V.

Conclusion

How children think is and should continue to be one of the major concerns of the teacher. Thinking here is understood to be intake and processing of information toward some goal of problem solving. Understanding children's thinking is necessary both because teaching children how to think is an important part of their schooling and because teaching any subject matter depends on the intermediate processing skills. At present, training to perform in some subject area and training to use intermediate processing skills are often undifferentiated; specific performance is emphasized and processing skills are expected to be learned along the way. We argue that, since processing skills are basic to educational success for each pupil, they deserve the major portion of the teacher's direct attention. The analysis of errors is a useful way, if not the only one, of viewing the elements of the process and refocusing instruction to produce thoughtful and reflective behavior.

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EDUCATIONAL PRACTICE AND THE ANALYSIS OF ERRORS.

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KINDS OF ERRORS, AND THEIR VALUE TO HELP TEACHERS KNOW AS MUCH AS POSSIBLE ABOUT HOW THEIR STUDENTS' MINDS WORK, ARE DISCUSSED. THE TERM "ERROR" IS USED TO REFER TO MISTAKES IN THE PROCESS OF REASONING RATHER THAN TO INCORRECT OUTCOMES OF REASONING. THE AUTHORS STATE, "CORRECT OUTCOMES MAY OR MAY NOT FOLLOW UPON ERRORS (OR MISTAKEN PROCESSES), BUT INCORRECT OUTCOMES ALWAYS INDICATE THE PRESENCE OF SOME SORT OF ERROR." THEY ALSO STATE, "THE INCORRECT OUTCOME IS USEFUL BECAUSE IT ALERTS THE TEACHER TO SEARCH FOR ITS SOURCE--THE ERROR OR ERRORS PRECEDING IT." A VARIETY OF ERRORS THAT FREQUENTLY OCCUR IN THE THINKING OF MANY DIFFERENT CHILDREN ON MANY TYPES OF PROBLEMS IS DISCUSSED. THE ERRORS ARE CLASSIFIED ACCORDING TO THE MENTAL OPERATIONS THE CHILD PERFORMS RATHER THAN IN RELATION TO THE KIND OF MATERIAL UPON WHICH HE OPERATES. THE AUTHORS CONCLUDE THAT CORRECT IDENTIFICATION OF ERRORS CAN GUIDE THE TEACHER IN SELECTING AN EFFECTIVE INSTRUCTIONAL STRATEGY FOR THE CHILD. (AL)

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EDUCATIONAL PRACTICE AND THE ANALYSIS OF ERRORS

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I.

The Importance of Understanding Children's Errors:

We begin with the assumption that teachers need to know as much as possible about how their students' minds work. How can a teacher arrive at such an understanding? She can, of course, read a book on the subject, and then attempt to apply what she has learned in her teaching. This approach, while obviously of some potential usefulness, is not always very fruitful. The ideas and data contained in a textbook on child development seem somehow remote from the daily problems of real teachers facing real children with immediate problems. The teacher must cope with immediate situations requiring immediate action. Where does she get the information on which to act in the here-and-now of the classroom?

We suggest that her best source of information is the students themselves. Books can tell her about The Child or The Typical Seven Year Old, but her concern is with a particular child, and it is this particular child who can best tell her how to proceed. The purpose of this paper, then, is not just to relay the findings of child psychology but to help teachers understand the messages implicit in their students' behavior. We shall be concerned here with only one segment of children's behavior -- the errors they make. We have chosen to focus on children's failures, rather than on their successes, simply because errors are more informative than right answers. To make this point clear, let us look first at the implications of success.

<u>The meaning of success</u>. Success may be less informative than failure simply because it is more ambiguous -- harder to interpret and easier to misinterpret. To take a very simple example, suppose that the teacher has asked, "Which state is larger -- Texas or Rhode Island?" One child answers "Texas," and the other answers "Rhode Island." We know that the second child needs help; somewhere, somehow, he has gone wrong. But what of the first child? Are we sure that he knows the right answer? Or was his reply a lucky guess?

When a child solves a problem, it is tempting to infer that he understands the principle of solution. The teacher has applied this principle in arriving at the right solution, and she may assume that the child has used it too. But this assumption is very often a delusion. For example, asked, "What is 2 x 2?" a child might answer "4." So far, so good, and if the teacher stops there she may assume that the child knows how to multiply. But if he goes on to say that 1 x 3 = 4, 2 x 3 = 5, and 5 x 5 = 10, it is clear that his first solution was the outcome of an incorrect strategy or that he was unable to use a correct strategy when more difficult or different problems are encountered. In one of our classroom observations, elementary school children were trying out a new science unit. They readily got the notion that if two objects of equal weight are placed at each end of a balance beam, the beam will balance. We assumed the children had learned that objects placed equidistant from the center would balance. But, in fact, they had not thought about distance from the center. When asked to balance an asymetrical beam, they still placed the objects at each end and were astounded by the results.

Sometimes children are able to handle problems they do not understand by invoking some very powerful strategy that is irrelevant to the structure of the particular problem but highly relevant to success over a wide range of problems. The following incident, reported by John Holt (1964), is an illustration:

> I once observed a class in which the teacher was testing her students on parts of speech. On the blackboard she had three columns, headed Noun, Adjective and Verb. As she gave each word, she called on a child and asked in which column the word belonged...There was a good deal of the tried-and-true strategy of guess-and-look, in which you start to say a word, all the while scrutinizing the teacher's face to see whether you are on the right track or not. With most teachers, no further strategies are needed. This one was more poker-faced than most, so guess-and-look wasn't working very well. Still, the percentage of hits was remarkably high, especially since it was clear to me from the way the children were talking and acting that they hadn't a notion of what Nouns, Adjectives and Verbs were. Finally, one child said,"Miss ________, you shouldn't point to the answer each time." After a while...I thought I saw what the girl meant. Since the teacher wrote each word down in its proper column, she was, in a way, getting herself ready to write, pointing herself at the place where she would soon be writing. From the angle of her body to the blackboard the children picked up a subtle clue to the correct answer.

Another reason for the limited information provided by successful performance is that there are situations in which one child's very limitations lead him to succeed where a more able child will fail. Sometimes we find that younger children are better than older ones in performing certain experimental tasks in the laboratory. On some perceptual judgments, for example, accuracy decreases with age. When a young child is asked to compare the size of a square put some distance away with the size of a square closer to his eye, he will tend to underestimate the far square slightly, but an adult's judgment will be much worse; he will greatly overestimate the size of the far square. The explanation may be that the older person has learned something about perspective. He knows that things far away appear smaller than they are, and he overcompensates for the illusion. His understanding is more subtle than the child's, but this very understanding leads him into more extreme error. We shall have more to say later about these "errors in growth."

The point we wish to make here is simply that success is not a trustworthy index of understanding. Too often, children are right for the wrong reasons, and their apparent success masks their mistakes and misunderstandings.

The uses of error. Our impression is that teachers often fail to profit by their students' mistakes. While observing proceedings in a number of classrooms, we have been struck by the degree to which error is simply ignored. Frequently the teacher asks a question and, receiving an incorrect answer from one child, passes on to the next child, and the next, until she elicits the response she is seeking. Where error is not ignored, it is often punished. It is treated as a sort of sin, the outcome of the student's shiftlessness or recalcitrance. It is not surprising that teachers handle errors in this way. A child who succeeds is a symptom of the teacher's success, while one who fails conveys the ominous implication that something has gone wrong in the teaching. The aversion to or suppression of error is understandable; yet it deprives the teacher of a rich source of intelligence about the operation of the child's mind.

What can a child's failure tell the teacher? At the very least, it suggests instruction is necessary. A child who succeeds on some task may or may not need further instruction, but one who fails clearly needs help. But what sort of help? The mere fact that a child fails does not tell us how to go about designing instruction that will help him. A common use of error is to count the number of mistakes a child makes on a test and, on the basis of the sum, assign him to a particular ability group; however, the sheer quantity of errors the child makes in reading, for example, does not tell us how to teach him to read. We need to know what it is that prevents him from reading well, and to find out, we must examine the <u>nature</u>, as well as the number, of his errors. For just as there are many correct and incorrect paths to success, there are many different routes to failure. If we treat all errors alike, we do not come to grips with the child's specific source of difficulty. Training in phonics will not help the child who systematically reverses his letters.

At this point it becomes necessary to refine our vocabulary somewhat. We have been using terms like <u>error</u>, <u>mistake</u>, <u>failure</u>, and so on interchangeably, generally to mean an incorrect solution. From now on, we shall adopt a distinction made by Margaret Donaldson (1963). The term <u>error</u> will be used to refer to mistakes in the <u>process</u> of reasoning rather than to incorrect <u>outcomes</u> of reasoning. For example, if a child says that 1 x 3 = 4, the incorrect outcome -- 4 -- is clear. It is not clear precisely what the child's error is, i.e., what mistaken process he is using. It may be that he substituted addition for multiplication or that he misread the 3 as 4. We just do not know unless we pursue the matter further.

Let us now restate an earlier point in this new language. Correct outcomes may or may not follow upon errors (or mistaken processes), but incorrect outcomes always indicate the presence of some sort of error. The incorrect outcome is useful because it alerts the teacher to search for its source -- for the error or errors preceding it. Sometimes the wrong answer contains a powerful clue to the error that produced it; for example, if a child announces that 1 + 6 = 16, we can make a good guess about the error which produced the incorrect solution. But often the error is not so obvious, and the teacher must probe hard to find it. The effort is worthwhile because it is, after all, not the outcome that demands correction but the process. We do not want the child simply to be able to emit the correct response -- "7" -- when fed the stimulus, "What is 1 and 6?" We want him to know how to add.

What other information is conveyed by error? Analysis of errors reveals not only what the child fails to do but what he does do in the process of reasoning. The child who says "1 + 6 = 16" or "I goed to the store" is not exhibiting random behavior. More likely, he is systematically applying a rule that happens to be inappropriate to the problem. That is, the failure to apply the proper mathematical or grammatical rule does not necessarily imply the absence of a rule. To the teacher the behavior may appear random, because the rules followed are not her rules. Analysis of error is a way of uncovering the rules the child actually uses.

Thus, the analysis of errors is an important guide to educational practice. The nature of the errors indicates to the teacher that the child is using rules inappropriate to the successful solution of the problem and that instruction is necessary. Errors can also guide the teacher in selecting an effective instructional strategy for the child. How can a teacher begin to analyze the errors her students make? The job may not be quite so formidable as it appears. Errors are not completely idiosyncratic. Certain types of errors appear frequently in the thinking of many different children on many different sorts of problems. We turn now to a consideration of some of these common errors.

II.

Some Common Errors in Problem Solving:

The mistakes or incorrect solutions children produce in school appear to be very diverse, but the errors of information processing that produce them are perhaps not so diverse. At least it seems possible that errors can be classified into some common types, and that this classification can help us to understand better the ways in which children think. On what basis can we classify errors? We could do it in terms of subject matter -- errors in arithmetic or in reading, or, more specifically, errors in addition or subtraction. But these categories seem both too broad and too narrow for our purposes. There are many operations involved in solving an arithmetic problem -- even one so simple as the addition of two digits -- and if we lump errors that occur in one part of the process with those that occur in another we are masking important distinctions.

The same reasoning suggests that a classification of errors in terms of subject matter is too narrow; we are interested in errors in the processing of information and it seems likely that many such processing problems cut across a variety of subject matters. Conceivably, a child may be plagued by an identical difficulty in both social studies and mathematics. Typically, curricula are compartmentalized, but minds are not. To sum up, then, a particular problem may contain the possibility of several different sorts of error, while a number of apparently diverse problems may contain a common pitfall.

Since our concern is with flaws in processing information, our categorization must reflect the mental operations the child is performing rather than the material upon which he is operating. There are several parts of the probelm-solving process that contain the possibility of error. The first part might be called intake errors. This has to do with the child's conception of the goals and the data of the task. When presented with a problem, he must ask himself -- "Where do I want to go?" and "What do I have to work with?" Once the child has assimilated the data, he must proceed to organize it. This involves the selection of a method or "program" for manipulating the available information in order to reach a solution. We use the term organizational error to refer to errors in this part of the process. Finally, the children may commit errors in executing the method or plan devised for problem solutions -- the actual performance of the operations demanded by the program. Thus, we shall discuss the errors children make under these headings:

A. Intake Errors:

1. Errors in Perceiving the Goal
2. Errors in Perceiving the Data
   a. Omissions
   b. Distortions
   c. Improper Additions

B. Organizational Errors:

1. Analysis and Synthesis
2. Sequencing

C. Executive Errors

We turn now to some illustrations of these several sorts of error. The ideas and illustrations in this section have come both from observations made in the classroom and from experimental investigations of children's thinking. It is sometimes easier to see what is going on in a child's mind when one is alone with the child, playing a fairly simple experimental game, than when one is trying to attend simultaneously to the multifarious pursuits of thirty children in a classroom dealing with relatively complex problems. Thus the errors that occur in the laboratory may be easier to see and understand. Furthermore, we feel these errors are relevant to life in the classroom, and we shall try -- after describing these errors here -- to illustrate their relevance to the classroom in the section which follows.

A. Intake Errors:

1. Errors in Perceiving the Goal.

Frequently, we suspect, there is a discrepancy between the purpose of a given lesson as conceived by the teacher and the purpose as perceived by the child. Errors in perception of the goal may occur through misunderstanding the instructions. For example, the teacher asks the child to add 4 and 3. The child, thinking he is to subtract one number from the other, answers "1". As a more subtle example, consider the game of "Twenty Questions." The aim of the game is to discover what object or event someone is thinking of, by asking questions answerable by "Yes" or "No," using as few questions as possible. The most intelligent questions are those that systematically narrow the range of possible solutions. But John Holt (1964), who has played the game with children in his fifth grade classroom, points out that for many children the aim is not to ask an intelligent question but just to ask a question -- any question -- that gets them off the spot. Consequently a child will ask the kind of question that looks like a wild guess. Knowing only that the item in question is a number between 1 and 100, he may say, "Is it 5?" For another child the aim may be to ask a "safe" question -- one that other children will not laugh at. Such a child often makes a "disguised guess." Holt (1964) quotes another example: Asked to identify an historical figure, a child inquired, "Was he killed by Brutus?" A disguised guess gets the child no closer to solution than an honest guess, but it spares him the ridicule of his classmates.

It is tempting to assume that children who ask unintelligent questions are for some reason unable to frame intelligent ones. But it is possible that the child is not <u>trying</u> to ask an intelligent question. Blurting out a question that "gets by" may constitute success for the child; it satisfies his aims, though not his teacher's. Before concluding that the child is unable to play the game, the teacher should make sure that the child is not playing a different game -- one with an implicit goal different from the teacher's aim.

What causes a discrepancy between the goals of the teacher and those of the child? One obvious possibility is that the teacher fails to make her own version of the goal clear to the child -- perhaps because she is not clear about it herself. The more ambiguous the teacher's instructions, the more likely the child is to pursue some private purpose. Most teachers are aware of this danger and try to avoid it by preparing carefully detailed "lesson plans," setting forth with precision the goals and sub-goals for each task. (Such a procedure contains dangers of its own, however. The teacher may be so intent on the goals she has defined that she is blind to the children's reasonable deviations from that goal.)

Even when the teacher has done her part to make her aims clear, a child sometimes substitutes his own goal for hers. It may be that the project at hand sets up a competing goal too powerful to be resisted. Thus, for example, in the Twenty Questions situation the child's need to maintain status with his friends may be so strong as to obliterate the stated purpose of the task or to cause him to lose sight of that purpose.

One way to avoid the situation where a child substitutes his own purposes for those of the teacher -- or the "lesson" -- is to allow the child to generate his own goals. Provide him with a balance beam and let the behavior of the beam generate the question. By exercising control over the data surrounding the child (as Rousseau did with Emile) the teacher can feel fairly confident that many questions she would like to ask will be asked by the child himself. And if he asks them himself, they are apt to be meaningful to him. There is no problem of input, if the question is the output of the child rather than of the teacher or of the text.

Once again, there are perils in such an approach. It is difficult to arrange conditions that will generate sensible and useful questions from the child. Also, the teacher must be able to understand the goals generated by the child -- this may involve as many risks as the child's understanding of the teacher's goals. Not only must the teacher understand the child's goals, she must tolerate them; she must be clear about the range of questions she will allow. Few teachers are honestly able to allow full freedom to their students in setting goals. We once watched a teacher hand each member of the class a piece of paper with a meal worm on it. The children asked what they were supposed to do with the meal worms, and the teacher replied, "Whatever you like." One little boy promptly picked up a pencil and began to draw on the paper whereupon the teacher indignantly demanded, "Did I tell you you could draw?"

While teachers cannot allow their students to pursue irrational goals, they can learn about the children's purposes and enlist and exploit them. One way to learn about a child's goals is to examine the way he distorts the stated goals of the task. If the teacher can use the child's disguised guess in the Twenty Questions game, for example, as a clue to how his mind is operating, rather than as a deviation to be punished, she may be able to set up situations where the motive implied by the guess is not aroused.

2. Errors in Perceiving the Data.

The errors a child makes in perceiving the data are the outcome of an interaction between characteristics of the data and the child's cognitive processing. These errors often take the form of omissions, distortions, or improper additions. A particular type of error does not arise solely from the presentation of the data (whatever its degree of adequacy) but also from the particular capabilities of the child. We turn now to some illustrations of errors in perception of the data, together with some of their common causes -- causes that reside both in the data and in the child's mind -- or, more precisely, in the encounter between the two. The sources of error mentioned below are not meant to be exhaustive but merely suggestive of the sorts of interaction that can take place.

a. Errors of Omission.

Sometimes a child fails to perceive some necessary piece of information. In such cases, although the data that the teacher or textbook has set forth are sufficient to solve the problem, the data from which the child is working are not. Often the teacher is able to ferret out such errors. Tests of reading comprehension, for example, where the child must read a paragraph and then answer questions about it, may tell the teacher what the child has not assimilated. But often when the child is given a body of data to use in solving some problem, his perception of the data is not tested. The assumption is that he has at his command all the information supplied; when he fails to solve the problem, there is no attempt to trace the fault back to omissions in the original perception.

What causes a child to omit portions of the data? One possibility is that the child is a victim of informational overload. The amount of information in the paragraph, for example, exceeds his processing abilities, and so some of it fails to enter his mind. He sees the words but fails to register the information. There would seem to be at least two cures for such informational overload. The first and obvious solution is for the teacher to present the child with less information, or present it in smaller doses. The second is somewhat more subtle. This is to give the <u>child</u>

some means of reducing the overload -- techniques for reducing and summarizing a body of data so as to make it assimilable.*

Omissions in perception do not always lead us astray. In fact, some omissions are inevitable. There are always more stimuli impinging upon our senses than we can absorb. If we tried to attend to everything, we would end by attending to nothing. Perception, then, is a matter of selection. We learn to attend to the things that matter and to ignore those that do not. The child goes astray not just because he omits, but because he omits necessary information. Now, what do we mean by necessary? One way of distinguishing necessary from unnecessary data is to separate the <u>informative</u> from the <u>redundant</u>. Informative data reduce our uncertainty and add to our ability to solve the problem. Redundant data tell us nothing new.

Redundancy is related to omissions in perceiving data. When an identical message is presented to a naive listener and a listener who is well informed, the message will contain more new information and less redundancy for the naive listener than for the informed one. Although the messages are objectively the same length, they are subjectively longer for the naive listener. Because he has more information to assimilate, there is more danger of overload and omission. Omission is not only more likely

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*We shall return to a discussion of these summarizing techniques later when dealing with errors in organizing the data. We can observe the child's techniques for organizing data when we watch him manipulate the data in an attempt to solve a problem. It is likely that these same organizing techniques operate to help or hinder <u>perception</u> of the data, as well, though invisibly and perhaps automatically and unconsciously.

for the naive listener, it is also more costly. Every word omitted means a loss of information. The informed listener, however, can lose a word with impunity, since it is likely to be redundant.

We do not know very much about the reasons for children's omissions, but we do have some indirect evidence that omissions are more damaging for children than for adults. There are a number of studies that show that the older the child, the easier it is for him to recognize a stimulus on the basis of insufficient cues. For example, in an experiment by Gollin (1960), young children were shown a series of drawings of a single object beginning with a very fragmentary sketch. Each picture in the series contained a few more lines until, on the final card, the representation was complete. The children were asked to guess the identity of the picture as they looked at each card. In general, the older the child, the fewer lines were required for recognition. In another study, Mary Potter (1966) showed children a series of photographs of the same scene in increasingly clear focus beginning with a highly blurred representation and ending with a clear one. In this instance "redundancy" was clarity of focus rather than number of lines, but the results were similar to Gollin's -- the older the child, the earlier the recognition.

The "omissions" in these studies were in the stimulus rather than in the child's perception. But the results, in comparing the child and adult, suggest that omissions (whether <u>of</u> the data or <u>in</u> the data) damage the child's perception to a greater degree. If the child takes only a hasty glance at a picture, perceiving only a few lines, it is unlikely that he will recognize the picture.

These experiments were concerned with pictorial redundancy. Young children may well require more cues when the data are in the form of a verbal representation rather than in a pictorial form. In most verbal messages there is a built-in redundancy that protects us from misunderstanding. Usually we need not hear every word in a sentence in order to grasp its meaning. Consequently, we are free to ignore certain elements of the message. Suppose that the teacher has embarked on a lesson in American history. She begins, "The first President of the United States, George Washington...." For those of us with some knowledge of American history, "George Washington" is redundant. We already know the name of the first president of the United States, and so we can tune out at that point with impunity. But for anyone without that knowledge, the name is not redundant; it is informative. It is information he needs in order to comprehend the sentence.

Sometimes when the child is operating on the basis of insufficient data, the reason is not that he has omitted data but that the data as presented were insufficient. Here the child's difficulty is not failure to perceive but failure to interpolate information to fill in the gaps in the presentation. Suppose that just as the teacher reaches the words "George Washington," a jet plane zooms overhead, or the teacher mumbles, or the child drops his pencil and dives under the desk to retrieve it. In any of these situations, "George Washington" is apt to be lost. Although none of the students may hear the words, some will be able to add them out of their own heads. The uninformed child, however, will not be able to supply them.

This example suggests that the reason older children can interpolate and extrapolate more easily than younger ones is that they are more familiar with the content of the incomplete message. But besides knowledge of facts, there is probably also an important increase with age in knowledge of certain formal rules which make interpolation and extrapolation easier. People who know how to count, for example, have no trouble extending the series "1, 2, 3, ..." Or, to take a more complicated example, it is sometimes said that a syllogism is redundant. Given that all men are mortal and that Socrates is a man, the conclusion -- that Socrates is mortal -- is redundant in the sense that it is contained in the premises. If you know the premises, you know the conclusion. But this is true, of course, only if you can perform the necessary inferential operation.

Whatever the reasons for the decrease in need for redundancy with age, it must be remembered that the need is not perfectly correlated with age. If every child in a given classroom possessed the same need for redundancy, the teacher's job would be easier. But in fact there is likely to be considerable variability in need for redundancy among children of the same age. This was true of subjects in the Gollin (1960) and Potter (1966) studies. Even for a single child, requirements will vary, depending upon the child's familiarity with the particular matters being discussed. This means that the teacher who is careful to build in considerable redundancy in order to make the data assimilable by some of the students risks boring the rest of them.

The teacher is hampered not only by the discrepancies among her students but also by the discrepancy between herself and the students.

Jerome Bruner (1966) has said that the relation between instructor and student "is a relation between one who possesses something and one who does not" and that as a result "there is always a special problem of authority involved in the instructional situation." There is also, for the same reason, a special problem of communication. Because the teacher knows more than the child knows, she may say less than is necessary to ensure his understanding. Anyone who has ever given a lecture -- even to sophisticated graduate students -- has found that his lecture conveyed less information than he thought it contained. The lecturer, knowing the subject well, tends to leave out essential facts -- facts that "go without saying" for one who understands the subject, but that need to be said to those who understand it imperfectly.

Given, then, that data presented in the classroom are likely often to be ambiguous, and given that children have difficulty in dealing with ambiguity, errors of omission are apt to be both frequent and damaging.

b. Errors of Distortion.

When data are ambiguous, the child, instead of failing to register them, may distort them. In the preceding section we noted how the experienced person could compensate with his own knowledge for incompleteness in the data. But a person's knowledge and expectations can also lead him astray. He may shape the data in terms of his expectations and, if these expectations are inappropriate, the result is distortion.

The Swiss psychologist, Jean Piaget (1959), has asserted that new information is not simply registered "as is" on a child's mind. Rather, it is "assimilated" -- moulded to conform to the child's existing knowledge. Piaget had performed an experiment in which one child -- between 6 and 8 years old -- had to tell a story or explain the working of some mechanism (e.g., a bicycle) to another child of the same age. The result that is of interest to us here is that the listener, as Piaget says, "instead of taking the explainer's words at their face value, selects them according to his own interest and distorts them in favor of previously formed conceptions." When the listeners were asked to repeat what they had heard, they gave much better renditions of the mechanical explanations than of the narratives, even though the mechanical explanations were less adequate than the narratives. Piaget believes that the listeners shaped both the mechanical explanations and stories to fit what they know; they made fewer errors on the mechanical explanations because they knew something about the operation of bicycles. The stories were new to them, and so they distorted them beyond recognition.

Piaget describes the children in this experiment as "egocentric," meaning, in part, that they are less respectful of the data. They do not sufficiently distinguish between the thought in their minds and the words that are said to them. Egocentrism wanes, he thinks, around seven or eight years of age. This may be so. But it is also true that, all our lives, we continue to shape what we see and hear to fit what we know. In fact it is this ability to "go beyond the information given," as Bruner (1966) puts it, that enables us to act on the basis of incomplete data. It is an ability that is at the core of intellectual activity, but it is also, as Bruner points out, a major source of error. If the child errs more frequently, it is partly because his expectations are less accurate. *

Beyond fluid expectations, another feature of the young child's thinking contributes to his tendency to distort. He has, as Potter (1966) says, relatively "lax criteria" for recognition and comprehension. Potter notes that her young subjects (5-7) appeared to be equally satisfied with their wrong and their right hypotheses. There was no sudden "click of recognition" when the picture was finally identified, as there was for the older groups. In general, she says, younger subjects seemed to have a less definite boundary between certainty and uncertainty. They also were less likely to express doubt about their judgments. They did not do the sort of cycling back and forth between their judgments and the pictures -- checking their guesses by looking -- that the older subjects did. Similarly, in Piaget's (1959) study, the children as listeners were usually confident that they had understood the incomprehensible narratives of the explainers. The result of these lax criteria is that the young child

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* There is, however, at least one sense in which fitting data to expectations can lead to more errors in adults than children. In Potter's (1966) study, high school and college students tended to cling to the hypotheses they had formed. For example, once they had "seen" a cow, they had trouble seeing anything else, even though no cow was pictured. Young children, on the other hand, switched with abandon from cows to fire-hydrants to umbrellas. They too were governed by their expectations, but their expectations were short-lived and fickle. So, while errors of distortion generally become less frequent with age, there still are many ways in which adults distort data -- in terms of their needs, their values, their "sets" or expectations.

does not feel the need to check and recheck his perception of the data and so eliminate his initial errors.

c. Errors of Improper Addition.

Sometimes the child's error is not in omitting part of the data or distorting it but in importing data which appear to the teacher to be irrelevant. Occasionally, the difficulty is that the child cannot remain within the constraints set by the problem. Among the tests that Margaret Donaldson (1963) administered to children was one in which the task was to assign three boys (Dick, Jack and Jimmy) to each of three schools (Red, White and Blue). In addition to knowing that each boy attends a different school, the children were given the following data:

(1) Dick does not go to Red School.

(2) Jimmy does not go to White School.

(3) Dick does not go to White School.

One little girl, after tussling with this problem for a time, finally announced, "It depends on the district they live in." This child does not yet know the rules governing admissibility of evidence. It can be argued that she does have some notion of relevance; this child does not, for example, bring in random observations pertaining to the color of the walls. The irrelevance of her response is that the little girl is playing an empirical game, while the teacher is playing a logical game. Knowledge about school districts is not irrelevant to the empirical truth about Jack and Dick and Jimmy -- only to the logical truth of the propositions.

Children sometimes focus on details in an array of information that the teacher, in designating the array, is hardly aware of. For example, in the balance beam problem mentioned earlier, some children seized upon a small piece of sandpaper -- glued to the underside of the beam to secure the beam to its base -- as a possible source for the eccentric behavior of the asymetrical beam.

Improper additions to the data are often the result of the child's inability to distinguish relevant from irrelevant information -- to sort the signal from the noise. There are two difficulties here. The first is not limited to children but applies to anyone faced with an unfamiliar situation. It is that the child does not <u>know</u> which aspects are relevant and which are not. To the naive observer, what is relevant is by no means clear. A piece of sandpaper is obviously irrelevant to the problem of balance only if you have some tentative grasp of the variables likely to affect balance. The teacher may inadvertently include distracting details in her presentation of the problem. Precisely because she knows the solution herself and has screened all irrelevant details from her mind, she may fail to screen irrelevancies from her presentation.

The second difficulty is this: even when the child knows which aspects are significant, he may be unable to suppress the irrelevant aspects. As reported by Gibson (1966), Eleanor Maccoby studied children's resistance to distraction by placing earphones upon children and asking them to tell her the contents of a message coming into one ear, while distracting information is played into the other ear. Older children are more able than younger ones to shut out the irrelevant information and give an accurate account of the relevant information. This experimental analogy may seem far-fetched, but it seems likely that a child is frequently in this sort of state in school; the teacher or the book sending "signals" from one direction, while all sorts of distracting "noise" come from many other directions.

Once we assume that the child has understood all the data presented to him, i.e., that no intake errors have occurred, we are ready to move on to the category of organizational errors.

B. Organizational Errors:

Organizational errors occur when the child attempts to organize the data available to him in such a way as to generate the required solution. Organizing data involves at least three processes: isolating certain elements of the data (analysis), combining these elements in new ways (synthesis), and appropriately ordering a series of such operations (sequencing).

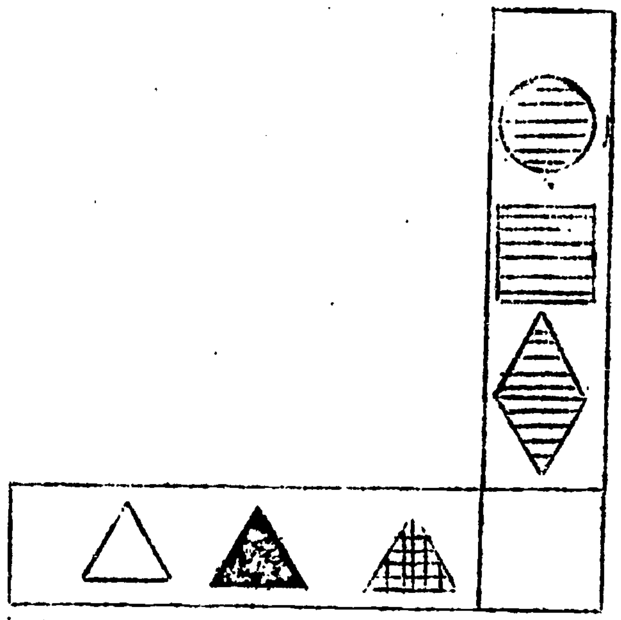
1. Errors in Analysis and Synthesis.

A child can encounter difficulty both in analyzing the properties of an object and in recombining or synthesizing these properties into a new object or concept. The following example used in an experiment by Inhelder and Piaget (1964) illustrates such a task.

Example:

Inhelder and Piaget asked children to fill in the cell marking the intersection between a row and a column of pictures. Below is an illustration of this problem (Figure 1).

Figure 1.



The correct answer -- a striped triangle -- requires the abstraction of the properties of stripeness and triangularity from the objects to which they are attached, and the synthesis of these two attributes. Young children tend to choose an object identical to (not abstracted from) one already presented in the row or column -- a failure both in analysis and synthesis.

Another example of an experimental exploration of this organizational process is presented by Vygotsky in Thought and Language (1962). One of Vygotsky's interests was to see how children, in successive stages of development, analyze, synthesize and otherwise organize the data presented to them. The task, not unlike the intersect problem described above, was to sort a series of blocks into categories which involved a development of a concept combining several elements (tall-large *vs*. tall-small, flat-large *vs*. flat-small). The child's task was to isolate the appropriate attributes, put together the two blocks which would yield the desired dimension, and then sort the array of blocks accordingly.

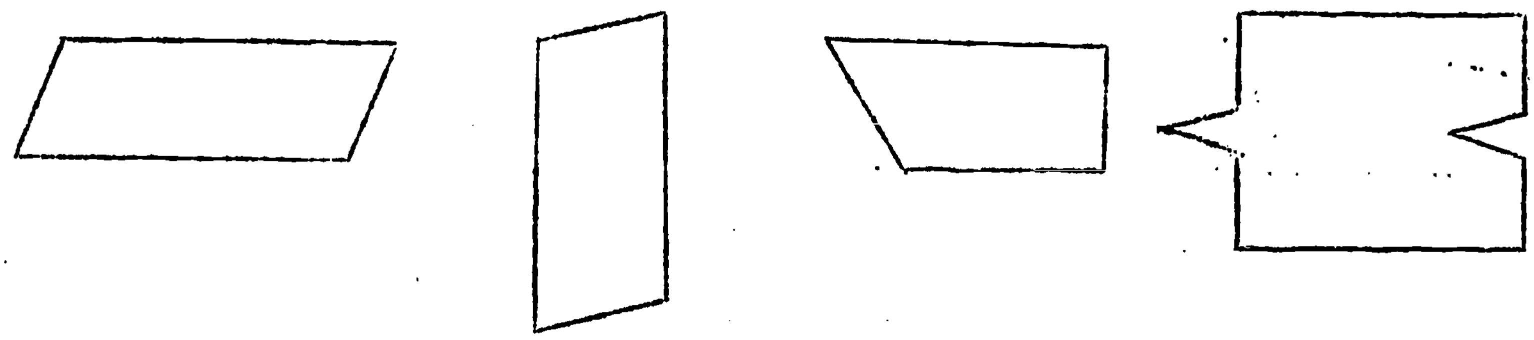
Vygotsky (1962) found that a very young child begins the task by forming "heaps" of blocks, arbitrary groupings with vague and shifting criteria. He concluded that the child was sorting according to subjective or idiosyncratic principles. The older children made a more consistent use of a single, clearly perceptual category, such as color or shape. Of still greater difficulty were the comprehension and application of the less immediate, more complex concepts including combined attributes such as tall-large vs. tall-small. To arrive at correct solutions using combined attributes, the child must be able to abstract the essential elements in the objects (e.g., their size, shape, color) and then synthesize these elements into the correct concepts.

Thus not only must a child be able to isolate the attributes and analyze the problem into its components, but also to decide for each given task which attributes can be ignored and which must be focused on. Transfer of learning involves precisely this ability to consider the crucial attributes in an entirely new setting. We have observed that not enough practice has been provided in the classroom for this analytic task. Children are often told what the relevant data are and provided with formulas which guide them through a sequence of steps in using these data. They are not forced to select independently the relevant aspects of the data nor to decide upon the appropriate sequence of logical operations. Consequently they are deprived of the opportunity to make errors which would clarify the distinction between relevant and irrelevant information.

Holt (1964) recorded an example of how this failure to analyze and synthesize the aspects of the data most relevant to a given problem gives rise to certain common classroom shortcomings. He frequently found that the instant a problem was put on the board a chorus groaned: "I don't get it." What the children meant was that the steps to solution did not spring immediately to mind because the problem could not be automatically fitted into a prescribed formula. They complained they hadn't had that problem, and were lost as to how to go about relating this new problem to the ones they had mastered in the past.

Wertheimer (1959) watched a class in which a teacher enumerated with great precision the steps required in arriving at the area of a parallelogram shaped like the first one in Figure 2 below, given the knowledge of how to find the area of a rectangle: "Drop a vertical here...etc... etc...." The teacher assumed that the children would themselves extrapolate the principle behind these steps when solving problems involving other parallelograms. Thus it was never explained that the purpose of the steps they went through was to transform the parallelogram into a rectangle while preserving its area. After the children had "learned" the method (i.e., memorized the steps), Wertheimer asked them to find the area of other shapes (below).

Figure 2.



Many of the children "didn't get it." They had never had the need, nor the practice, for abstracting the underlying principle.

In addition to failing to discriminate relevant from irrelevant data, few children are skilled in discriminating sufficient from insufficient data. Rarely do young children "know what they don't know" -- i.e., few children know when they do not have all the relevant and necessary information for problem solution.

For example, Lesser (1966) showed five, seven and nine-year-old children a picture of the sun on the horizon, asking if the sun was rising ("coming up") or setting ("going down"). No clues were provided in the picture to identify either choice as correct. Almost all children -- operating on the assumptions that they were given all the necessary and relevant information (don't teachers always do this?) and that the adult must have a correct choice in mind or he would not have asked the question in the first place (don't teachers always do this too?) -- chose one alternative or the other, despite the absence of sufficient information. Very few, even among the older children, gave the only reasonable reply, "I can't tell." Those few who were aware that they did not know enough, however, were then in a position to seek the further information necessary to answer the question.

Some evidence suggests, then, that few children rapidly develop skills in discriminating relevant from irrelevant data or sufficient from insufficient data. These errors of analysis seriously interfere with learning and problem solving across a wide range of content areas.

2. Errors in Sequencing.

Another aspect of organizing information is the integration of information over time. Many problems require the use of a systematic plan for arranging information in a coherent sequence.

This problem of sequential ordering is demonstrated by Mosher (in Bruner et al., 1966) in an experiment modeled after the game of Twenty Questions. Children had to discover the cause of a given event -- for example: an automobile accident -- by asking a limited number of questions answerable by "Yes" or "No." Younger children tended to use a strategy Mosher called "hypothesis scanning." They asked a series of unrelated, highly specific questions, each of which, if answered "Yes," would give them a solution at once; e.g., "Did a bee fly in the window and sting him so that he lost control of the car and went off the road into a ditch?" Older children were more apt to use a more effective "constraint seeking" strategy. They systematically eliminated whole classes of solutions by asking more general questions, and they were more likely to follow one question with another, subordinate one -- e.g., "Is it something to do with the weather?", "Was it raining?"

Thus, the older child analyzes the possible solutions into alternative sets, ordering these sets with some notion of probability of offering a solution. He is then able to ask questions which retain or eliminate a whole set of data at a time, rather than questions which only confirm or disconfirm discrete bits of information.

Of all types of errors, it is probably these organizational errors which interest psychologists most. Psychologists are interested in how children reason, how they organize information in any given situation to arrive at conclusions and solutions. A close observation of those types of errors can be one of the richest sources of clues to the workings of the developing mind.

Teachers, too, could learn much about the operations of the child's mind from observing his methods of organizing data. Suppose, for example, that the child is merely given a parallelogram and told, "You know how to find the area of a rectangle. How would you find the area of this object?" Will he see the similarity and the difference between the two figures? Will he be able to grasp the compensatory relation between a triangle added to one side of the parallelogram and detached from the other? How he comes to such a solution and develops his strategy will give the teacher information about how and when she should intervene in the problem-solving process.

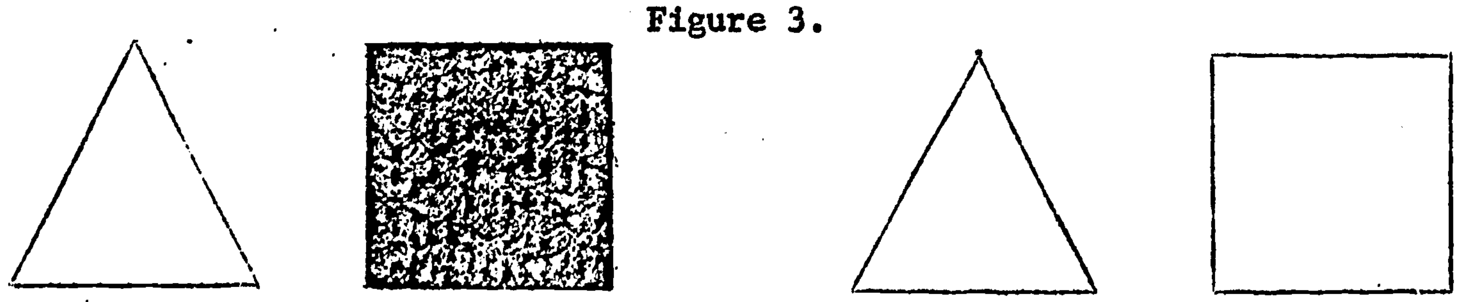
C. Executive Errors.

Once the child has devised (or been given) a strategy for solving a problem, he must execute that strategy. It is errors of execution, we suspect, that are most easily seen and understood in the classroom.

Donaldson (1963) defined executive errors as those which "arise not from any failure to understand how the problem should be tackled, but from some failure in the actual carrying out of the manipulations required." The difference between organizational and executive errors is the difference between the child who does not know how to add and the child who forgets to carry the 2. The executive error which occurred most frequently in Donaldson's (1963) study was "loss of hold on one's own reasoning." Executive errors appear when the child discovers something and then forgets what he has understood.

If we assume that the child has all the given information at hand, that the goal has been understood, and that an effective strategy for solving a problem has been selected, then difference in success between children of different ability will probably be due most often to executive errors. In a study by Huttenlocher (1964), subjects between 6 and 16 years old were shown two figures, each in either black or white, such as those in Figure 3. They are told that one of these figures in one of its colors will turn a light on. If they are shown the figures in A and the light goes on, then shown those in B and the light does not go on, they should conclude that the black square turns the light on. All the subjects seemed to grasp the principle of solution. The younger ones failed more frequently, Huttenlocher reported because they "often failed to take in or retain the instances presented." Typical comments were "I forgot what you showed me first time," or "I was trying to remember what color the triangle was and then I forgot what I just figured out."

Figure 3.



A B

Huttenlocher's (1964) young subjects failed more frequently on problems where many steps were required to arrive at the solution. Executive errors seem to increase with the length of the sequence of reasoning required. The longer the inferential chain, the more likely the child is to lose hold. This suggests that a teacher might be able to diagnose whether a child's difficulty is executive or organizational by giving him a problem similar in structure to the one he has failed to solve, but requiring a shorter sequence of steps.

Executive errors -- being most clearly observable in the classroom -- are often overstressed. From the point of view of our interest in cognitive development of the child, they seem trivial compared to other types of errors, since they reveal little about the mental operations of the child. It is the distinction between organizational and executive errors that is important for the teacher to keep in mind. If the teacher treats an executive error as though it were an organizational one, lecturing the child on the principles involved in long division, when his difficulty is just an executive error in addition, she wastes the child's time and her own. Conversely, if she treats an organizational error as an executive one, the consequences are more severe: the child may learn to execute the steps in a geometrical proof with exactness (as in the case of the parallelogram) but may have no idea of the rationale behind the steps.

Although it is important to recognize the distinction between executive and organizational errors, it is also important to see the relationship between the two. An error in organization may lead to an error in execution.

It often happens that a child makes executive errors not so much because he lacks executive ability, but because the method he is using is so cumbersome that even an Einstein could not execute it without error. For example, a strategy of hypothesis scanning in the Twenty Questions game is likely to lead to loss of hold, because the child is forced to hold in mind a large number of unrelated outcomes. The constraint-seeking child integrates the information he acquires; he need remember only a few inferences derived from the integration and can shed data as he goes. Once he has ascertained, say, that the cause of the automobile accident had something to do with the weather, he goes on to ascertain that it was raining when the accident occurred. At this point, he can forget the outcome of his previous, more general question. If your attack on a problem is orderly, rather than haphazard, it is much easier to keep track of where you are and where you've been.

In fact, it is analyzing and synthesizing that makes it possible to retain information. We "chunk" or recode the data in more economical form, reducing its bulk while preserving the information it contains. Suppose, for example, that the task is to memorize the following set of digits in the sequence in which they are given:

91215172124

The job is close to impossible if each digit is seen as discrete. One must store and retrieve a sequence of eleven separate pieces of information. But if one discovers the rule governing the arrangement of the digits, and organizes the sequence as:

9 12 15 18 21 24

it is necessary to remember only the rule and the initial and final digits in order to reconstruct the list. One effect of a good method is to reduce cognitive strain; it makes the problem manageable.

On the other hand, a child may fail to organize material in the most efficient fashion for fear of committing executive errors. That is, he may recognize that one method is superior to another, but because he is more adept at the inferior method -- perhaps he has practiced it more -- he sees it as a sure, even though slow, route to success. Thus, he may continue laboriously to count on his fingers when given a problem in multiplication, rather than relying upon his still shaky grasp of the multiplication tables. The time spent in solution is longer when counting on the fingers, but the cognitive strain is perhaps less. Similarly, a child may continue to ask specific questions in the Twenty Questions game although he recognizes that general questions are more appropriate. The constraint-seeking strategy requires planning and so imposes cognitive strain on the child at the start, even though it lessens the burden of memory later. The child may distrust his ability to construct a sequence of steps and so may hesitate to employ such a strategy. If the teacher puts the premium on successful solutions and errorless execution of tasks, the child may be driven into more cumbersome methods than he is capable of.

We shall see more examples of the configuration of errors in the next section, for it is this configurational quality that makes it difficult both to diagnose and to treat the error.

III.

The Teacher's Response to Children's Errors:

We cannot emphasize enough the importance for classroom practice of attempting to decipher and understand the child's processing of information. This understanding is essential as a guide to several educational activities: the preparation of materials for classroom use, the identification of levels of achievement for individual children and the development of training programs most appropriate to the specific deficits responsible for a child's failure.

In the usual classroom situation a child is rewarded for the speed with which he can deliver a correct response. As we shall see later, a child's attempt to use a more complicated and mature, though perhaps less practiced, strategy mitigates against his success; he gives answers more slowly and less accurately and receives less reinforcement for his behavior. Yet it is this child who is moving ahead of his peers and needs to be encouraged in his progress. If the teacher understands the process underlying his behavior she can let him know that his incorrect or delayed answer is of lesser importance than the appropriateness of the sequence of reasoning steps he is developing. Thus, when a child appears to be falling behind, the teacher must be able to discriminate among the several sources of possible error.

Example:

The nature of the error may indicate either that the child is actually moving ahead of his peers or that he is falling behind in a specific area requiring specific remedial training. Obviously, the distinction is crucial. When a child who has been able to do simple addition becomes lost because the complexity of the numbers increases, he is usually considered not ready for the advance and made to continue practicing the simple problems. His error might stem from an inability to perform the necessary analysis of numbers into their component units (analytic error, page 24) or a failure to hold a series of numbers in his memory (executive error, page 30).

If analytic errors are involved, the most useful training might emphasize skills of number analysis; for executive errors, training of memory span might help. In either case, sending the child back to do more problems in simple addition makes the special practice he really needs only minimally or accidently available.

We shall discuss here how the specific errors outlined in Section II may be observed and handled in the classroom.

A. Intake Errors:

1. Errors in Perceiving the Goal.

The teacher must be alert to errors in the child's conception of the goals of the task facing him. A child's performance often will suffer because he has not grasped what it is the teacher expects him to do. Obvious classroom occasions for such errors are when the goals expressed explicitly by the teacher are different from the goals understood by the child. More subtle occasions arise when both the goals expressed explicitly by the teacher and those understood by the child are different from the implicit goals by which the teacher evaluates the child's performance.

Such misunderstandings can be illustrated by again drawing upon the experience in the science class previously mentioned (page 13).

Example:

In a recent summer session of a science class for children from "culturally-restricted" background, the children were handed boxes containing a meal worm and a few other objects and told that at the end of the session they would be asked to give as many facts about the mealworm as they were able to discover by themselves. Some children immediately poked, handled and otherwise actively observed the mealworm; others, simply sat glancing listlessly about the room and half-heartedly stared at their worm. The latter, of course, had little to say when their turn came to describe their discoveries. One might have assumed from the observation of these children, that the latter group was less intelligent, creative, or at least less interested. On subsequent questioning, however, it became clear that some of the less creative children, coming as they did from a more restricted background, were not sure that it was indeed permissible for them to handle the mealworm and otherwise use the materials freely. Insecure about what behavior was permissible, they merely sat pondering the answers which they would be expected to give, desperately searching for possible cues to success.

The implicit goals of the teacher could have been stated as follows: "Ask yourself as many questions as possible about the mealworm and perform experiments designed to answer these questions; then have the answers to your questions ready to share with your class." Stating the goal explicitly allows the children to see their active examination of the worm as very necessary to the assigned task and, if unsure about the rules, to question the teacher about them. As it happened, some children never understood what it was that they were expected to do.

Example:

A similar error in perceiving the goal of the task appears in the usual teaching of arithmetic. A child is given a series of examples to solve which are so arranged that they illustrate

> some principles of equivalence or commutability (e.g., 2 x 3 = $\square$, 3 x 2 = $\square$; 7 x 5 = $\square$, 5 x 7 = $\square$). Understanding this principle is the implicit goal of the exercise. The explicit and more immediate goal, however, as the child often understands it, is simply providing the correct answer for each question in turn. Each question is seen as a distinct unit, and if the child is intent enough on providing each in the series of correct answers, he may never perceive the pattern of these exercises. Although the purpose is to teach a general principle, the pupil is expected to deduce that principle for himself. Supplying correct solutions to individual problems actually may distract the child from perceiving the true goal of the task: understanding the principle of commutability.

This example also illustrates again that while error is a good indication that something has gone wrong, a correct answer alone is not a sufficient assurance of correct processing. When a pupil acts as if he is following a rule which we are trying to teach him, we must still investigate further in more direct ways whether this rule is indeed operating in his thinking.

The common school exercise of writing essays provides a further example of error in the conception of the goal of the task.

> Example:
>
> A child may be assigned a topic designed to draw on the events around him and his personal experience, e.g., "Our Town" or "My Summer Vacation." If he assumes that the teacher wants his essay to contain a large number of facts or hold the reader's interest with important detail, the pupil will try to satisfy these criteria. The teacher's implicit interest, however, is in the student's ability to organize his ideas and express them coherently. She may brand as stilted or unoriginal the work of students who have thus misdirected their efforts. In instances when some or most of a group of students meet the teacher's implicit goal, she is likely to assume that those who do not are unable to do so. Instead, they have misperceived her criteria for success.

Thus, classroom failures may result from children's general and often correct perception that the teacher has some unrevealed criteria which they must try to guess. Much of the child's creative energy goes toward discovering what the real goal might be: by peeping over the shoulder of the acknowledged best pupil, by copying from recognized authorities, or by asking the teacher "leading" though often irrelevant questions, instead of concentrating on the task at hand. The brighter child is not only better able to perform the task, but he is likely to be better able to "psych out" the teacher particularly if he shares the teacher's background and attitudes. The slower student, then finds himself at a double disadvantage: not only is it more difficult for him to complete the assignment once he has understood its goal, but it is also more difficult for him to guess the teacher's implicit goals. We all have seen this kind of child, a hard worker who somehow always misses the point and shakes his head, saying, "I just don't get what she wants."

This analysis suggests that a sort of guessing game is played in the classroom. Some might argue that this is as it should be. The ability to make these inferences about the teacher's real goals may be regarded as a measure of intelligence, and making explicit what is expected of the child might blur the differences among pupils. In fact, intelligence test items (e.g., "circle the object in this array which does not belong with the others") measure this same ability to infer rules. If this argument is indeed true, and if success in school depends partly on this ability to guess the real expectations of the teacher, it is no wonder that children who share the teacher's social background excel at this game.

We argue, however, that the function of the school is to instruct the child rather than to test him (testing being only a step toward better instruction), and if the ability to infer rules of behavior or logic is an important functional skill, it should be taught directly in the beginning school years. The child must not be prevented from performing adequately simply by a lack of understanding of the goals toward which he is expected to work, thus giving the appearance of lacking ability.

There are several ways in which a teacher may help to bring such errors into the open. One is simply to question the child closely on what the latter assumes to be the purpose of the assignment, by asking how he plans to go about fulfilling his assignment, and why. Still another technique for diagnosing intake errors might be to have the pupil play the role of the teacher: One pupil might be asked to explain the goals of the assignment to his classmates and the teacher would have the opportunity to observe what _he_ considers to be the important elements of this explanation. Or a pupil might grade the efforts of others, not merely by checking the right answers provided by the teacher, but by evaluating essays, creative endeavors and ways of attacking problems. The pupil is thus forced to apply the criteria related to his understanding of the goals of the task, to make qualitative judgments and to defend them on the basis of his implicit understanding of the task. If the information gathered by the teacher in these or other ways indicates that the child's understanding differs widely from her own, the teacher must make an effort to be especially clear and explicit in informing the children of her real goals.

We do not intend to imply that whenever a child misperceives the goals it is because he has been misinformed by the teacher. In most cases, the child has distorted or ignored the stated goals for idiosyncratic reasons.

No matter how clearly the goals are stated, the child may be unable to comprehend them or be confused by inappropriate transfer from other situations. Conflicting goals may also interfere with a child's performance. For example, it might be difficult for a child who has always been praised for his fastidiousness to become as free and expressive with finger paints as his art teacher might wish. An understanding teacher would try to make the child see the specificity of the situation, indicating that his behavior in the art class has a special and distinct purpose and does not conflict with desirable behavior elsewhere.

2. Errors in Perceiving the Data.

In addition to understanding the goals, the child must have the information needed to solve a problem -- and here too, errors are frequent. In school the child learns to expect that no questions will be asked of him unless he is presumed to have means to answer: this means that he does not need to wonder whether the information he needs for a particular solution is at his disposal. Woe to the substitute teacher who poses an unknown or unrecognizable problem to the class. The cries of "we haven't had this" or "we're not supposed to know this," are not only justifications for not attempting a solution, they are cries of injustice and betrayal. This expectation of always being on safe and familiar grounds also serves to make the child feel that if he can not answer an assigned problem the fault is his alone, each failure reflecting adversely on his ability and unrelated to the sufficiency of the data available to him. Then, when some of the necessary data are in fact absent, he does not conduct the kind of scanning or inquiry which would tell him what is missing.

It seems likely that children could profit from specific training in the recognition of redundancy, irrelevance and insufficiency of information. For example, the child could be given a question to answer, along with passages containing a body of data. They would then be asked to eliminate what they do not need to answer the question and to state what further information is necessary.

One teacher used an ingenious technique in order to disabuse her fourth grade students of the notion that they had all the requisite information at their command. *

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* The question posed to the children was: who can get the coldest cup of water using the least amount of ice? Shortly, all children arrived at a temperature of 32°, using one ice cube and thermometer. Meanwhile, the teacher had unobtrusively added salt to her cup of water and compared her reading of 28° to that of the class. The children then proceeded to add a second ice cube to their cups but still could not lower their temperature below 32°. With the addition of more salt, the teacher's reading became 24°. At this point, the students personally wanted to check her reading and substitute their own thermometer. Upon confirming her reading, they were completely stumped. The class neither asked questions nor did they offer ideas in an attempt to account for the difference. In response, the teacher suggested that the students were not investigating the difference in results with sufficient care. She asked them to think about the possible introduction of new or alternate materials and procedures. In other words, they could have been more thorough as "detectives" and discovered the fact that she had added an additional substance, namely salt.

Even after this experience, the children continued to expect that all necessary data would be presented by the teacher. In a subsequent lesson, the class had established the fact that ice was lighter than water through observations of ice floating on water. Consequently, the teacher dropped an ice cube into a jar of clear liquid whereupon it immediately sank to the bottom. When asked to account for this discrepancy, the children proceeded to act as though all the necessary data had been given and thus offerred unreasonable explanations such as, "You threw it so hard, it stuck to the bottom," or "The pressure of the water held it down." Finally, one student asked whether or not the teacher had added salt again. Though incorrect, this particular question initiated a line of questioning by the children which eventually led to their questioning the sufficiency of the data provided.

The younger the child, the less experience he has with actual selection of data. Even when all that needs to be known has been presented to him by the teacher, he may disregard some of the information, include other irrelevant facts, or change some data in idiosyncratic ways.

It is difficult for the teacher to know through examining the child's answers whether he is omitting information, distorting it or adding what he regards as relevant to the problem at hand.

When data are distorted, it often appears as a simple misperception on the child's part -- he looks at an addition sign and thinks it's multiplication or vice versa. These misperceptions, however, often are not random. What may appear as an error due to inattention is in fact due to an incorrect expectation by a child; expectation, when correct, helps the child to get on with the task more quickly but at the same time allows him to relax his attention to details since he already knows what they are. This means that a multiplication problem placed in the midst of additions is much more likely to be treated, incorrectly, as an addition by a child who can both add and multiply with little difficulty. Here, again, the distortion is reinforced by the school practice which usually labels clearly for the child what it is he is doing and when the task is about to change. Since there ordinarily is little need or opportunity to practice checking his expectations, there is little need for the child to practice attending to details or staying alert.

At this point, a teacher may justly object that we have posed a double-edged problem: on the one hand we argued for a clear and explicit presentation in order to reduce intake errors: on the other hand, we now argue that such a presentation does not give the child necessary opportunity to practice certain types of attention. Though on the surface these two positions seem incompatible, we believe (and we will discuss this in greater detail later) that the teacher at different times might wish either to minimize or maximize (invite) the possibility of error. The choice of either procedure will be based on her understanding of what is transpiring in the classroom and on her evaluation of which strategy, at a given time, will be most advantageous for providing both teacher and pupil with maximum insight into the pupil's thought process.

Distortions of the data may arise from specific expectations that a child has in a specific situation or from expectations arising from his past experiences or lack of experience. The younger the child, the more likely he is to be tied to the most immediate and familiar experiences. When given information about dogs, for example, he is most likely to remember the information which fits his own dog and most likely forget the information which does not fit with his earlier observations of dogs.

A certain amount of redundancy or correspondence between school learning and the child's outside experience will serve to reinforce or fix the learning. Thus, it is important that the teacher recognize which data fit within some framework the child already possesses and which data present him with an entirely new set of experiences. Entirely new experiences may need focusing and repetition in order to assure that they are not quickly lost to the child and omitted from his subsequent reasonings.

Improper additions to the data are also likely to be the result of attempting to fit new information into past experiences. In her book about children's thinking, Donaldson (1963) cited instances where a child was asked to evaluate the incorrect syllogism: Tom is a good boxer, Tom is a red head, therefore all good boxers are red heads. Children may recognize this as a fallacy on the empirical grounds that they know someone who is not a red head and is still a good boxer rather than recognizing that the true issue is the analysis of the logical fallacy.

Finally, any type of distortion of data might be maintained by a child in spite of the incongruity which would be apparent to an adult. In other words, the child's criteria for what obviously does fit or doesn't fit with the rest of his knowledge are far more lax than those of an adult. The criteria must be lax to allow for that accelerated expansion of his very limited world which is learning. There are many stories like the proverbial one of the mis-heard Psalm: "Gladly the cross-eyed Bear" where the child goes on for a long time without being disabused of his distortion.

The fact that the teacher known that she has presented all the data to the children is not adequate proof that they have actually perceived these data in their intended form. The teacher must be cognizant of the possibility of distoritons, omissions and additions, and must institute some procedure by which she can obtain adequate feedback from the children to assure herself that they are in fact in the possession of necessary information.

B. Organizational Errors:

1. Analysis and Synthesis.

Once it can be assumed safely that no errors in the intake of data have occurred, the child must proceed to use these data to arrive at a solution to the problem. The data must be ordered and combined. An inability to see certain data as relevant to the problem will make the child disregard these data and seek clues to the solution in unexpected areas. This again if often a function of the young child's personal view of experience. Information he "knows" intellectually may never be really incorporated into that personal world.

> Example:
> We have recently talked to an eight-year old who was avidly reading about ancient Greece and had good understanding of history in general. We asked him what he thought was the oldest city in the world and with little hesitation he answered "Boston." When asked why he thought that, he justified it on the basis of its many landmarks from the Revolutionary War - the beginning of American history. When the connection between his knowledge of ancient civilization and the original question was pointed out to him, his reaction was not an immediate "aha!" as might be expected; instead he understood that his answer was wrong and tried to guess what the desired answer might be. It was clear that the connection, even though pointed out, was not enrirely understood. The ancient world he knew about was part of a quasi-fictional world; he had been told that it had really happened, but this information obviously lacked the kind of personal immediacy which would place it on a continuum of time with his own home town.

This is not an isolated example. A group developing social science curricula had very similar experiences. After the pupils completed a program where they mastered facts designed to give them an historical perspective, they still gave their home town, or events of immediate relevance to themselves, when questioned about historically important places and events.

In New York, for example, "New York City" was the most common answer given to the question about the oldest city by children who had learned about both American and world history.

If we are aware that this kind of personal interpretation of data is the rule for the young child, we must be careful not to rely on him to make the connections which seem obvious to an adult, but must either explain these connections to him carefully or structure the instruction so that these demands are not left entirely to the child.

Thus, the teacher, who has given all the relevant information or knows that the children possess these facts, is not assured that the children actually are considering the facts. Often, under the pressure of answering in class, a child will desperately try to make connections or remember cues, sometimes going further and further away from the problem. We describe this kind of behavior as "guessing" but guesses are not entirely arbitrary but are a function of some body of information that the child considers most relevant for dealing with the problem.

2. Sequencing.

Our analysis of errors might be regarded with reservations by the practitioner for its apparent oversimplification. Any attempt at close analysis of behavior and cognitive process gives this appearance. Error in its behavioral context can rarely be seen so clearly and in such isolation. As mentioned earlier, the child's everyday behavior is more likely to present us with a configuration of errors lending themselves to a variety of interpretations.

Errors in the intake of data are often difficult to distinguish from errors in organizing data. Indeed the two are closely related. Obviously the content of the information will influence the decisions the child will make about organizing it in solving a problem.

<u>Example</u>:

In multiplying 9 x 4, a child who knows only simple addition will laboriously add 4 nine times; a more sophisticated child who knows about commutation might instead add 9 four times -- a somewhat shortened task; someone with more extensive understanding of the process of multiplication may multiply 4 by 10 and subtract 4 -- applying perhaps a general rule about multiplying by nine; and finally, one whose store of data includes the answer "36" merely supplies it from memory without intervening mental operations.

Thus, the kind of information most readily available to the individual will influence the way he organizes it. Conversely, the way a child analyzes the problem will influence the choice of data he considers relevant. Simple examples of this reciprocal relationship between the characteristics of the data and problem-solving process are assignments in which students are asked to compare two events or time spans in history. The dimensions on which the events are compared will determine the facts examined, and the information most obvious to the students about the two events will influence the grounds for comparison. Success depends upon close matching of the method of attacking the problem with the available data.

Bringing in extraneous data as a shortcut to a solution often detracts from the child's development of analytic skills in dealing with school problems. A child may become skillful in "reading" the teacher, a technique facilitated by certain types of teaching methods. He learns to respond to subtle gestures or voice inflections which tell him what is in the teacher's mind. If a child is successful with this technique, he can ignore all the structural intricacies of the problem which might provide him with a solution. He has thus found a shortcut to getting the right answer. Although his class performance is satisfactory, he is not practicing important cognitive skills and may find himself suddenly left behind when the teacher alters her methods or when such cues are not available.

Errors in processing of information -- failures to correctly analyze, synthesize or order the material -- are often seen as failures to understand a particular subject matter and the remedial training provided consists of reviewing some more simplified information about the subject which the child must master. This technique fails to come to grips with the real source of the error: incorrect analysis, synthesis, or organizing. Once this source is identified, remedial training can be directed to the specific processing skill and would cut across subject matter. We expect that a child so treated would make intellectual gains across a wide range of school endeavors. The specific content of this training could be chosen in an area of great interest to a particular child capitalizing on the strengths and involvements he already has. Thus, a child who is doing poorly in history and whose failure is traced to his inability to analyze could be taught analytic skills through his favorite subject (e.g., English or arithmetic), and it could be assumed that these skills would transfer to his handling of historical material.

C. Executive Errors.

We have already discussed executive errors which occur when the child presumably knows what his goals are and understands how to proceed, but cannot yet do so smoothly. He becomes confused by a long list of numbers, loses a step in a sequence he has planned, or forgets to use the outcome of his own reasoning. Since no incorrect processing has occurred, these errors require no explanation by the teacher other than simply pointing them out. The child is capable of eliminating these errors with more practice, time, or attention.

Whereas a child who is caught in an intake or organizational error may need redirection, the one whose error is executive needs only to be reassured that he is proceeding correctly and encouraged to try again. A possibility of distinguishing among different types of errors allows the teacher to distinguish the child who is forging ahead but needs practice in using his newly developed abilities from the child who is left behind because he has not yet grasped some essential rules of procedure. Both children may fail to come up with the correct solution, but each needs to practice something different. Obviously, indiscriminate ignoring of or reprimanding for these executive errors, no matter how subtle, is of little educational value to the pupils. It might have the detrimental effect of discouraging a child from using a more advanced, and therefore harder to execute, strategy which in the long run would accelerate his development. Concerned only with being correct, the child will use the safest and least challenging way of obtaining his answers.

IV.

Errors, Education and Growth

A. Errors and Teaching Method:

We hope that the identification of specific errors will give the teacher insight not only into the thought processes of an individual child, but also into the value of her methods of presentation. An abundance of errors of a particular type might call for a restructuring of material to eliminate the source of this failure. Material which is appropriate for one age group may be a source of error for children of a different age and appropriate restructuring may allow the maximum amount of information to be transmitted to both groups.

Our commitment to an analysis of errors entails a commitment to teaching methods in which the emphasis is shifted from correct answers to the process by which the children arrived at these answers. Many children we have observed are not at all interested in explanations. The challenge is to get the right answer. Once they have gotten it, by whatever means, they are not willing to stop and examine it. They are anxious to get on to the next question or to "get off the hook." Thus, the child sees school as a place where he is being continually tested rather than a place where he comes to learn. If reflecting upon the answers, the 'how' and 'why' of each solution, could become the main activity in the classroom, the children would soon become involved in such a procedure.

Anxiety over the product of their reasoning causes many children to find ways to shortcut or circumvent the very procedure which the teacher is trying to teach, for the sake of arriving at their answer quickly. The bright impulsive child is able to use shortcuts successfully while his less intelligent counterpart is more often guessing desperately. A good guess still gets him ahead of the slow reasoners. This temptation to shortcut would be eliminated if the description of the process itself became the main task of the classroom. The children would be free to experiment with a variety of ways of dealing with problems, some less immediately efficient but perhaps opening new avenues of thought. The resultant awareness of the thinking processes, and freedom to explore divergent forms, would result in producing more thoughtful and perhaps more creative individuals. The child's use of alternative ways of reaching solutions and of dealing with problems would present the teacher with much information about the range of the children's abilities. This information is the teacher's most effective guide to her own organization and presentation of classroom materials.

B. Inviting Errors:

Reducing the errors of children usually is considered to be the legitimate function of the teacher. Our present analysis, however, suggests that at times the teacher may choose to maximize or invite error.

One way of exposing the error which the teacher suspects is at the root of a child's inability to perform is to structure the material in such a way that the possibility of the error is maximized, thus helping the child to discover for himself the source of his difficulty. An alternative approach minimizes the possibility of error, thereby helping the child over the hurdle. If such changes in presentation give the expected increase or decrease of errors we can assume that the processing weakness has been correctly pinpointed and the teacher can address herself directly to it. Whether the teacher in fact chooses to maximize or minimize the possibility of the error will in turn depend on her evaluation of the difficulty of the problem for the child in question.

A teacher who is sensitive to the types of errors that her students are likely to commit would begin to have clearer expectations for her students. That is, she distinguishes when a certain type of error indicated that a child was lagging behind his peers and required remedial attention from an instance in which the child faltered because he had forged ahead to a new and yet unpracticed strategy. In the latter case, she would ignore the error and encourage the child to practice his new processing skills. Similarly, errors may appear under stress of classroom competition or a test situation in a child whose performance is better under more relaxed circumstances. Here, handling the stress reaction seems more appropriate than treating the child as if his failure were in the realm of the subject matter. Although it may be true that an overpracticed response will not be quite as vulnerable to stress, there is some question as to the usefulness of this kind of overlearning, particularly when the time comes to abandon it for a more advanced procedure.

C. Errors and Developmental Stage:

Obviously differences in developmental stage affect the frequency of children's errors. There is a difference both in what is expected of a child and what the child expects of himself at various points of his development.

A younger child may not be expected to perform certain operations or understand certain relationships and thus his ability to do so never comes into question. On the other hand, as was indicated earlier an older child may fail on a simple task because he expects a greater complexity and structures the task accordingly. Viewed this way, error may be one way of determining the point of development for a child. If we decide that a child has not reached a point in development when he can be expected to perform certain necessary operations, we might provide him with aids designed to compensate for the specific shortcomings.

> Example:
>
> If a child of certain age and experience is likely to lose hold of all the steps he must take to reach a solution to a problem even though he knows well what these steps are, we might provide him with a notational system which will allow him to keep track of the outcome of his reasoning and relieve him of relying on his, as yet weak, short term memory. At the same time, however, we might want to provide means by which he could train his memory so that eventually he would not need the aids.

Not enough systematic analysis of error has been conducted either experimentally or in the classroom to provide us with cues as to their specific developmental source. In other words, we cannot as yet tell which type of error indicates that a child is at a higher (or lower) level of development than his peers who do not commit this error. We can assume that executive errors are a function of experience and thus indirectly age, and that a child who can handle a complex series or sequences, can also handle simpler ones. Errors dealing with perception of goals and data appear to have some relation to development: younger children are more distractable, less attentive, and more prone to an egocentric view of the world around them. Increased knowledge of developmental trends would provide insight about the relative developmental standing of individual children and allow us to alter our instruction, as well as our demands and expectations, for individual pupils.

V.

Conclusion

How children think is and should continue to be one of the major concerns of the teacher. Thinking here is understood to be intake and processing of information toward some goal of problem solving. Understanding children's thinking is necessary both because teaching children how to think is an important part of their schooling and because teaching any subject matter depends on the intermediate processing skills. At present, training to perform in some subject area and training to use intermediate processing skills are often undifferentiated; specific performance is emphasized and processing skills are expected to be learned along the way. We argue that, since processing skills are basic to educational success for each pupil, they deserve the major portion of the teacher's direct attention. The analysis of errors is a useful way, if not the only one, of viewing the elements of the process and refocusing instruction to produce thoughtful and reflective behavior.

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