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MAKING PRIMARY ARITHMETIC MEANINGFUL TO CHILDREN.

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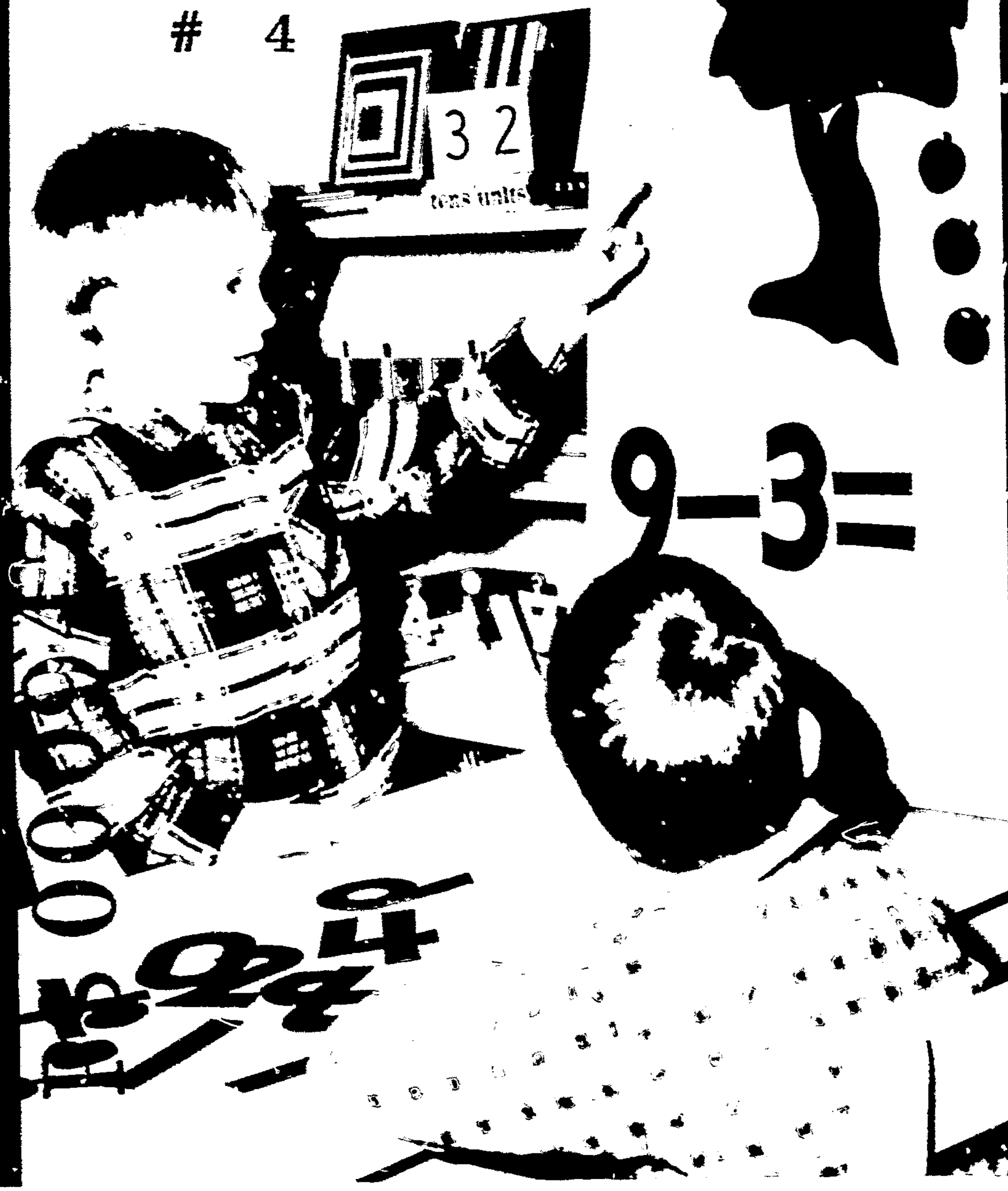
PUB DATE 61

EDRS PRICE MF-$0.25 HC NOT AVAILABLE FROM EDRS. 35P.

DESCRIPTORS- *ARITHMETIC, MATHEMATICAL EXPERIENCE, MATHEMATICAL CONCEPTS, *ELEMENTARY SCHOOL MATHEMATICS, MATHEMATICS CURRICULUM, *MATHEMATICS INSTRUCTION, *MATHEMATICS EDUCATION, PRIMARY GRADES,

MATHEMATICS EDUCATION IN THE PRIMARY GRADES HAS TRADITIONALLY CONSISTED OF THE ROUTINE AND REPETITIVE SOLVING OF GREAT QUANTITIES OF NUMERICAL PROBLEMS AND THE ROTE LEARNING OF THE NUMBER COUNTING SEQUENCE, OR LATER, THE ADDITION OR MULTIPLICATION TABLES. THIS EMPHASIS IS CHANGING, HOWEVER, FOR IT IS BECOMING EVIDENT THAT FOR SUBJECT MATTER TO BE LEARNED WELL, IT MUST BE PRESENTED TO THE STUDENT IN A MEANINGFUL WAY. PROBLEM SOLVING MUST ACCOMPANY NUMBER-CONCEPT COMPREHENSION, AND THE ROTE LEARNING OF NUMBER SEQUENCES OR TABLES MUST BE ACCOMPANIED BY AN UNDERSTANDING OF THE RELATIONSHIPS BETWEEN NUMBERS. AN UNDERSTANDING OF MATHEMATICAL CONCEPTS AND PROCESSES IS VERY IMPORTANT FOR THE PUPIL'S YEAR-BY-YEAR SUCCESS IN MATHEMATICS COURSES. THE DEVELOPMENT OF THIS UNDERSTANDING SHOULD BEGIN IN THE PRIMARY GRADES. TO ACCOMPLISH THIS UNDERSTANDING, A MEANINGFUL MATHEMATICS EDUCATION MUST BE OFFERED THE PUPIL. IT SHOULD INCLUDE (1) VARIED EXPERIENCES IN DEALING WITH NUMBERS AND THEIR MANY USES, (2) THE USE OF MATHEMATICAL CONCEPTS AND PROCESSES IN PROBLEM CONTEXTS THAT ARE FAMILIAR AND RELEVANT TO PRIMARY-AGE PUPILS, (3) DEMONSTRATION OF THE PURPOSES AND INTENT OF MATHEMATICS, THAT IS, THE AIMS OF ALL THE NUMBER MANIPULATIONS CHILDREN ARE REQUIRED TO DO BEYOND JUST THE AIM OF OBTAINING ANSWERS, AND (4) ORGANIZING THE PRESENTATION OF THE MATHEMATICAL CONCEPTS AND PROCESSES IN A LOGICAL SEQUENCE, IN AN EFFICIENT BUILDING-BLOCK FASHION. THIS DOCUMENT IS AVAILABLE FOR $0.75 FROM NEA, 1201 SIXTEENTH STREET, N.W., WASHINGTON, D.C. 20036. (WD)

ESTHER J. SWENSON



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Library of Congress Catalog Card Number: 61-9989

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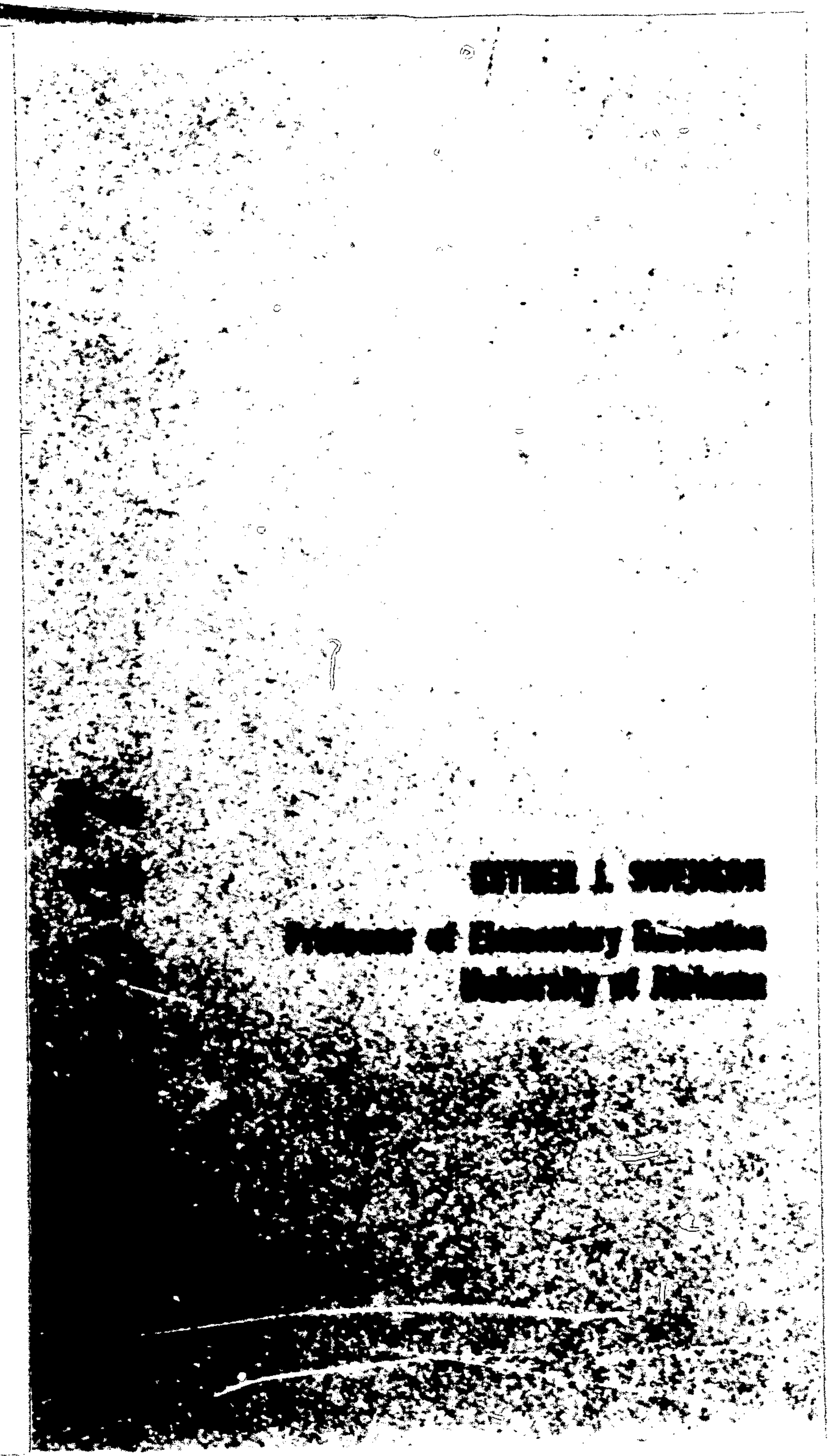


Making primary arithmetic meaningful to children

—a thoughtful consideration of "meanings" in arithmetic for young children

FOREWORD

Arithmetic concepts are vital to living. Too often teachers of young children assume an attitude that arithmetic in the kindergarten and the first three grades is unimportant. Many of them say, "I never could do arithmetic," as if it were something one learns separate and apart from living. Esther Swenson points out that primary arithmetic is fundamental, basic, and of first importance. Through effective illustrations and a deep understanding she presents a philosophic attitude toward arithmetic which is needed by all teachers.

The word "meaningful" is often used by teachers but with little thought toward really giving arithmetic meaning. Dr. Swenson develops four major concepts for meaningful arithmetic: meaning is experience, meaning is context, meaning is intent, and meaning is organization.

With so much emphasis placed on mathematics at the present time, teachers of young children will welcome help from a person who understands children, teachers, and arithmetic. Our thanks go to Esther Swenson for this timely and useful bulletin.

LULU PALMER, *President*,
Department of Kindergarten-Primary Education,
National Education Association

Life is a search for meanings. This is true not only for the philosopher, the intellectual, the mature man, but also for the average man, the average adolescent, the average child. The young child, in a peculiar sense, spends his waking hours trying to find out meanings: What is this? Why are you doing that? What do you call it? May I try it? What will it do? What can I do with it? Where did it come from? Whose is it? I want to touch it (or taste it, or smell it, or listen to it, or see it, or handle it). The seemingly endless curiosity of the young child is a search for meanings in his everyday life.

The school has a special role to fulfill in helping children acquire accurate, clear, and rich meanings. Unfortunately, teachers sometimes get so involved in teaching the words in which meanings have been expressed by others that they forget the danger that the words may not carry the intended meanings into the life of the child. Consequently, there is much talk today about making learning meaningful and about teaching meaningfully.

Talking intelligibly about the meaning of meaning is not easy; the words sound a little like double-talk. Nevertheless, it is important for us to ask ourselves seriously what we mean when we talk about making any school subject meaningful. The primary-grade teacher needs to ask himself two questions: First, what is meant by meanings and how are meanings acquired? Second, what particular meanings should be acquired by children during the early elementary-school years? In this bulletin, our concern is with arithmetic.

MATHEMATICAL CONCEPTS AND PROCESSES

The arithmetic meanings to be acquired by most children in the primary grades are, briefly, certain mathematical concepts and processes. We often make a serious error in assuming that primary arithmetic consists entirely of a routine counting, reading, and writing of numerals, and routine oral or written repetition of such arithmetic facts as $2+2=4$ or $15 \div 3=5$. Some teachers complain bitterly that there is not enough content in primary-grade arithmetic books. (They may sometimes be right; but this is a separate matter.) What often *seems* to be a limited coverage *is* at times limited because of the teacher's narrow view of the meanings involved. Teachers in the primary grades have the responsibility to teach much more than the oral and written forms. They need to recognize their responsibility to teach the underlying and

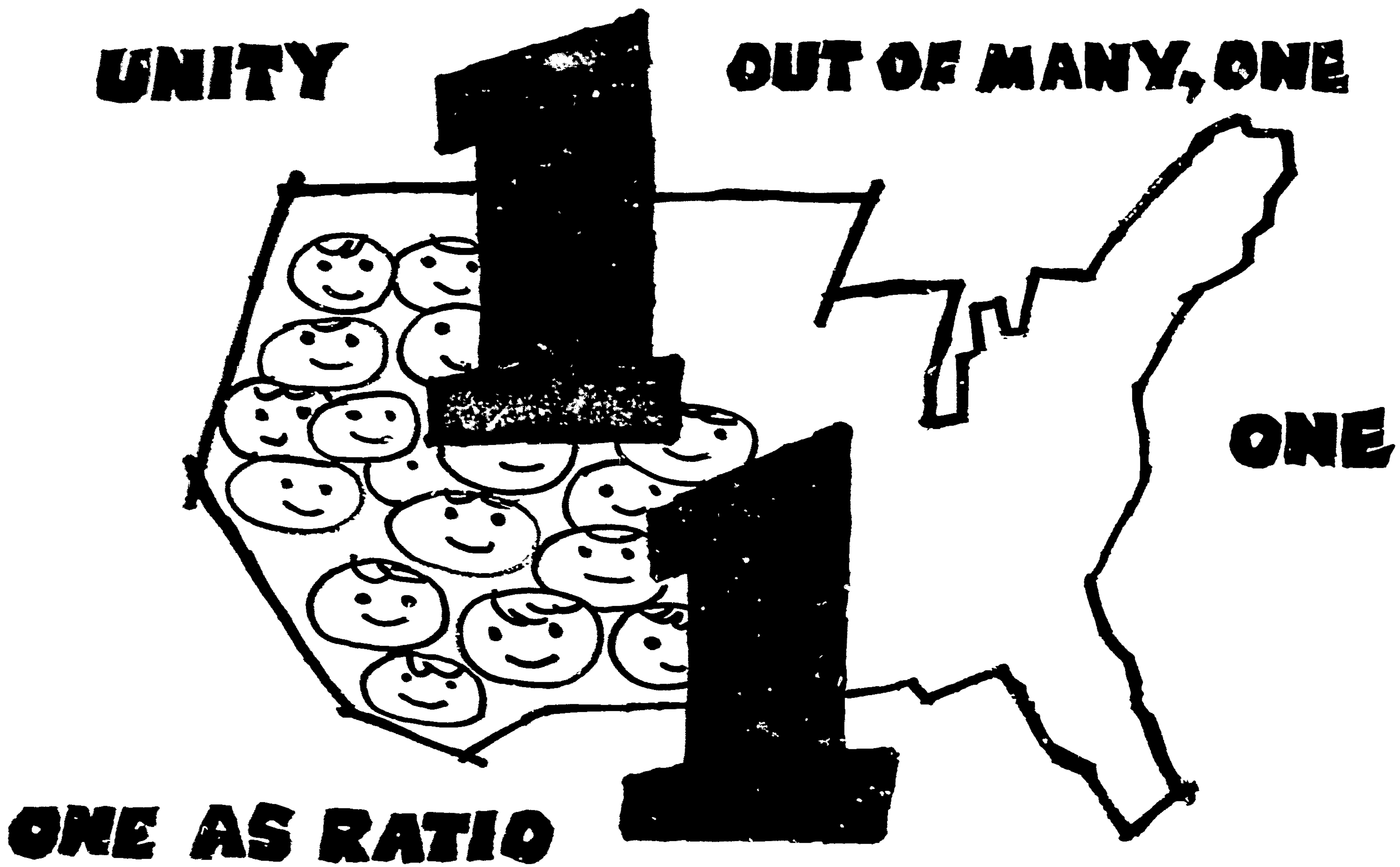
surrounding meanings of the concepts and processes which are merely signified by the forms.

The concepts and processes which undergird the *whole* arithmetic curriculum should be introduced and practiced in the primary grades. The primary teacher sets the foundation, be it sound or shaky, on which the whole arithmetic structure rests. These concepts should not be thought of as primary in the sense that they are just easy ideas for immature learners. They should be considered primary in the sense of being *fundamental* or *basic* ideas which are of first importance. Some of these primary concepts are *one-to-one correspondence*, *rational counting*, *place value*, and *base*. The processes of addition, subtraction, multiplication, and division, usually referred to as the four fundamental processes of arithmetic, sometimes are not adequately considered in their fundamental sense of various forms of *grouping* and *regrouping*.

As an illustration of a concept which is often passed over too lightly, consider the meaning of the number 1. Some people assume it to be the easiest of all numbers since it is the first in the whole-number sequence. The word or the numeral for the meaning "one" is not necessary in the child's early experience with quantity until he acquires workable concepts for the idea of more than one. If he has a penny or a ball or a flower, he can use the article *a* or he can say simply *penny*, *ball*, or *flower*. It is only after he experiences plurality of objects in contrast to the single object that he needs a name for the quantity "one." For instance, his brother has two nickels and he has only one, or, he needs to put on two shoes, but he can find only one. The meaning of "one" should be brought in as a contrasted idea in the development of the concept of a "group."

"One" has another meaning which is very difficult to comprehend. This is the idea of unity, a concept which has always intrigued and puzzled mature men and those with superior mental powers. For instance, we might consider the complexity of thought involved in Wendell Willkie's "one world"; the psychologist's "unity of the self"; a conception of our federal government as "out of many, one"; or the mathematical definition of "one" as the ratio of two equal quantities.

The concept "one" is certainly not the easiest number concept, even though it represents the smallest whole number. Many an elementary-school child has been baffled by the meaning of such a superficially easy fact as $3 \times 1 = 3$ or $1 \times 3 = 3$, even though many adults probably learned those facts as the beginning items in the tables of 3's. (Whether as children they really learned the facts or just learned to say the facts is per-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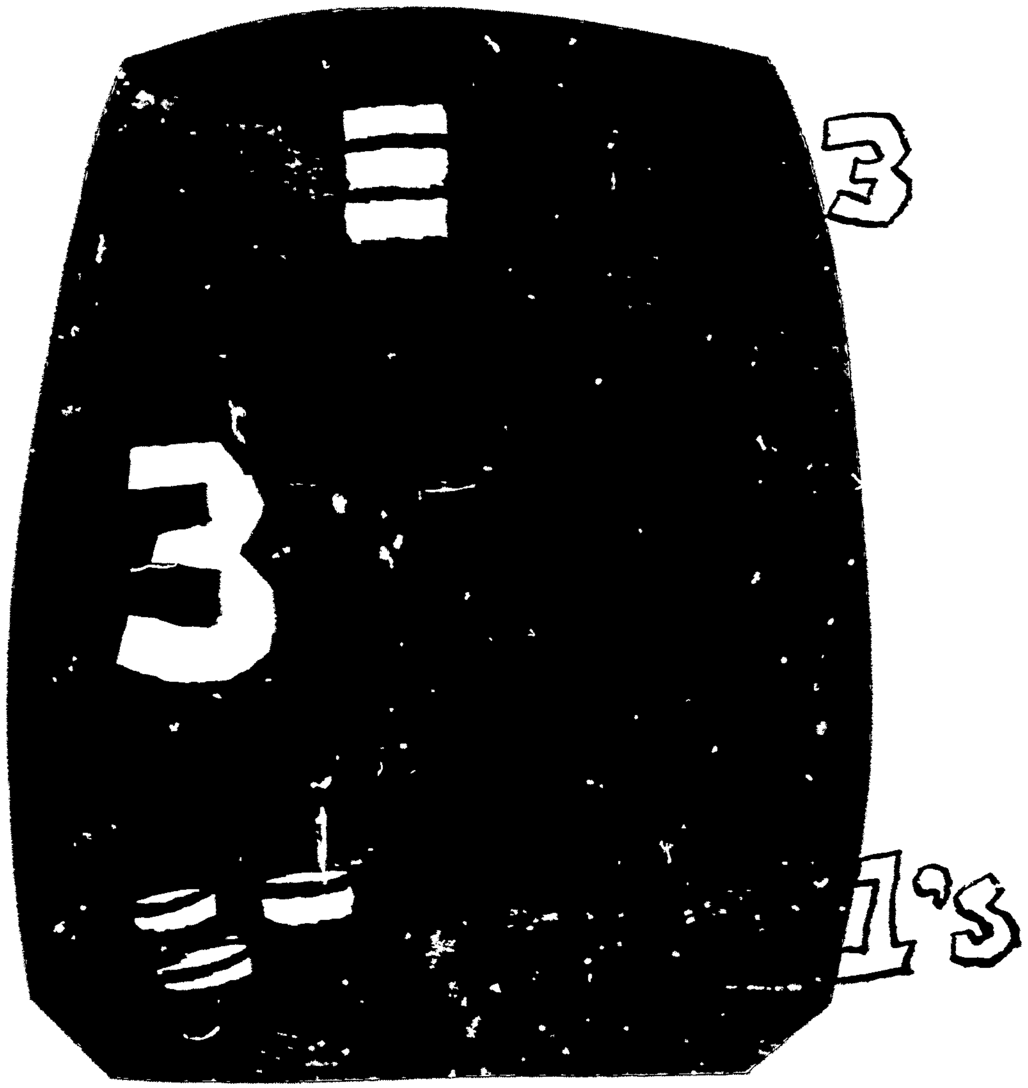


haps debatable.) As beginning multiplication facts, they are out of place. If a child buys one pencil for 3¢, he knows that he pays 3¢. He does not need the multiplication fact that $1 \times 3¢ = 3¢$. If he has one three, he has three; that is all he needs to know. On the other hand, if he buys three sticks of gum at one penny each, he has to pay three pennies. Three ones are obviously three, since that is what "3" means. In a sense, these facts are so easy that they become difficult. The child recognizes the truth of the situation as he experiences it; he does not need a formal statement about it in order to deal with it. Such statements as $3 \times 1 = 3$ or $1 \times 3 = 3$ are not required until he comes to a situation in which he must multiply two-place numbers. By that time, he has had additional experience on which to base his development of the multiplication meaning involved. The tables of 3's might better begin with "three 2's are 6" or "two 3's are 6."

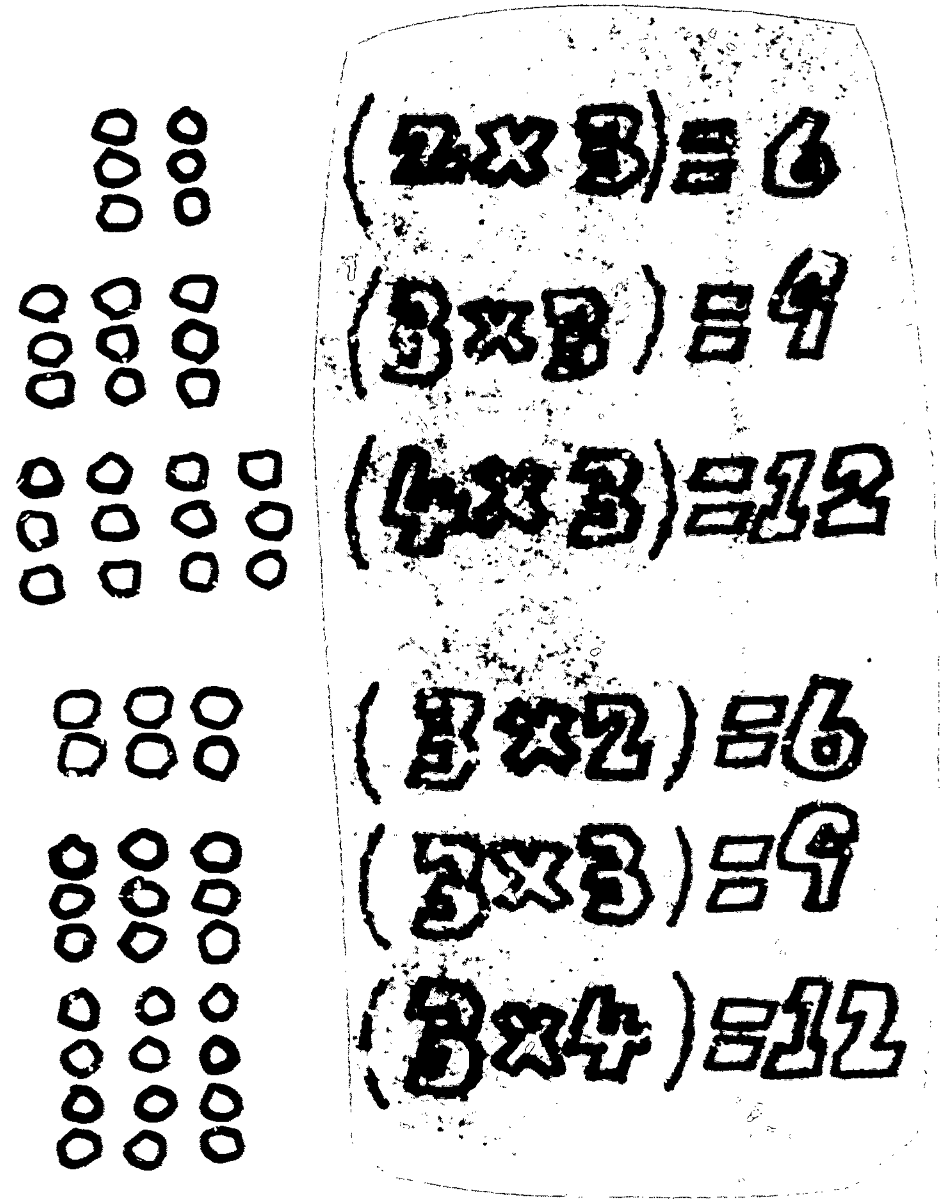
Multiplication tables should never be assigned to children ready-made. The facts should be developed meaningfully. Then the children can and should build their own tables as an organization of their learning.

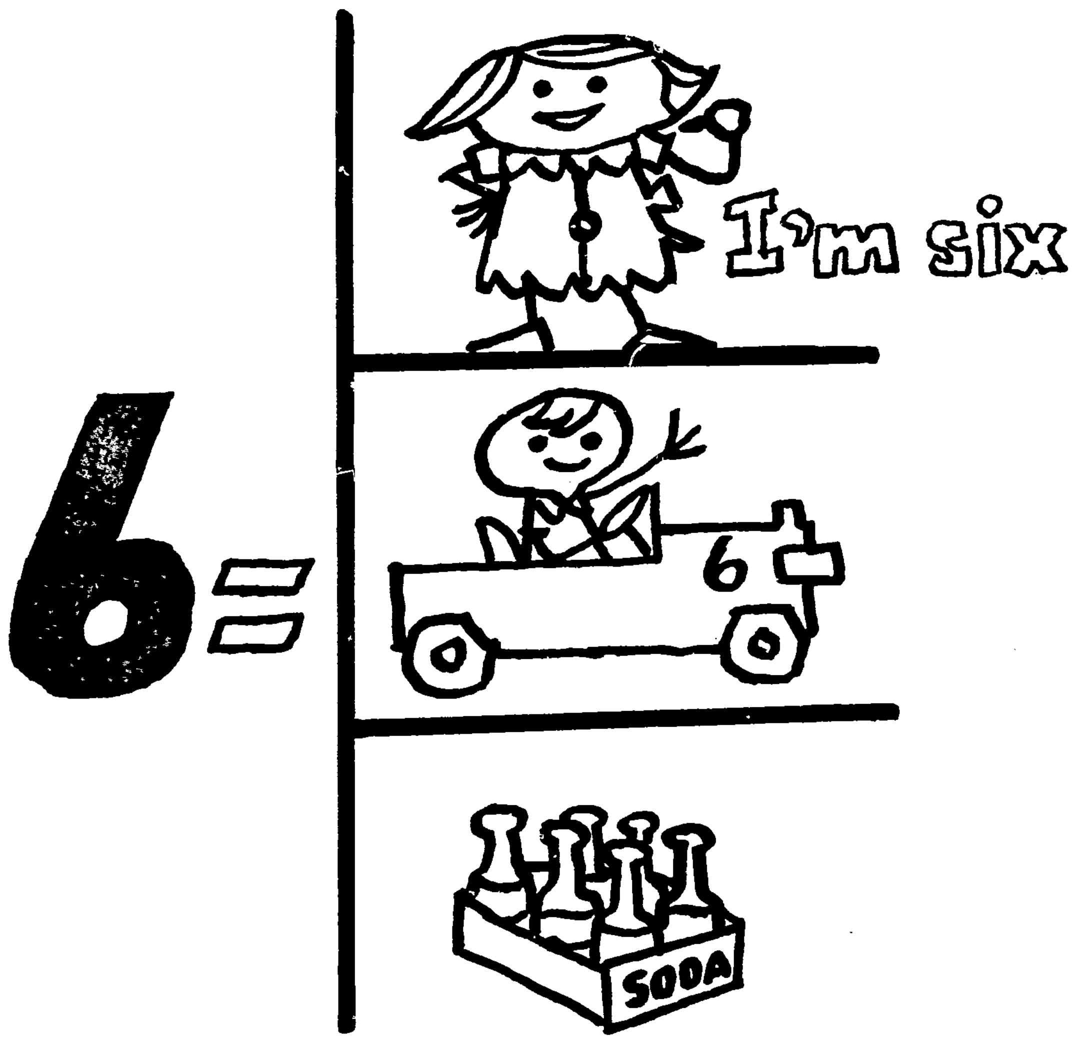
In what follows, the author has tried to weave a sample pattern in which the warp consists of some threads of discussion about meanings and how they may be acquired and in which the woof consists of

illustrative arithmetic concepts and processes to be meaningfully acquired by children. In no sense is this intended as a complete design for the teaching of primary-grade arithmetic. The analogy of the weaving of a pattern is not accidental. The teaching and learning of arithmetic should be such a pattern, woven so skillfully that meaningfulness is an integral part of each child's learning, not something superimposed or patched on the original cloth. Both teacher and learner participate in the weaving.

MEANING IS EXPERIENCE

To a child, the meaning of a word is what the word has denoted in his experience. An object means only what his experience with it reveals to him. A wool jacket means comforting warmth on a cool evening or hot discomfort when worn on a hot day; it can be an article of clothing in one situation and a rolled-up pillow for his head at another time. The word "teacher" to the child in the kindergarten or first grade means *his* teacher, the only teacher he has had; to the older child the word "teacher" has a composite meaning made up of his experiences with different teachers he has known. To the first-grader, the symbol "6" means





whatever experiences he has had involving that symbol —how old he is, the numeral painted on the winning racer in the local soapbox derby, the number of pennies he had to pay for a candy bar. Meaning is experience, and meaning can be built only through experience.

Meanings cannot be transmitted ready-made from one person to another. Real meanings are built within the learner's experience. The teacher cannot give the children his meanings; he must guide them into experiencing their own meanings. Because a child can be taught to say the arithmetic words and write the same arithmetical symbols as the teacher has said or written, it cannot be safely assumed that he has the same meanings as the teacher.

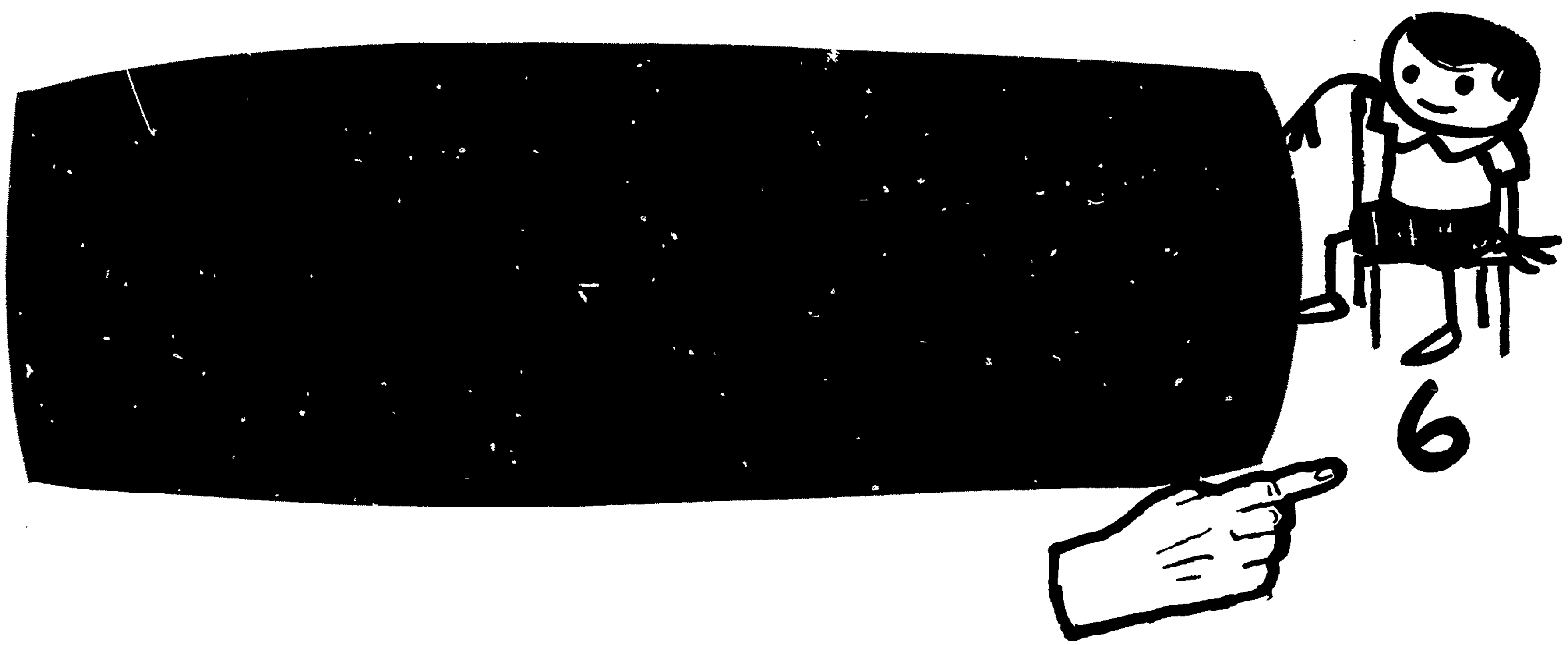
No matter how easily a child can recite the names of the numbers—even in their correct order—he does not know what counting means until he has had numerous experiences with the counting process in varied situations. It is not enough that the child can say the names in their right order; he must have had the experience of correctly matching objects and number names. He must be experienced in finding how many pennies he has in his hand by matching the pennies with the number words. He must have been able to find the sixth book on a shelf by matching books and number words. He must know that the number word *six* comes

after the number word *five*, not merely by rote, but by observing that a group of six chairs has one more chair in it than does a group of five chairs, and that the sixth chair is the next one after the fifth chair. The child must know from experience with dates and days that if yesterday was October 14, today is October 15, and tomorrow will be October 16. He must know that a stick 11 inches long is longer than one which is 9 inches in length; if he is not sure, he should find the answer himself by measurement of the two sticks against each other and against a ruler. The child must know that if his team in a game has eight players and the other side has nine players, his side needs one more player to have an equal number. He does not actually know the meaning of any number word or number symbol until he has experienced it in enough situations and under enough different circumstances to recognize it whenever and however it occurs.

Teachers of young children would find it profitable to gather all the typical uses of each number in primary-grade arithmetic. The following list may suggest some of the applications which can be utilized in building the meanings of specific numbers:

two: pair of shoes, child's eyes or ears, hands on a clock, twins

three: feet in a yard, clover leaf, triangle, tricycle



four: legs on a chair or table, square, baseball diamond, dog's legs

five: fingers on a hand, toes on a foot, package of gum, school days in a week

six: half-dozen egg carton, carton of soft drinks, 6-inch ruler, strings on a guitar

seven: days of a week, Seven Dwarfs, red stripes in the American flag

eight: Santa's reindeer, musical octave, 8-ounce measuring cup (with ounce markings)

nine: baseball team, 9-hole golf course, innings in a baseball game.

A very important word meaning for primary-grade children to learn as part of their arithmetic work is the word "group." Let us review some of the experiences Mary, a first-grade child, has with the word—experiences which become not only the basis but also the very substance of her expanding meaning of the word.

1. On the first day of school, Miss Allen, Mary's teacher, asks Mary to bring her chair and join the group. Miss Allen talks with the children about the group. She says they are a nice, quiet group. The teacher tells them that every morning when she takes her seat in her big chair, she wants them to take their chairs and join the group. She asks Mary if she has anything she wants to tell the group. *Mary knows that the teacher means that all the children gathered together are a group.*

2. Later in the day, Miss Allen asks Mary to help put away the picture books on the table. She suggests that she should gather them all up and put them in a neat group in the middle of the table. *Mary finds out that not only children but also books can be a group.*

3. One day Joe is displeased because one of the other children got to the supply cabinet before he did and took the puzzle that he wanted to put together. He sits in a chair away from the other children, pouts, and refuses to join the other children at his table. Mary tells him he belongs in her group. Joe says, "I'm in my own group." Billy laughs and says, "You can't be a group all by yourself. It takes more than one to be a group." *Mary listens and wonders about that. She is learning more about the meaning of group. It means more than one, according to Billy.*

4. On the playground, Miss Allen teaches the children a new game. She helps them form two groups, but some of the children do not stand still. Jim stands out in the middle between the two groups. Miss Allen asks, "What group are you in?" Jim says, "I'm in Sam's

group." The teacher tells him that he should stay with the other children in Sam's group so that everyone will know he belongs in that group. *Mary understands the importance of having all members of a group together.* (The "together" quality for immature learners is a physical togetherness. Later, the concept of being *considered* together can be developed, e.g., all the women in the United States might be *considered* together as a single group while being widely scattered as to physical location.)

5. One day Miss Allen shows the children a large box of toy cars, dolls, and animals. She suggests that Mary, Jim, and Sue put the toys on the table in groups. Mary takes all the dolls out of the box and makes a group. Jim puts all the cars in a group. Sue puts all the animals together.

Miss Allen asks, "How did you know which toys to put in each group?"

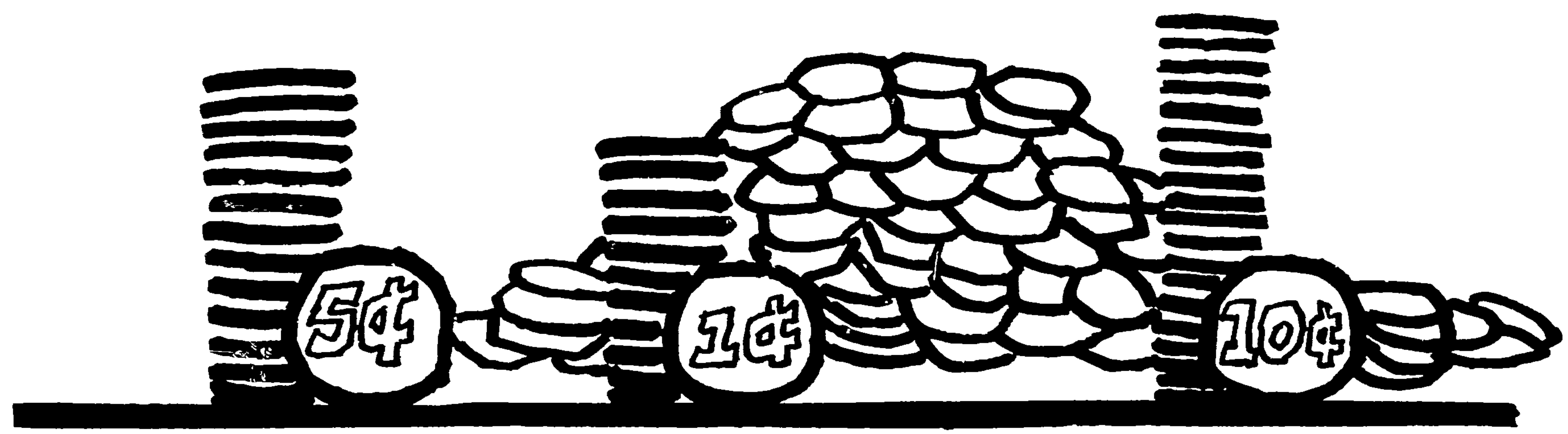
Mary answers, "I put all the dolls together because they are alike." Jim replies that he put the cars together because they are all cars and because dolls and animals are not cars and do not belong in his group. Then Alice says, "But all the animals aren't alike. Some are dogs, some are cats, some are horses, and some are cows. I think we should have one group for each kind of animal."

Miss Allen and the children talk this over, and finally come to the conclusion that it would be all right to have only dogs, only cats, only cows, and only horses in groups; but that it is also all right to have a group of all animals together so long as they are called a group of animals, not a group of cows.

Thus, Mary learns not only that things in a group must be alike in some way, but also that you can put different groups together if they are still alike in another way.

6. On later occasions, Mary hears her teacher tell the children to group themselves around the tables, to group the books on the shelves of the bookcase, to group the pennies, the nickels, and the dimes when counting the lunch money. *Mary has a chance to extend her meaning of the word, group. "Group" is not only the name of more than one thing put together because of some similarity, but is also something you do to things when you put them together to make a group.*

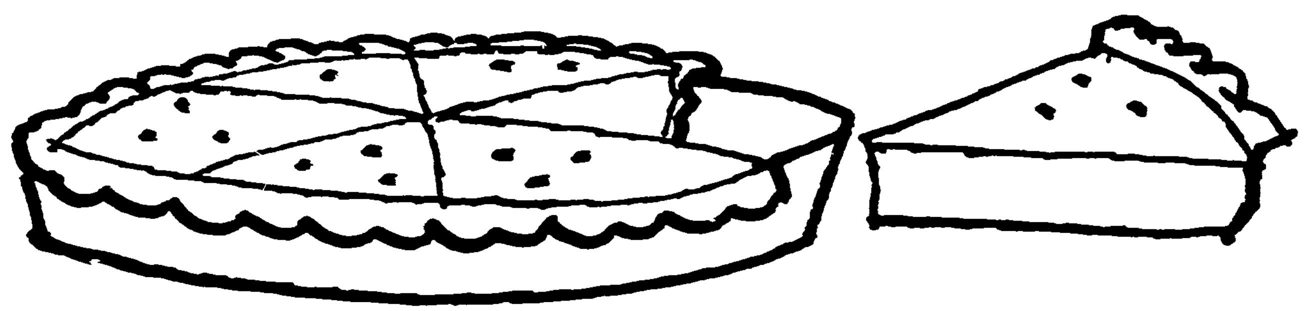
This is by no means a complete story of Mary's learnings through experience about the meaning of the word, group. However, it indicates how she had an opportunity to improve and expand her understanding of the meaning of the word. Her learning came about not by definition but by *experience* with the word



applied correctly in situations which were understandable to her. She even began to see a distinction between the use of the word as a noun and as a verb, but, of course, with no mention of parts of speech. She learned the uses of the word—that is, its meanings—through meaningful practice under the guidance of a teacher who herself was conscious of the importance of this word's meaning in arithmetic.

It may be asked whether or not this is really arithmetic. Have we not strayed far from arithmetic? No, not at all. The idea of the group which Mary acquired through experience in a variety of situations is one of the basic concepts of arithmetic. Understanding of numbers rests on understanding of groups in their quantitative aspects. Understanding of addition, subtraction, multiplication, and division is of necessity based on understanding of the verb "to group," for these fundamental processes of arithmetic are themselves processes of grouping and regrouping. Even the understanding of *one* at later stages of arithmetic has a group meaning, as in the case of one pie being equal to a group of six sixths.

Six situations were mentioned in which Mary had an opportunity to experience the meaning of "group." Six is not enough, but these several instances serve to illustrate the variety and succession of opportunities



which are necessary for a child to acquire the meaning of a term or an idea. The acquisition of a concept, whether in mathematics or in another area, is a many-sided process. Teachers of primary-grade children need to be particularly aware of the significance of such simple situations in concept building. At times Miss Allen used the opportunity provided in a present situation; at other times she deliberately provided the situation. In all cases she intentionally led Mary and the other children to benefit from the learning potential in the situation.

MEANING IS CONTEXT

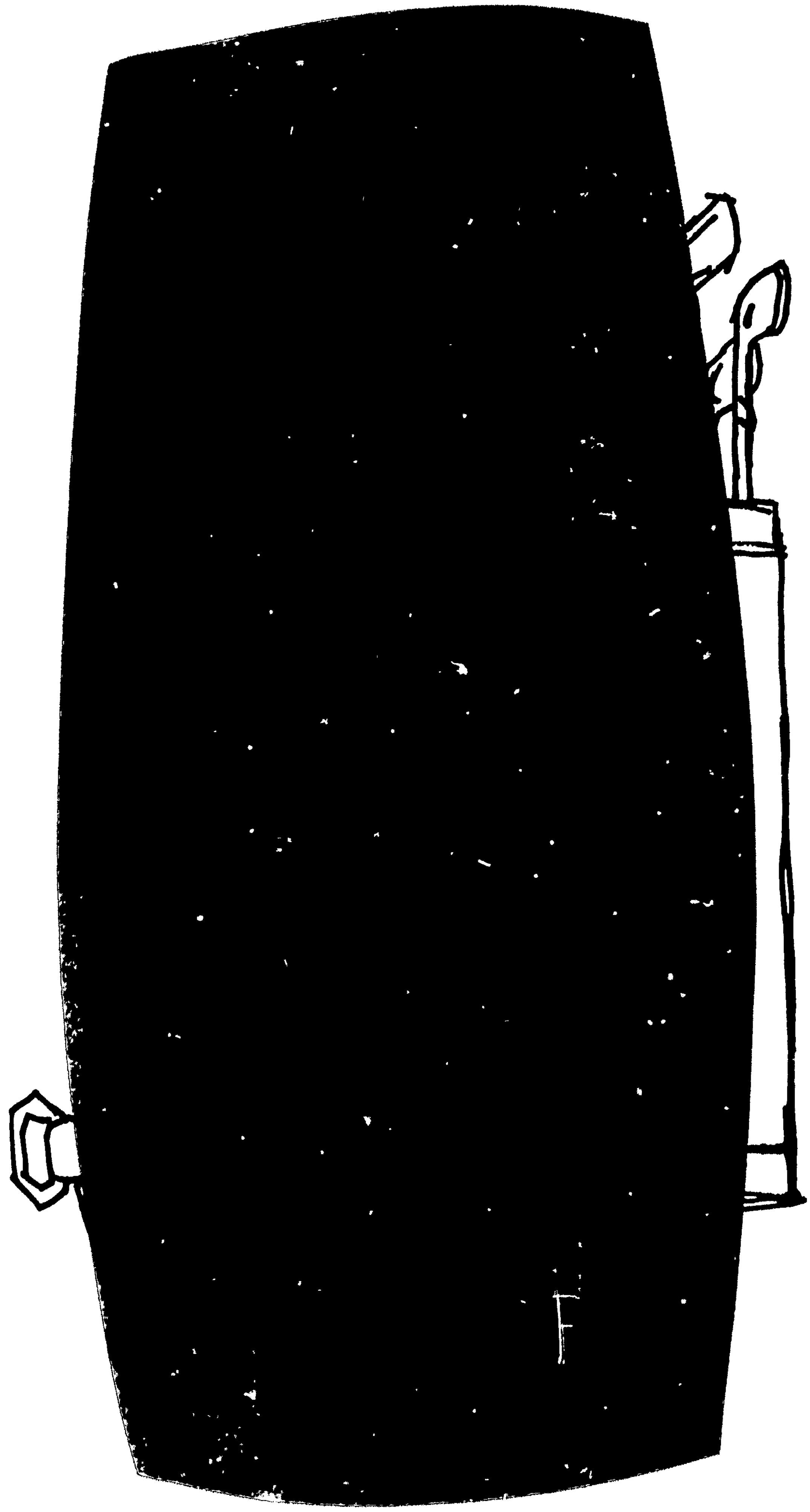
In a real sense, meaning *is* context. Further, meanings are *derived* from context. Meaning accrues through experiencing a thing, event, or situation in relation to other things, events, or situations. Therefore, the types of meanings and the quality of meanings vary a great deal in terms of the quality of the environment in which they are learned.

Some examples may clarify this point. Words have different meanings in different contexts. Consider the meanings of the word "set" in these sentences:

The table was *set* for four people.
The alarm clock was *set* for six o'clock.
For this play, the stage crew had to put up a new *set*.
John hoped to get a new *set* of golf clubs for Christmas.

The *Oxford English Dictionary* has more than 20 pages devoted to definitions of the word "set." This illustrates that there are hundreds of different contexts in which it occurs. Imagine the variety of meanings it may have, depending upon the context in which the learner hears the word used.

A woman told how she had learned the meaning of right and left. She said she first learned which was her right hand and which was her left when her older sister was teaching her to play "My Bonnie Lies Over the Ocean" on an old-fashioned organ. Her sister repeatedly told her to play certain notes with her right hand or her left hand and soon discovered that the young child did not know which hand was left or right. The difference was pointed out and the child was taught to touch certain keys with the fingers of the right hand, other keys with fingers of the left hand. When that little girl's first-grade teacher asked the children to raise their right hands, the child thought of how she had sat at the organ and of which hand was called "right" by her sister; then she raised that hand. As an adult she looked back upon this experi-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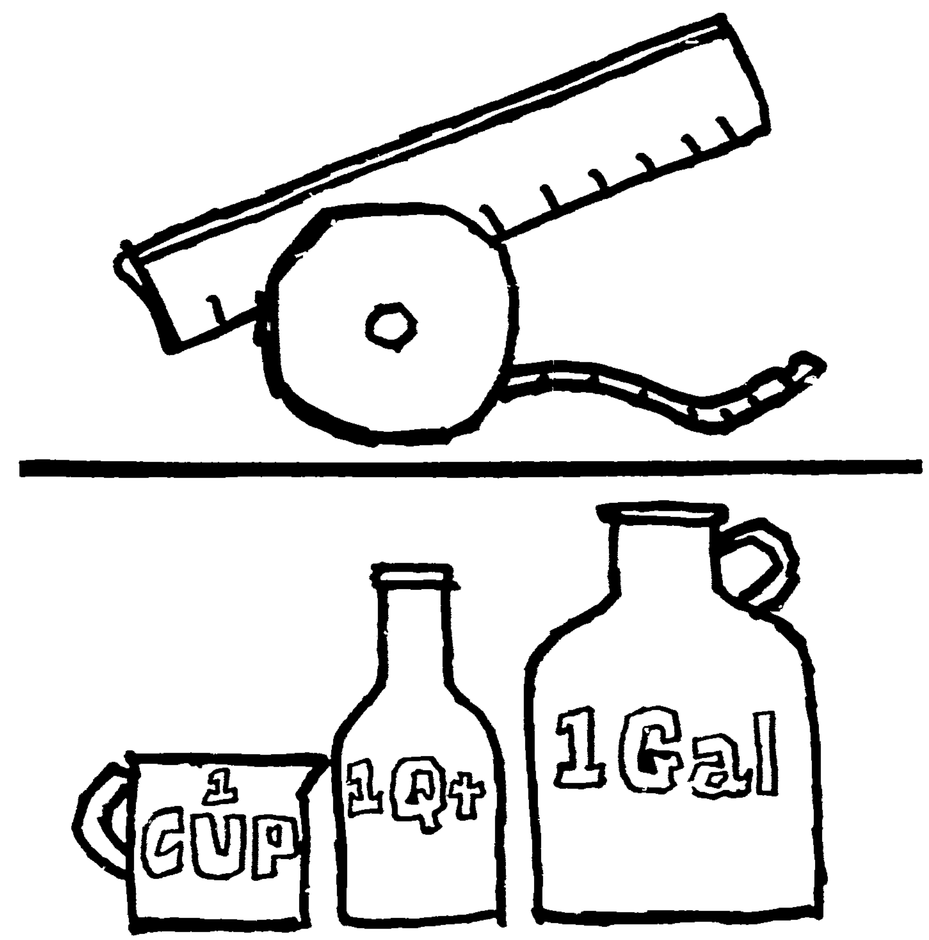
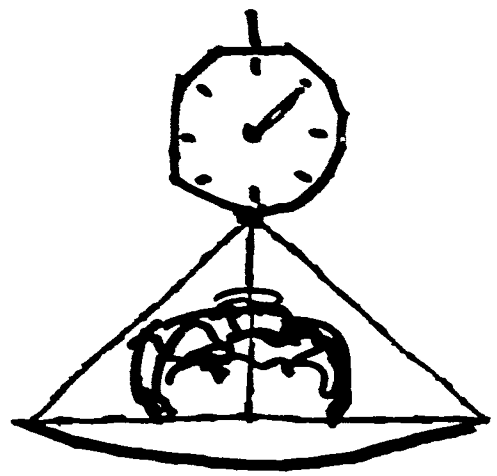
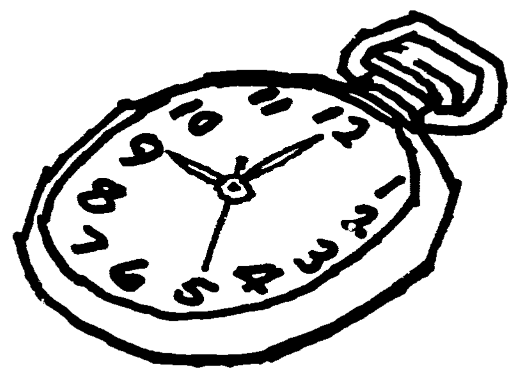
ence and said that for a few years after learning right and left at the organ, she always thought of that situation in connection with the use of her right or left hand. The context in which she learned left and right was part of the meaning. Left and right are necessary concepts in teaching place value in arithmetic.

The teacher should realize that the preschool child or the primary-grade pupil learns arithmetic meanings in relation to a certain context or to the various contexts in which those meanings are experienced. The richer the learning environment is with respect to the particular meanings, the better are the chances of acquiring accurate and full understandings.

Consider, for example, the learning of the meanings of standard measures. Too often, perhaps, children learn in a strictly verbal context that 12 inches = 1 foot, 3 feet = 1 yard, and 5280 feet = 1 mile. What they learn by reading and repeating those statements is a limited meaning of those measures. How much better it would be if they could learn the meaning of inches, feet, and yards in a context of use in and out of the schoolroom. Perhaps the children are helping to plan the placement of shelves in a new bookcase. They measure the tallest book to go into the bookcase to find how wide to make the space between shelves.







They measure other books and count those of various sizes to see how to space other shelves. In so doing, they are acquiring meanings of inch and foot in a context which is bound to contribute to better understandings of these measures. Or, they may estimate the width of the window sill and the length of a window box they are considering placing on the window sill. They check their estimates by measuring, using a foot ruler or yardstick. They compare the two measurements and decide whether or not the window box will fit on that window sill *before* they carry the heavy box to the window.

Perhaps a committee of children helps to measure the height of the windows as part of a project to find out whether or not funds allotted for drapes will be enough to buy the amount of cloth needed. The meaning of 15 yards is more accurately understood and appreciated by children who have had to make an important decision regarding that amount.

Primary-grade children are not too young to compile their own lists of distances between places, or of weights or capacities of objects in their own environment which correspond to standard measures. Much time may be used in the exploration necessary to find an object which is exactly one foot long, something else which is one inch wide, something that weighs

one pound or five pounds, containers which hold one pint, one quart, or one gallon. This exploration, however, need not be done all at once; it can be a continuing project which becomes an adventure in estimating and checking by actual measurement. The time will be well spent if the teacher guides the children's explorations wisely; the meanings which result will have lifelong value for the children. Even the teacher may sharpen his own concepts of measures from this use of the environment as a source of quantitative meanings.

The quality of children's meanings of numbers and the number system as a whole are also heavily dependent on the help they receive in discovery of the *relationships among numbers within the system.* Any quantity has many meanings in relation to other quantities. The other numbers are parts of its context. For example, what is 12? It is the number that comes between 11 and 13; it is 3 fours, 2 sixes, 4 threes, 6 twos; it is the sum of 8 and 4, 9 and 3, 5 and 7, 6 and 6, 2 and 10, 1 and 11; it is half of 24; it is one-third of 36; it is 3 less than 15; it is the number of inches in one foot; it is the number of months in one year; it is the number of eggs in one dozen. That is only the beginning of a list for the meanings of 12 which can and should be built by children as they are guided into meaningful use of this number and other numbers in relation to one another. What a pleasant game it would be for those children who are ready for it to see how long a list of meanings of a particular number they can make, adding to it from time to time as they observe and discover unlisted relationships of this number to other numbers and as a measure of quantity.

The typical primary-grade child probably gets less experience than he should in solving arithmetic problems. A problem is a felt difficulty for which a solution is sought. Much of what is called "problem solving" is not problem solving in its true sense. The so-called word problems in arithmetic books or those originated by the teacher are often merely computation exercises. A problem is present only if the learner faces an unresolved difficulty for which he really needs or wants an answer. Thus we return to meaning as context.

More actual problem solving would be done in primary-grade classrooms if children were faced with more real problems for which they needed or wanted answers. The context of a true solution in which the answer to the problem is important to the learner not only has a stimulating, motivating effect on the young learner, but that real context also adds immeasurably to the meaningfulness of the arithmetical features of

the situation and of the arithmetical processes required for solving the problem.

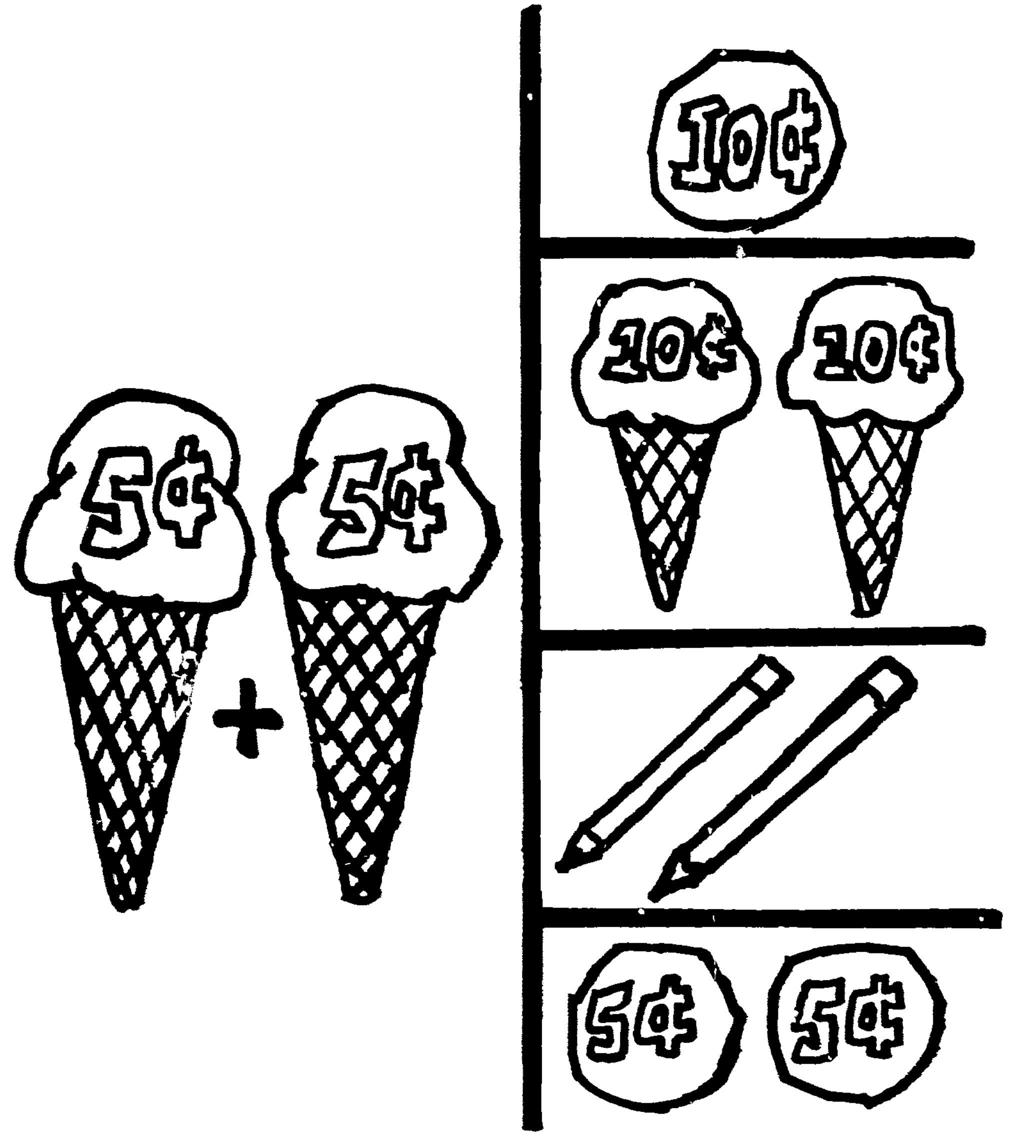
Suppose a group of third-graders is asked, "If ice cream cones cost 5¢ each and you buy 2 of them, how much will both ice cream cones cost?"

To Johnnie that is not a problem because he already knows the answer; he has no felt difficulty except perhaps a need to please the teacher by giving the expected answer.

To Willie, no problem exists because he is distracted by the price of the ice cream cones; instead of even thinking of the answer or how to obtain it, he voices his opinion that ice cream cones do not cost 5¢ but 10¢ each.

Susie does not really think of the multiplication fact $2 \times 5¢ = 10¢$; but she knows that she bought 2 pencils yesterday at 5¢ each and paid 10¢. She volunteers 10¢ as her answer and gets credit for doing some problem solving and for knowing the fact the teacher assumes she used, whether or not she actually did.

Alice pays no attention to the problem statement as a whole; she listens only for the numbers and a word clue. She selects "5¢ each," "2 of them," and "how much . . . both . . . cost?" She thinks "2 fives are 10," and



supplies a correct answer with little thought or concern for ice cream cones or the realism of their given price.

This is not the ideal problem-solving situation. Willie, the child who was really thinking most clearly about the context of his real world, including the price of ice cream cones, was the only child of the four who did not come up with a "right" answer. He was, however, the only one who recognized and was disturbed by a real felt difficulty: he knew the answer would not be true to fact. He knew that if he wanted two ice cream cones, he would not be able to buy them in his community for 5¢ each. If teachers and children derived more problems from the actual environments of the learners, employing problems for which the children really needed to know the answers and situations in which part of the problem solving was finding out the real facts of the situation, we would have better problem solving, more problem solving, and increased transfer of problem-solving skill to new situations.

The second grade was taking its turn at operating a school supply store. This involved selling pencils and notebook paper for a half-hour each morning to children from their own and other rooms in the building. Any profit made was for use in their room library. Many real problems existed in such a situation: how much to charge for pencils in order to make a reasonable profit; how much to charge for notebook paper; how much profit was earned each day; keeping and checking the inventory; how much money was presently owed to the bookstore which was supplying the pencils and paper at reduced rates.

These are real problems. The children really needed to know the answers. In solving them, they had to discover for themselves what facts were needed to solve each problem, seek out these facts, and then choose the correct processes for computing to find correct answers. Accuracy of computation became a matter of profit and loss, not a score written by the teacher on the paper handed in by each pupil.

The learning context is very much a part of the meaningfulness of problem-solving activities.

MEANING IS INTENT

Meaning is also intent. Meaning is instrumentality. Meaning is agency. *Meaning represents a means to an end.* Meaning indicates purpose or aim.

We may ask,"What did he mean?" We could just


COST
1¢
1¢
1¢
1¢
1¢
1¢
1¢
PROFIT

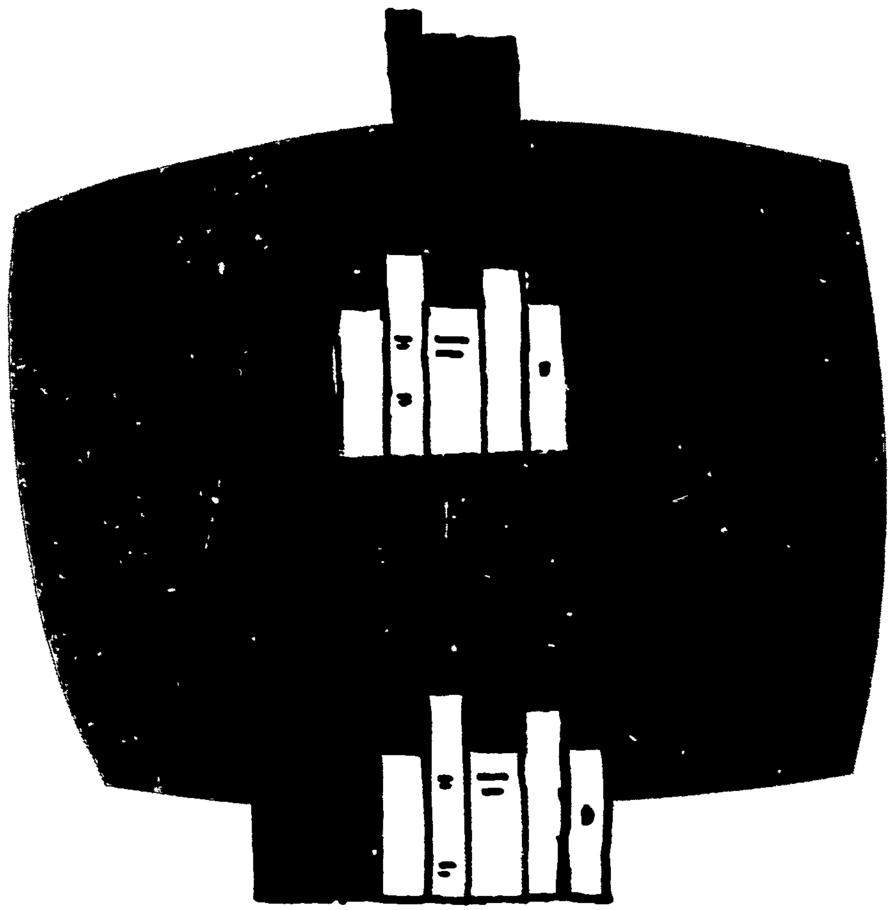
as well have said, "What was his intention?" "What was he driving at?" "What was his aim?"

When arithmetic processes are taught meaningfully, they are taught so that they serve as means to ends. A child who knows the meaning of addition, subtraction, multiplication, and division can use these processes as means for achieving certain aims, intents, or purposes. If they understood subtraction, the children selling the school supplies knew that they used the subtraction process to find out how many pencils they sold in a day. They knew that if they subtracted the number of pencils they still had after the sales period from the number they had at the beginning of that period, the difference would tell them how many were sold. They knew that one purpose for which subtraction is used is to find differences in quantities, differences in the sizes of groups. They should know also that addition, by contrast, is a means of finding a total of two or more subgroups which are being or have been combined into a single group.

Children will not use the terminology just mentioned, but they can and do learn such meanings if the processes of arithmetic are introduced and practiced as means to certain ends. Children would see much more meaning in arithmetic activities if we had more experiences centered around grouping and regrouping actual things in order to accomplish certain purposes.

At the risk of seeming to accentuate the negative, let us consider not only the absence of meaningful use of arithmetic processes, but also let us recognize the fact that sometimes children are even taught wrong meanings of processes. Occasionally, they are told that adding is "getting more." In other words, the purpose of adding is to get more. This is not true. The child who puts a group of four books on the shelf with another group of five books is not using the addition process for the purpose of having more books. He is merely trying to put two groups together to make a new total group. If he knows the addition fact $4+5=9$ and understands it, he uses it to mean that putting a group of four books and a group of five books together will result in having a new group of nine books. The same nine books were there before he added and after he added. The difference is in the way they are grouped. The equal sign should express *equality* in the child's thinking as well as in oral pronunciation.

One can hardly overemphasize the importance of clarification of process meanings in the thinking of teachers of young children. If they are taught misconceptions or even half-truths about process meanings, how can they become skillful users of arithmetic



as means to proper ends in situations involving problems of quantity?

Teaching children to look for certain clue words in verbal statements of problem situations is not teaching children what they will really need to know when they have to figure out the means by which they can solve their everyday problems involving number relations. This may give false ideas of what arithmetic really is and may make the fundamental processes a mystery instead of an open door to purposeful control of problem situations.

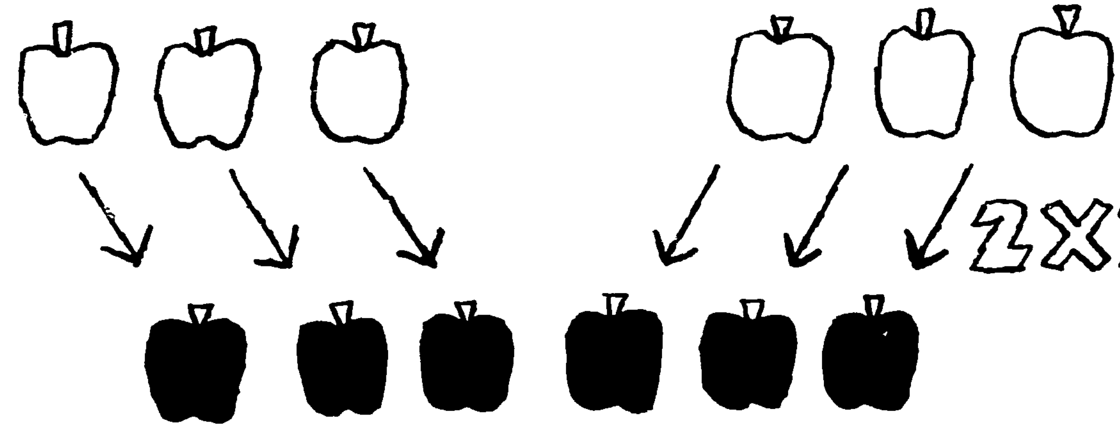
We must not forget that meaningful arithmetic is a *means to an end*. Processes really understood serve children well in achieving their aims in the area of experience with numbers.

MEANING IS ORGANIZATION

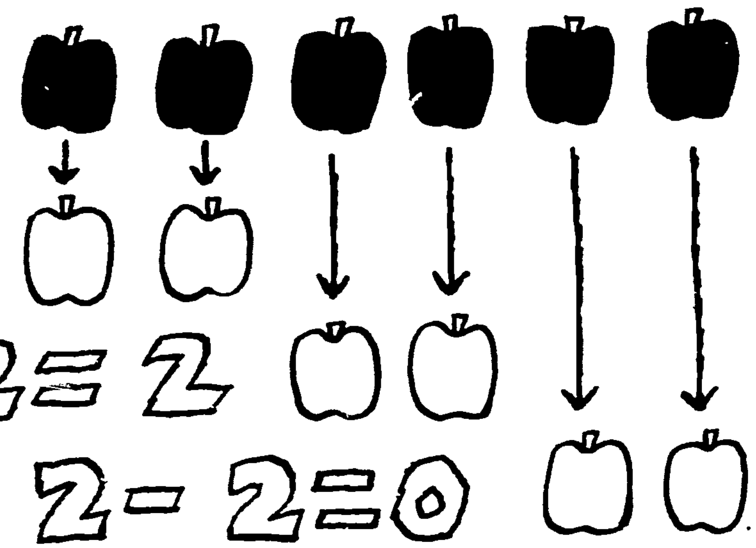
Clear meaning *results* from effective organization and effective organization *results* from clear meanings. Children as well as adults need to see how the pieces fit together.

Not only do children need to see each arithmetical process as a means to an end, as serving a peculiar purpose, but they also need to see how the various processes relate to one another. They need to experiment

$3 + 3 = 6$



$2 \times 3 = 6$



$6 - 2 = 4$

$4 - 2 = 2$

$2 - 2 = 0$

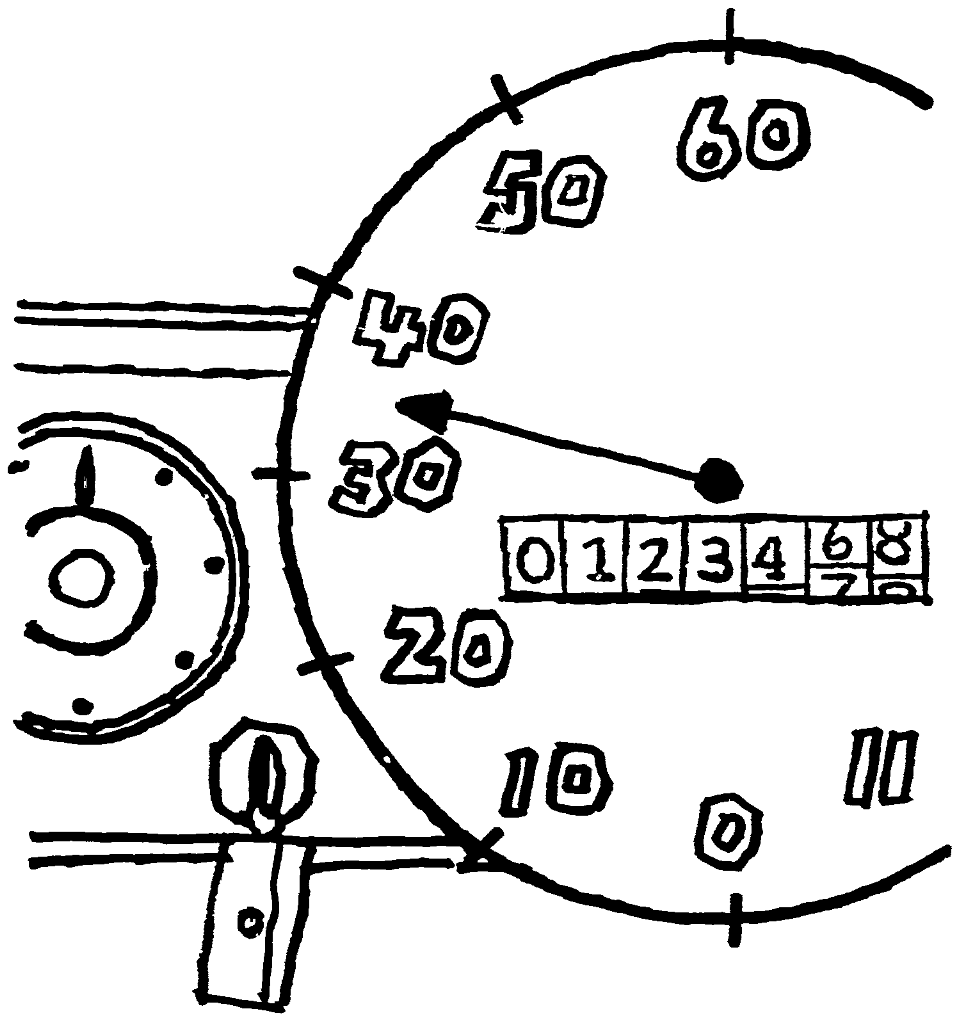
$6 \div 2 = 3$

with groups and with grouping of objects, taking groups apart and putting them back together, adding groups to make a new total group, separating that total group into new subgroups. Only in this way can they really understand the relationships among addition and subtraction facts and between the addition and subtraction processes. They need to try out solutions to problems by different procedures or different sequences of procedures, discovering for themselves various ways of accomplishing the same purpose. They need to learn how addition and multiplication are alike and how they differ; this applies also for subtraction and division.

No one who is realistic about the abilities of primary-grade children in general, or about the mathematical and teaching competencies of primary-grade teachers in general, expects that all children leaving the third grade will have achieved a mature, complete, and highly organized pattern of relationships among the fundamental processes of adding, subtracting, multiplying, and dividing. However, they can all learn this type of meaning to some extent, each to his own level of competence and his opportunity for learning. Further, if primary-grade teachers are conscious of children's eventual capacity to understand the meanings of arithmetic processes in their over-all relations one to another, they will do a better job of helping young children meaningfully organize the relationships with which they do have some experience. A few meaningful relationships, clearly seen, clearly understood, and clearly used, are better preparation for what follows than any amount of unrelated and disorganized miscellaneous repetition of routine memorized statements and forms.

One further point must suffice. Meaning *is* organization, and meanings *stem* from organization. New meanings depend upon other organized meanings already grasped by the learner, and they depend also upon the organization of experience planned by the teacher who knows how learning takes place and who also knows arithmetic content. Two basic arithmetical concepts which must be taught well if arithmetic is to be meaningful are the concepts of *base* and *place value*. Of course, they are not taught completely in the primary grades, but to omit emphasis upon them in the early years of schooling is inexcusable.

The meaning of a "group" has already been emphasized. It is essential as a forerunner of the idea of the basic collection or group which we call the *base* of our number system. Our number base happens to be ten; possibly an understanding of the base of ten is sufficient for most primary-grade children. That is not



enough, however, for their teachers. A teacher who does not understand the concept of the number base with numbers other than ten probably lacks the clarity of understanding which will make him the best guide in teaching this concept. The good teacher always needs to know more than he teaches in order to teach well that which he expects his pupils to learn.

While the concept of *place value* is closely related to the concept of *base,* they are not the same. We have a decimal number system; i.e., we have a base of ten. The Romans, too, had a base of ten (actually in combination with a base of five). To that extent, the Arabic and Roman systems are similar. But we use place value in our system in a way that is not present in the Roman system, one of the great superiorities of our system over theirs. The fact that both base and place must be taught if our number system is to be understood makes the teacher's planning more complex. It becomes necessary to consider sequence of learning activities for the meaning of the base and for the meaning of place value, each by itself and both in relation to each other. The sequence from use of concrete objects toward increasingly abstract representations of concepts is also involved in good planning for sequence of learning activities.

At the risk of obscuring some of the intended

meanings by too much condensation, let us take the meaning of a set of arithmetical symbols which we read as "one dollar and eleven cents" and which is written as $1.11. Children (and adults) vary widely in their level of understanding of these oral or written symbols. The individual levels of understanding achieved depend, of course, upon intellectual ability, backgrounds of experience, and all other factors which affect learning in general. They also depend to a great degree upon the teacher's organization of the basic concepts in presenting them and upon the aids provided by the teacher to help the learners do their own organizing and fitting together of number concepts. Only three levels of meaning will be described; many others might intervene.

Level A

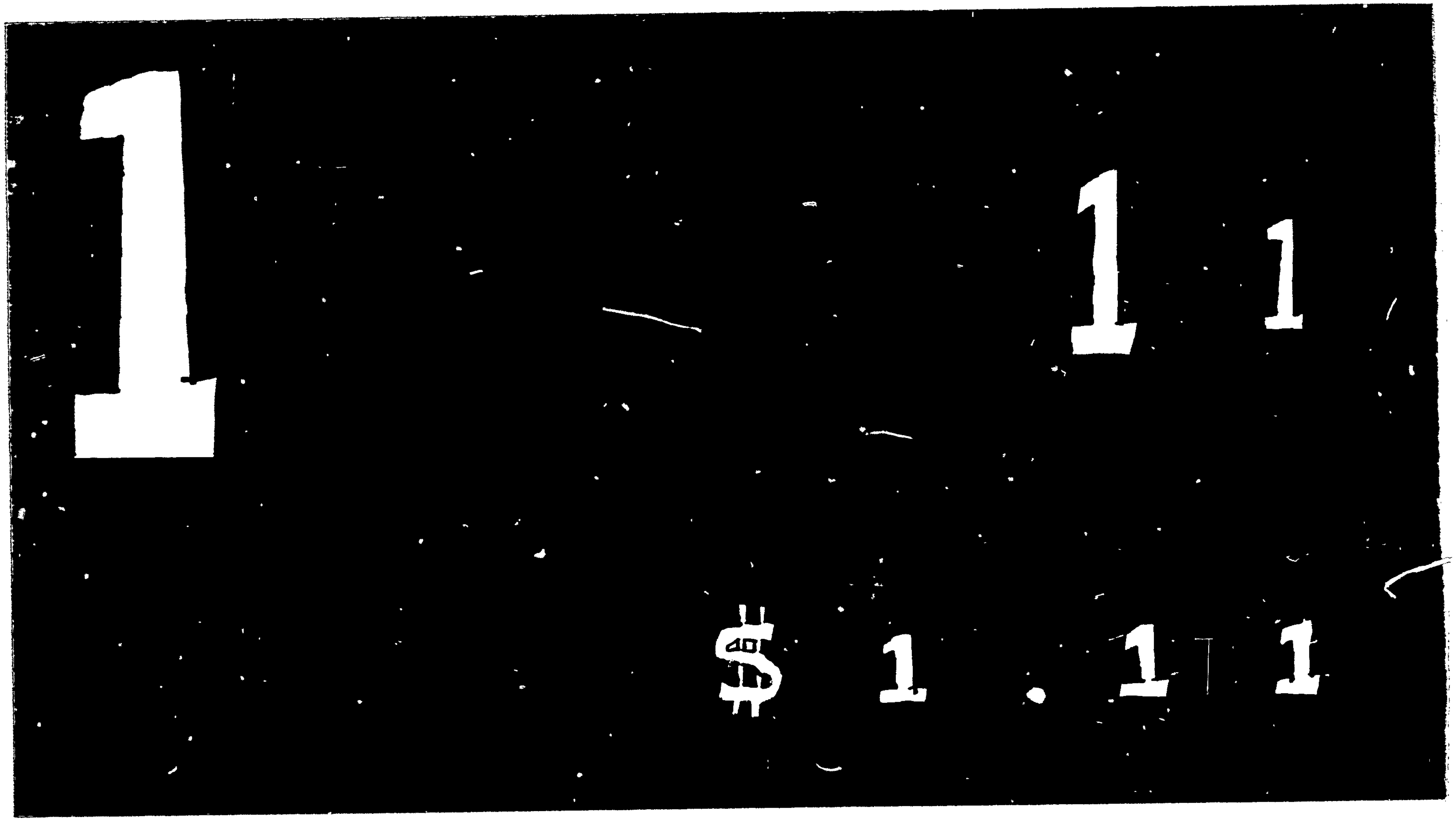
The meaning of our base of ten is shown here by having the groups shown clearly. At the right is the one penny corresponding to the simple meaning of the symbol "1." In the middle is a stack of *10 times as many* pennies, also shown by the symbol "1," but written to the left of the "1" which meant only a single penny. At the extreme left is a collection of collections, 10 stacks of 10 pennies each, or 100 pennies, also indicated by a symbol "1." Children can see that the written "1's" are exactly alike, but they have very different meanings.

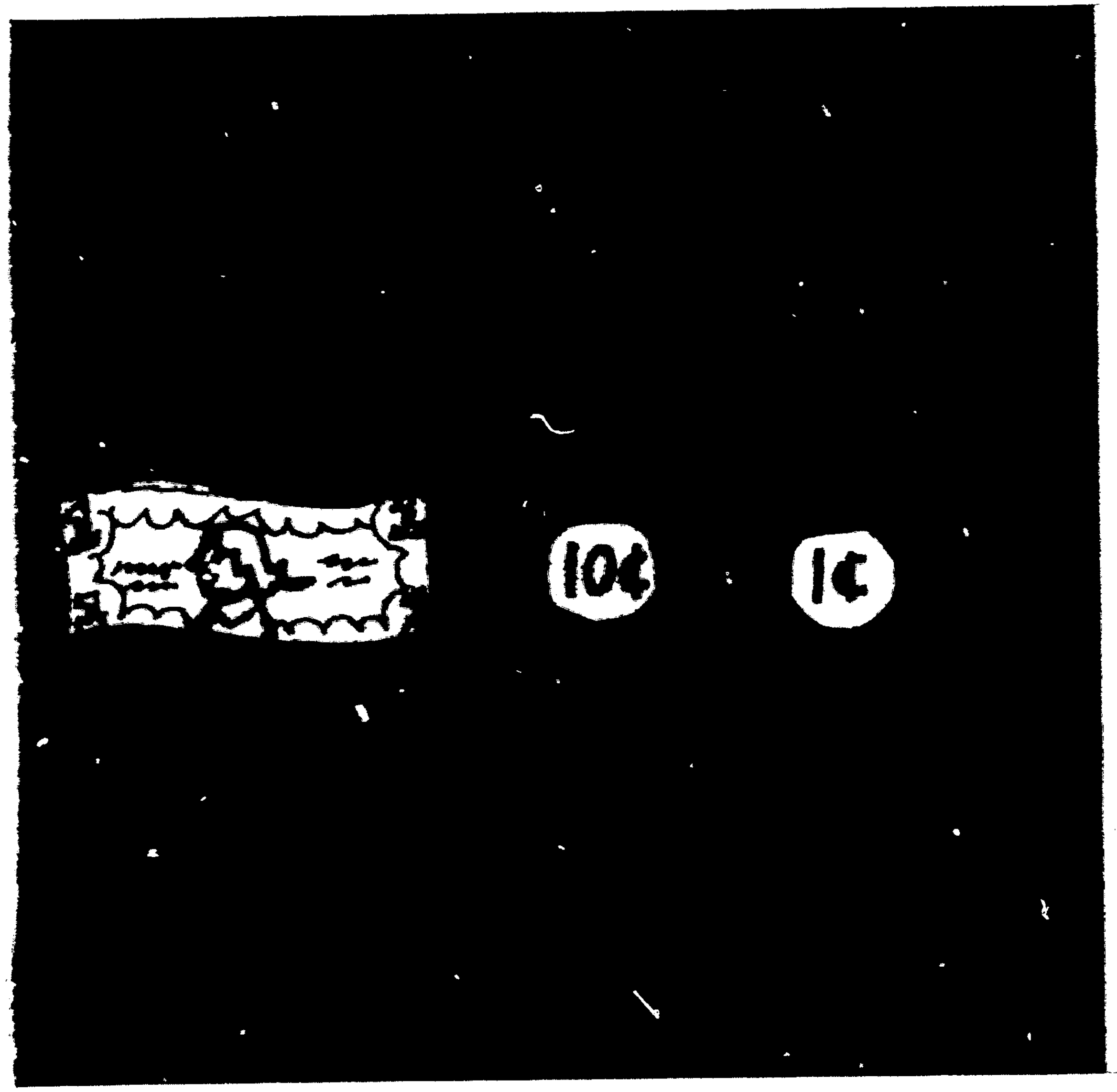
The use of identical written symbols shows that they all have the same face value, the value of one something. The difference in meaning is indicated by where those symbols are written. The "1" in the middle means *one group of 10;* the "1" at the extreme left means *one collection of ten 10's*. The numeral always has a place value 10 times that of the numeral immediately to its right.

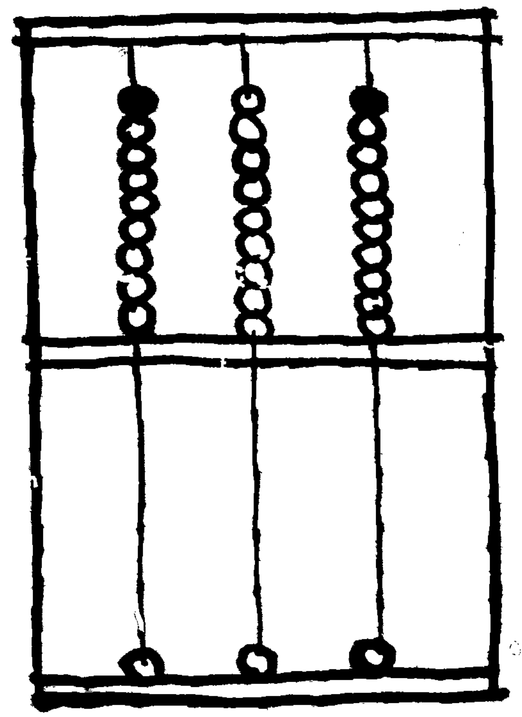
This presentation is given first because the *groups* are clearly shown. The base represents a collection idea, and the aids used show these collections clearly.

Level B

Here base and place values are shown at a more abstract level. The "1" at the extreme right has the same meaning it had in the first illustration, that of one penny or one cent. The middle "1" is shown as one dime; that is, the "1" refers not to pennies, but to another coin worth ten times as much as a penny. This requires a higher level of abstraction for the young child. He may not see clearly that this "1" is worth 10 times as much because he sees only one coin (a dime).


1
1 1
$1.11





$1.11

C

The concept "worth ten times as much" is involved. Whether or not this is meaningful to him depends greatly upon the experiences he has had with purchasing items and getting ten times as much for a dime as he gets for a penny. Similarly, the "1" at the extreme left is a one-dollar bill. The meaning of "1" is shown by the single denomination of our money, one object. The meaning of "100 cents" or of "10 dimes" must be developed through experiences; it must not be assumed. It certainly is not visually apparent.

Level C

This illustration demands a still higher level of abstraction than does the preceding illustration. Here each symbol "1" is shown by a single bead; but in this situation all the beads are exactly the same. There are no different visual quantities of cents or different varieties of coins to go with the different place values. In fact, we do not have any representations of money at all; wooden or plastic beads are used in place of coins. Our concepts of place value for this object-symbol representation of an amount of money depend more heavily than ever on our understandings (that is, our meanings) of place value. We can only be guided by the placement of each bead on the rods to show place value.

These examples have been given to suggest that when teachers in the primary grades use concrete aids or set up learning situations for teaching such fundamental concepts as base and place, careful thought should be given to the sequence in which they present various aids. Because the adult sees the same basic concepts demonstrated in each of these three presentations, he must not assume that young children do also. Children need time and opportunity to get the concepts well in hand, first in relatively obvious and simple presentations, followed by gradual introduction of higher levels of abstraction.

Concrete aids to learning in arithmetic are often misused. They are used too often for themselves alone rather than truly as aids which should lead from the concrete to the abstract, that is, to the maturity of understanding which makes them unnecessary.

Our number system is a highly organized pattern of relationships. Its simplicity in representing complex ideas is its beauty. But young learners can hardly be expected to come to appreciate that beauty of structure and organization except as they are given well-planned sequences of experiences by which they may discover this meaningful organization for themselves.

SUMMARY

Teachers of primary-grade children will do a better job of helping their pupils acquire correct, clear meanings in arithmetic if they will remember and seek to apply these ideas:

Meaning is experience. Children learn to know and understand what they have experienced. Telling them about numbers and arithmetic processes is not enough. They must live the meanings of numbers, the number system, and the processes with numbers.

Meaning is context. Children learn best within a rich context of meanings. The teacher helps learning to proceed more quickly and effectively to the degree that he provides an environment within which children can readily discover and practice understandings.

Meaning is intent. Children's purposes and intentions are as important as adults' purposes and intentions for them. One mark of a good teacher is the ability to guide children to see and to use the purposes for which they need arithmetic meanings and the purposes for which they need to be able to solve arithmetic problems.

Meaning is organization. If the results of learning are to be useful, they must be organized. The number system cannot be called by that name unless number meanings are organized within the comprehension of the child who uses the system. The processes cannot be well understood except in relation to one another. The good teacher seeks to help children achieve such organization.

The teacher who ponders these ideas and seeks to apply them in guiding children's learning activities will not be disappointed. The children who have such a teacher will not only search for meanings, they will also find much of what they seek.

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