Jabberwocky: The Complexities Of Mathematical English

There is no doubt that some students find the language of mathematics dense and difficult to understand. **MERILYN CARTER** and **LORNA QUINNELL** explore the complexities of mathematical language and offer some useful suggestions for helping children make sense out the mathematical text.

'Twas brillig, and the slithy toves
Did gyre and gimble in the wabe;
All mimsy were the borogoves,
And the mome raths outgrabe.
Lewis Carroll (Jabberwocky, 1871)

For some students, mathematical English appears as unintelligible as this verse by Lewis Carroll (an English cleric who was a mathematician in his spare time). Students find it hard to interpret mathematical problem texts. Ability in basic interpersonal communication does not necessarily result in proficiency in the use of mathematical English. It is important that teachers understand the challenges of mathematical English because it is on the basis of these understandings that students can be helped.

Mathematics is a unique language with its own symbols (grapho-phonics), vocabulary (lexicon), grammar (syntax), semantics and literature (Bechervaise, 1992; Sharma, 1981). As in any other language, to make meaning of the text, the student must learn: signs and symbols (for example: ÷, ×, ≠); lexicon (for example, coefficient, square, similar); syntax (for example, multiplication precedes addition or the meaning of the absence of a symbol); and semantics (for example, variables in some situations are likely to be rational, whereas in other situations are likely to be irrational). Moreover, mathematics is a creole language, that is, it is a hybrid of English and mathematical language and, as in many creole languages, some words and symbols have a redefined status (Bechervaise, 1992).

According to Hater, Kane and Byrne (1974), "reading mathematics includes the reading of words and symbols with differing relations and orders that are placed sometimes in a line, but at other times in charts, graphs, pictures, or algorithms" (p. 662). So, when the normal burden of reading is complicated by the language, symbols, tables, charts and diagrams of mathematics, it is little wonder that students find it challenging.

Lexical features of mathematical English

Cardinal numbers are adjectives when used in ordinary English (*three students*), but nouns when used mathematically (*the answer is four*) (Munro, 1979). Pronunciation of the numbers thirteen to nineteen are very similar to *thirty, forty*… to *ninety*, causing problems, especially to children new to English or with hearing loss. Whilst the English words for whole numbers generally follow the decimal system, *eleven* through to *nineteen* are exceptions. In the cases of the numbers 13 to 19, the order in which the symbols are read does not follow the normal pattern of reading from left to right (Park, 2003; Perso, 2005). For instance, when children read the word *thirteen* they may be tempted to write 3 first.

The naming of very large numbers is also confusing. A large number with seven digits such as 1 000 000 is called a *million*, but *milli-* is also used as a prefix for a small measurement with three decimal places. The prefixes *bi-*, *tri-*, etc. appear to have little to do with the size of the numbers described as billion, trillion, etc.

Ordinal numbers are generally formed by adding a suffix of -*th* to the word for the cardinal number, but there are exceptions. If the numeral ends in a 1, 2, 3 or 5, the ordinal number becomes *first, second, third* or *fifth* (respectively): e.g., *thirty-first, fifty-second, ninety-fifth*. In the cardinal number system, 2 is higher than 1, but in the ordinal number system, first is higher than second. A third form of numbers is nominal numbers where the number is used as a label or code (for instance, the number on a football jersey or the bar code on an item for sale).

When moving beyond whole numbers, things become more complicated. In the case of common fractions, the words used to describe numerators follow the pattern of cardinal numbers. However, when describing denominators, the words follow the pattern of ordinal numbers, but with exceptions for the words *half* (plural *halves*) to describe a denominator of two and *quarter* for the denominator four. Finally, there is an alternative system of describing fractions in words such as "five over six." With these complexities of verbalising the symbols, it is not surprising that many students find fractions difficult.

In the case of decimals, the system used to describe the whole number part is not used to describe the decimal part, although the numerals are written in the same way. Any numerals written after the decimal point are described using the names of the single digit numbers, that is *zero* through to *nine*. So, the number 345.678 is described in words as "three hundred and forty-five point six seven eight." Further, the word *oh* may be used instead of the word *zero*, so 4.03 could be described as "four point zero three" or "four point oh three." Failure to verbalise decimals correctly can be linked to a misunderstanding of place value.

Finally, when describing ratios, students are presented with many choices. One option is to describe a ratio in fractional form. However a ratio of 3:5 can also be described as "three is to five," "three in five" or "three to five" (understanding that the latter form is not a time).

Other words can be used to indicate number. Examples are *pair* (implying two), *dozen* (implying 12), *initially* (implying zero) and *alone* (implying one). Prefixes can also show number, in words such as *century*, *tetrahedron, pentominoes, bilateral.* On the other hand, although *none* or *no* can both be thought of as meaning *zero,* there can be subtle differences—consider the difference between *no result* and a *zero result.*

The mathematical vocabulary has three components. First, it includes many ordinary English words such as *above, more, profit, dollar,* and *increase* (Munro, 1979; Newman, 1983). Mathematics teachers must check that students understand and can correctly use these words. There is a second group of words where ordinary English words change meaning, including *variable, similar, square, power, rational,* and *equality* (Pierce & Fontaine, 2009). Students need help to understand the contexts in which the meaning of these words change. Finally, there are words that have meaning only in mathematics, for example, *rectangle, coefficient, per cent, median, hectares, binomial, denominator,* and *vinculum* (Pierce & Fontaine, 2009). Further, some of these mathematical words such as *square, scale, range, polygon* have different meanings in different areas of mathematics (Spanos, Rhodes, Dale & Crandall, 1988). Mathematical homonyms such as *two/too/to, sum/some, pi/pie, sign/sine, y/why* (Durkin & Shire, 1991) can also create confusion.

The meaning of some words can be considered from several perspectives. An example is the word *more.* It can be part of an expression requiring addition (*3 more than 5*), subtraction (*How many more is 5 than 3?*), multiplication (*3 times more than 5*), or even division (*How many times more than 3 is 15?*). "*More*" can be part of an inequality such as, "5 is more than 3," or a synonym for *extra* or *again,* for example, "some more cake." Left is another confusing word. Consider the following problem:

> When John left home he had five dollars in his left hand. The bus fare cost him three dollars. How much did he have left?

It has been suggested that some students subtract every time they see the word *left,* regardless of its context. There are a very large number of synonyms for the arithmetic operations (see attached table) (Rothman & Cohen, 1989; Spanos et al., 1988). Students need to understand the meaning of all of these words. However, care must be exercised. The use of words such as "how many," "how much" or "how many times" can require the inverse operation.

Negation or the opposite can be implied by the use of several words, including *not, never, complement, converse, all but,* and also by a host of prefixes including *ir-, un-, a-,* and *anti-* (Mestre, 1988; Saxe, 1988). One area in which mathematics and English agree is that the use of a double negative implies the positive. Other words such as *barely, just, merely, scarcely,* or *seldom* may have the effect of diminishing the impact of the remainder of the sentence.

Mathematical English has inherited a great many words from Latin and Classical Greek. Many of these words have retained their original plurals forms (e.g., *radius/radii, datum/data, axis/axes, index/indices, polyhedron/polyhedra*). In some cases the anglicised form, taking on the normal -s ending, has become an acceptable alternative (e.g., *formula/formulae/formulas*). These plurals require explicit teaching.

Mathematics can create many words from the one stem, such as *divide, division, divisible, indivisible* (Newman, 1983). Students should consider both the similarities and differences in meaning of such words, and the length of the word. Mathematics can also use lengthy and sometimes unusual strings of words to convey a single meaning, for instance, *lowest common denominator, simple interest* (Spanos et al., 1988). On the other hand, complex meanings can be concealed by apparently simple words such as *mean* and *surface area.*

Teachers can assist students to learn the mathematical vocabulary by using the same techniques and activities as are used in the teaching of English and other languages. They include:

- students developing their own definition of a word—by examining what the word

means (using words, symbols or visuals), what it does not mean, and contextual examples of usage of the word);
- matching games such as concentration and dominoes;
- loop card activities;
- classifying activities such as card shuffle; and
- the explicit teaching of spelling—which would also help to prepare students for the spelling tests in the National Assessment Program – Literacy and Numeracy (NAPLAN) tests.

Some of these require the preparation of reusable sets of cards that suit the vocabulary being taught.

Syntactic features of mathematical English

Word order is possibly the syntactic feature that causes the most confusion for students. Often the written and symbolic forms of an operation are written in different orders. For example, *take 3 from 8*, and *the difference between 8 and 3* are both written 8 – 3 (Abedi & Lord, 2001; Newman, 1983). Students may attempt to deal with this confusion by always subtracting the smaller number from the larger number, regardless of the order in which they are presented, leading to reversal errors. A further complication is that word order is crucial in some situations and not in others, for example *3 multiplied by 7* can be modelled as *3 times 7* or *7 times 3*. Other situations can appear to be ambiguous, for example *four plus five divided by three*.

Prepositions are often short words that can be overlooked by students. However, they may be critical to the interpretation of a mathematical statement. Consider the difference between *from the house to the car* and *to the house from the car*. Alternatively, consider between *8 divided by 2* and *8 divided into 2* (Munro, 1979). When examining a problem text, students should be encouraged to focus on the prepositions by underlining them.

Many mathematical problems are expressed in abstract and impersonal forms or in passive voice. A typical statement might be, "The difference in the ages of two students is six years." This is more complex than "Sandra is six years older than Peter." Passive voice also affects word order. For example, when the passive form of "a *sample* of 25 was *selected*" is converted to the active form "he selected a sample of 25," the order of the noun sample and verb select is reversed. Passive voice and abstract and impersonal forms make the interpretation of problem texts more challenging for students (Abedi & Lord, 2001).

Mathematical texts often contain several different ideas packed into a relatively small number of words. This can be quantified as lexical density—the mean number of lexical words (nouns, verbs, adjectives and adverbs) per clause. Texts with higher counts, described as lexically dense, are considered to be more difficult to read. Spoken English may be as low as two lexical words per clause (Halliday, 1990). To illustrate this concept, the lexical densities of two items selected from the 2010 Year 7 NAPLAN Reading test were calculated. An item of 272 words on a scientific topic had 5.6 lexical words per clause whereas a fictional passage of 236 words had a lexical density of 4.2 (Carter, 2011). In mathematical writing (like scientific writing) conciseness is valued, resulting in texts with high lexical densities. An example taken from the 2010 Year 7 NAPLAN Numeracy Non-Calculator test is:

> Ben has 2 identical pizzas. He cuts one pizza equally into 4 large slices. He then cuts the other pizza equally into 8 small slices. A large slice weighs 32 grams more than a small slice. What is the mass of one whole pizza? (Australian Curriculum, Assessment and Reporting Authority [ACARA], 2010, p. 10)

The question has a lexical density of 7.75. In a recent study, seven out of ten Year 7 students who were observed whilst working on this problem failed to make meaning of the problem text, overlooking the importance of the words “more than a small slice”. As the individual words were not complex at the Year 7 level, it is suggested that the combination of nine separate mathematical ideas packed into a total of 44 words made the interpretation of the entire text beyond the reach of most of the students (Carter, 2011).

Another problem that arises from the concise nature of mathematical writing is the lack of contextual clues. Good readers who encounter an unfamiliar word whilst reading will often read on in the hope that the meaning of the word can be gleaned from the context of the text. In everyday English texts, this is often a successful strategy. For this reason, in other learning areas, students are encouraged to skim-read a text to gain an overall impression of its meaning. Often this is sufficient to make meaning of the text. However, in mathematics, the failure to decode a single key word can prevent an understanding of the entire text. Mathematical texts contain few contextual clues to assist in making meaning (Munro, 1979; Newman, 1983). The use of skim-reading techniques with mathematical texts may result in a student missing words that are crucial to the interpretation of the text. In mathematics, students must use a close-reading strategy, that is, to focus on every word in the passage.

The unusual use of some spatial words in everyday English can add to students’ confusion in interpreting the same words when used in the context of mathematics (Gough, 2007). Brisbane residents may talk of travelling *down to Sydney* (presumably meaning further south) or *up to Toowoomba* (meaning higher altitude). *Higher latitudes* could mean either further north or further south, *sub-Sahara* does not mean underneath the desert and *moving the tables to make more room* does not mean that we enlarge the floor space.

But wait, there’s more…

This article focuses only on the use of words. Mathematical language also includes a vast array of symbols, tables and visual images. Space prevents an examination of these. However, it is clear that mathematical English is more difficult to understand than ordinary English. It may impact upon success in written assessment where the ability to make meaning of the question is crucial. Ability in basic interpersonal communication does not necessarily result in proficiency in the use of mathematical English.

What to do?

Having established that the language of mathematics is unique, it follows that its use can only be taught within the discipline of mathematics. Teacher awareness of the complexities of mathematical English is an essential first step. These complexities must then be explicitly taught to students.

Some activities used in English for the teaching of language that can also be applied to mathematical language have already been described. Additionally, teacher modelling of the process of unpacking (deconstructing) a problem text at every possible opportunity is essential. For example, identifying nouns can assist in locating facts. Verbs often indicate the processes or operations that must be applied. Prepositions are important in determining mathematical relationships. The use of prompts and/or graphic organisers to assist in the deconstruction process can help to remind students of the steps in the process. If the teacher deconstructs the text every time the solution of a worded problem is modelled, students will learn to follow the same process when working independently.

Further, students can be assisted in dealing with more complex language forms by practicing the simplification of passive, abstract and impersonal forms of language, (if necessary, by introducing names for the players in the problem). Such practice can happen in English lessons as well as mathematics lessons

The use of language in mathematics assessment must be carefully considered. There are many studies, reported in Abedi (2009), that suggest "that mathematics test performance of some students has been affected by differences in the syntactic complexity of the language of word problems" (p. 171) and that even minor changes can make them more accessible to students. However, this article is not proposing that the language of assessment be simplified to remove all of the challenge in interpreting a problem. Students must become proficient with the methods used by mathematicians to communicate. However, as Abedi stated, "there is a difference between language that is an essential part of the content of the question and language that makes it incomprehensible to many students" (p. 173). There is a distinction between necessary and unnecessary linguistic complexity and the use of complex language in assessment items must be the result of a deliberate decision.

The complexities of mathematical English are such that there is no 'quick fix' for students or teachers. The development of confident readers of mathematical texts requires the use of planned and explicit learning opportunities in mathematics lessons in all year levels. The aim is to develop students who do not think of mathematical English as Jabberwocky.

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Synonyms for the arithmetic operations

Addition	Subtraction	Multiplication	Division
accumulate	backwards	array	half, third, etc
add	decrease	by	distribute
altogether	debit	commission	divide
all told	debt	double, triple, etc.	divisible
and	deduct	twice, thrice, etc.	divisor
another	difference	square, cube, etc.	factor
augment	diminish	two-fold, three-fold, etc.	fraction
bigger (than)	discount	factor	groups
credit	down	groups of	left
deposit	exceed	lots of	over
extra	fall	magnify	out of
faster (than)	fewer (than)	multiple	parts
forward	from	multiply	per
further (than)	gone	of	per cent
gain	leave	product	portion
greater (than)	left (over)	repeated	rate
grows	less (than)	taxation	reciprocal
heavier (than)	lighter (than)	times	remainder
higher (than)	lose		quotient
increase	lower (than)		share
longer (than)	off		split
more (than)	narrower (than)		
older (than)	nearer (than)		
positive	net (e.g., income)		
plus	minus		
rise	negative		
sum	reduce		
taller (than)	remaining		
thicker (than)	remove		
together	reverse		
total	thinner (than)		
up	shorter (than)		
wider (than)	slower (than)		
with	subtract		
	take (away)		
	withdraw		
	younger (than)		