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SOME BEHAVIORAL OBJECTIVES FOR ELEMENTARY SCHOOL MATHEMATICS PROGRAMS.
BY- SHARPE, GLYN H.
COLORADO STATE DEPT. OF EDUCATION, DENVER
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THIS PUBLICATION OUTLINES SOME OF THE TERMINAL BEHAVIORAL OBJECTIVES OF THE ELEMENTARY MATHEMATICS INSTRUCTIONAL PROGRAM. INSTRUCTIONAL OBJECTIVES WHICH SPECIFY EXPLICITLY WHAT SKILLS PUPILS HAVE MASTERED ARE INDICATED FOR MANY OF THE TOPICS OF MATHEMATICS. FOR EACH OBJECTIVE, AT LEAST ONE EXAMPLE IS GIVEN TO CLARIFY THE BEHAVIORAL CRITERION WHICH DETERMINES WHEN THAT OBJECTIVE HAS BEEN REACHED BY THE PUPIL. CHECKLISTS OF COMPETENCIES FOLLOW OPERATIONAL DEFINITIONS OF MATHEMATICAL CONCEPTS TO SHOW THE RELATIONSHIP BETWEEN THE BEHAVIORAL OBJECTIVE OF AN EXERCISE AND THE TASKS REQUIRED OF THE CHILD. EACH CHECKLIST IS DESIGNED FOR EVALUATING GOAL ATTAINMENT IMMEDIATELY AFTER EACH EXERCISE. (RP)

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Some Behavioral Objectives for Elementary School Mathematics Programs

COLORADO STATE DEPARTMENT OF EDUCATION

BYRON W. HANSFORD, COMMISSIONER

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Some Behavioral Objectives
for
Elementary School Mathematics Programs

Prepared by
Glyn H. Sharpe, Section Head
Title III NDEA Program

COLORADO STATE DEPARTMENT OF EDUCATION

Office of Instructional Services
Leo P. Black, Assistant Commissioner

Division of Elementary and Secondary Education
Raymond A. McGuire, Director

Title III NDEA Program
Glyn H. Sharpe, Section Head

Denver, Colorado 80203

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FOREWORD

As we enter the second decade of the reform movement in mathematics, attention is being given to defining instructional objectives in terms of pupil behavior. Several samples of this type of objective for each grade level for many of the topics of mathematics are included here as examples of what is being considered.

These examples attempt to satisfy instructional criteria, such as the description of outcomes, terminal pupil behavior, and the intent of the teacher in this activity.

Instructional objectives which specify explicitly what pupils will be **doing** when objectives have been reached should prove to be of much use to the elementary teacher. The very act of careful teacher observation of pupil behavior should help make teachers more alert to the needs and achievements of their pupils.

We hope this publication will aid in the ongoing improvement in education.

Byron W. Hansford
Commissioner of Education

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INTRODUCTION

The reform movement in elementary school mathematics is now in its second decade. Results of this movement have been felt in almost every classroom in the nation. This effort has brought about a change in approaches to teaching mathematics in both the topics being taught and in grade placement of these topics. The period has been marked by experimental projects in mathematics education. Some of these projects, like the School Mathematics Study Group (SMSG), have produced sample materials, spelling out the content by grade level. Other projects, like the Madison Project, have produced new materials that are supplemental in nature and emphasize an inductive mode in teaching these materials.

Groups have been active during this period to bring about improved preservice training for prospective teachers and to institute inservice training programs for teachers presently in the field. Efforts to provide adequate materials for the low achievers, advanced placement, and mathematics for the future have begun and are continuing.

One of these groups has set up "Goals for School Mathematics."[1] These goals take us into a second phase of curriculum reform pointing perhaps thirty years into the future, discussing what the mathematics programs might be like then.

There is also attention on the rationale for teaching mathematics. Developing concurrently with the reform movement in mathematics has been a movement spearheaded by such people as Bruner and Piaget, who have challenged many assumptions of educators as to appropriate maturity levels for certain kinds of learning activities, **as well as** some of the approaches to teaching the materials. Their point of view emphasizes that we must know the nature of the child in his early years, as well as what materials and ideas are appropriate for his study. Implied in this, too, is the idea that we must also know the long-range skills and concepts that he will need in the future if we are to determine what topics, materials, and procedures are appropriate to use at each grade level.

All of this indicates that we must indeed know the answers to three questions that Mager[2] proposes:
What is it we must teach?
How will we know when we have taught it?
What materials and procedures will work best to teach what we wish to teach?

If we are even partially to answer these hard questions, the implication is clear that we are going to have carefully spelled-out objectives in terms of pupil behavior as observable terminal actions of students.

Historically we have stated our objectives in broad general terms such as the following:

1. To teach problem solving.
2. To understand the fundamental processes of arithmetic.
3. To really understand operations with fractions.
4. To have an appreciation of the social values of arithmetic.
5. To solve quadratic equations.

We have also stoutly maintained that many of the things we teach are intangible, esthetic, and cannot be measured. Unfortunately, this may well place the teacher in the position of having difficulty in establishing that he teaches anything at all. It also tends to place us in the position of not really knowing whether we have reached our objectives.

The purpose then of this publication is to spell out some of the objectives of the elementary mathematics instructional program in **behavioral terms** and to indicate at least one example for each objective showing the behavioral criterion that will determine when each objective has been reached by the pupil.

Gagne[3] in discussing the psychological issues in science education points out that goals of science education may be approached from three points of view. If we apply these criteria to mathematics education, the following occurs:

1. **The content viewpoint:** The best way to learn mathematics is to start to study geometry, algebra, or calculus in the early grades.
2. **The creativity viewpoint:** This point of view would emphasize that since mathematicians are creative individuals, one should deliberately undertake to teach creativity.

[1] Educational Services, Inc., "The Report of the Cambridge Conference" Boston: Houghton Mifflin Company, 1963.
[2] Mager, Robert W., **Preparing Objectives for Programmed Instruction** San Francisco: Fearon Publishers, 1962
[3] Gagne, Robert M., "Psychological Issues in Science—A Process Approach" A lecture delivered at Chicago and San Francisco, 1965.

3. **The process approach:** This approach might be considered somewhat middle-ground between the other approaches. It attempts to incorporate some of the best features of both.

The process approach is perhaps symbolized best by The American Association for the Advancement of Science in their elementary science project[4] "Science—A Process Approach." The AAAS elementary science materials are being used with a large sample of elementary pupils in grade levels K-6. Some of the key ideas of this program include:

1. A highly complex set of intellectual activities can be analyzed into simpler activities.
2. The intellectual activities (processes) usually can be generalized across several disciplines.
3. The intellectual activities may be learned from the simplest. This is then used in building towards the more complex activities.
4. A reasonable sequence of instruction can be constructed which aims to have pupils acquire process skills which have the following order:

Simple kinds of observation → classifying
measuring
communicating
quantifying
organizing through space and time → making inferences and prediction
making inferences and prediction → students learn to make operational definitions

Such an approach implies that teachers must be very sure of where they are going and how they will know when they have arrived. Of supreme importance is the necessity of carefully stating instructional objectives in behavioral terms in such a way that the actions of pupils at the termination of an exercise or activity will indicate that these objectives have been met.

The following performance descriptions are used to indicate this pupil behavior:

Performance Descriptions[5]

The action words which are guides in the construction of statements of the AAAS **Science—A Process Approach** instructional objectives are:

Identifying—The individual selects (by pointing to, touching, or picking up) the correct object of a class, in response to a class name. For example, upon being asked, "Which animal is the frog?" when presented with a set of small animals, the child is expected to respond by picking up, or clearly pointing to, or touching the frog.

Distinguishing—Identifying under conditions in which the objects or events are potentially confusable (a square and a rectangle whose lengths are almost equal), or when two contrasting identifications (such as right and left) are involved.

Constructing—Generating a construction or drawing which identifies a designated object or set of conditions. For example, beginning with a line segment, the request is made to "complete this figure so that it represents a triangle."

Naming—Supplying the correct name (orally or in writing) for a class of objects or events. For example, when asked, "What is this three-dimensional object called?" to respond, "A cone."

Ordering—Arranging two or more objects or events in proper order in accordance with a stated category. For example, "Arrange these moving objects in order of their speeds."

Describing—Generating and naming all of the necessary categories of objects, object properties, or event properties that are relevant to the description of the designated situation. For example, upon being asked to "describe this object," the child should respond in sufficient detail so that there is a probability of approximately one that any other individual who hears the description will be able to identify the object or event.

Stating a Rule—The child makes a verbal statement which conveys a rule or principle (the language used to convey the rule or principle need not always be in technical terms, but must include the names of the proper classes of objects or events in their correct order). For example, upon being asked, "What is the test for determining whether this surface is flat?" the child would respond by mentioning the application of a straightedge, in various directions, and to determine whether the straightedge is touching all along the edge.

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[4] American Association for the Advancement of Science, "Science—A Process Approach" Maryland University, 1964.
[5] Ibid.

Applying a Rule—Using an acquired rule or principle to derive an answer to a question. The answer may be a correct identification, the supplying of a name, or the construction of a figure. For example, when asked, "Which of the following figures satisfies the definition of an angle?" the child's response should identify two rays with a common end point.

Demonstrating—Performing the operations necessary to the application of a rule or principle. For example, if the child is asked to "show me how you would tell whether this surface is flat," then the response requires that the individual use a straightedge, determine whether the edge touches the surface at all points along the edge, and repeat this touching in various directions.

Interpreting—The child should be able to identify objects and/or events in terms of their consequences. There will be a set of rules or principles always connected with this behavior.

Objectives of Operational Definitions 3, "Force and Work Energy"

At the end of this exercise the child should:

1. be able to **distinguish** an operational definition from a nonoperational definition.
2. if asked for a definition of a physical concept, attempt to **state** it in operational form.
3. be able to **distinguish** between situations in which force is applied, but no work is done; and situations in which force does work.

The objectives of this exercise are described in terms of three of the ten available action words: **distinguishing** between force and work as well as distinguishing between operational and nonoperational definitions, **applying a rule** for making an operational definition, and **demonstrating** the making of an operational definition. The reader is now invited to carry out a systematic examination of these objectives. Recall that the test of acceptable statements of objectives is one which requires the reader to be able to see the child carry out some observable performances. Read the objectives for this exercise again. Do they satisfy this criterion? Can you imagine a task which could be associated with the first objective? Can you imagine a task which could be associated with the second objective? Does the third objective suggest some particular task to you? The author of this exercise suggested appraisal tasks for these objectives, from which the following checklist of competencies was formed.

CHECKLIST OF COMPETENCIES

Exercise: Operational Definitions 3, Force and Work Energy

I. Stretch a rubber band between the right and left hand, grasping it in the fingers of each hand. Ask the child to **describe** all the forces operating in this situation.
   1. Force exerted on rubber band by right hand.
   2. Force exerted on rubber band by left hand.
   3. Force exerted by fingers grasping rubber band.

   If the child does not respond or stops after **describing** only one force, ask him if that is the only force. This may be done only once.

II. Ask the child to form an operational definition of temperature.
   4. One **yes** check should be given if the child's response includes some observable effect of temperature such as expansion of a liquid or solid, rise of a column of mercury, etc.
   5. One **yes** check should be given if the child's response includes some provision for measurement of the above: expands so many units in the tube, for example.

III. Present the child with the following definitions and ask him to identify those that are operational definitions. Each statement should be given twice and the child should then be asked to decide whether it is an operational definition.
   6. Alpha is the distance in meters divided by the time in seconds. One **yes** check should be given if the child **identifies** this as an operational definition.
   7. Beta is the heaviness of the things. One **yes** check should be given if the child **identifies** this as not an operational definition.
   8. Gamma is the amount of stuff in particles. One **yes** check should be given if the child **identifies** this as not an operational definition.
   9. Delta is the object pushed times the area being pushed. One **yes** check should be given if the child **identifies** this as not an operational definition.
   10. Epsilon is the change in speed divided by the time between two observations. One **yes** check should be given if the child **identifies** this as an operational definition.

IV. Ask the child to raise a ball a few feet above the table and let it drop. Now ask: "What part of the exercise you just performed was work being done against the force of gravity?"
   11. One **yes** check should be given if the child's response is, "When the ball was being raised."

If one examines the tasks described in the checklist with care, it becomes evident that the behaviors mentioned in the objectives are related to one or more of the eleven checklist items. This relationship between the behavioral objectives of an exercise and the tasks required of the child by the checklist items is one of the general characteristics of this measure. There are three other characteristics of the checklist instrument: (1) there is a checklist which accompanies each exercise; (2) each set of checklist tasks is designed to measure the behavioral acquisition immediately after each exercise; (3) the checklist is constructed as an instrument to be administered to one child at a time.

The tasks described in the checklist are purposely designed **not** to test for the particular content of an exercise or to use the particular materials of that exercise. Rather, the set of behaviors supposedly established by the exercise is the focal point and it is the acquisition of these behaviors which is being assessed. An effort was made in the development of each checklist to design tasks which involve changes of context and stimulus from the activities of the exercise and at the same time to appraise the particular behaviors specified by the exercise objectives.

In addition to the individual assessment instrument, the checklist of competencies, a group appraisal instrument is included at the end of each exercise. The appraisal for Operational Definitions 3 can be found with the description of this exercise in the first Commission **Newsletter.** The reader is again invited to relate the objectives of the exercise to this appraisal. The group instrument—the exercise appraisal—is also constructed in such a way that each of the behavioral objectives of the exercise is measured by the exercise appraisal.

Why use two measures of immediate acquisition of behavior? Each of these behavioral measures makes a unique contribution to the body of evaluation data for this set of instructional materials. The checklist provides data on behavioral acquisition in a well controlled and individually observed measurement environment. However, because of this characteristic of the checklist, only a sample (approximately ten percent) of the children is observed. The exercise appraisal lacks the measuring refinement of the checklist, but provides a means by which the teacher can decide whether the behavioral acquisition of the large instructional group—the class—is at an acceptable level. These two immediate assessment instruments do share one vital feature. Both are intended to be measures of reliably observable behavior.

In addition to the two behavioral measures—the checklist and the exercise appraisal—which are measures of immediate acquisition, the tryout teacher is required to complete a "teacher feedback form" for each exercise she teaches. The contribution of teacher feedback to the successful revision of an exercise can be a significant one. The significance of the teacher's comments is, however, directly proportional to her ability to describe the directed and nondirected instructional experiences which she observes. Therefore, an effective feedback form should enable the teacher to record what could be heard or what could be seen happening during these situations. That is, the feedback form must provide to those who were not present a means of viewing what happened, and the teacher must attempt to be an objective reporter of these occurrences. The AAAS feedback form attempts to provide such a device. This report does admit "teacher feelings" to the domain of teacher feedback data, provided that the teacher describes what it was she saw or heard that shaped those feelings.

The following chapters will indicate what are felt to be desirable objectives for elementary mathematics. Some objectives will apply to several grade levels concurrently and will be referred to as "stated under this strand in level K,"[6] for example. The action words used by the AAAS have been used to indicate terminal behavior for pupils in elementary mathematics.

[6] Ibid.

Present Objectives Of Teaching Mathematics

A summary of the objectives which are common to most elementary mathematics education programs would include:

Social aims of the mathematics program: **to develop proficiency with certain skills and the application of these skills in solving problems** that students will encounter in real life situations (a common expectancy).

To develop reading ability, precision of expression, and accuracy.

To develop desirable attitudes in pupils towards understanding mathematics and mechanical procedures.

These objectives are reflected in evaluation procedures which measured primarily only the proficiency of skills and application of these skills.

With the reform movement in mathematics came other general objectives. Among the most important of these were:

A thorough understanding of numeration systems, axioms, properties of operations on numbers; emphasis of proofs and language of sets received a great deal of attention.

Early introduction of pupils to the real number system.

Understanding of induction, as well as deduction, concerning mathematical proofs.

Some instruction in geometry from an intuitive and non-metric viewpoint in the primary grades.

The content of the elementary school mathematics program at present certainly reflects these objectives, as well as those of the mathematics curriculum prior to the reform movement.

It is the purpose of this publication to look carefully at these topics and to write sample objectives in terms of pupil behavior for each grade level.

Grade Level Strands:

The following strands are those usually taught at the kindergarten level and spiraled throughout the six-grade mathematics program.

The Concept of Sets
Numbers and Numerals
Systems of Numeration
Geometry
Properties and Techniques of Operations on Numbers
Inequalities
Measurement
Probability and Statistics

Although these topics are not all-inclusive, they are felt to be representative of what is usually taught in the mathematics classrooms of today. This listing may seem at first to indicate a so-called **"new arithmetic curriculum."** However, recent revisions in almost every major publishing company's offering for elementary arithmetic certainly reflect these topics to some degree of treatment.

A. **Grade K**

1. **Concept of Sets**
   Comparing familiar objects, i.e., chairs to pupils, boys to girls, children to parents.
2. **Numbers and Numerals**
   Ordering numbers
   Counting numbers
   Recognizing sets by using cardinal numbers
   Meaning of **first, second, . . .** etc., and **last**
   Counting by ones
3. **Systems of Numeration**
   Counting
   One-to-one correspondences
   Equal objects in different sets matching by drawing lines
4. **Geometry**
   Recognition of shapes—intuitive geometry
   Rubber band
   Shapes in pegboard
5. **Properties and Techniques of Operations on Numbers**
   Union of sets as a model for addition
   Simple number addition up to sums of 9
   Commutative law of addition
6. **Inequalities**
   Visualization of **larger, smaller, greater than, less than**
   Using patterns to show **one more, one less**

7. **Measurement**
   Concepts of temperature
   Warmer or cooler
8. **Probability and Statistics**
   Picture graphs

B. **Grade 1**

1. **Concept of Sets**
   One-to-one correspondences
   Equivalent sets, non-equivalent sets
   Union and set separation
   Subsets
2. **Numbers and Numerals**
   Simple fractions, like ½, ⅓, ¼ etc.
   Naming sets of cardinal numbers
   Ordinals (1st-10th)
   Understanding of the numbers (0-100)
   Names for numbers (numerals)
   Counting by ones, twos, fives, and tens
3. **Systems of Numeration**
   Proficiency in using the 10 digits (0-9)
   Place value (0-100)
   Some preliminary work on expanded notation, i.e., $54 = 5$ tens $+ 4$ ones
4. **Geometry**
   Recognition of shapes
   Definition of simple closed curves
5. **Properties and Techniques of Operations on Numbers**
   Adding whole numbers
   Use of zero as the additive identity
   Two-digit addition and subtraction combinations through 20
   Expanded notation
   Vertical notation
   Subtraction using zero and a number from itself
6. **Inequalities**
   Comparison of two numbers
   Relation symbols—$>$, $<$, $=$, and $\neq$
   Recognition of signs of operation
   Finding addends (missing), e.g., $3 + \square = 7$, $5 + 2 \bigcirc 6 + 2$, etc.
   Some word problems
7. **Measurement**
   Money
   Time
   Unit measure
   Linear measure
   Temperature
8. **Probability and Statistics**
   Picture graphs
   Simple bar graphs to compare two objects
   Tabulation of trials

C. **Grade 2**

1. **Concept of Sets**
   Set notation $\{1, 2, 3, 4\}$, empty set $\{\quad\}$
   Union of sets as addition where members are disjointed
   Regrouping sets
   Equivalent subsets

2. **Numbers and Numerals**
Fractions to include 2/3, 3/4, 1/6
Reading and writing numerals (0-1000)
Counting by (3's, 4's, 6's . . ., 10's)
The relation of ordinal numbers to counting numbers

3. **Systems of Numeration**
Place value numerals (0-1000)
Expanded notation to thousands

4. **Geometry**
Line segments
Number lines: (1) order relation, (2) model for addition
Negative number line

5. **Properties and Techniques of Operations on Numbers**
One, two, three digit numbers
Missing addends $3 + \square = 9$
Missing sums $3 + 4 = \square$
Missing operation signs, e.g., $3 \bigcirc 4 = 7$
Parts of wholes
Subtraction as inverse operation

6. **Inequalities**
Sentence inequalities $3 + 4 > 3 + 2$
Missing inequality signs $4 \bigcirc 6$

7. **Measurement**
Money, time, year, modular arithmetic
Linear measure
Volume measure
Weight

8. **Probability and Statistics**
Picture graphs, graphs, or number lines
Tallying data

D. **Grade 3**

1. **Concept of Sets**
Union of sets, intersection of sets, solution sets
4 sets of 3 = 3 sets of 4

2. **Numbers and Numerals**
Fractions ¾ as $3 \times$ ¼, etc.

3. **Systems of Numeration**
Place value through millions
Expanded notation through millions

4. **Geometry**
Line segments
Number lines
Negative numbers
With number line
Including fractions on the number line
Ideas of separation, e.g., a point separates a line into two parts

5. **Properties and Techniques of Operations on Numbers**
Inverse of multiplication: $3 \times \triangle = 12$, $12 \div 3 = \triangle$
Repeated subtraction as model of division
Addition and subtraction up to and including 7-digit numbers
Natural numbers, whole numbers, fractions, integers
Zero property of addition and multiplication of simple products

6. **Inequalities**
   Include multiplication and division: $(3 \times 4) \bigcirc (3 \times 5)$
   Missing signs $\frac{12}{3} \bigcirc \frac{12}{3}$, etc.
7. **Measurement**
   Money, use of $ sign and decimal point
8. **Probability and Statistics**
   One dimension line graph including fractions, circle, bar, and coordinate graphs

E. **Grade 4**

1. **Concept of Sets**
   Extension of ideas using subsets
   Union as addition
   Intersection of sets of points and lines
2. **Numbers and Numerals**
   Sets and the number line
   Diagrams
   Fractions
   Other names for numbers, e.g., $3\frac{1}{2} = 4 - \frac{1}{2}$ or $3 + \frac{1}{2}$
3. **Systems of Numeration**
   Systems of numeration in bases other than 10
   Expanded notation in other bases, e.g., 321 (four) $= (3 \times 16) + (2 \times 4) + (1 \times 1)$
   Modular or clock arithmetic
4. **Geometry**
   Paths, planes, curves, polygons, pyramids, cylinders, cones, and spheres
   Line and plane separation
5. **Properties and Techniques of Operations on Numbers**
   Binary operations defined
   Properties of operation emphasized
   Inverse relationship of (1) addition and subtraction, (2) multiplication and division
   Addition of fractions with like denominators
   Common denominators
6. **Inequalities**
   Story problems like: 3 more than $\frac{1}{3}$ of a number is 6
   Missing sets for all $n>3$, or $n<2$
7. **Measurement**
   Dry measure
   Area and perimeter measurement for simple geometric figures
   Liquid measurement
8. **Probability and Statistics**
   Venn diagrams
   Line graph
   Line and Venn diagrams to show union and intersection of sets of numbers
   Coordinates

F. **Grade 5**

1. **Concept of Sets**
   Sets of numbers, points, lines continued
2. **Numbers and Numerals**
   Prime numbers
   Composite numbers
   Equivalent fractions
   Decimal fractions
   Percents
3. **Systems of Numeration**
   Base 2 notation
   Continued work with systems in other bases
   Modular systems

4. **Geometry**
   Review of forms studied in Grade 4
5. **Properties and Techniques of Operations on Numbers**
   Products and factors
   g. c. f.
   L. C. M.
   Addition and subtraction without common denominator
   Multiplication of fractions
6. **Inequalities**
   Review of missing sets for $n>3$
   2 more than ½ of a number is 5, etc.
7. **Measurement**
   Area as square units
   Area of square and rectangle
   Angle measurement units
8. **Probability and Statistics**
   Scale reading
   Ratios in two dimension scales
   More line graphs

C. **Grade 6**

1. **Concept of Sets**
   Review of subsets
   Infinite and finite sets
   Definition of geometric figures in set language
2. **Numbers and Numerals**
   Equivalent numbers: $4 = (2\times2) = 2^2$
   Numerals for all the integers
3. **Systems of Numeration**
   Place value in other bases
4. **Geometry**
   Separation of sets of points by closed figures
   Interior, exterior sets of points
   Ideas of congruence
   Similar triangle
   Quadrant separation of a plane
   Coordinates as ordered pairs
5. **Properties and Techniques of Operations on Numbers**
   Review of Grade 5 ideas
6. **Inequalities**
   Sentences dealing with inequalities
7. **Measurement**
   Volume cubic measurement
   Geometric solids
   Relationships of dry and liquid measure
8. **Probability and Statistics**
   Organizing data, rate, and statistical graphs
   Scale drawings and readings
   Plotting points from ordered pairs

We have not attempted to develop a scope and sequence chart, but have listed the concepts that are common to most elementary mathematics programs, so that sample behavioral objectives might be given for each concept under the eight stra' ds at each grade level.

Teachers should consider these objectives as samples only. Pupil behavior other than the examples listed is probably important and will become part of a teacher's checklist of pupil performance in mathematics.

SAMPLE OBJECTIVES FOR GRADE K

There are many things that could be listed as desirable outcomes for kindergarten pupils. Some of the more important of these might include:

1. The pupil can identify and classify objects as to:
   a. size of objects
   b. shape of objects
   c. what do these objects do under certain specified conditions?
   d. similarities and differences
   e. greater than, less than, and the same as (equal)
2. The pupil is able to make gross distinctions concerning time intervals and how we measure these intervals.
3. The pupil is able to count at least to ten, assign numbers to an object or group, and distinguish between a number and the symbol for the number.
4. The pupil is able to order objects by several schemes such as color shades, size, weight, length, etc.

With reference to the strands that seem to run throughout the elementary mathematics program, the kindergarten pupil should be able to perform as indicated by each specific objective given. The list of objectives is not comprehensive and should be considered an example by the user. At best, it comprises a minimum list of objectives. Some of the objectives may prove to be ambiguous in practice, and the teacher certainly should have in mind at all times their improvement.

Objectives for Grade K

Strand—Concept of Sets

Objectives:

1. Given two familiar sets such as the set of boys and the set of girls present that day, the pupil can **describe** which set has the most members or whether the sets have the same number of members.
2. Given a set containing four members such as four rocks, cubes, or pebbles, a pupil can **describe** how many objects are in the set and **distinguish** between those that are alike and those that are different.
3. **When presented with two sets such as** $\{1, 2, 3, 4\}$ and $\{\square, \triangle, \ast, \triangledown\}$ the pupil can **construct** lines that show a one-to-one correspondence between the two sets.

Strand—Numbers and Numerals:

Objectives:

1. Given a set of unordered numerals (0-9) on cards made of felt, paper, or plastic, the pupil can **order** these by placing them on a flannel board or desk top in the order, $0 \longrightarrow 10$ and $10 \longrightarrow 0$.
2. When asked questions like "How many chairs are there in this room?" the pupil can supply the correct **name** for this number.
3. After being given a set of objects such as cubes or rods, the pupil can write or verbally state the cardinal number that **identifies** the set.
4. After observing an event such as a candle which is lit, then extinguished, the pupil can **order** the events by verbally **describing** the things that happened first, second, . . . last.
5. Using a number line, cubes, rods, or pebbles, the student can **name** the number of objects obtained by adding 1 more to objects grouped ones through nines.

Strand—Systems of Numeration:

Objectives:

1. Given two sets having the same number of members, the pupil can **construct** lines or place yarn to show one-to-one correspondences between members of the two sets.
2. Given objects like pebbles, cubes, or rods the pupil can count objects up to 20 by grouping objects in sets of one through twenty members and verbally **naming** how many objects are contained in each group.

Strand—Geometry:

Objectives:

1. The pupil who has been given a set of geometric solids containing a cube, cylinder, rectangular solid, sphere, and cone can **identify** these by **naming** members verbally when asked to do so.
2. Using a pegboard the pupil can use rubber bands to **construct** a square, triangle, and rectangle. He can **name** these geometric shapes when the constructions have been made by the teacher or another pupil.
3. Using a pencil and paper or the chalkboard the pupil can make rough **constructions** of a circle, triangle, square, and rectangle.

Strand—Properties and Techniques of Operations of Numbers:

Objectives:

1. Using a number line, cubes, or rods the pupil can **name** the sum of two numbers through sums of 9 by **verbally stating** the result they obtain from using the manipulative device.
2. Pupils can **name** the sum of 2 numbers like $2 + 3$ and **design** simple experiments to show that their answer is correct by using such ideas as the jumps on a number line, union of two sets, or placing rods and cubes end to end.

Strand—Inequalities:

Objectives:

1. Given several objects including some that are alike and some different the pupil can **identify** and hold up for inspection an object that is the same, less than another, or greater than another.
2. Using unit cubes or proportional rods, the pupil can show what "one more than" means by **naming** first the smaller number, adding a unit cube, and **naming** the resulting number.
3. The pupil can use the materials as described above to **name** "one less" than the assigned number.

Strand—Measurement:

Objectives:

1. After some experience with reading scales on a thermometer, the pupil can **record** temperatures inside and outside the classroom and answer such questions as "Is it cooler or warmer today than yesterday?" or "Is it cooler outside or inside?"
2. Given a ruler or meter stick the pupil can perform **crude measurements** on desk length, sides of geometric figures, heights of other pupils, etc., when asked to do so.

Strand—Probability and Statistics:

Objectives:

1. Given picture graphs showing comparisons of the populations of two things pupils can **identify** the larger, smaller, or sameness of the populations being compared.
2. Using a record of daily temperatures the pupil can **identify** grossly the trend of temperature and perhaps estimate the temperature for tomorow.

An excellent way for teachers to determine whether or not each objective has been reached would be to prepare a criterion checklist for each separate objective stated.

SAMPLE OBJECTIVES FOR GRADE ONE

The pupil who has completed grade one should be able to write out as well as name objects that he used during the kindergarten year. Some general activities might include:

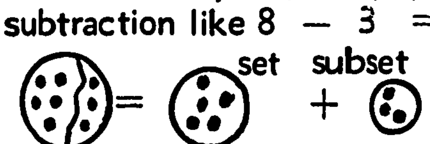
1. **Distinguishing** big and little objects from a series of objects.
2. Reading and **interpreting** symbols both in letter and number.
3. Assigning symbols to reflect comparisons.
4. Adding and subtracting number combinations 0 through 20; distinguishing angular direction in space; identifying lines, planes, and angles; and devising his own units for measuring things.
5. Constructing simple bar graphs to compare two quantities.
6. Devising models to identify fractions such as proportioned rods or parts of a circle.
7. Using zero both as place holder and the additive identity.

The following specified objectives are suggested again for the eight strands of mathematics as sample behavioral criteria. The list is not comprehensive and at best would be a minimum listing.

Strand—Concept of Sets:

Objectives:

1. Same as objective No. 3 for grade K.
2. Given two sets like $\{1, 2, 3, 4\}$ and $\{a, b, c, d\}$ the pupil can **identify** this as an equivalent set.
3. Given a set of heterogeneous objects such as pebbles, the pupil can **recombine** these on the basis of size, texture, or some other basis into subsets.
4. Given a set of objects, the pupil can separate or recombine this set into two sets to show a model for subtraction like $8 - 3 = 5$.



Strand—Numbers and Numerals:

Objectives:

1. Using proportioned rods or unit cubes the pupil can identify and **construct** models for the fractions 1/2, 1/3, 1/4, 1/5, ... 1/10. When asked to order these from smaller to larger the pupil can do so.
2. The pupil can use cardinal numbers to **name** or denumerate sets.
3. The pupil can **identify** by naming the numbers $(0 \longrightarrow 100)$.
4. When asked to count by twos, fives, and tens the pupil can verbally respond by counting or **writing** the required numbers.

Strand—Systems of Numeration:

Objectives:

1. Given numbers like 63 the pupil can distinguish place value by naming the numeral that is in the tens place and the numeral that is in the ones place.
2. Given numbers like 49 the pupil can **write** this number in expanded notation in the following form: 49 = 4 tens and 9 ones, **or** 49 = 4 tens + 9 ones.
3. The pupil can **write** all the numerals for numbers $0 \longrightarrow 99$.

Strand—Geometry:

Objectives:

1. Replication of objectives Nos. 1, 2 and 3 of objectives under this strand for grade K.
2. When given a set of curves like the following $\{\infty, \supset, \square, \overline{A\ B}, O\}$ the pupil can **identify** the simple closed curves and the open curves.
3. The pupil can **construct** special kinds of simple closed curves like squares, circles, rectangles, and triangles and label these with the correct names.

Strand—Properties and Techniques of Operations on Numbers:

Objectives:

1. The pupil can find sums of two numbers through 20 by **writing** these sums to problems like $8 + 6 = \square$ or $\begin{array}{r} 8 \\ +5 \\ \hline \end{array}$

2. The pupil can find the difference of two numbers through 20 by **writing** these differences when given problems like $13 - 10 = \square$, or $8 - 6 = \square$.

3. The pupil can **write** the answer to find a missing addend for problems of the type $8 - \square = 5$, or $7 = 2 + \square$.

4. The pupil can write sums and differences using both vertical and horizontal notation for problems like $\begin{array}{r} 10 \\ +4 \\ \hline \end{array}$, $8 + 6$, $7 - 6$, $13 + 5$, $\begin{array}{r} 12 \\ +1 \\ \hline \end{array}$, etc.

5. The pupil can use the additive identity zero to **write** sums, differences, and missing addends for problems like $8 + 0 = \square$, $7 - 7 = 0$, $3 + \square = 3$, etc.

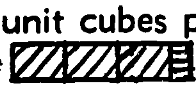
Strand—Inequalities:

Objectives:

1. Pupils can **identify** relations between two numbers and write the relation symbol ($>$, $<$, or $=$) that shows the relation between numbers like 4 ⧁ 3, 5 ⧀ 8, 6 ⊜ 6, etc.

2. Pupils are able to **name** the missing symbol for problems like $5 + 2$ ○ $6 + 2$, $8 - 1$ ○ $9 - 1$, etc.

3. Given word problems like "John has 3 apples and Mary has 5 apples. Who has the greater number of apples?" pupils can **verbally** respond to the correct answer.

4. Using rods or unit cubes pupils can "make up a story" and **verbally tell** their story. An example story might be  three and one is four.

Strand—Measurement:

Objectives:

1. Objectives No. 1 and No. 2 for grade K are reinforced for grade 1.

2. When given coins, pupils can **identify** a coin that is worth 5 pennies, 2 nickels, 5 nickels, 5 dimes, 2 quarters, etc.

3. The pupil can **read** and **record** temperature measurements and time measurement.

4. The pupil can use simple measuring instruments such as the ruler or meter stick to **measure** lengths, distance around simple geometric figures. He can **measure** volume of simple solids by counting unit cubes.

Strand—Probability and Statistics:

Objectives:

1. Replication of objectives No. 1 and No. 2 for grade K.

2. The pupil can **construct** simple bar graphs to compare heights of objects, growth of plants, etc.

3. The pupil can record results of such things as how many red sides as compared to white sides do we get when we drop discs that have different colored sides. (We could compare heads and tails of pennies, or thumb tacks that stand on end or lie on their sides.)

SAMPLE OBJECTIVES FOR GRADE TWO

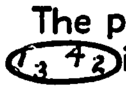
By the time that the pupil leaves grade two he should know how to use many measuring instruments for linear measure, area, and volume. He should know how to read scales and angles, and he should have some descriptive knowledge of points, angles, and planes. He should know how to make rough constructions in two dimensions and know how to tabulate data.

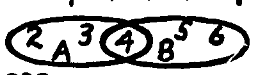
The experience with operations on numbers should be extended to include a more formal understanding of the order and grouping properties of addition and multiplication, as well as the relation of inverse operations to addition and multiplication.

Relationships including the tabulation and graphical plotting of these relationships should be mastered by this time. Specific objectives in terms of the strands are suggested again as samples of what behavioral performance might be considered important.

Strand—Concept of Sets:

Objectives:

1. Replication of objectives for grade 1.
2. The pupil can use set notation to designate a group of objects by **writing** the notation of a set like 

in the following way: $A = \{1, 2, 3, 4\}$ or $B = \{1, 2, 3, 4\}$.
3. The pupil can **construct** Venn diagrams to show the union and intersection of two or more sets like $A = \{1, 2, 3, 4\}$, $B = \{3, 4, 5, 6\}$.
4. Given problems like 

the pupil can identify members of sets A and B using set notation to **name** these.
5. Given proposals like $\square + 3 = 5$, the pupil can **identify** and write the truth set.

Strand—Numbers and Numerals:

Objectives:

1. Given proportioned rods or unit cubes, the pupil can **construct** and identify models for the fractions 1/2, 1/3, 2/3, 3/4, etc.
2. The pupil can **name** and **write** numerals through 1000.
3. The pupil can **write** numerals through 50 that result from counting by threes, fours, sixes, and tens.
4. The pupil can **construct** a model of sets to show the relationship between the ordinal numbers and the counting numbers.

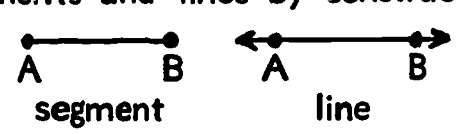
Strand—Systems of Numeration:

Objectives:

1. Given a number like 864 the pupil can **name** numerals that are in the ones, tens, and hundreds place.
2. The pupil can **write** expanded notation for three-place numerals in the following way: 863 = 8 hundreds + 6 tens + 3 ones.
3. The pupil can read from a thermometer below zero or a number line that uses negative numbers.

Strand—Geometry:

Objectives:

1. The pupil can **distinguish** between line segments and lines by constructing models for each and labeling them in the following way 

2. Using string, or another measuring instrument, and a set of line segments, the pupil can **identify** and name those that are congruent to each other.
3. The pupil can **name** or **write** symbols for a line, ray, and line segment.
4. The pupil can **construct** a number line that goes both directions from zero.

Strand—Properties and Techniques of Operations on Numbers:

Objectives:

1. Replication of objectives for grade one.
2. The pupil can **write** missing addends or sums for problems of the following type: $\frac{1}{2} + \square = 1$, $\frac{1}{4} + \frac{1}{4} =$
3. The pupil can **write** some other names for the same number. For example, he may write $3 = 2 + 1 = 4 - 1 = 1 + 1 + 1$.
4. The pupil can **construct** arrays of the type 2  4 and write names for the area in square units.

Strand—Inequalities:

Objectives:

1. Replication of objectives for grade one.
2. Pupils can **write** inequality statements word problems like "sixteen is seven more than nine" $\begin{pmatrix} 16>9 \\ 16=9+7 \end{pmatrix}$, or five is two less than seven $\begin{pmatrix} 5<7 \\ 5+2=7 \end{pmatrix}$

Strand—Measurement:

Objectives:

1. The pupil can use measuring instruments to **measure** length, area, and volume of objects.
2. The pupil can **construct** angles and use a protractor to **measure** the angle.
3. Objectives are to be replicated as listed for grade one.

Strand—Probability and Statistics:

Objectives:

1. The objectives No. 2 and No. 3 under this strand for grade one should be replicated.
2. The pupil can **construct** circle graphs to show relations of parts of a whole and to show relationships of quantities or populations.

SAMPLE OBJECTIVES FOR GRADE THREE

By the end of grade three pupils should be able to discriminate quantities of small and large size, great distances and small distances. They should be somewhat knowledgeable about time measurement. They should be able to group and regroup sets of objects, to understand the inverse relationship of addition to subtraction as well as multiplication to division.

Pupils should be able now to view things somewhat more objectively and to record information carefully. They should be able to devise schemes of displaying relationships using line graphs, circle graphs, bar graphs, and be able to interpret these.

Pupil proficiency in operating with numbers should include the ability to determine simple products and quotients, as well as the ability to add and subtract two seven-digit numbers.

Specific objectives suggested as appropriate for this grade level follow. (Teachers should remember that these are samples. At best they would make up a minimum listing.)

Strand—Concept of Sets:

Objectives:

1. The pupil can **write** solution sets for problems like $\square > 3$, $3 + \square = 8$, $\triangle < 6$, etc.
2. The pupil can regroup 4 sets of 3 objects using cubes, pebbles, or other objects, and can **construct** 3 sets of 4 objects or 2 sets of 6 objects, etc.

Strand—Numbers and Numerals:

Objectives:

1. The pupil is able to locate and **name** negative numerals on a number line. He can **determine** answers to addition problems like $-3+4=1$ on the number line.
2. The pupil can **construct** crossed number lines and can locate points in the plane, **naming** these with answers like "over 2 and up 1."
3. When given a fraction like ¾ the pupil can **identify** 3 × ¼ or 1 × ¾ as other names for the same number.

Strand—Systems of Numeration:

Objectives:

1. The pupil can write numbers through millions, **naming** the place value for each numeral.
2. Given a number like 87243, the pupil can write this in expanded notation in the following way: $87243 = (8 \times 10000) + (7 \times 1000) + (2 \times 100) + (4 \times 10) + (3 \times 1)$.
3. The pupil can **write** decimal notation for simple fractions like 1/10, 1/12, and 1/5.

Strand—Geometry:

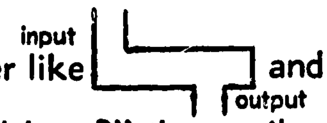
Objectives:

1. Replication of grade two objectives for geometry.
2. The pupil can **construct** a line, a point that separates the line into 3 sets, and **name** these.
3. The pupil can **construct** a plane, a line that divides the plane into 3 sets, and **name** these by labeling line $\underset{AB}{\longleftrightarrow}$, part of plane to right of $\underset{AB}{\longleftrightarrow}$, and part of plane to left of $\underset{AB}{\longleftrightarrow}$.
4. Given a set containing a cube, rectangular solid, sphere, cylinder, and a cone, the pupil can **identify** these and **write** names for each.

Strand—Properties and Techniques of Operations on Numbers:

Objectives:

1. Given a problem like $3 \times \triangle = 12$, the pupil can **write** another equation, namely $12 \div 3 = \triangle$ to indicate the inverse operation for multiplication.
2. Given a problem like $3+8=11$, the pupil can write two other equations, $3=11-8$ and $8=11-3$, to show the inverse operation for addition.
3. Replication of objectives for grade two using the identity properties of addition and multiplication.
4. Given a problem like $8 \times 3 = \square$, the pupil can **construct** an array to show a model for this product and prove his product by counting squares.

5. Using a number grinder like 

and asking a question like "If I put in 3 and get out 5, what kind of a machine do I have?" the pupil can **name** it as an "add 3 machine."

Strand—Inequalities:

Objectives:

1. The pupil can **write** missing symbols for relations between numbers and sentences of the following types:

$$3 \bigcirc 2$$
$$5 \bigcirc 8$$
$$(2+3) \bigcirc (2+2)$$
$$(12\times2) \bigcirc (12\times1)$$
$$\frac{12}{3} \bigcirc \frac{12}{2}$$

The pupil writes $>$ or $<$ in the place hold that makes the sentence true.

3. The pupil can **name** solution sets for whole numbers in problems like 3 in the following ways: T = 4, 5, 6, 7, . . .; or **states** "The truth set contains all whole numbers that are greater than three."

Strand—Measurement:

Objectives:

1. A pupil can use metric measuring devices to **measure** length. He can **name** the standard unit for length in the metric system.
2. A pupil can **name** coins with decimal notation, such as one quarter equals $.25.
3. The pupil can **construct** number lines showing fractional parts between 0 and 1. He can **identify** and point to locations like ½, ¼, or ¾ on the number line.

Strand—Probability and Statistics

Objectives:

1. The pupil can **construct** his own circle graph or bar graph when given data to display.
2. Given spring scales or balances, the pupil can weigh unit objects and groups of the units. Having performed an experiment or been given data from an experiment, he can **estimate** what ½, 1½, 2¼, etc., of the unit weights would be.

SAMPLE OBJECTIVES FOR GRADE FOUR

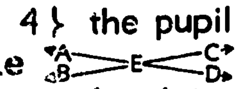
The pupil who has completed grade four should be able to understand systems of numeration in bases other than 10. He should be able to understand simple metric geometry and be able to identify common geometric solids. He should be proficient with multiplication and division of three-digit numbers and be able to understand the relationship of these inverse operations to addition and multiplication. He should be able to construct Venn diagrams to show union and intersection operations with sets.

He should be able now to show relations of time and distance, weight and volume, weight and stretch of springs, and equations like $\square + \triangle = 8$.

Specific objectives might include the following suggested samples:

Strand—Concept of Sets:

Objectives:

1. Given a set like $A = \{1, 2, 3, 4\}$ the pupil can **name** and **write** all the subsets.
2. Given two intersecting lines like  the pupil can **identify** the intersection set as point E.
3. The pupil can **construct** two lines that intersect and two lines that are parallel.

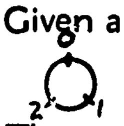
Strand—Numbers and Numerals:

Objectives:

1. Replication of objectives for operations with numbers listed under this strand for grade three.
2. The pupil can **write** numerals in bases other than 10.
3. When given numbers like 321 (four), the pupil can **write** in expanded notation 321 (four) = $(3 \times 16) + (2 \times 4) + (1 \times 1)$.
4. The pupil can **identify** common demoninators for fractions like $\frac{1}{2} + \frac{1}{4}$ and **write** these sums.

Strand—Systems of Numeration:

Objectives:

1. Given a set of numbers $\{0, 1, 2\}$ the pupil can construct a modular system  determine a direction for modular addition, and construct an addition table for this system.
2. The pupil can **identify** place value for numerals in base 10 through billions and for numerals in other bases.
3. Using the classroom clock, pupils can **write** sums for modular addition on the clock like $10 + 4 = 2 \bmod 12$.

Strand—Geometry:

Objectives:

1. The pupil can **construct** and **label** paths, polygons, pyramids, and cones.
2. The pupil can **construct** a line segment, angle, or polygon congruent to a given segment, angle, or polygon.

Strand—Properties and Techniques of Operations on Numbers:

Objectives:

1. The pupil can **identify** from a given set of equations like the following:

   $2 + 7 = 8 + 1$
   $3 + 4 = 4 + 3$
   $5 \times 4 = 4 \times 5$
   $(2 + 3) + 5 = 2 + (3 + 5)$
   $(2 \times 1) \times 2 = 2 \times (1 \times 2)$
   $(2 + 5) > (1 + 5)$
   $3 + 5 = 4 + 4$
   $5 \times 1 = 1 \times 5 = 5$
   $3 + 0 = 0 + 3 = 3$

   those that show the

   a. commutative property of addition
   b. commutative property of multiplication
   c. associative property of addition
   d. associative property of multiplication
   e. identity property of addition
   f. identity property of multiplication
2. The pupil can use parentheses to identify names for the same number from the following equations,

for which two examples are given:

$3 + 4 + 5 = 3 + 9$

$3 + 5 - 2 = 3 + 3$

3. The pupil can use the number line to add two numbers like $3 + 2 = \square$. He can **identify** the sum on the number line or **write** answers to problems like this.
4. The pupil can **write** the equation for problems of the type $5 - 3 = \square$ in the form $5 = \square + 3$ as a check for addition.
5. Pupils can use the inverse property of addition to **write** answers to problems like:

$8 + 5 = \square$

$\square + 8 = 13$

$\square + (8 - 8) = 13 - 8$

$\square = 5$

Strand—Inequalities:

Objectives:

1. The pupil can **write** answers to problems like 3 more than a number is equal to 6, by writing a number sentence $3 + n = 6$ and solving the equation using properties of addition and multiplication in the following way:

$3 + n = 6$

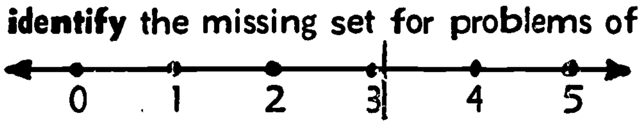
$3 + n = n + 3$

$n + 3 = 6$

$n + (3 - 3) = 6 - 3$

$n = 3$

2. The pupil can **identify** the missing set for problems of the type $m > 3$ and can show this using a number line:



Strand—Measurement:

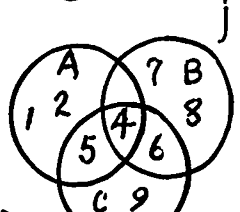
Objectives:

1. The pupil can **construct** three-dimensional diagrams to indicate models for space and volume.
2. The pupil can use a three-dimensional array as a model for volume and can **state a rule** for finding the volume like "$V = l \times w \times h$" for rectangular solids.
3. The pupil can **state a rule** for finding the perimeter of simple geometric figures like a square, triangle, and rectangle.
4. The pupil can **demonstrate** his ability to measure length, area, volume, liquid, and dry measure by defining the unit he would use and the instrument used to do the measurement.
5. The pupil can **apply rules** for definitions of geometric figures by **identifying** from a set of objects the one described. An example would be:

Given $\left\{ \begin{matrix} C \\ \triangle \\ A\ B \end{matrix},\ \begin{matrix} 2D\ C \\ \square \\ A\ B \end{matrix},\ O,\ \boxed{} \right\}$, find the figure that is defined as the union of three line segments.

Strand—Probability and Statistics:

Objectives:



1. Given Venn diagrams showing the relations of sets A, B, and C, the pupil can **interpret** and **describe** at least 10 of the following relationship:

(a) A = { }

(b) B = { }

(c) C = { }

(d) AUB = { }

(e) AUC = { }

(f) BUC = { }

(g) AUBUC = { }

(h) A B = { }

(i) A C = { }

(j) B C = { }

(k) A B C = { }

2. The pupil can **interpret** graphically relations like $\square + \triangle = 8$, by tabulating the solution set and plotting ordered pair in the coordinate plane.
3. From a line graph of problems like $\triangle = \square + 3$, the pupil can **interpret** values other than integral for $\square$ and $\triangle$.

SAMPLE OBJECTIVES FOR GRADE FIVE

Upon completion of grade five, pupils should have definite understanding of set notation concerning natural numbers, rational numbers, and geometry. During this year the pupil should have much experience with products and factors, greatest common factor, least common multiple, and multiplication of rational numbers.

Metric geometry should be given some attention along with scale drawings and ratios. The pupil should have more experience with angle measurement and line graphs.

The pupil should be able to **describe** number operations in terms of properties of operations. He should be able to construct angles, bisectors, and line divisions. He should be able to describe equivalent fractions, prime numbers, composite numbers, and decimal fractions.

Suggested specific objectives are indicated as sample behavioral performances by pupils:

Strand—Concepts of Sets:

Objectives:

1. The pupil can **describe** the following sets of numbers by writing:
   A. Natural numbers are the set described by $n = \{1, 2, 3, \ldots\}$
   B. Whole numbers are the set described by $w = \{0, 1, 2, 3, \ldots\}$
   C. Integers are the set described by $j = \{\ldots -2, -1, 0, 1, 2 \ldots\}$
2. The pupil can **construct** Venn diagrams to show the relation of the sets described above. He can **describe** the subset relation of:
   A. Natural numbers to integers.
   B. Whole numbers to natural numbers.
   C. Whole numbers to integers.
   D. The relations of all these sets.
3. The pupil can **state** rules for defining lines, angles, polygons, planes, open curves, and closed curves in set language.
4. The pupil can **apply** rules to **construct** geometric figures when asked questions like "What is the union of three segments?"

Strand—Numbers and Numerals:

Objectives:

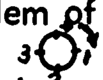
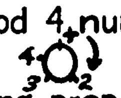
1. The pupil can **construct** a sieve and name the prime numbers through 97.
2. Given composite numbers like 64 the pupil can **identify** another name for 64 in terms of a product of prime factors.
3. Given two numbers like 48 and 64 the pupil can **apply the rules** for determining the least common multiple and greatest common factors of the given numbers.

   Example: $48 = 2 \times 2 \times 2 \times 2 \times 3 = 2^4 \times 3$
   $64 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^6$
   $LCM = 2^6 \times 3$
   $GCF = 2^4 \times 3$
4. The pupil can **identify** and write equivalent fractions for numbers like 1/2, 4/6, 3/8, etc.
5. Given a common fraction like 1/2, the pupil can **identify** and write the equivalent decimal fraction.
6. Given a decimal fraction like .5, the pupil can **identify** the equivalent common fraction or the per cent of a whole.
7. The pupil can **write** answers to addition and subtraction problems through millions and multiplication and division problems through thousands.

Strand—Systems of Numeration:

Objectives:

1. The pupil can **describe** a number like 3246 in expanded notation in the following way:
   $3246 = (3 \times 10^3) + (2 \times 10^2) + (4 \times 10^1) + (6 \times 10^0)$
2. The pupil can **describe** a number like 324 (five) in expanded notation in the following way:
   324 (five) in expanded notation in the following way:
   $324 \text{ (five)} = (3 \times 5^2) + (2 \times 5^1) + (4 \times 5^0)$

3. Given the problem of **describing** a mod 4 numeration system, the pupil can **construct** a reference circle like  or  , define one binary operation (perhaps clockwise addition), and show that the following properties of the operation hold.
   A. closure
   B. commutative property of the operation
   C. associative property of the operation
   D. name the identity element
4. The pupil can describe base 10 numeration in terms of:
   A. closure
   B. commutative property of binary operations of addition and multiplication
   C. associative property of binary operations of addition and multiplication
   D. the distributive law of addition over multiplication
   E. identity elements of addition and multiplication
   F. inverses of numbers with operations of addition and multiplication
5. The pupil can describe place value of numbers like 0.135718 by expanded notation.

Strand—Geometry:

Objectives:

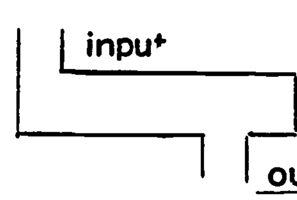
1. Review and reinforcement of objectives listed under grade three.
2. The pupil can **describe** in terms of solution sets the following:
   A. Intersection of two lines
   B. Intersection of a line and a circle
   C. Intersection of a line and a plane
3. The pupil can **construct** the following:
   A. a right angle
   B. the perpendicular bisector of a line
   C. the perpendicular bisector of an angle
   D. an equilateral triangle
   E. an isosceles triangle
   F. an angle equal to a given angle
4. The pupil can **state** and apply rules for determining congruency of line segments, angles, and other polygons.
5. The pupil can start with a line segment, **construct** a triangle, then **construct** another triangle congruent to this triangle.
6. The pupil can replicate a given polygon and **state rules** to show they are congruent.

Strand—Properties and Techniques of Operations on Numbers:

Objectives:

1. Given a number grinder (a machine having 2 numbers going into the input and a third number as the output) and some information like the following table:

input	output
2, 3	7
3, 4	10

the pupil can **state** the rule for combining numbers that the machine uses 

in the following manner:

Let $\square$ = 1st number
$\triangle$ = 2nd number
∇ = 3rd number
then $2\square + \triangle = \nabla$

5. Given a line graph relation between $\square$ and $\triangle$, the pupil can **state the rule** of how $\square$ is related to $\triangle$, and **write** the formula.

Strand—Inequalities:

Objectives:

1. The pupil can **order** a set like $\{1/2, 1/3, 2/5, 1/8, 7/9\}$ from smallest to largest or largest to smallest.
2. The pupil can **describe** sets like the set of all n, such that $n>3$ or $n<2$. He can **demonstrate** his understanding by showing this set on a number line or coordinate plane.
3. He can **interpret** word problems like, two more than one half of a number is 9, by **writing** a symbolic sentence and determining the value of the number.

Strand—Measurement:

Objectives:

1. The pupil can find total surface area of a cube and a rectangular solid by **applying** a rule for finding the area of a face and adding up the faces.
2. The pupil can **construct** arrays that serve as models for total area and can count square units to **demonstrate** that the rules for finding areas of squares and rectangles are given by the rule $(a = b \times h)$.
3. Given a protractor, a student can measure angles, **distinguishing** whether they are acute angles, right angles, or obtuse angles, as determined by the measure of the angles.
4. The pupil can **demonstrate** volume measurement by constructing a rectangular solid using unit cubes.

Strand—Probability and Statistics:

Objectives:

1. Given a map, the pupil can **describe** the scale used and can make map measurements which he can convert to actual distances.
2. Given a ratio of two numbers like 1/2, the pupil can **show** how this would appear with two-dimension scales.
3. The pupil can **construct** scale drawings of a floor plan or map.
4. The pupil can tabulate information and **interpret** trends like population growth of a county.
5. Using a bar, circle, or line graph, the pupil can **demonstrate** data he has collected.

SAMPLE OBJECTIVES FOR GRADE SIX

The pupil, after completing the sixth grade, should have some appreciation of scientific notation, approximation, and indirect measurement.

His experience with numeration systems should enable him to write numbers in expanded notation using exponents. The work with cancellation laws of multiplication and addition should provide him with useful tools in writing solution sets for number sentences.

He should now be familiar with introductory aspects of both non-metric and metric geometry. He should be able to organize data into tabular and graphical form to show relations between variables. Some experience with true and false statements is appropriate.

Specific objectives suggested as sample pupil performance criteria for grade six are as follows:

Strand—Concept of Sets:

Objectives:

1. The pupil can **distinguish** finite from infinite sets and can write examples of each.
2. Objectives 2, 3, and 4 of grade five, replication.
3. Given a set of numerals and a defined operation, the pupil can **apply** rules to determine:
   a. Closure for the operation
   b. Commutative and associate properties of the operation
   c. If there are inverses
   d. Identity elements of the operation

Strand—Numbers and Numerals:

Objectives:

1. Given a number like 4, the pupil can **describe** equivalent numbers by writing other expressions like:
   $$4 = 2 + 2, 4 = 4 \times 1, 4 = 2^2, 4 = 4^1, \text{or } 4 = 2 \times 2 = 2^2.$$
2. The pupil can **apply** the rules using $\square$, $\triangle$, ∇ notation or literal notation to **describe** properties of operation like:
   a. $\square + \triangle = \triangle + \square$ Commutative property of addition
   b. $\square \times \triangle = \triangle \times \square$ Commutative property of multiplication
   c. $\square + (\triangle + \nabla) = (\square + \triangle) + \nabla$ Associative property of addition
   d. $\square \times (\triangle \times \nabla) = (\square \times \triangle) \times \nabla$ Associative property of multiplication
   e. $\square + 0 = 0 + \square = \square$ Identity property of addition
   f. $\square \times 1 = 1 \times \square = \square$ Identity property of multiplication
   g. $\square \times (\triangle \times \nabla) = (\square \times \triangle) + (\square \times \nabla)$ Distributive property of addition over multiplication
   h. $\square + {}^{-}\square = {}^{-}\square + \square = 0$ Inverse of addition
   i. $\square \times \frac{1}{\square} = \frac{1}{\square} \times \square = 1$ Inverse of multiplication
   j. The set of natural numbers are closed under addition and multiplication.
   k. The set of whole numbers are closed under addition and multiplication.
   l. The set of integers are closed under addition and multiplication.
   m. The sets of natural numbers, whole numbers, and integers are not closed for subtraction and division by using one counter example.
3. Objectives 2, 3, 4, and 5 listed under grade five are repeated.
4. The pupil can **write names** for all integers.
5. Pupils can **construct** a coordinate system and locate points in the plane as ordered pairs of numbers.

Strand—Systems of Numeration:

Objectives:

1. Given a number in a base other than 10, the pupil can describe the number in expanded notation using powers of the base.
2. The pupil can **write** the name of a number like 2,700,000 or 0.000035 in scientific notation in the following way:
   $2\,700\,000 = 2.7 \times 10^6$
   $0.000035 = 3.5 \times 10^{-5}$
3. Replication of objectives 4 and 5 in grade five.

Strand—Geometry:

Objectives:

1. Replication of objectives 2, 3, 4, and 5 of grade five.
2. The pupil can **construct** a simple closed curve and identify the following by labeling
   a. Set of exterior points
   b. Set of interior points
   c. The curve
3. The pupil can **construct** a quadrant separation of a plane
4. The pupil can **identify** given points in the plane by writing the ordered pairs that give the location of the points.

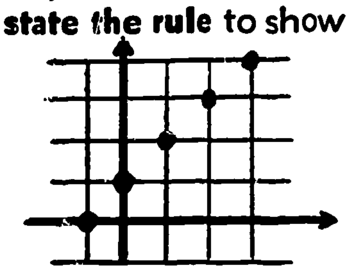
Strand—Properties and Techniques of Operations on Numbers:

Objectives:

1. Replication of objectives 3 and 4 for grade five.
2. The pupil can **state the rule** to show relationships between variables when given tables like:

$\triangle$	$\square$
0	3
1	4
2	5
.	.
.	.
.	.

or graphs like:



3. When given problems like 943×25, the pupil can use the distributive property of multiplication over addition to **name** the produce in the following way:

$$943 = 900 + 40 + 3$$
$$26 = 20 + 6$$
$$(20 + 6) \quad (900 + 40 + 3) = 20 \times (900 + 40 + 3) + 6 \times (900 + 40) + 3$$
$$= 18000 + 800 + 60 + 5400 + 240 + 18$$
$$= 24518$$

4. Given a problem like 54×65, the pupil can **write** these numbers as products of prime numbers, **identifying** the lowest common multiple and the greatest common factor of the two numbers.

Strand—Inequalities:

Objectives:

1. Given problems like $10 > \square > 5$, or $5 < \square < 8$, the pupil can **name** solution sets by using the number line to identify this solution set.
2. The pupil when given problems like $\triangle > \square + 3$ can **identify** the solution set graphically by plotting $\triangle = \square + 3$ and indicating that part of the plane that is the solution set for $\triangle = \square + 3$.

Strand—Measurement:

Objectives:

1. Pupils are able to **state the rules** and apply these rules to determine volume of cubes, rectangular prisms, and cylinders.
2. Pupils can **determine** area and circumference of circles given the radius or the diameter.
3. Using a compass and straight edge, pupils can **construct** the following:
   a. The circumscribed circle of a square.
   b. The inscribed circle of a square.
   c. Congruent angles, circles, polygons, and planes to those given.
4. A pupil can **apply rules** for testing congruency of two given geometric figures.
5. When given two lines, the pupil can **construct** another line that intersects the given line and can **determine** whether the lines are parallel.

Strand—Probability and Statistics:

Objectives:

1. Replication of objectives 2, 3, 4, and 5 listed under grade five objectives.
2. Given a table of known measurements such as—then the pupil can **interpret** locations between known measurements. For example, he can say that after 2½ seconds the distance would probably be 6 feet.

(Seconds) Time	(Feet) Distance
0	0
1	3
2	6
3	6
4	9
5	12

DESIGNING A CRITERION TEST TO MEASURE OBJECTIVES

Much of the emphasis of this publication has been in terms of stating objectives in pupil behavioral terms stated with action verbs. This, of course, does not answer such questions as the following:

A. How well do we want the pupil to do something in order to determine that he has reached the particular objective?

B. Does every pupil have to be able to complete requirements of all objectives?

Each teacher will perhaps have different minimum acceptable levels of pupil performance. Several techniques can be implemented to determine lower acceptable levels of performance. Suggested techniques are:

A. Time limit specifications. Examples might be:

1. Given all the exercises: $\begin{array}{r} 23 \\ +41 \\ \hline \end{array}$ $\begin{array}{r} 16 \\ -12 \\ \hline \end{array}$ $\begin{array}{r} 18 \\ +12 \\ \hline \end{array}$ and $\begin{array}{r} 42 \\ -14 \\ \hline \end{array}$, a third grade pupil can **write** at least 4 of 5 correct answers in five minutes.
2. Given 6 groups of four objects, a second grade pupil can regroup these into 4 groups of six objects in three minutes.
3. Starting with unit cubes, first graders can **construct** models for numbers 1 through 10 in four minutes.

B. Number specifications. This indicates the minimum number of correct responses that the teacher will accept. Example #1 above has this requirement. Another example might be to ask pupils to match names with geometric figures and set a minimum number of correct responses as the criterion.

Example: A second grade pupil can **name** at least 3 of the following figures: a. ○ b. △, c. □ d. ▭

C. Percentage specification. This might serve as a minimum specification level in the following way:

1. A certain percentage of the pupils can write correct names for at least 80% of the test items on a particular test.
2. Ninety percent of the first grade pupils can reach objective #2 under geometry.

The point is that not only **must** teachers state the objective in behavioral terms, but they **must also establish a minimum performance level** that is acceptable to them as evidence that the objective has been reached by pupils. It is probably inevitable that these levels will vary from teacher to teacher because of ability of pupils, background of teachers, materials for pupil use, and mode of teaching. Still, until such time as teachers are willing to establish these performance levels, some doubt must remain as to "What am I teaching?" and "How do I know I have taught it?"

D. Ability Specification. The question of "Does every pupil have to be able to complete requirements of all objectives?" has not been answered. Only during recent years have we worried seriously about the low achiever in mathematics. The national curriculum studies have been directed toward the average and above average pupils.

Some suggestions to the problem of dealing with the low achievers might be:

1. Specify lower minimum levels of proficiency for low achievers. An alternative approach might be to expect completion of fewer objectives or an extension of the time for completion of a particular objective.
2. A partial individualization of teaching.
3. Additional staff to give these pupils more adult attention.
4. Deliberate training for teachers in dealing with low achievers in mathematics.

Appendix C of a recent publication[7] dealing with low achievers in mathematics lists the following guidelines for teaching this type of pupil:

1. Low achievers should be taught by able and well trained teachers.
2. Modern educational technology should be exploited.
3. Classroom activities should be purposeful and varied.
4. Particularly for the low achiever, the need for mathematics comes from experience in the physical world.
5. The teacher should be receptive to questions.
6. A laboratory setting is especially effective for low achievers.
7. Special help should be provided for beginning teachers.

Hopefully, these guidelines should give some direction for teacher who are concerned with desirable progress for this type of pupil.

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[7] **The Low Achiever in Mathematics,** U. S. Office of Education, Bulletin 31, 1965 Washington, D. C.