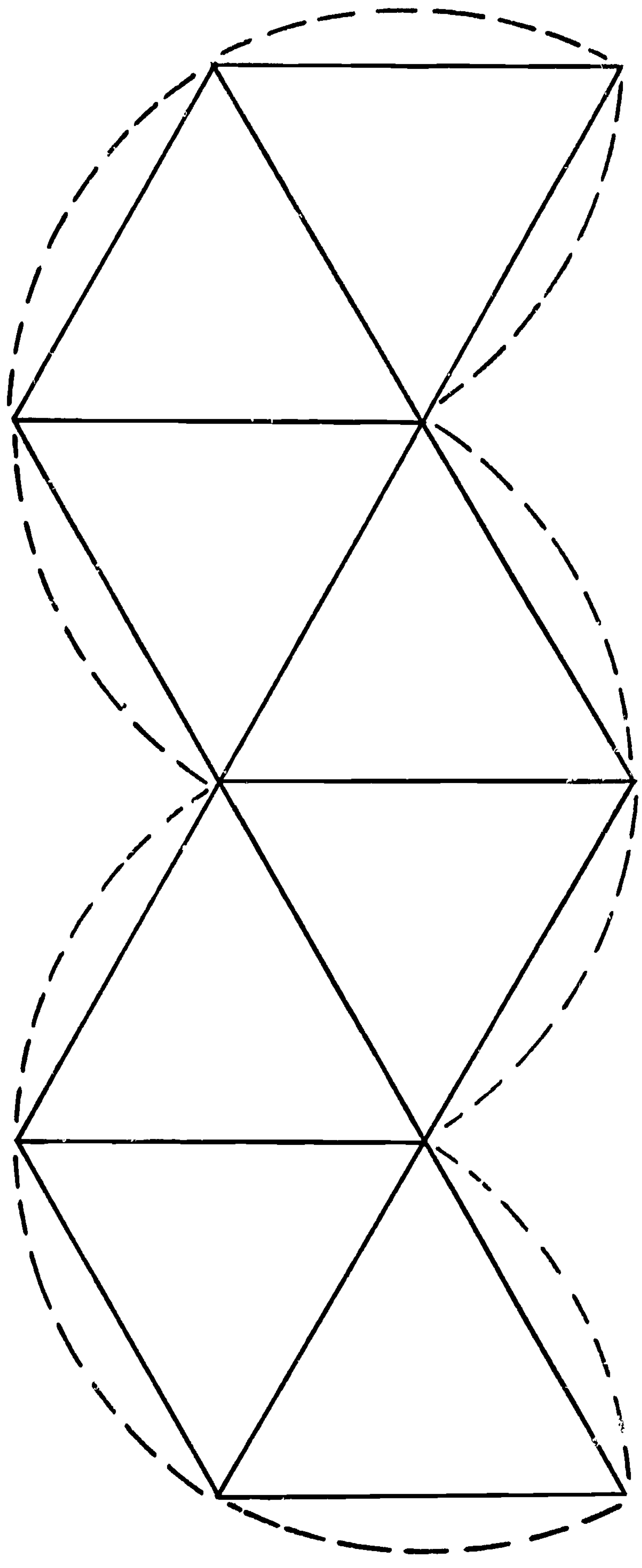
If cut out, folded, and taped, each drawing will form a particular geometric solid. Fold along the solid lines and use the dotted lines as tabs.



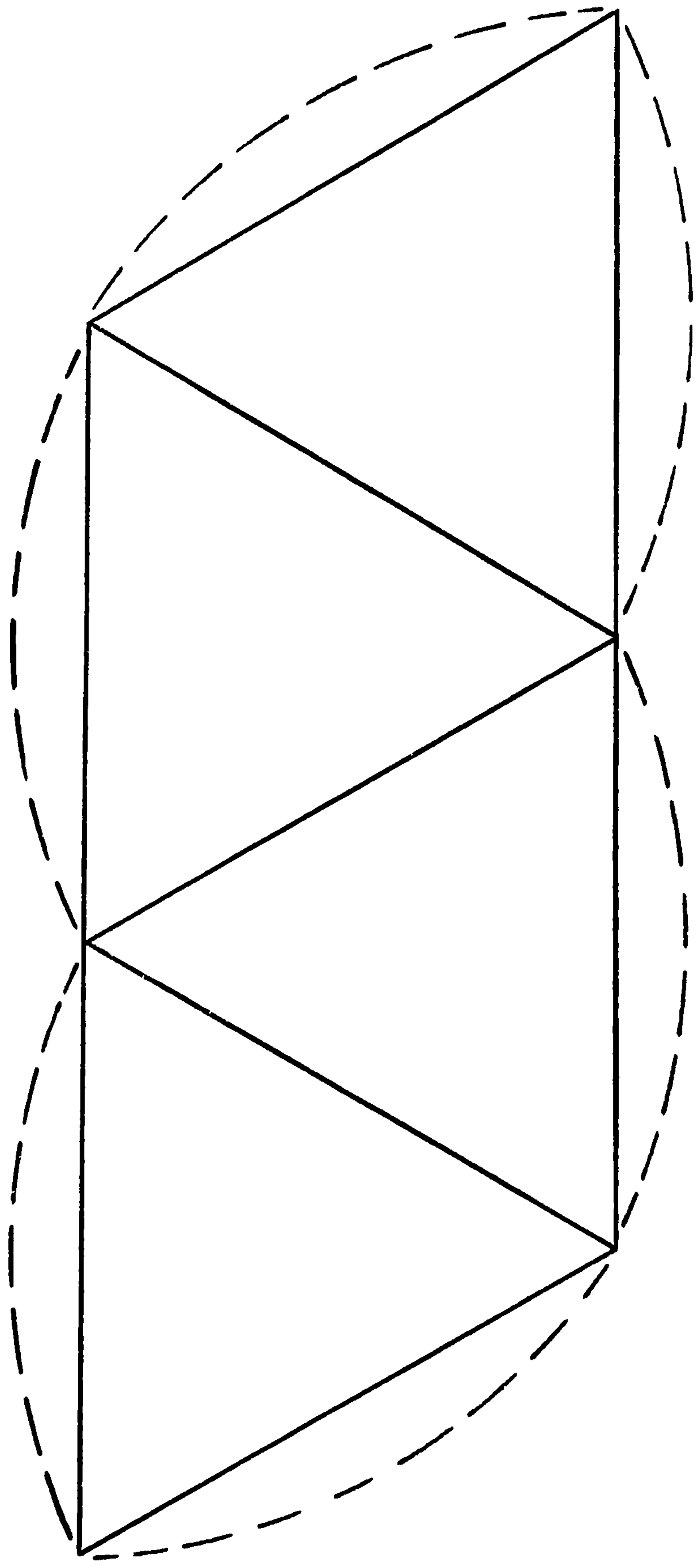
HEXAHEDRON



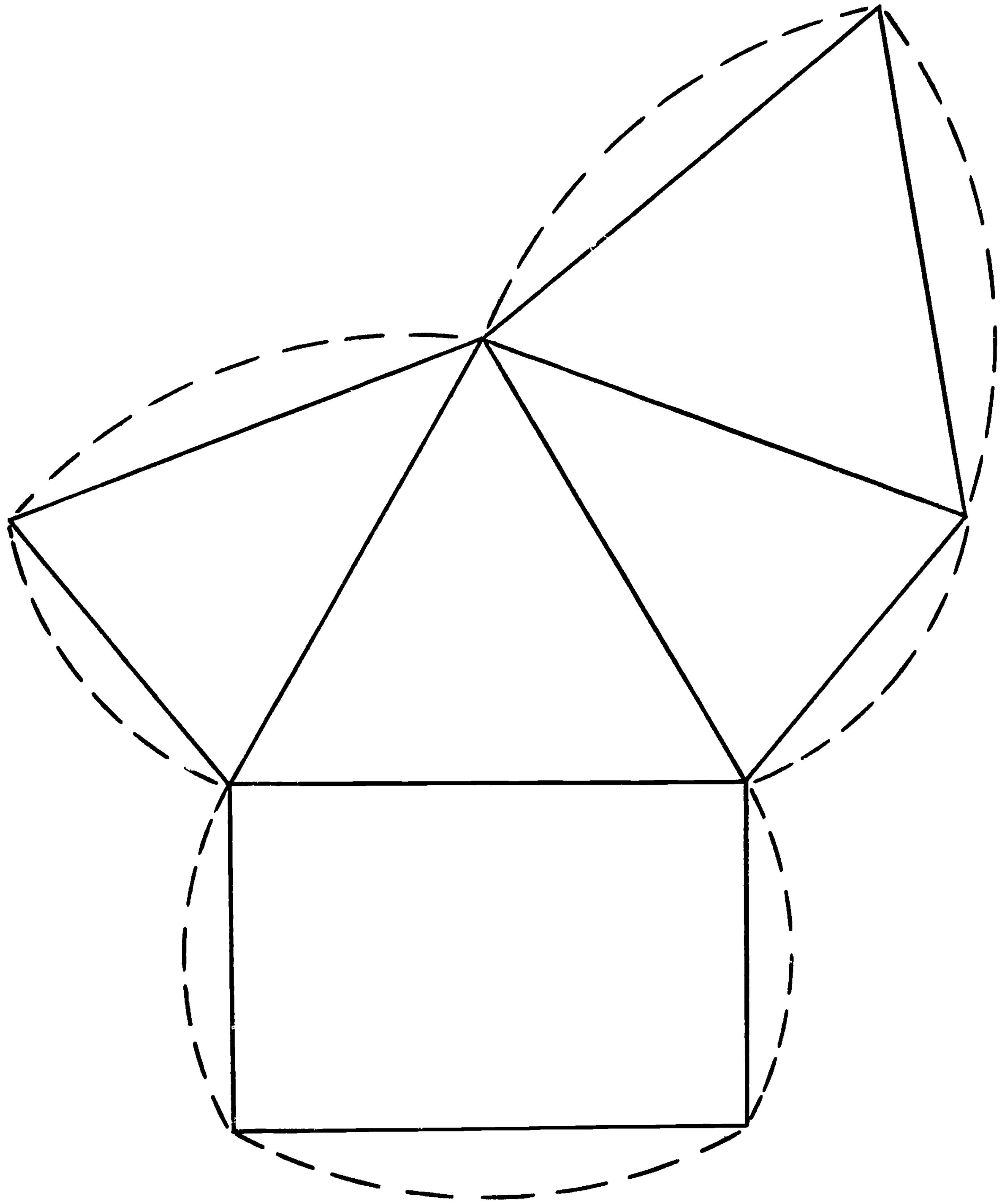
ICOSAHEDRON



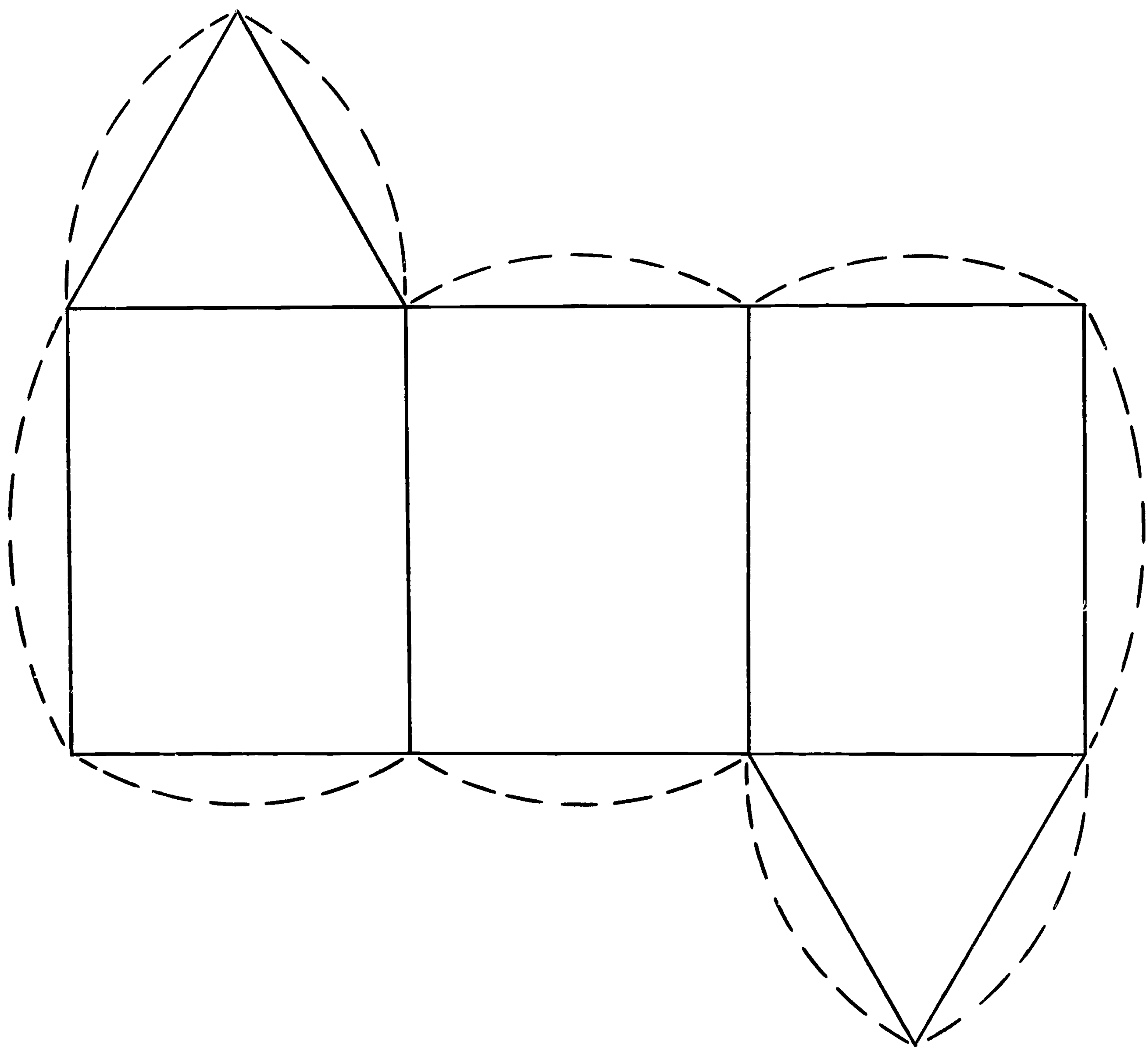
OCTAHEDRON



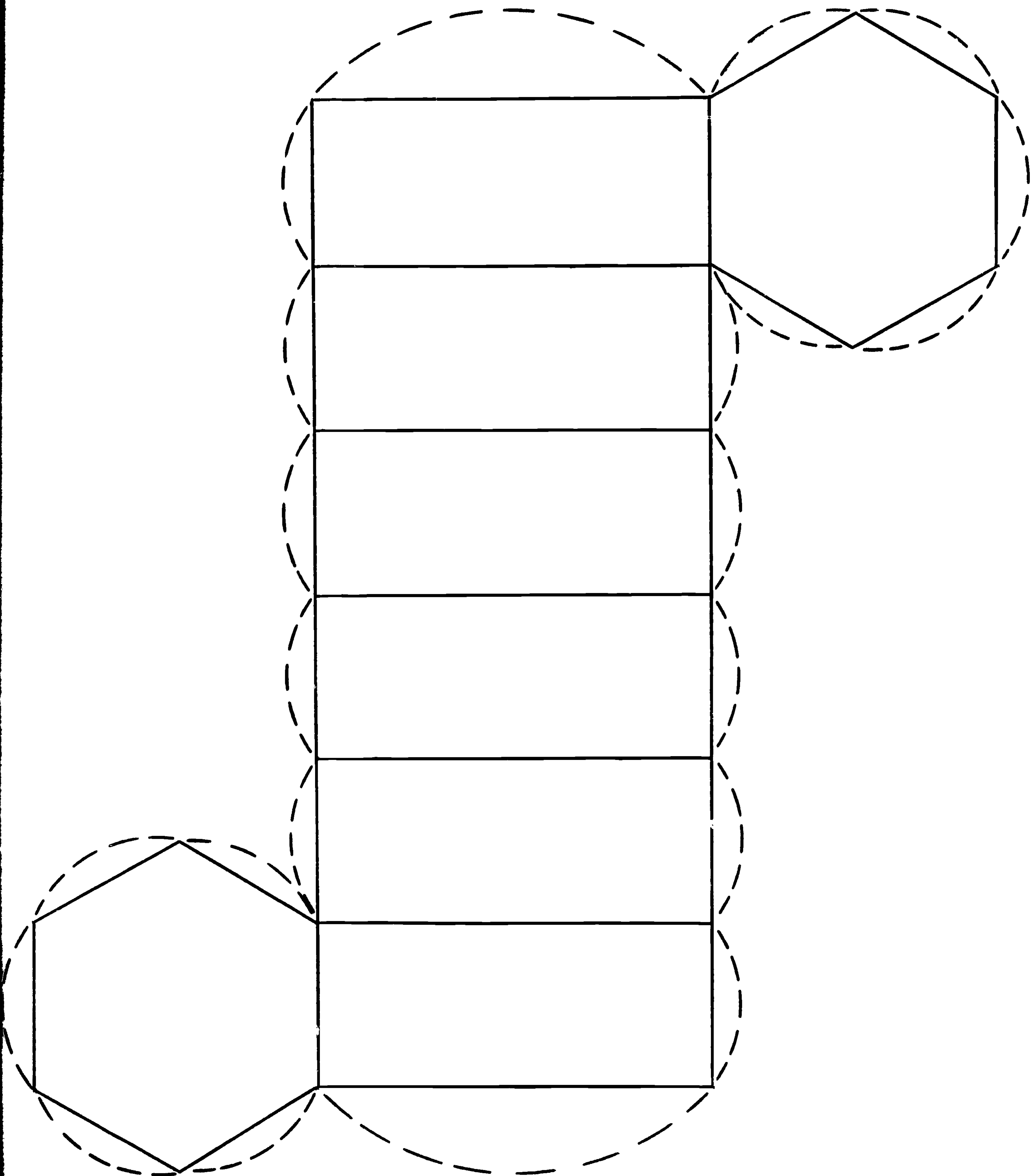
TETRAHEDRON



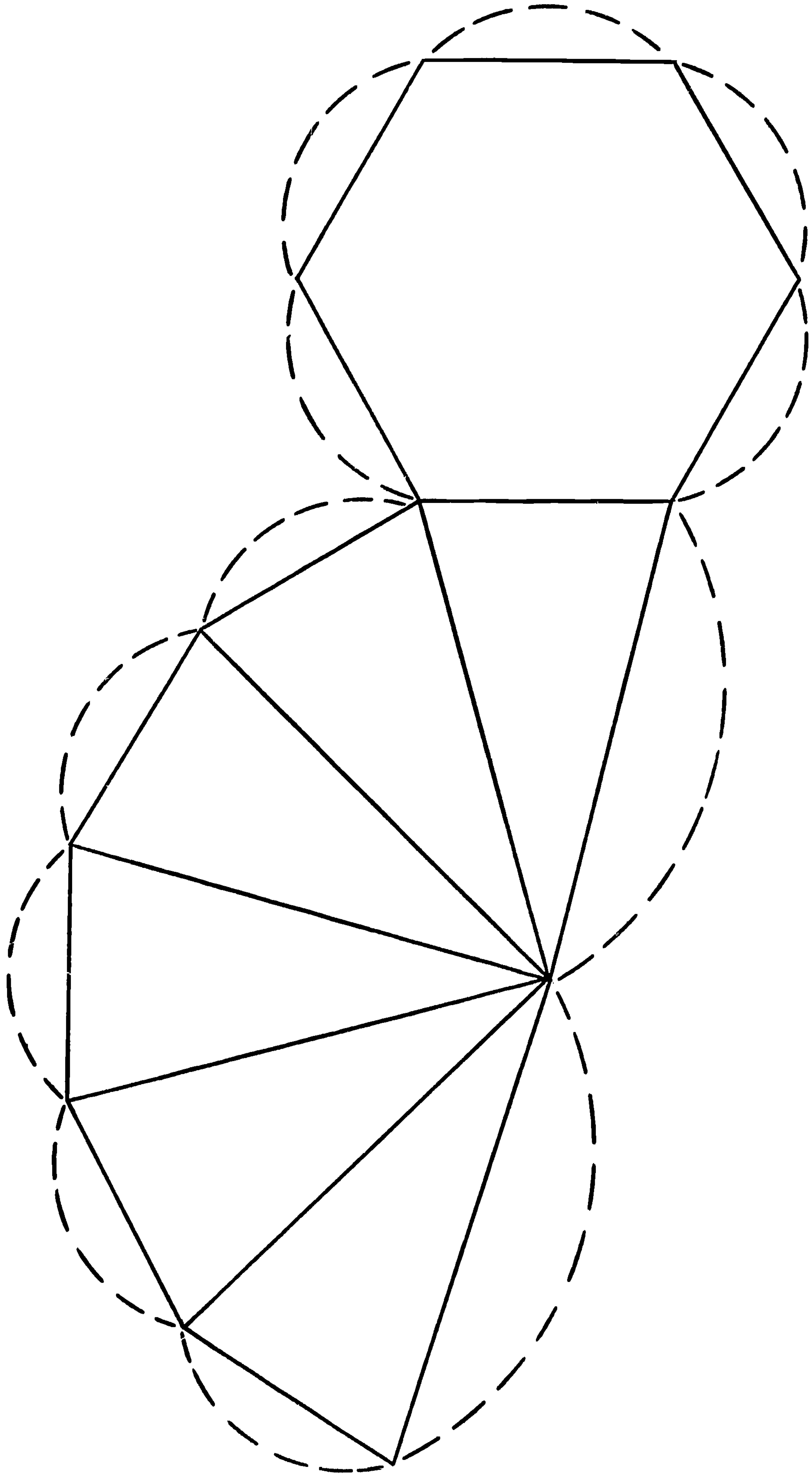
RECTANGULAR RIGHT PYRAMID



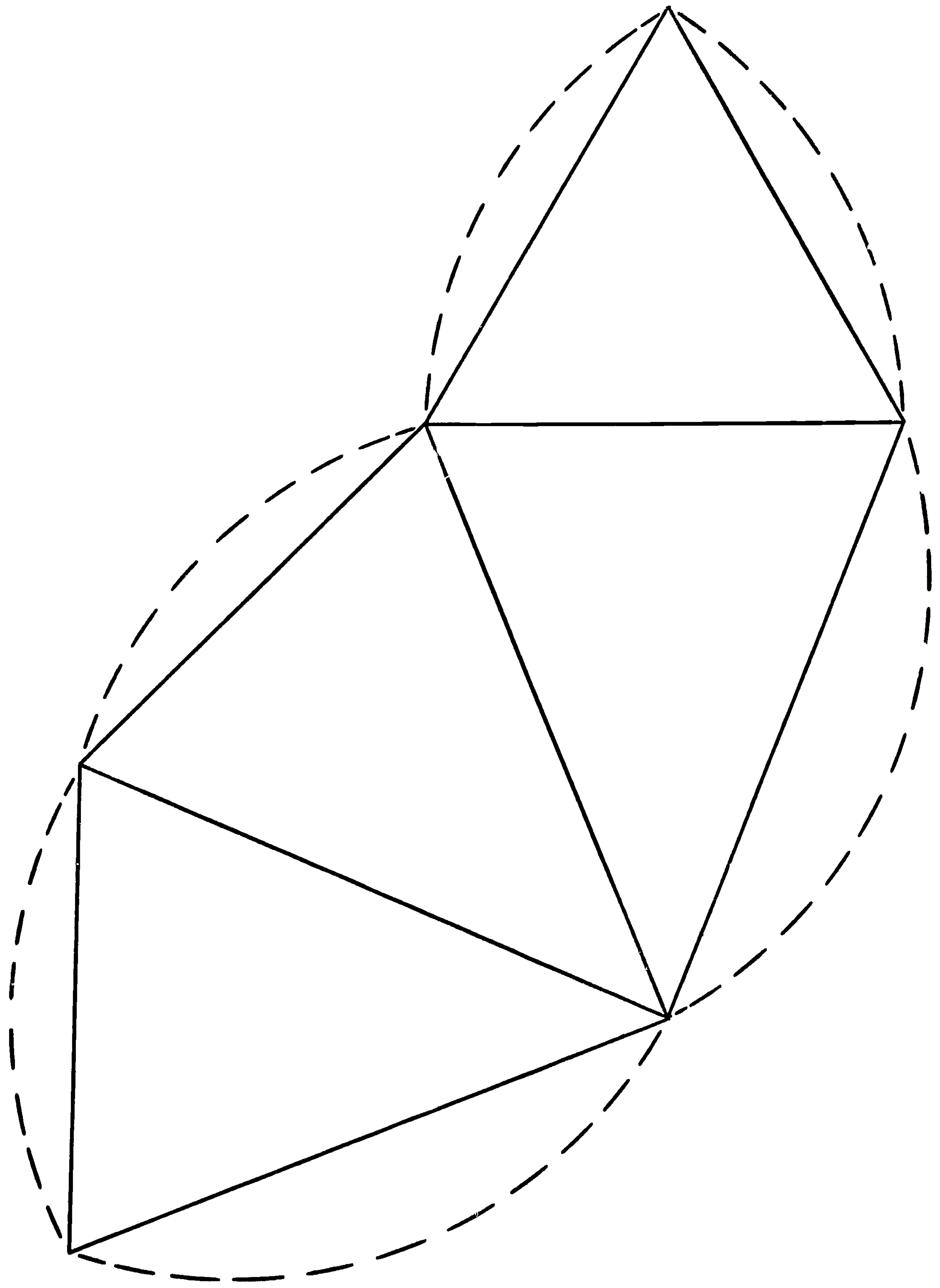
TRIANGULAR RIGHT PRISM



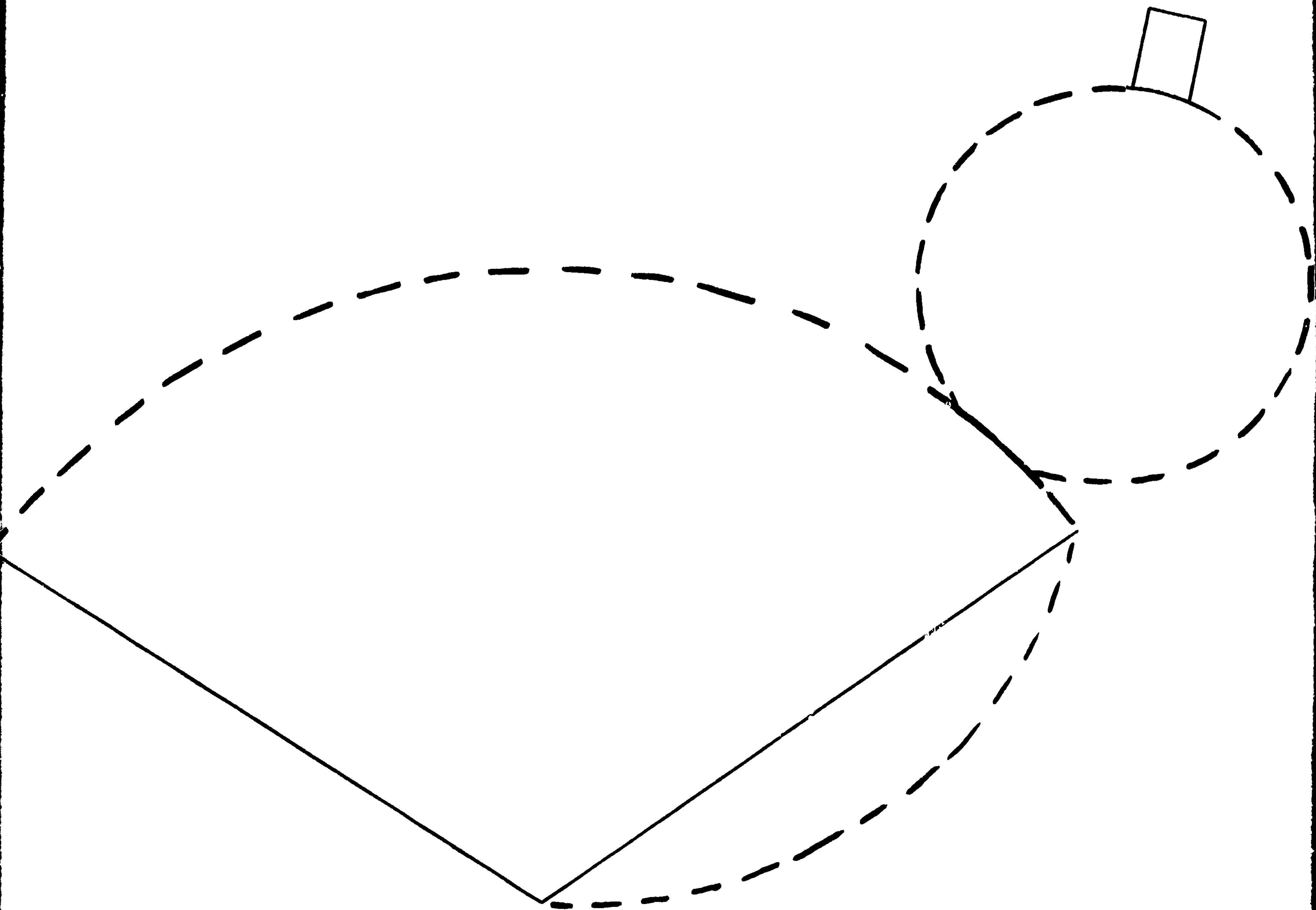
HEXAGONAL RIGHT PRISM



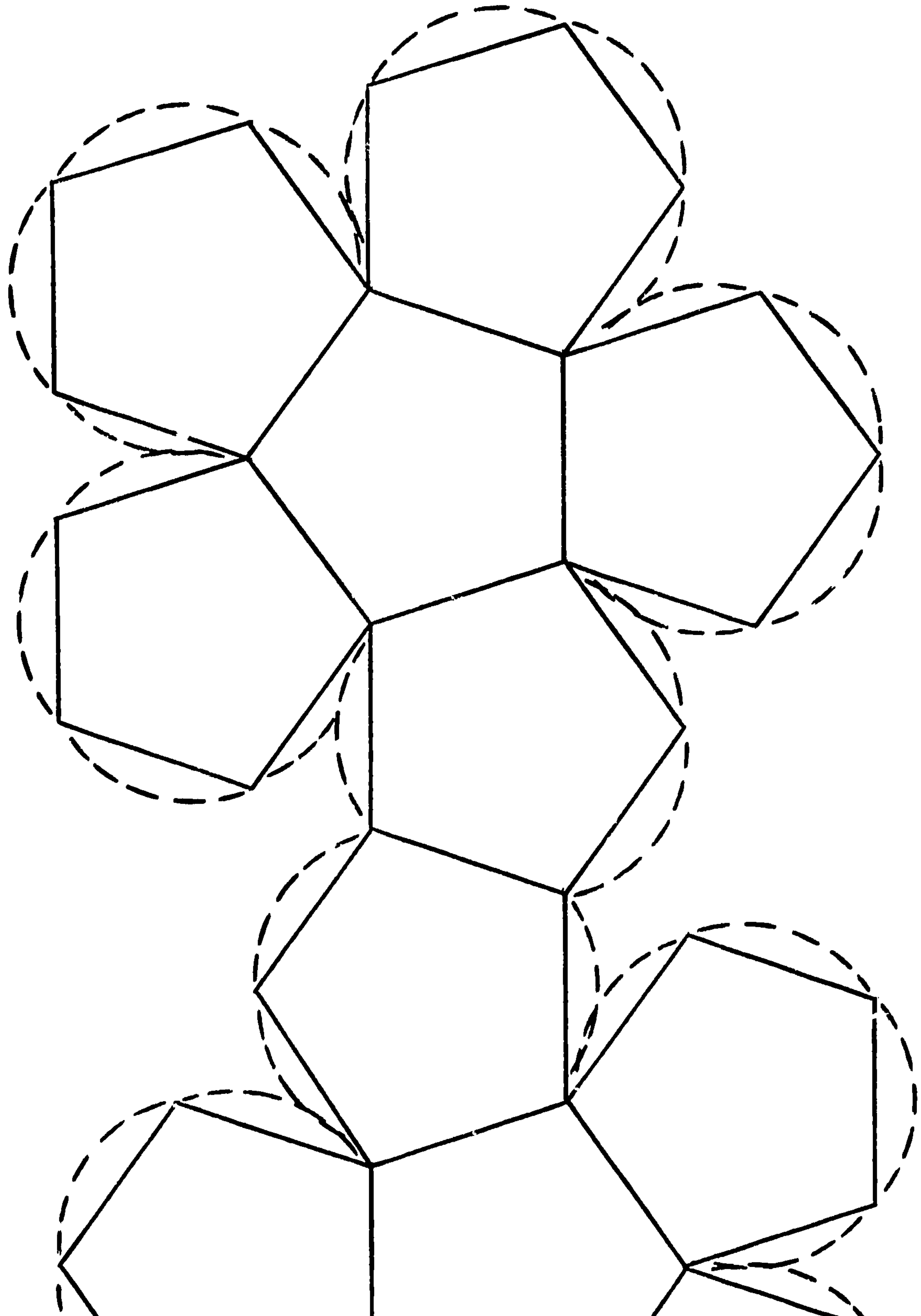
HEXAGONAL RIGHT PYRAMID



TRIANGULAR RIGHT PYRAMID



RIGHT CIRCULAR CONE



DODECAHEDRON