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HOW TO USE THE OVERHEAD PROJECTOR IN MATHEMATICS EDUCATION.

BY- KRULIK, STEPHEN KAUFMAN, IRWIN

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THE OVERHEAD PROJECTOR IS A SIMPLE AND EFFECTIVE TEACHING AID IN MATHEMATICS INSTRUCTION. MOST TRANSPARENCIES FOR PROJECTION CAN BE MADE EASILY AND INEXPENSIVELY BY THE TEACHER USING READILY AVAILABLE MATERIALS. ALTHOUGH EXPENSIVE, SOME COMMERCIALLY PREPARED TRANSPARENCIES ARE INDISPENSIBLE, BECAUSE OF INVOLVED PREPARATION TECHNIQUES OR EQUIPMENT. USE OF THE OVERHEAD PROJECTOR EFFECTS SAVINGS IN TIME, ENABLES EXAMPLES OF PROBLEMS TO BE STORED AND REUSED AT WILL, AND ENRICHES THE TEACHING OF OTHERWISE DIFFICULT AND COMPLEX IDEAS. REFERENCES OF OTHER PUBLICATIONS ABOUT VISUAL AIDS ARE LISTED AS WELL AS SOURCES OF READY-MADE TRANSPARENCIES AND SOURCES OF SPECIALIZED SUPPLIES FOR PREPARING PROJECTUALS. THIS DOCUMENT WAS PUBLISHED BY THE NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS, 1201 SIXTEENTH ST., N.W., WASHINGTON, D.C. 20036 (CS)

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HOW TO USE

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HOW TO USE the Overhead Projector in Mathematics Education

By Stephen Krulik and

Irwin Kaufman

U.S. DEPARTMENT OF HEALTH, EDUCATION & WELFARE
OFFICE OF EDUCATION

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INTRODUCTORY NOTE

The authors, Stephen Krulik and Irwin Kaufman, are associated with Lafayette High School in Brooklyn, New York, and are authors also of a more elaborate text, Multi-Sensory Techniques in the Teaching of Mathematics, *Prentice-Hall Education Series. This includes some additional examples and illustrations for use with mathematics classes.*

HOW TO USE THE OVERHEAD PROJECTOR IN MATHEMATICS EDUCATION *is one of a series of "How to . . ." pamphlets published by the National Council of Teachers of Mathematics and supervised by an editorial board of which Dr. Samuel L. Greitzer is the present chairman. These pamphlets are devoted to providing practical assistance to the teacher with his everyday duties in the mathematics classroom.*

A complete list of titles in this series is included in the current publications brochure of the National Council of Teachers of Mathematics. This is available on request, without charge.

1 THE OVERHEAD PROJECTOR

PROGRESS IN EFFECTIVE TEACHING depends in part upon the utilization of new techniques that will yield better results in the same given period of time. Of the many aids to effective communication that have been offered to teachers, and especially to teachers of mathematics, the overhead projector* seems to be one aid that may radically improve the teaching of mathematics.

The overhead projector gives a large image at a short distance from the speaker. There are many projectors of this type on the market, manufactured by almost as many companies, and this book will not attempt to explore the technical details of the machine itself. We assume that those teachers who have a machine available for use will have an instruction booklet to explain its operation. Our concern is the most *effective* use of this learning aid.

The basic principle is simple: Light from below a horizontal platform of heavy plate glass magnifies and projects whatever image is on the platform, through a mirrored projector head, onto a vertical screen that is usually placed above and behind the speaker. The teacher may use the platform itself as he would use a chalkboard, drawing on it with grease pencil or other instrument, or he may place on it a transparency or a projectual (the words are identical in meaning) which has been prepared in advance. A transparency may be com-

* Although in classroom use the projection is onto a screen paralleling a side wall, it is called an "overhead" projector because of its origin. This type of mirrored projector was first designed for bed patients, and the ceiling served as a screen.

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pletely prepared ahead of time, or it may be prepared in part, to be completed during class discussion.

MAJOR ADVANTAGES

The overhead projector affords the teacher several major advantages. These include the following:

1. *Simplicity of operation*

2. *Position of the teacher*

By placing the projector in front of the room, the teacher can look at his students and adjust his presentation as he goes along. He speaks directly to his students, eliminating possible loss of rapport often experienced while writing on the chalkboard. At no time need he turn his back to the class.

3. *No change in classroom lighting*

There is no need to darken the room or turn off the lights. The overhead projector is operated with full classroom lighting. This enables the students to take notes as the teacher presents the material. The machine may be turned on and off with a flick of the switch to allow for class discussion. In fact, the teacher *should* turn the machine on and off at various intervals throughout the presentation. In this way, the teacher can focus the class attention and control the discussion.

4. *A horizontal writing surface*

The "stage" of the machine is usually a large, horizontal surface. This allows the teacher to write or draw as he goes along, on the platform itself or on a projectual, without turning away from the class. It also allows the teacher to use projectuals that are large enough for easy handling and give adequate space to do any needed drawing or writing without working in miniature. (The customary size is 8½ by 11 inches, but the authors have used them as large as 10 by 12 inches.) The teacher can also draw attention to various

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parts of the projectual as he goes along, using a pen or pencil as a pointer.

5. *Advance preparation of projectuals*

Many of the diagrams used in spatial geometry are complex. It is very difficult to draw diagrams of lines and planes in space, especially at the board. Using transparencies prepared in advance is a great advantage.

Simple, basic art skills are all that is needed for very effective preparation, although greater ability and more complicated techniques can, of course, add much of beauty and interest. A teacher who does not wish to make the projectuals himself can often obtain what he wants by directing the work of a student who is artistically gifted, or one who is taking a mechanical drawing course.

Once made, the transparencies can be used, stored away, and reused at will.

6. *Utilization of dynamic effects*

Use of "overlays" in addition to an original transparency can lead to dramatic and colorful presentations. These overlays are made in a series of projectuals with a part of the diagram on each successive projectual. Each is added at the appropriate time as the teacher proceeds in his presentation, thus providing a cumulative effect. [A sneak preview of Figure 2, page 13, will show an example of overlays to illustrate the familiar $(a + b)^2 = a^2 + 2ab + b^2$.]

A reverse approach, that of starting with a series of overlays and removing them one by one to simplify a concept, is very effective in some situations.

PHYSICAL CONSIDERATIONS

We must add at this point that there are several considerations in the use of the projector that depend on the physical layout of the classroom. The screen, for example, must be large enough so that everyone can see

it without effort. Its lower part must be high enough so that the teacher, whether sitting or standing, does not obscure it. The projector must be placed in such a position as to provide an image large enough to fill the entire screen. One technique that has been found helpful is to place the screen in a corner of the room and have the students turn their head slightly to view it, as shown in Figure 1.

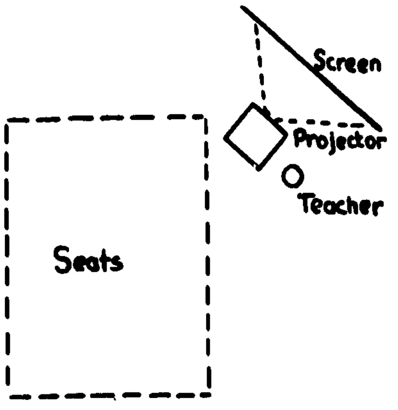


Figure 1

In moving the screen into position, the teacher should be careful to allow for the "keystone effect." This is a distortion that projects the top of a picture in a wider size than the bottom. In some cases, this effect is so minor that it can be ignored. If the effect is bothersome, however, it is a good idea to tilt the top of the screen a bit forward. A good rule of thumb is to have an angle of about 90 degrees between the screen and the center of the projected light beam. Tiltable screens that can minimize the keystone effect are available.

If the teacher wishes, for special effects or for convenience, no screen need be used. The intense light of the projector beam allows the image to be projected on a flannel board or a chalkboard. This latter lends itself well to geometry, where the basic diagram of a theorem can be projected on the board and the construction lines added in chalk. For example, in proving that two triangles are similar when the measures of the three angles of one triangle are respectively equal to the measures of the three angles of the other triangle, the teacher might project the two original triangles onto the chalkboard and then use chalk to show how we move the smaller triangle on top of the larger.

2 MAKING PROJECTUALS

THERE ARE UNLIMITED OPPORTUNITIES for preparing your own projectuals for use in the classroom. Your own imagination and ingenuity serve as almost the only boundaries. The essential materials need not be expensive. The basic requisites for good transparencies are that they be made of clear material, usually a plastic acetate, and that they be easy to store.

HANDMADE PROJECTUALS

The type of projectual most often used is a single sheet of clear plastic, usually about 8½ by 11 inches. Commercially prepared sheets are available from many sources. There are several substitutes that cost considerably less, and for most purposes are equally satisfactory. For example, reprocessed X-ray films are sometimes available and are very inexpensive, about two cents a sheet. The film is durable, clear, easily written on with a grease pencil, and it can be stored easily. Its low price allows each student to have a sheet of his own for presenting homework problems to the class, and we shall discuss this in more detail in a later section on homework assignments.

Some teachers use the clear plastic used in the manufacture of seat covers for home furniture or automobile seats. This is a flexible, soft kind of plastic, stronger than most of the plastic films available to the housewife. This type of soft, pliable plastic can be used to demonstrate side-to-side motion. It can be written on with a ball-point pen. On this material, the pen "cuts" into the plastic, and will usually project in a variety of colors as drawn. The colors do not project true, however. (That is to say, the colors as projected are not the same as those drawn on the plastic.) Also, the softness of the

plastic does not lend itself to storage or uniformity. Its cost is very low.

As a protective measure, some teachers attach their finished projectuals to a frame or mask made of light cardboard. This simple device is not essential for projection purposes, but it should be utilized when the projectual is to be saved for future reuse. The frame prevents curling of the edges of the plastic, and provides a space for short notes, and for identification. It also allows for convenience of storage, since the uniform frame makes the projectuals all of one size.

The basic device used to prepare projectuals is a grease pencil (also known as a china marker), laundry marking pen, or even a simple eyebrow pencil. All will give a satisfactory line when used directly on the plastic sheet. The lines will project clearly as drawn. The teacher can add to a projectual in this way during class time, drawing as he talks to the students. Lines drawn with these instruments can be erased with a piece of tissue.

Transparencies or projectuals can be traced directly from an illustration in a book by placing the plastic sheet over the illustration and using a soft pencil to copy it. Go over the pencil lines with the grease pencil, and the projectual is completed.

Transparencies can be prepared with the help of a typewriter. The teacher simply inserts a sheet of paper, a sheet of fresh black carbon paper, and a sheet of plastic for the copy in place of the usual onionskin. When the sheet of paper is typed on, as usual, the clear plastic receives the impressions from the black carbon paper. This process requires a fresh sheet of carbon paper for each projectual. (If the teacher

has access to a typewriter with extra large type, as used in sight-conservation classes, the projectual will be more easily read by the class.)

Another method is to use a fountain pen and a black, water-soluble ink. There is, however, a special acetate India ink that is better adapted to writing on the plastic. It can be used with steel-point pens or the pens with special nibs or points for drawing and lettering. In either case, when using plastic acetate, the teacher must remember to write with a slightly slower speed than on paper. If the teacher wants to shade in large areas of dark color, a cotton-tipped swab may be used, with straight-line motion, to fill in the areas.

The spirit duplicating machine may be used to give a good projectual, and this is a special convenience when students are to be given copies of the same material, since it provides students and teachers with the same diagrams and makes it possible to project the material on the screen for easy class reference.

The teacher makes the master for the spirit duplicating machine as usual, depending on the machine used. After running off several sheets of paper, he inserts a piece of plastic in place of the next sheet of paper, the frosted side (if a commercially prepared acetate) receiving the image—and then finishes the run. This is one instance in which it is usually better to use commercially prepared acetate, if possible, for the transparency will be a little bit clearer and sharper.

We have found the transparencies from the spirit duplicating machine especially useful for discussions of statistical material and graphs.

If the teacher has taken a photograph that applies to the lesson (this can happen with bridges, applications of locus theorems, etc.), the negative itself may be used as a projectual, directly on the stage of the projector. Of course the small size of the negative may be a drawback, so the teacher should experiment to learn whether it will

be sufficiently clear. If it will not, he can have it enlarged at a photographic store. Color negatives will project only in black and white by this method, however. If color is desired in the projectual, a different method of making the transparency should be considered.

Color can be added to homemade projectuals for a more effective presentation. Points of emphasis such as the exponents in the expansion of the binomial theorem can be indicated, and color contrast effectively used.

One method of adding large blocks of color is to use the transparent sheets of a color-projecting adhesive material. The teacher cuts out a piece of this material in the exact size and shape of the area he desires to have projected in color. The protective backing is removed, and the adhesive side applied directly to the front of the transparency. After the wrinkles are smoothed out, the edges can be trimmed and the lines sharpened with a razor blade or a stylus. We have found that a wood-carving knife makes an excellent tool.

Felt-tip marking pens are also available to add color to the projectual. These pens come with their own ink supply, and the choice between permanent or water-soluble ink depends on whether the transparency is to be temporary or will be saved for future reuse.

Another way to apply color is by means of colored tapes. These come in widths of 1/8 and 1/4 inches, are pressure-sensitive, and project in true colors. They can be used to prepare graphs in advance of a lesson, or they can be laid out on a grid projectual as the teacher proceeds. The different colors make comparative studies quite easy, and the tapes allow for straight lines by simply stretching them along the lines of the grid. These grid lines will show faintly beneath the tapes, making it a simple matter for students to read the graphs.

There are also pressure-sensitive tapes in other widths, broken lines, dotted lines, graph grids, sets of parallel lines suitable for shading, arrowheads, and even lettering in different sizes. These are intended for use by draftsmen to simplify the preparation of blueprints, but can be used for projectuals as well.

MACHINE-MADE PROJECTUALS

Very often the teacher of mathematics will come across some material in a textbook or a magazine that he would like to reproduce as a projectual. Manufacturers of copying machines have now developed processes that make copying material of this type onto transparent sheets relatively simple.

A representative of one of these companies will be happy to illustrate the method used by his company, thus permitting the teacher to see the advantages and disadvantages of the system. It is advised that the teacher try any proposed system for himself as well as having the salesman demonstrate it.

PRINTING AND LETTERING

In planning for lettering for projectuals for the overhead projector, the teacher will wish to consider the advantages and disadvantages of several types of lettering. Of course, many teachers prefer to add letters by hand with ink or grease pencil.

The teacher who is not artistically inclined may prefer to use, for permanent transparencies, a form of "instant lettering" which comes prepared on a translucent sheet of paper and is sold by many art supply houses. This alphabet is composed of transfer-letters which adhere to almost any surface. The teacher places the sheet on the projectual and goes over the needed letter with slight pressure. (We use a ball-point pen for this step.) He then removes the sheet of lettering, and transfers the letter to the projectual. This is a permanent type of lettering, but it can prove quite expensive since each letter

can be used only once. This lettering can also be applied before the material being copied is made into a projectual.

Another effective type of "professional" lettering can be made with a scriber and a template of letters. While the initial investment here is rather costly, about ten to fifteen dollars, the materials can be used indefinitely. As a result, the process becomes inexpensive over a period of time. The scriber is something like a pair of compasses used for geometric constructions. The template, usually made of a heavy plastic, is a lettering set, with the letters cut out. The teacher places the point of the scriber in the outline of the proper letter of the template, and moves the point along the outline cutout. As he does this, the pencil traces the letter onto the projectual. After it is traced, the letter can be gone over with a permanent ink. Templates are available in varying sizes, shapes, and forms, including capitals and lower-case letters, italics, Old English, numbers, Roman numerals, etc. These can be purchased at draftsmen's supply stores and at most art supply stores.

OVERLAYS The word "overlay" is used to mean a series of transparencies used in combination: transparencies either placed atop one another in sequence or removed in sequence. It is also used to mean any one of the transparencies that make up the set.

It is a useful device in building important concepts where each transparency is a subset of the entire picture. The pieces of an overlay must be matched with the original diagram, since each carries a portion of the overall picture. Each piece can be colored differently, for better effect.

The overlay makes it possible to discuss each section of a process or development as it occurs and as it relates to the whole.

It can be used, for example, to illustrate most effectively the consecutive steps needed for a construction in geometry, by having each step on a separate transparency and placing these transparencies atop one another in the desired order. Each piece then shows the next arc or line being added to the construction.

Subtractively, an overlay can be used to simplify a complicated concept and reduce it to its elements. Unnecessary lines in a diagram or unnecessary terms in an algebraic expression can be removed until only the essentials remain.

To make the separate transparencies required for an overlay, the teacher first prepares the complete diagram, and decides on the number and nature of each of the separate transparencies he will need. The master diagram is best drawn on a translucent material rather than on heavy opaque paper or cardboard. If this is placed on a stand with a light under it, like a mimeoscope, it will be possible to see the diagram at all times.

Now transparent blank sheets can be placed on the diagram, due care being taken to see that they are properly aligned, and each portion of the diagram drawn on its blank. That is, the first part is drawn on the first sheet, the ink allowed to dry, the next sheet laid on the first one and the next part of the diagram drawn on it, and so on until the whole overlay is finished. The first sheet can now be enclosed in a cardboard frame (available from producers of the transparent sheets), and the others attached to the same frame by means of hinges. Advice and materials are usually procurable from the same companies that produce overhead projectors.

Manufacturers of projectors have developed methods of producing overlays in color. These methods are rather

good, and produce permanent overlays in a variety of colors.

As an example, suppose it is desired to illustrate the identity

$$(a + b)^2 = a^2 + 2ab + b^2.$$

The overlay might be made in color as follows:

1. A master is prepared showing a square with sides $a + b$, Master 1. This might be reproduced in black.

2. A second master is prepared showing a small square with side a, reproduced in gray as it is shown in Master 2 of Figure 2. This overlay is arranged so that it coincides in position with side a of the first sheet.

3. A third master is prepared, this one showing a square with side b. A different color is used for this, as indicated in Figure 2 by the use of solid color.

4. Still another color (indicated by a darker shade) is used for the final master, showing two rectangles with sides a and b.

The basic projectual, Master 1, is taped to the back of a cardboard frame, face down. The succeeding overlays, in proper order, are hinged to the front of the frame so that they can be placed on top of the original as needed. By bringing the series of overlays into view one at a time, the teacher can build up the concept of the binomial $(a + b)^2$ actually consisting of a square with side a (a^2), a square with side b (b^2), and two rectangles with sides a and b ($2ab$).

There are other color techniques that require more artistic ability and more complex equipment. For example, the teacher who has an artistic touch can use an airbrush with acetate inks to spray large or small areas with different colors. This technique gives a professional touch and a very vivid image.

HINGED OVERLAYS

In using overlays, there are some occasions when it might be desirable to keep separate the basic projectual and the overlays to be used with it. This is the case, for example, with projectuals that involve sets, with those for discussion of various positions for Euler circles, and with those involving areas where an area is to be moved from one position to another.

However, the majority of compound projectuals are not of this type. For most overlay projectuals, like the one shown in Figure 2, it is advisable to keep the basic slide and all the overlay sheets together so that they will be readily available. This is done by fastening them together with hinges. These may be made from masking tape or Scotch tape, and professional hinges are also available.

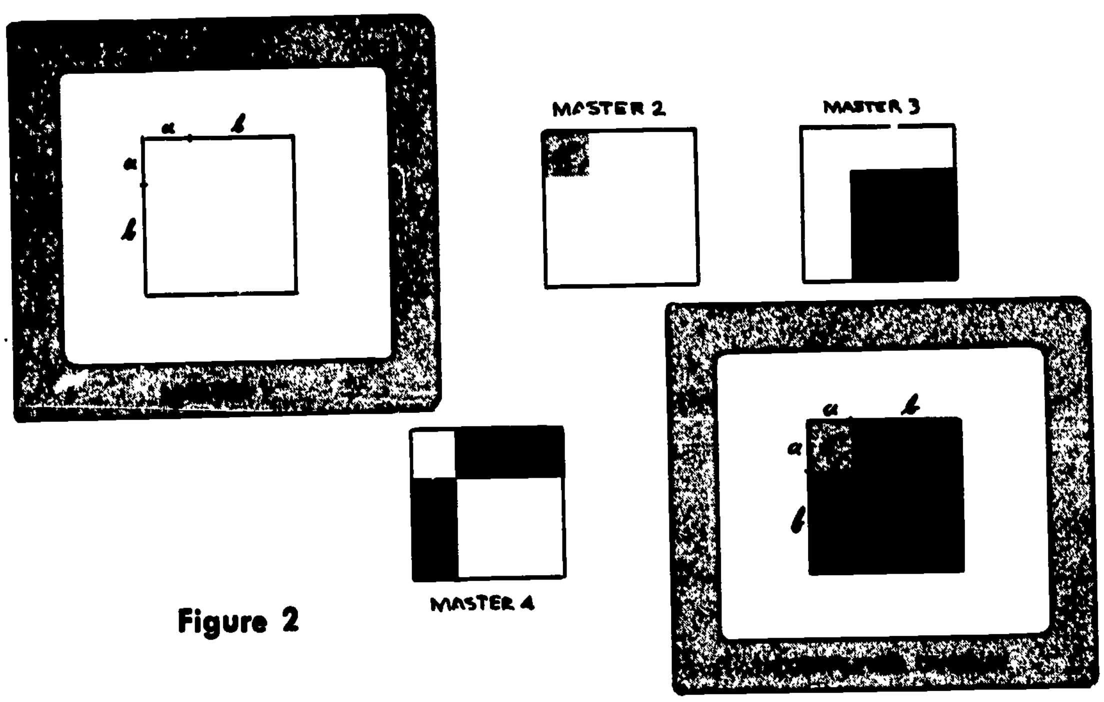


Figure 2

In Figure 3, a basic slide and an overlay are shown fastened by means of two hinges made from masking tape. These should not be so large as to impinge on the image when it is projected, nor so small as to be too

weak for the job. A 2-inch strip for the hinge is recommended.

In addition to the hinges, a small tab of masking tape should be added to the overlay. Applied to the free end, this tab makes it easier to pick up the overlay and turn it into place.

If two or more overlays are to be used with a basic slide, each can be fastened to the basic slide with hinges, using two or more edges.

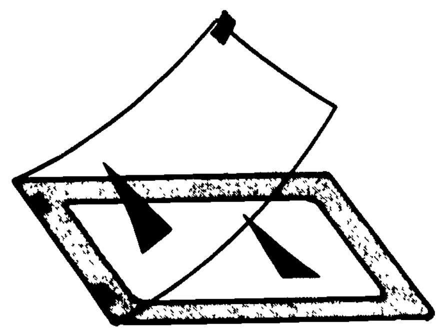


Figure 3

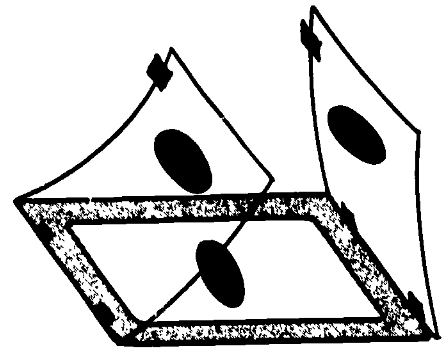


Figure 4

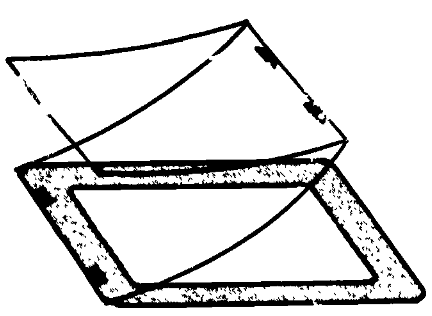


Figure 5

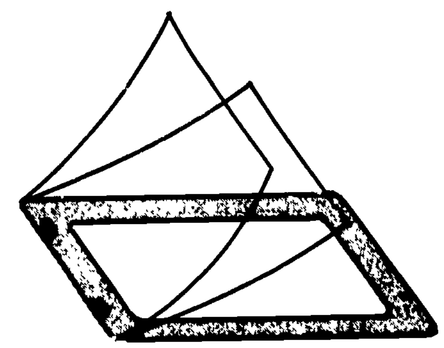


Figure 6

Figure 4 shows how two overlays, each with its own hinges and tabs, are fastened at opposite edges of a basic slide. If each edge is used, four overlays can be attached to one basic slide. When this is done, it is advisable to number each tab to make certain of the order in which each overlay is to be turned into position.

It is also possible to hinge one overlay onto another, as shown in Figure 5, although this limits the motility of the projectual.

Figure 6 illustrates another possibility, that of hinging several overlays to one edge.

We would like to emphasize that overlays can be subtractive as well as additive in nature—that is, one can start with a complete figure and remove overlays to reach a fundamental result. The methods for hinging are the same, but the use is reversed.

Finally, one can use poster board or opaque drawing paper either to "black out" section after section of a figure or to reveal section after section of a figure. The paper is hinged to the basic slide in the manner described, and used just as one would use the customary overlay. These processes can be very effective.

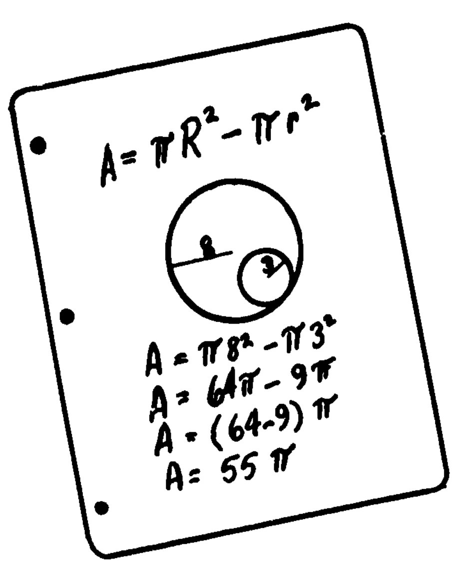
3 SOME SPECIAL PROJECTUALS

IN PREVIOUS SECTIONS, WE HAVE discussed how to use the machine and how to make some different types of projectuals, as well as how to add color. In this section, we will describe some of the projectuals we have made and used with our classes in mathematics. The list is by no means complete, but it will enable you to see the versatility of the overhead projector. We hope that it will also suggest ways in which you can adapt your own lessons to this machine.

HOMEWORK ASSIGNMENTS

The overhead projector can be used in the daily discussion of homework.

With use of inexpensive clear plastic sheets such as the reprocessed X-ray films already mentioned, it is possible for each student to have a sheet of his own to use for the term in connection with homework assignments. The plastic is punched with an ordinary hole-punch so that it can be held secure in the student's notebook.



Each day, we assign examples to different students to place on the plastic sheet for the next day's class. The student develops a step-by-step solution to his problem; when this is presented to the class, he comes to the machine, places his example on the stage, projects the image for the class to see, and discusses his solution. This procedure saves a great deal of time, since there is not the usual delay that occurs when problems are placed on the chalkboard. A further advantage lies in the fact that an example

can be recalled later, since the student does not erase his problem until the next time he is assigned "projector homework."

TESTS

Another time-saver is to prepare test questions on projectuals and have them ready to be thrown on the screen as soon as the class comes into the room. Since the problems do not have to be written on the board on class time, the pupils have more time for the test itself without encroaching on other classtime activities. (Since the space on the projectual is limited, this technique is not recommended for long tests.)

In case the teacher wishes to save his tests from one term to the next, the projectual is easily labeled and stored. This also makes it possible to refer to the tests at a future time.

GRAPHS FOR 9-12

This topic of graphs is one that can be carried through the ninth, tenth, eleventh, and twelfth years of mathematics. At each level, there are several applications of the overhead projector that greatly enrich the teaching of these topics.

Ninth grade

Here we can use the overhead projector to discuss the solution common to two simple linear equations. By drawing each line on a separate sheet of acetate, the teacher can easily illustrate parallel lines (no common solution), intersecting lines (one solution), or coincident lines (all common solutions). The projector can also be used to discuss the number line, as we extend our system from integers to negative numbers (one overlay), to fractions (another overlay), and to irrational numbers (another overlay).

Tenth grade

Here, the entire unit of coordinate geometry can be taught by having a prepared grid as a base and using overlays drawn on clear plastic sheets.

Eleventh grade

The unit on conic sections can be most easily adapted to the overhead projector with a grease pencil and separate sheets of acetate. If we draw, for example, a parabola and a circle on separate sheets, we may illustrate how we obtain 0, 1, 2, 3, or 4 points of intersection of the two quadratic graphs. If we use a parabola made of a piece of string, we can bend and stretch the string to illustrate the role played by each coefficient in the equation $y = ax^2 + bx + c$.

Twelfth grade

In this grade, we can continue our study of the conic sections and also introduce our students to graphs of equations in polar coordinates. A polar grid can be placed on the platform of the overhead projector, and another sheet of clear plastic placed over it. Using different sheets for the overlays, we can thus draw, on the same sheet of graph paper, the graphs of many different equations.

In the twelfth grade, too, we can introduce students to the method of finding the areas beneath curves by approximating rectangles through a series of overlays. We can also discuss how the secant becomes the tangent to a curve as the limit of $\frac{\Delta y}{\Delta x}$ as $\Delta x \to 0$.

OTHER EXAMPLES

There are many isolated topics, also, which lend themselves to more effective teaching through the use of carefully designed projectuals. For the balance of this section, we shall try to show how we have used several projectuals in our daily teaching in the various grades

of mathematics. Let us repeat that this is by no means a complete list of those we have used, or of the possibilities.

When we teach the use of trigonometry tables, some pupils find great difficulty in reading the tables. The same is true when we discuss the logarithm tables, etc., found in most textbooks. It is possible for the teacher to prepare a projectual, by one of the processes described in the previous section, exactly like the one in the student's book. By pointing on the projectual as he talks, the teacher can instruct the students in the intricacies of the tables.

In the modern mathematics introduced at the secondary level, we can use projectuals to illustrate the use of Venn diagrams for the intersection of two sets. We construct a basic transparency showing a rectangle, in black, and two overlapping circles, one in red and one in blue. It is quite easy for the student to see the entire set of objects (the red and/or blue circles) as well as the area of intersection (the overlapping part).

When the student finds himself faced with geometric constructions with compasses and straightedge, the overhead projector can be used to illustrate each step of the way. Another overlay can be made to illustrate each construction line as you proceed. The teacher who has trouble using the "string" for a compass can prepare these projectuals in advance.

The overhead projector lends itself quite naturally to the topic of loci. For example, when we discuss the locus of points that satisfy two conditions, we make use of overlays. Take the typical problem that follows:

a) What is the locus of points at a distance d from a given line, m?

b) What is the locus of points at a distance r from a point P that is on m?

c) Of how many points does the locus common to *a* and *b* consist under the following conditions?

(1) $r < d$.

(2) $r = d$.

(3) $r > d$.

If we construct a basic projectual showing the line *m*, and the point *P* on it, we can illustrate this theorem with two common soda straws and some coins of different sizes. The lines (straws) can be moved as required, and the circles (coins) changed as needed.

Again, for those theorems in the tenth year where we often make use of superposition (such as the theorem involving the similarity of two triangles by angle-angle = angle-angle), we can easily illustrate the actual moving of one triangle onto the other, and the coincidence of the equal angles.

One topic in the eleventh grade which often confuses the students a great deal is the topic of proportions in liquid or chemical mixtures.

To illustrate these problems, we have made use of the overhead projector and brightly colored overlays. For instance, if the problem discusses a mixture that is 5 percent iodine and 95 percent alcohol, and the student is to add only alcohol, we can illustrate what is happening with the projectuals in Figure 7. The basic one shows a semicircular area in one color to represent the alcohol and a "tube" of another color within it to represent the iodine. The overlay shows the additional alcohol with a ring of the alcohol color added to the original.

The student can thus see that the *amount* of iodine does not change, but the *percent* of the mixture that is iodine does.

In the twelfth year, the entire unit on solid or spatial geometry can be illustrated with great detail on brightly colored projectuals prepared in advance. These can

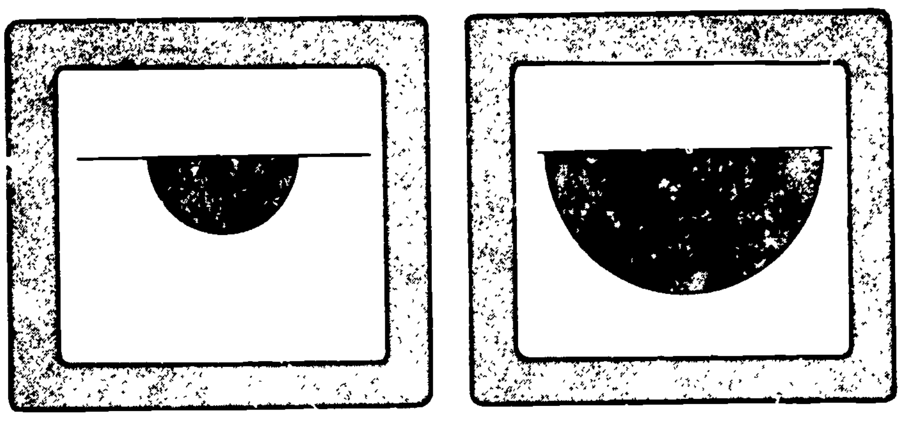


Figure 7

save much time, and perhaps a lot of embarrassment on the part of the teacher when he attempts to draw them in class.

They can be used effectively to illustrate problems like this one: Can a gun that is 3 feet long be placed into a box that is a cube in shape and that has an edge of 2 feet? To discuss this problem thoroughly, we need a good projectual of a cube. The first overlay can show the diagonal of a face $(2\sqrt{2})$, and the second overlay the space diagonal. If we wish, a third overlay can be put down showing the right triangle standing through the cube. The hypotenuse of this triangle gives us the solution to the problem $(2\sqrt{3})$.

Also in the twelfth year, we can quite effectively illustrate the idea of a tree diagram—in coin tossing, for example. We make a basic projectual, as pictured in black and color in the first diagram of Figure 8. We add an overlay, using a different color (indicated here by dashed lines). We conclude with a second overlay, using still another color for the dramatic effect. Of course we could add additional overlays if the work were done on a smaller scale, but we find that we have usually made our point by this time.

Again, let us emphasize that these are only some of the many things that can be done with projectuals. Using

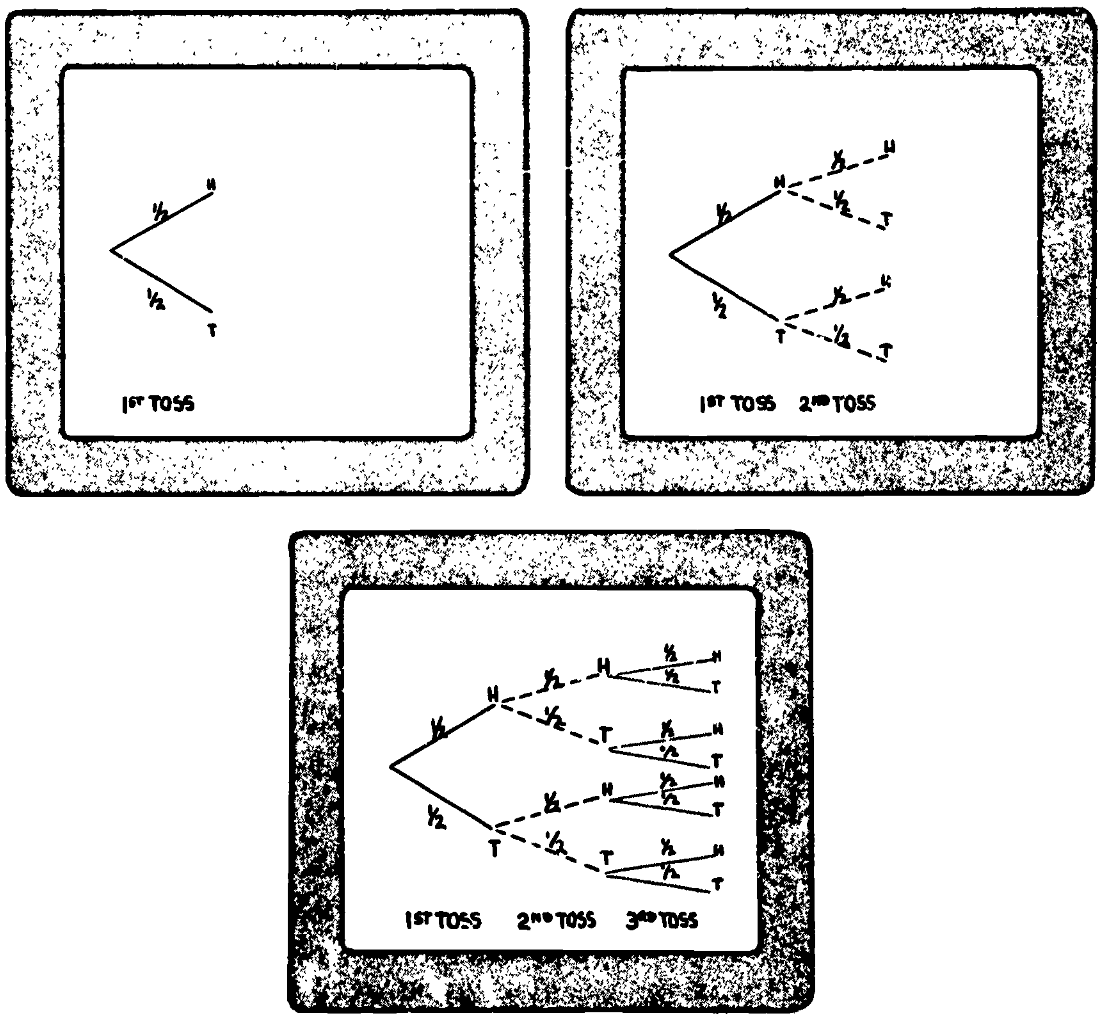


Figure 8

other materials, such as rubber bands, eyelets (to allow motion), etc., in connection with transparencies, we can produce an almost endless number of dramatic and effective presentations for use with mathematics classes. The imagination of the individual teacher is almost the only boundary.

Some additional examples and illustrations of projectuals for use with mathematics classes are given in *Multi-Sensory Techniques in the Teaching of Mathematics,* by Krulik and Kaufman, Prentice-Hall Education Series.

4 MORE ABOUT EQUIPMENT

READY-MADE TRANSPARENCIES

MANY ORGANIZATIONS HAVE BEgun to produce projectuals for the classroom teacher in mathematics, and most of these are quite good. The artwork is sound and well done. The quality of the materials is of the best. The mathematics involved is correct, since the companies use teachers as consultants and advisers. However, in many cases the expense involved is quite prohibitive, and we have found that the best projectuals we own and use are those which we have made as the need arose. It is usually possible to find an art student who can make the needed drawing; and, if the equipment is available, he can turn out quite professional-looking transparencies at relatively small expense.

Several techniques which some of these companies can provide, however, the teacher will find hard to duplicate. For example, the use of polarized projectuals is usually beyond the scope of the average teacher. Here, a special pair of lenses are used on the overhead projector, and the material to be displayed has a special type of material affixed to it. The combination gives an effect of motion, making an impressive (though expensive) display.

Color pictures such as magazine covers can be "lifted" onto a projectual. Again, equipment and techniques involved are beyond that available to the average teacher.

BASIC EQUIPMENT

What is the basic equipment needed to start a library of projectuals? The list that follows does not include all the equipment we have described previously, and it takes into account only those items the average teacher

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can purchase without denting his budget, but it does include all that is needed as basic equipment.

First of all are the plastic sheets needed to make the transparencies. These, and the frames for them, should be purchased in reasonable quantity by the local school.

As a start on individual purchases, we suggest buying a plastic or wooden cutlery tray, divided into sections for storing the equipment, some of which you may already have on hand:

Four or five grease pencils (also known as china markers)

Acetograph pen and ink, optional

Felt marking pens in various colors, both water-solubie and permanent

Wood-carving knife, although a single-edge razor blade can be substituted

Ruler

Compasses

Scissors

Assortment of colored pencils, Venus #200

Tapes

Masking tape

Transparent gummed tape

Lettering set, either templates or instant letters

Small pencil sharpener (hand type)

Another optional item would be a set of what are known in art stores as "reservoir pens." These are fountain pens made for use with India ink. They eliminate the need for dipping and filling as you work.

There are, of course, many other items which would make the work easier: a T-square and a drawing board on which to work, for example. However, the list we have given will cover almost any situation that will come up in the making of basic projectuals, and the entire cost should be no more than fifteen dollars.

WORD OF CAUTION

Finally, let us caution the teacher that the overhead projector can become a monster. In devising projectuals for the machine, teachers can get so carried away with the clever and intricate ideas they have devised that they lose sight of the basic premise behind this aid.

Remember that this machine is an *aid!* Its purpose is to help increase the teacher's capacity to present his material to students. It is never to be used as an end unto itself.

5 SOURCES

AS THE AUTHORS HAVE EXplained, this booklet is not intended as an exhaustive treatment of the subject. Readers desiring to investigate further may refer to other publications concerning visual aids. Some that are largely concerned with the use of the overhead projector are the following:

REFERENCES

HARTSELL, HORACE CLAY, and VEENENDAAL, WILFRED L. *Overhead Projection*. Buffalo, New York: Henry Stewart, 1960.

KEMP, JERROLD E. *Planning and Producing Audiovisual Materials*. San Francisco, California: Chandler Publishing Co., 1963.

KAUFMAN, IRWIN, and KRULIK, STEPHEN. *Multi-Sensory Techniques in the Teaching of Mathematics*. Englewood Cliffs, New Jersey: Prentice-Hall, 1963.

SCHULTZ, MORTON J. *The Teacher and Overhead Projection*. Englewood Cliffs, New Jersey: Prentice-Hall, 1965.

UHL, RONALD. *How to Prepare Your Own Transparencies*. Washington, D.C.: Visual Arts Press, 1963.

READY-MADE TRANSPARENCIES

There are many sources of ready-made transparencies. A partial list includes the following:

Admaster Prints, Inc., 425 Park Avenue South, New York, New York 10016.

American Map Company, Inc., 3 West 61st Street, New York, New York 10023.

Robert J. Brady Company, 3227 M Street, N.W., Washington, D.C. 20007.

John Colburn Associates, Inc., 1122 Central Avenue, Wilmette, Illinois 60091.

Colonial Films, Inc., 71 Walton Street, N.W., Atlanta, Georgia 30303.

Creative Visuals, Inc., Box 310, Big Spring, Texas 79721.

DCA Educational Products, Inc., 4865 Stenton Avenue, Philadelphia, Pennsylvania 19144.

Encyclopaedia Britannica Films, 1150 Wilmette Avenue, Wilmette, Illinois 60091.

General Aniline and Film Corporation, 140 West 51st Street, New York, New York 10020.

John W. Gunter, Inc., P.O. Box G, San Mateo, California 94402.

C. S. Hammond and Company, 515 Valley Street, Maplewood, New Jersey 07040.

Hubbard Scientific Company, 2855 Shermer Road, Northbrook, Illinois 60062.

Instructo Products Company, 1635 North 55th Street, Philadelphia, Pennsylvania 19131.

State University of Iowa, Bureau of Audiovisual Instruction, Extension Division, Iowa City, Iowa 52240.

Keuffel and Esser Company, 300 Adams Street, Hoboken, New Jersey 07030.

McGraw-Hill Book Company, Inc., 330 West 42nd Street, New York, New York 10036.

RCA Educational Services, Camden, New Jersey 08102.

Tecnifax Corporation, 195 Appleton Street, Holyoke, Massachusetts 01042.

Tweedy Transparencies, 321 Central Avenue, Newark, New Jersey 07103.

United Transparencies, Inc., P.O. Box 888, Binghamton, New York 13902.

Visual Materials, Inc., 980 O'Brien Drive, Menlo Park, California 94025.

Visualcraft, Inc., 2737 West Union Street, Blue Island, Illinois 60406.

MATERIALS AND SERVICES

Materials for homemade projectuals can usually be purchased at the local shopping center without fuss and fanfare or, at the very most, by making a special trip to the nearest art supply house. More specialized supplies, and assistance, may be secured from the following:

Admaster Prints, Inc., 425 Park Avenue South, New York, New York 10016.

Charles Beseler Company, 219 South 18th Street, East Orange. New Jersey 07018.

Best Plastic Products, Inc., 325 West 25th Place, Chicago, Illinois 60616.

Bourges Color Corporation, 80 Fifth Avenue, New York, New York 10011.

Robert J. Brady Company, 3227 M Street, N.W., Washington, D.C. 20007.

Instantype, Inc., 6551 Sunset Boulevard, Los Angeles, California 90028.

Keystone View Company, 865 Market Street, Meadville, Pennsylvania 16335.

Keuffel and Esser Company, Audiovisual Division, 300 Adams Street, Hoboken, New Jersey 07030.

Minnesota Mining and Manufacturing Company, 2501 Hudson Road, St. Paul, Minnesota 55119.

Ozalid Division, General Aniline and Film Corporation, Johnson City, New York 13790.

Stik-A-Letter Company, Route 2, Box 1400, Escondido, California 92025.

Tecnifax Corporation, 195 Appleton Street, Holyoke, Massachusetts 01042.

The Wood-Regan Instrument Company, Inc., 184 Franklin Avenue, Nutley, New Jersey 07110.