# Word Problem Solving

## a Schema Approach in Year 3

**EDUARDA VAN KLINKEN** shows how research on reading and solving word problems may be applied to help year three students interpret and solve word problems. A set of questions is provided so you may apply what you learn from reading this article to your own classroom.

This article outlines how a Brisbane independent school, Clayfield College, improved the ability of its Year 3 students to solve addition and subtraction word problems by utilising a schematic approach. We had observed that while students could read the words in the text of a written problem, many had difficulty identifying the core information and were unable to derive a relevant number sentence. We drew on research that suggested a schematic approach to problem solving was promising (Jonassen, 2003) and suitable for Year 3 (Griffin & Jitendra, 2008). In this paper I briefly outline the most pertinent research and provide a short summary of how the unit unfolded. The word problem solving unit consisted of two 40-minute lessons per week throughout an eight-week block and was undertaken in two co-educational Year 3 classes.

There is a difference between how successful problem solvers go about their task when compared to weak problem solvers (Xin, Wiles & Lin, 2008). Students who solve problems successfully are able to look beyond the superficial surface features of a story and are able to analyse the underlying structure or *schema* of the problem. They recognise that although word problems may have different storylines, they are represented by a limited number of mathematical relationships. In contrast, students who are weak problem solvers are likely to be distracted by irrelevant

information such as characters or setting. They fail to recognise the schematic similarities between different stories, treating each new story as a completely isolated problem (Schiff, Bauminger & Toledo, 2009).

Research over a number of decades shows that weaker problem solvers can successfully be taught to solve problems in the same way as successful problem solvers by helping them to look at problems in a 'top-down' schematic way (Marshall, 1995). Carpenter and Moser (1983) classified word problems into three broad schemas: *Change*, *Difference* and *Combine*. In our unit we adopted this classification. The characteristics of each will shortly be defined.

We limited the word problems in this unit to problems that required a simple one-step mathematical calculation—the stories contained only one action. Nevertheless, even within this single-step parameter, a number of factors can contribute to difficulties for students. According to Reusser (2000), the reasons why some students fail to solve word problems successfully are that: (a) the scenario represented is outside the experience of a student; (b) the vocabulary is unfamiliar; or (c) the story uses a difficult sentence structure. Hannell (2005) noted that relatively simple problems become complicated when they include amounts or numbers that are larger than the student is able to compute mentally. We often observed this in our unit; as soon as numbers above 20 were included in the word problem, many students were unable to identify the underlying schema of the problem.

The position of the unknown quantity in the number sentence must also be considered when analysing the difficulty of a word problem (Garcia, Jimenez & Hess, 2006). In one-step problems, the position of the unknown quantity occurs in three possible positions. The easiest is 'result unknown' (7 – 3 = ?), followed by 'change unknown' (7 – ? = 4) with the most difficult being 'start unknown' (? + 4 = 7). Our pre-test results clearly reflect students' increasing confusion when the unknown quantity occurs in the more challenging positions (Figure 1). In our unit, the position of the unknown quantity was one of the most important factors when simplifying or complicating problems.

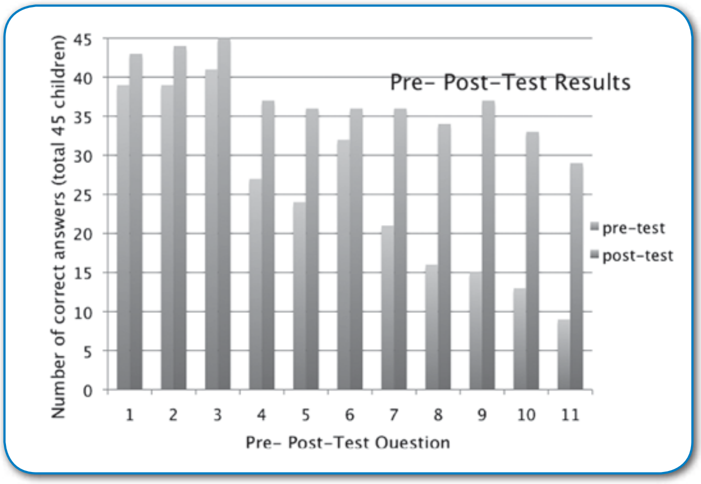


Figure 1. Graph comparing pre- and post-test results.

During the eight weeks, we were able to frequently observe the impact of children's number sense on their ability and efficiency in solving word problems. Even the least able students were able to solve 'result unknown' number sentences correctly because they were able to use a 'count all' approach, using fingers or hastily drawn illustrations to keep track. In 'change unknown' problems, many of our students counted forwards or backwards from the starting amount, persevering with counting until the desired number was reached. Although this strategy was effective for most questions because we utilised small numbers, the better students used their knowledge of number facts to solve these problems more efficiently. For the most difficult 'start unknown' problems, many students used a trial and error method. The more able students were able to rework the problem into another form (for example, ? + 4 = 7 can easily be solved by reworking it into 7 – 4 = ?). Despite the fact that our students had been taught about 'number fact families,' many were unable to apply this when solving number sentences.

## The schematic diagrams

In our unit, the use of the *schematic diagram* became integral to the teaching and learning process. A schematic diagram is a visual tool to assist students in paring down information so that only the important structural information remains. We found it to be the single most important means to assist weaker students to successfully analyse the schemas in the stories.

Van Garderen (2007) has shown that the ability of students to represent information visually is related to their ability to grasp the schema of the problem. Weak problem-solvers tend to include information about characters and setting, but only make superficial reference to the dynamic nature of the relationships and quantities in the story. Successful problem solvers, by contrast, are able to successfully represent the critical mathematical connections. Figures 2 and 3 demonstrate how contrasting schematic understanding is reflected in students' visual representations.


Figure 2. Visual representation of Question 1 in pre-test. "The girl was happy because she had money in the bank." There is no evidence of schematic understanding.

We based our three schematic templates loosely on those suggested by Marshall (1995). Each was introduced as a visual reflection of class discussions as we explored the essential mathematical relationship of each schema.

The translation from word story to schematic diagram was challenging for our weaker students. They had to learn to think in a new way, looking beyond the superficial story elements to identify the relevant schema. We quickly became aware that these students needed considerably more time and individual assistance to make this critical conceptual step.

## Introducing the schemas

We introduced the *Change* schema first because of its dynamic nature and the many day-to-day examples that could be found in the classroom. Verbs are critical to the action in the story and we explored them in considerable detail. Students discovered that verbs such as "bought," "found," "baked" or "earned" were clues to increasing amounts, whereas verbs such as "lost," "stolen," "wilted" or "eaten" were clues to decreasing quantities. During this first phase of the unit we introduced the Change schematic diagram as a means of isolating the key information, describing it as a "picture that shows only the really important information" (Figure 4).

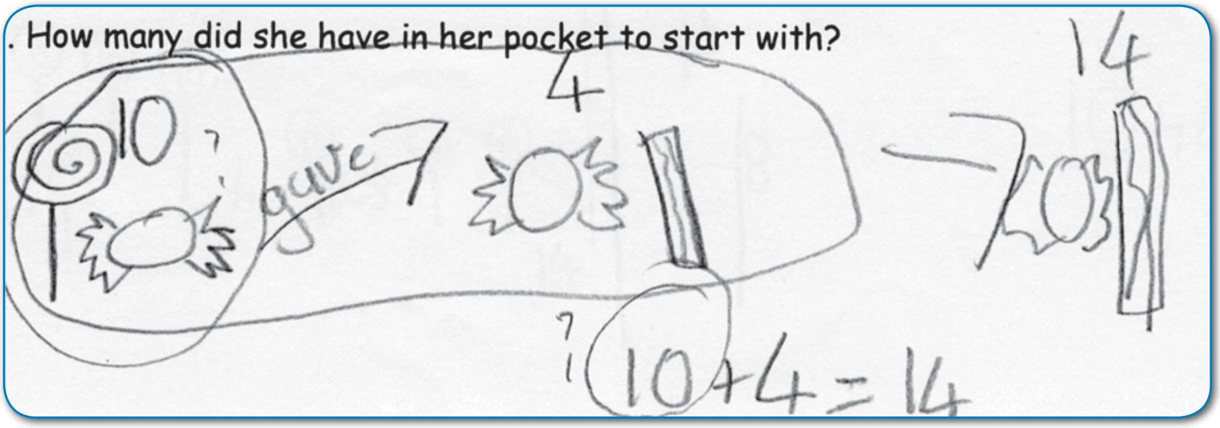


Figure 3. Visual representation of Question 6 in post-test. The student represents the important information and is able to write a matching number sentence. See Appendix A for details of the questions.

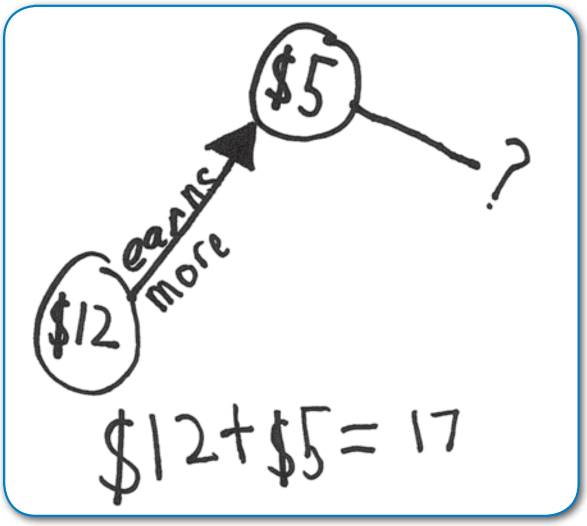


Figure 4. Change problem: Jenny has $12 in the bank. She earns $5 doing some jobs. How much money does she have now?

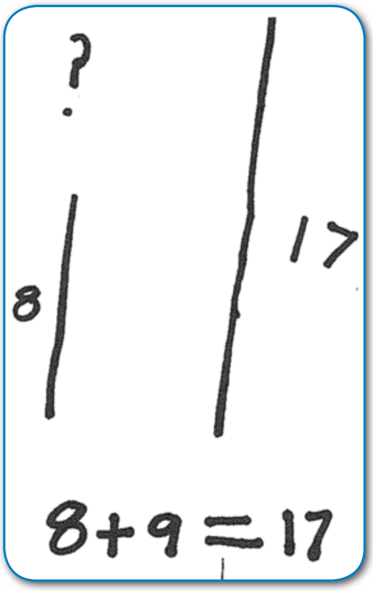


Figure 5. Difference problem: Mark is 8 years old and his cousin is 17. How much older is his cousin?

We began by presenting students with stories that had no unknown amounts; e.g., "Chloe planted nineteen flower seedlings. Over the weekend six died. Now there are only thirteen left." We then worked with the students to help them derive both a matching schematic diagram and number sentence. We discussed what the original quantity was and whether it had been increased or decreased to give the final amount. Gradually we introduced stories with a 'result unknown', followed by 'change unknown' problems and finally 'start unknown' problems. After three weeks, during which time varying approaches and ideas were shared and discussed, most children were comfortably recognising, solving and writing Change problems with the unknown amounts in all three positions.

*Difference* problems involve finding the difference between two amounts (Figure 5). They include those that seek to equalise two amounts by adding or subtracting an amount until the two quantities are the same (see question 10 in pre/post test). Of the three schemas, Difference problems are the most difficult for students to solve (Garcia et al., 2006). This was certainly the case in our unit where Difference problems generally scored poorly in the pre-test. Because of this, we spent four weeks early in the unit discussing practical scenarios that compared amounts. Activities included the comparison of ages, temperatures and height, as well as comparing how long it took children to complete a task.

The *Combine* schema was the last to be introduced. Though similar to the Change schema, it is distinguished by the fact that it does not involve action, but is the linking together of two static groups. By the time we introduced this last schema, students had considerable experience in identifying structures and we were interested to see whether they could apply their knowledge to identify this third schema. We gave them a number of Combine word problems and asked them to identify the new mathematical structure. Pleasingly, many students identified the 'combining of two amounts' as the essential feature of this schema. The schematic diagram was then developed to reflect joining, the essence of this schema (Figure 6). Unfortunately, due to time constraints, the Combine schema could not be explored in any detail.

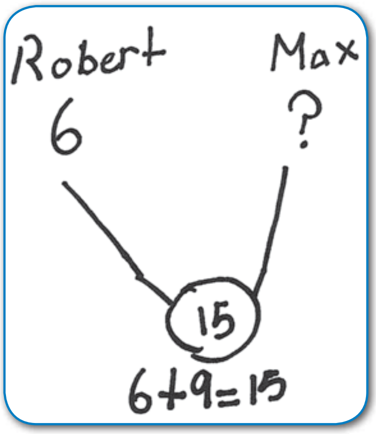


Figure 6. Combine problem: Robert has six rocks. Together Robert and Max have fifteen rocks. How many rocks does Max have?

## Conclusion

Students and teachers enjoyed this unit immensely because it provided an opportunity to look at mathematics in a deeper way, other than that provided for in the regular curriculum. The schema approach gave teachers an alternative and successful path to approach a difficult curriculum topic. It provided numerous opportunities to link 'real life' with 'classroom' mathematics. Additionally, it engaged students constantly in mental calculation. The schematic diagram, which was a new concept for our teachers, proved to be a pivotal tool in revealing the 'bare bones' of a problem. Throughout the unit there were many opportunities for students to articulate, compare and clarify their thinking through class and informal discussions.

The success of the unit in quantitative terms is easily measured by comparing pre- and post-test results. We were encouraged by the fact that our students made the most dramatic gains in the most difficult problems, solving word problems that most were unable to attempt at the start of the unit.

Finally, this small research project supports the results of other research showing that a schematic approach to teaching word problems can help Year 3 students to conceptualise word problems in a schematic way, thereby leading to a deeper understanding as well as greater flexibility and accuracy when solving word problems. Further details of the unit can be obtained from the author (evanklinken@clayfield.qld.edu.au)

## References

Carpenter, T. P. & Moser, J. M. (1983). The acquisition of addition and subtraction concepts. In R. Lesh & M. Landau (Eds), *Acquisition of mathematical concepts and processes* (pp. 7–44). New York: Academic Press.

Garcia, A. I., Jimenez, J. E. & Hess, S. (2006). Solving arithmetic word problems: An analysis of classification as a function of difficulty in children with and without arithmetic learning difficulties. *Journal of Learning Disabilities, 39*(3), 270–281.

Griffin, C. C. & Jitendra, A. K. (2008). Word problem-solving instruction in inclusive third-grade mathematics classrooms. *Journal of Educational Research, 102*(3), 187–201.

Hannell, G. (2005). *Dyscalculia: Action plans for successful learning in mathematics*. London: David Fulton Publishers.

Jonassen, D. H. (2003). Designing research-based instruction for story problems. *Educational Psychology Review, 15*(3), 267–296.

Marshall, S. P. (1995). *Schemas in problem solving*. Cambridge: Cambridge University Press.

Reusser, K. (2000). Success and failure in school mathematics: Effects of instruction and school environment. *European Child and Adolescent Psychiatry, 9*(2), 17–26.

Schiff, R., Bauminger, N. & Toledo, I. (2009) Analogical problem solving in children with verbal and nonverbal learning disabilities. *Journal of Learning Disabilities, 42*(1), 3–13.

van Garderen, D. (2007). Teaching students with learning difficulties to use diagrams to solve mathematical word problems. *Journal of Learning Disabilities, 40*(6), 540–553.

Xin, Y. P., Wiles, B. & Lin, Y. (2008). Teaching conceptual model based word problem story grammar to enhance mathematics problem solving. *Journal of Special Education, 42*(3), 163–178.

**Eduarda van Klinken**
**Brisbane Girls Grammar School**
**<evk@uq.net.au>**

## Appendix A. Pre- and post-test questions

| Question | Word problem | Problem type | Number sentence |
|---|---|---|---|
| 1 | Jenny has $12 in the bank. She earns $5 doing some jobs. How much money does she have now? | Change | 12 + 5 = |
| 2 | There are 14 girls and 10 boys in Year 2 this year. How many children? | Combine | 14 + 10 = ? |
| 3 | There are 25 children in this class. Yesterday five children were away sick. How many children were in the class yesterday? | Change | 25 – 5=? |
| 4 | Mark is 8 years old and his cousin is 17. How much older is his cousin? | Difference | 8 + ? = 17 |
| 5 | Mum baked 12 cupcakes on Monday. On Tuesday she baked some more. Now she has 20 cupcakes. How many did she bake on Tuesday? | Change | 12 + ? = 20 |
| 6 | Kim had some lollies in her pocket. Her father gave her four more lollies. Now she has 14 lollies. How many did she have in her pocket to start with? | Change | ? + 4 = 14 |
| 7 | Robert has six rocks. Together Robert and Max have fifteen rocks. How many rocks does Max have? | Combine | 6 + ? = 15 |
| 8 | Joe had 19 marbles. He gave some to his brother. Now he only has 13 marbles left. How many did he give to his brother? | Change | 19 – ? = 13 |
| 9 | Anna had a bunch of flowers. Four died and she put them in the bin. Now there are only six flowers left over. How many were in the bunch to start with? | Change | ? – 4 = 6 |
| 10 | Len has fourteen fish. Emma has only six fish. How many more fish does Emma need to have the same number as Len? | Difference (equalise) | 6 + ? = 14<br>14 – 6 = ? |
| 11 | Charlie is 12 years old. He is 8 years older than his brother. How old is his brother? | Difference | 12 – 8 = ?<br>8 + ? = 12 |