

Secondary students' descriptions of "good" mathematics teachers

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Introduction

There are many recent studies which have explored the notion of high quality or effective mathematics teaching but relatively few have sought the views of students. Student perspectives regarding effective teaching are important for many reasons (Kane & Maw, 2005; White, Barnes, Lawson, & Johnson, 2009), and are arguably particularly significant in mathematics given the international trends of declining engagement and participation in this curriculum area (McPhan, Morony, Pegg, Cooksey & Lynch, 2008). Studies that have asked students to identify reasons for their disengagement from mathematics have found a lack of confidence with mathematics, a lack of understanding of mathematics, a lack of enjoyment of mathematics, and/or dissatisfaction with mathematics teaching to be major factors (Watt, 2004; Brown, Brown, & Bibby, 2008; McPhan et al., 2008; Nardi & Steward, 2003).

Research that has directly considered students' perspectives about effective mathematics teaching has yielded valuable insights. Kaur (2008, 2009) undertook a series of studies in Singapore, in which students were asked to describe the qualities of "good mathematics lessons" and "best mathematics teachers". He found consistent themes across the studies. Students deemed a mathematics lesson as a good one when some of the following characteristics were present: the teacher "explained clearly the concepts and steps of procedures"; "made complex knowledge easily assimilated through demonstrations, use of manipulatives, real-life examples" and the teacher provided "feedback to individuals or the whole class" (2008, p. 343). The students' descriptions of a good mathematics lesson also included the teacher "moving from desk to desk" (2009, p. 960). The most frequently offered descriptions of best mathematics teachers were: patient, understanding, caring/kind, good at mathematics, explains clearly, ensures students understand, and provides individual help. Kaur concluded that "good mathematics teaching in Singapore is student focused but teacher-centred" (2008, p. 346).

These characteristics of good mathematics teachers are similar to student descriptions of good teachers in general. In a questionnaire-based study of over 800 Australian secondary students from Years 8 to 12, White, et al. (2009) asked students to identify the aspects of teaching that they thought helped their learning. Teachers “explaining things well” (p. 4) and getting students “interested in the material” (p. 4) were identified by large numbers of students. Other highly rated aspects included teachers “being approachable”, “encouraging students to achieve”, “providing useful feedback”, “checking on understanding”, “being passionate and energetic about teaching” and “talking to students as individuals” (p. 4). Another Australian investigation found that students thought good teachers explained material well with a focus on understanding; made learning relevant; took into account individual differences; and had positive personal characteristics such as being kind and respectful (Strikwerda-Brown, Oliver, Hodgson, Palmer & Watts, 2008).

The aim of the present study was to identify students’ views regarding “good” mathematics teachers.

Method

Sample

A total of 90 students from Years 10 and 11 participated in the study. They were from two comprehensive public secondary schools in regional towns in New South Wales. Both schools have classes from Years 7 to 12. There was a high participation rate for the study, 78 and 87 per cent for Years 10 and 11 respectively. Students were randomly selected from those who had agreed to participate in the study. The final sample is summarised in Table 1.

Table 1. Characteristics of the sample

Year group of participants	Year 10	n = 38 (42%)
	Year 11	n = 52 (58%)
Gender of participants	Female	n = 51 (57%)
	Male	n = 39 (43%)
Mathematics choices for Year 11 (includes Year 10 and Year 11 students) (note: mathematics subjects are listed in order of level of difficulty)	None	n = 18 (20%)
	General Mathematics	n = 32 (36%)
	Mathematics	n = 27 (30%)
	Mathematics Extension 1	n = 13 (14%)
Current class grouping for Year 10 participants (A,B,C and D are the class labels used by schools)	A	n = 14 (37%)
	B	n = 7 (18%)
	C	n = 7 (18%)
	D	n = 10 (26%)

Interview

The interview format was semi-structured; that is, all students were asked the same question but additional follow-up questions or prompts that enabled elaboration or clarification were also posed by the researcher. The

interview question on “good” mathematics teachers was part of a larger study focusing on students’ perceptions of mathematics. The question asked of the students was:

Can you describe a really good maths teacher for me? This teacher doesn’t have to be a real person. It can be someone you are imagining. What would be the features of a really good maths teacher?

The question was carefully worded to be accessible for all students. It asked about a “good teacher” rather than “good teaching” on the assumption the former question might be easier for students to approach with confidence. It was also worded to allow students to go beyond their experience of actual maths teaching and teachers if they wished. All of the students responded thoughtfully to the question.

Data analysis

All interviews were audio-taped and transcribed. Students’ responses to the questions were coded iteratively, using an inductive approach and categorised into major themes (Thomas, 2006). Many of the themes are inter-related. Most students presented at least two features of good teachers in their descriptions. Within these descriptions of teachers, there were consistent findings—almost all of the features of good teachers mentioned were offered by significant numbers of students.

Responses were analysed using chi squared analyses to see if there were any differences according to student gender, school or year group. There were no significant differences according to these variables.

Findings and discussion

Participants’ responses are summarised in Table 2, in order of frequency of response.

Table 2. Students’ descriptions of “good” mathematics teachers.

Feature of “good” mathematics teachers	N (%)
Explains well	73 (81)
Provides one-on-one help	48 (53)
Positive characteristics – teacher is pleasant, kind, caring	36 (40)
Safe learning environment created - students feel they can ask questions, teacher does not “put students down”	22 (24)
Understands/knows students as individuals	22 (24)
Can control the class	16 (18)
Makes mathematics relevant	14 (16)
Does not rush	13 (14)
Knows how to teach mathematics (pedagogical content knowledge)	8 (9)

It is worth noting at the outset that students discussed mathematics teachers in generally positive terms. While they were given the opportunity to discuss a hypothetical “good” teacher, many in fact described their current mathematics teacher, or one they had been taught by in the past. They described inspirational teachers using powerful descriptions:

He can see on my face if I'm struggling with something and he'll re-explain it... he just explains things well... and you can tell he knows what he's doing when he's teaching. He has it planned in his head how he's going to do it. (Female Year 10, student no. 35)

Knows what they're doing and can help you out and can kind of, when they're teaching, they can actually teach you. It's like all our teachers, they teach us fine, but you still don't really get it. And then I'll have teachers and they teach you good and you just get it. It just really, it gets into your brain. (Male Year 11, student no. 52)

Explaining so that students understand

Overall, the picture which emerged of good mathematics teachers is reminiscent of Kaur's (2008) descriptions of "teacher-directed, student focused" learning environments. Of all of the factors involved in good teaching, overwhelmingly, students placed the most importance on teacher explanation. Students' emphasis on the importance of explanation echoes findings from previous research (Kaur, 2008, 2009; Shimizu, 2009; White et al., 2009; Wilson & Corbett, 2007).

Over 80% of students included the capacity to explain as a characteristic of a good mathematics teacher. Three quarters of these 80% used the word "explain" in their description:

He actually explained it down to pinpoint, like how to find it and, how can I say it? He explains it really good – other teachers do explain it but not in the way of understanding. (Female Year 10, student no. 8)

The notion of explanation is recognised in the literature as one of the central tasks of teaching (Leinhardt, 2004). Explanation was of value to students because they wanted to understand the concepts they were being taught. Of the 73 students who described a good teacher in terms of their capacity to explain, over half (51%) referred to the concept of understanding; that is, a good teacher explains ideas so that students understand them. Most of these students indicated that a good teacher persists; that is, they "keep on" explaining until students understand:

Well, clear, concise, and if you need help they'll explain it and they just, they'll just be persistent in trying to help you. Like if you don't understand something and you just can't get it, they'll try everything just to help you. (Female Year 11, student no. 63)

Almost one quarter of students who referred to teacher explanation stated that a good teacher will use various approaches when explaining:

There's no specific A, B, C to it. But basically be approachable and kind, supportive and if someone doesn't get it and they don't understand that method of explaining, try using a different method. So you've got to try and have some understanding of different ways of explaining the same thing in a sense. (Male Year 11, student no. 70)

I think a good maths teacher is where they can see that the way they're teaching it isn't working so they try a different way and try and explain it in a different way. (Female Year 11, student no. 61).

Related to the notion of teacher persistence and explaining concepts so that all students can understand them, is the idea that a good teacher "does not rush" through the material. Fourteen percent of students mentioned this idea:

And they don't go on when half the class has it and half the class don't, we're all the same level and we can all do it. I don't like teachers that just rush ahead when you don't get the concept. (Female Year 10, student no. 38)

Again, related to explanation, about one in six (16%) of the students stated that a good mathematics teacher makes mathematics relevant to real life:

Like a teacher who will give examples of something in a way that we will generally relate to. Like giving examples that, like link to modern day. If say a new building's going up somewhere and they need the area... or how much area on one side. (Male Year 11, student no. 68)

A small group (nine percent) of students elaborated on the concept of the teacher explaining well, to include the notion of subject-specific teaching expertise. These students felt that good mathematics teachers "really know" what they are doing:

That they know what they're doing but they can also relate to students. My maths teacher always says I hated this in maths, but then I was shown this way and this helped me and she researches heaps of methods because she knew when she was taught that way it didn't work so she's really good. (Female Year 11, student no. 90)

He teaches every technique that you can use to grab a line, and he also tells his favourite lines... he just, he knows what he's doing and explains it really well. (Male Year 10, student no. 17)

These students are describing various aspects of pedagogical content knowledge (Shulman, 1987) in that they take account of the interaction between teaching strategies, content knowledge, and ways in which students learn.

One-on-one help and recognising individual differences

While explanation was central to students' descriptions of good teachers, they also believed that effective mathematics teachers know and understand them as individuals, and will give them one-on-one help. One or both of these two features of good teachers (explains well and offers one-on-one help) were offered by almost every student in the study.

One-on-one help is related to explanation—it is another vehicle by which a good teacher can deepen or individualise explanations. From the students' point of view the reason for wanting one-on-one help was so that they could better understand the material and concepts:

They'll try and break stuff down into easy to understand chunks. They'll one-on-one with you. (Male Year 11, student no.76)

I just liked it because he came up to you and showed you what to do. Like one-on-one. And then he'd move on and if you had a question you'd put up your hand and he'd work with you, work with you one-on-one type of thing. So they kind of think about the overall class and then go to the people who need help. (Female Year 10, student no. 9)

While there is considerable literature that notes the desirability for teachers to take account of individual differences, the students in this study actually specified the need for teachers to work with them "one-on-one". This interesting finding resonates with Kaur's (2009) research which found that students value the provision of individual help and the teacher "moving from desk to desk" (p. 960).

Positive teacher attributes and creating a supportive learning environment

While explanation and individual help were dominant features in students' descriptions of good teachers, these features do not paint the entire picture. Social and emotional factors in teaching were also important to students. Forty percent of participants reported that good mathematics teachers have positive characteristics such as being caring, respectful and patient. Students went further to relate these attributes to their own learning. They felt that good teachers are approachable and do not make them feel stupid:

Someone who will like go out of their way to help you, you know try to understand something. Like if they think you're struggling will actually go through it and explain something and not make you feel stupid for not understanding it. (Female Year 11, student no.62)

I guess being approachable because if a student can't approach a teacher they're not going to ask a question when they need to and they're not going to understand what they're learning. So yeah, just generally be nice. (Male Year 11, student no.88)

The quotations above illustrate the importance for students of positive relationships with teachers, but also the reason why such relationships are important: good teachers enable them to seek help when they feel they need it.

While students see good maths teachers as creating safe environments, a significant percentage of them also mentioned that a good teacher "can control the class". Once again, these two ideas were often linked by the students. Good teachers command respect because of who they are and how they act:

I think someone who's really calm. Someone who is really patient and who takes the time and, at the same time as well as being calm, people respect them so that they can keep the class under control. Because if you don't respect your teacher then you push their boundaries and then that's when all hell breaks loose. So you need to respect them. (Female Year 10, student no. 33)

Conclusion

The current study explored the concept of a "good" mathematics teacher from an important perspective: that of the learner. In describing good mathematics teachers, the young people in this study revealed valuable insights. As White et al., (2009) have argued, these insights enable us to better understand how students perceive classroom life, how they feel about their learning, and their thoughts on how their learning can be enhanced. Such data can usefully inform the current quest to engage students and retain them in the study of mathematics.

The students' desire to understand mathematics is an encouraging finding. Despite varied experiences of mathematics, all of the students appeared to care about mathematics as a domain of learning. Studies across secondary school year groups show a decline in positive mathematical self-concept for males and females (Nagy, Watt, Eccles, Trautwein, Lüdtke, & Baumert, 2010), but for the students in this study, understanding mathematics was important to them.

Students' depictions of good mathematics teachers in relation to explanation, one-on-one help, and the achievement of understanding, can be

seen as an expression of some of the elements of pedagogical content knowledge. The study underlines the importance for students of teacher skills in explanation. However, the responses in this study are an important reminder that mathematics teaching also involves a different type of knowledge—of students' emotional and social reactions to mathematics and the classroom environment. The emotional dimension of mathematics learning has been the subject of increasing research (see, for example, Boaler, Wiliam & Zevenbergen, 2000), and the findings of this study indicate that it is an important line of enquiry.

The findings from the current research raise some interesting questions. Firstly, to what extent are the descriptions of good teachers specific to mathematics teaching and to what extent were students describing features of what they saw as good teachers in general? Some of the themes from the current study do align with student descriptions of good teachers in general. Other teacher attributes highlighted by the students in this study may in fact be important for mathematics teachers in particular. The students' emphasis on approachability, non-intimidating classroom environments and on not being "put down" or made to feel stupid, for example, may well be vital for students' confidence in mathematics (Mendick, 2005). Further comparative research could provide valuable insights into how students perceive and describe effective teaching within different subject disciplines.

Finally, the question of responsibility for learning is a theme that is worthy of further research: who do students see as being responsible for their learning of mathematics? Participants indicated the centrality of teachers to their learning of mathematics. Further, they appeared to place responsibility with teachers to a large extent: they considered that good teachers should have the skills to explain concepts well and ensure that students understand them, and to provide expert individualised help. However, most participants did not expect to be passive recipients of teacher expertise. They expressed the view that good mathematics teachers create classroom environments wherein students feel comfortable to actively seek help and enhance their own learning. Further research exploring how students see teachers' and their own roles and responsibilities in the learning process in mathematics would be of value.

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From Helen Prochazka's

Scrapbook

Trigonometry

At first, trigonometry was associated with only the sides of a triangle. Thales (c. 624–546 BC) used triangles to find the height of a pyramid and to determine the distance of a ship from the shore with a method known as 'shadow reckoning'. The concept of a tangent developed from this technique.

For many centuries astronomy and trigonometry were interlinked. Many mathematicians considered themselves to be astronomers first. Mathematics and trigonometry in particular, was the principal tool they used for their astronomical calculations.

In about AD 150, the Greek mathematician and astronomer, Claudius Ptolemy (c. 90–168) published the *Almagest*. This 13-volume work deals with the motion and position of the stars and planets. It was used as a standard astronomy reference for more than a millennium, in the East and the West.

In Chapter 6 of Book 1, Ptolemy states that in relation to the distance of the fixed stars, the Earth was so small that it should be treated as a mathematical point.

The *Almagest* included the equivalent of a table of sine values. Today a scientific calculator will provide such data.

The Hindu mathematician and astronomer Aryabhata (476–550) collected and expanded upon earlier Hindu advances in trigonometry. His sine tables were calculated to an accuracy of four decimal places.

During the 8th century, Islamic mathematicians translated the Indian works. The focus of study became the sides and angles of a triangle, and the first real trigonometry began to emerge. By the 10th century, they were using the sine, cosine and tangent functions and were applying them to problems in triangles. The method of triangulation was developed and applied to surveying and cartography.

At about the same time, Chinese mathematicians also translated the Hindu treatises and developed trigonometry independently.

The German mathematician and astrologer Johann Regiomontanus (1436–1476) was the first person to treat trigonometry as a distinct mathematical discipline. His book "On Triangles of all Kinds" deals with trigonometry on flat planes and spherical surfaces. In it Regiomontanus wrote:

You who wish to study great and wonderful things, who wonder about the movement of the stars, must read these theorems about triangles. Knowing these ideas will open the door to all of astronomy and to certain geometric problems.