

THE UNIFIED PHONETIC TRANSCRIPTION FOR TEACHING AND LEARNING CHINESE LANGUAGES

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ABSTRACT

In order to preserve distinctive cultures, people anxiously figure out writing systems of their languages as recording tools. Mandarin, Taiwanese and Hakka languages are three major and the most popular dialects of Han languages spoken in Chinese society. Their writing systems are all in Han characters. Various and independent phonetic transcriptions have been thus developed to be as the mapping mechanisms between Chinese mother tongue languages and Han characters. For teaching and learning facilitation purposes, we really require a convenient phonetic transcription system between daily Mandarin, Taiwanese and Hakka to speed Han characters data processing applications. The Roman spelling system is a universal tool that owns the one and only one spelling rule. By studying and analyzing the Roman spelling system, we have disclosed that 4135 Romanized phonetic transcriptions can be adequately applied to handle Han characters' mappings of Mandarin, Taiwanese and Hakka spoken dialects. In this paper, we propose a minimal perfect hashing function to process unified 4135 Mandarin, Taiwanese and Hakka Romanized phonetic transcriptions to their corresponding Han characters simultaneously. The unified phonetic transcription can be used to promote Chinese mother tongue languages applications and developments. Furthermore, it can be applied as a mechanism to popularize digital learning and teaching of Chinese mother tongue languages.

INTRODUCTION

People generally recognize that it is valuable to teach and learn mother tongue languages in today societies. People anxiously figure out writing systems of their languages to record and preserve their distinctive cultures. Mandarin, Taiwanese and Hakka languages are the three major spoken dialects in Chinese society. There are many speakers of the languages in China, Malaysia, Singapore, Philippine, Thailand and Indonesia.

Mandarin is the widest spoken language in the world and there are about 1300 millions people worldwide. Pinyin, more formally Hanyu Pinyin, is the most common Standard Mandarin Romanization system in use. Hanyu means the Chinese language. pin means "together, connection, annotate" and yin means "sound". Pinyin uses the Latin alphabet to represent sounds in Standard Mandarin. Taiwan has adopted Tongyong Pinyin on the national level since October 2002. Tongyong Pinyin is a modified version of Hanyu Pinyin. Based on the Chinese remainder theorem, Chang and Wu (Chang & Wu, 1988) designed the hashing function to process 1303 distinct Mandarin phonetic transcriptions of Han characters.

Minnan refers to a family of Chinese languages which are spoken in southern Fujian and neighboring areas, and by descendants of emigrants from these areas in diasporas. It is usually called Taiwanese by residents of Taiwan, and Hokkien by residents of Southeast Asia. Taiwanese can be written with the Latin alphabet using a Romanized orthography which was developed first by Presbyterian missionaries in China and later by the indigenous Presbyterian Church in Taiwan; use of the orthography has been actively promoted since the late 19th century. Taiwanese is one of the most used dialects spoken in Taiwan, and evolved from the ancient languages of China, the Ho-Lo language family. According to the traditional but representative and authoritative Taiwanese dictionary (Shen, 2001), Shieh (2003) developed the hashing function of 3028 Taiwanese phonetic transcriptions of Han characters.

Hakka dialect is one of the seven major spoken dialects in Chinese Society. The Hakka language has numerous dialects spoken in southern provinces of China, Taiwan, Singapore, Philippine and Indonesia. It is the 32nd widest spoken language in the world and there are about 100 millions Hakka speakers worldwide. Hakka is not mutually intelligible with Mandarin, Cantonese, Minnan and most of the significant spoken variants of the Chinese language. The Hakka dialects across various China provinces differ phonologically, but the Meixian dialect of Hakka is considered the archetypal spoken form of the language. Shieh and Hsu (Shieh & Hsu, 2007) proposed a minimal perfect hashing function for the 1428 Hakka phonetic transcriptions of Han characters from authoritative Meixian Hakka dialect dictionary (Lee, 1995).

Many researchers have enthusiastically endeavored to study related the spoken languages subjects such as language curriculums in multicultural society (Kilimci, 2010), typologies of spoken language learning aids (Kartal, 2005), mappings between spoken language and its writing system (Chang & Wu, 1988; Shieh, 2003;

Shieh & Hsu, 2007) (as depicted in Figure 1), etc. They are striving to protect and promote their individual native cultures, and make them widespread utilization.

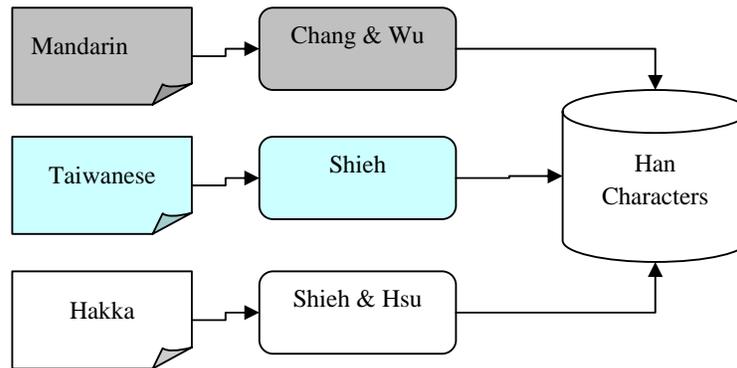


Figure 1: Various and Independent Han characters mappings for Chinese Languages

In Chinese societies, for teaching and learning facilitation purposes, we really require a convenient phonetic transcription system between daily Mandarin, Taiwanese and Hakka to speed Han characters data processing applications. These spoken languages are all with their respective Romanized phonetic transcriptions. Pleasantly surprised, the Roman spelling system is a universal tool that owns the one and only one spelling rule and can be generally and simultaneously applied to different languages applications. By studying and analyzing the Roman spelling system, we have disclosed that 4135 Romanized phonetic transcriptions can be adequately applied to handle Han characters' mappings of Mandarin, Taiwanese and Hakka spoken dialects. The 4135 integrated phonetic transcriptions are composed of 7 tones, 29 consonants, and 120 vowels at most. For language application purposes, it is much important for us to establish a mechanism to efficiently retrieve different Han characters and their corresponding pronunciations from its vocabulary repository, as illustrated in Figure 2. Many Chinese language learning applications, such as on-line or mobile dictionaries, translations, text-to-speech conversions, e-books, etc., can be further developed to help learners and teachers.

In this paper, we apply the Chinese remainder theorem to design a fast and efficient hashing function (Knuth, 1998) to map the unified 4135 phonetic transcriptions to corresponding Han characters of Mandarin, Taiwanese and Hakka languages. We also give a proof that the loading factor is more than 0.887, which is the best one when applying the Chinese remainder theorem to the design of hashing functions for the word sets.

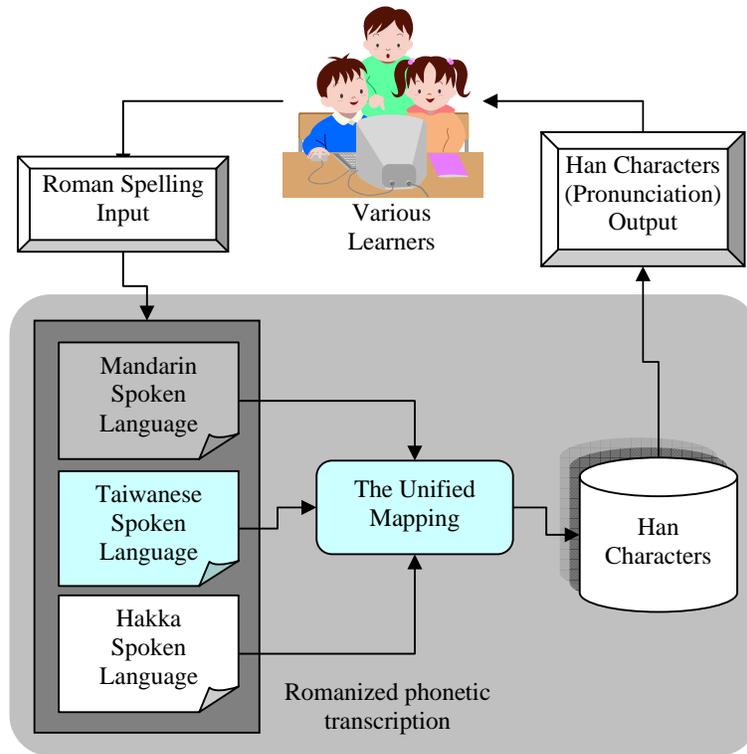


Figure 2: The Unified Mapping for Chinese Languages

Hashing Functions Based on the Chinese Remainder Theorem

In this section, we first introduce the Chinese remainder theorem and its application to hashing function designs of character data sets. Then we review the hashing function designs of Mandarin, Taiwanese and Hakka phonetic transcriptions based on the theorem.

The Chinese remainder theorem (Chang & Lee, 1986)

Theorem 1. Let r_1, r_2, \dots, r_n , be n integers. There exists an integer C such that $C=r_1 \pmod{m_1}, C=r_2 \pmod{m_2}, \dots, C=r_n \pmod{m_n}$, if m_i and m_j are relatively prime to each other for all $i \neq j$.

For example, let $r_1=1, r_2=2, r_3=3, r_4=4$ and $m_1=4, m_2=5, m_3=7, m_4=9$. Here m_i and m_j are relatively prime for $i \neq j, 1 < i, j < 4$. By the Chinese remainder theorem, there exists an integer $C=157$ such that $C \pmod{m_1}=157 \pmod{4}=1=r_1, C \pmod{m_2}=157 \pmod{5}=2=r_2, C \pmod{m_3}=157 \pmod{7}=3=r_3, C \pmod{m_4}=157 \pmod{9}=4=r_4$.

The following theorem results easily from the Chinese remainder theorem.

Theorem 2. Given a finite integer key set $K=\{L_1, L_2, \dots, L_n\}$. If L_i and L_j are relatively prime to each other for all $i \neq j$, there exists a constant C such that $h(L_i)=C \pmod{L_i}$ is a minimal perfect hashing function (Chang & Lee, 1986).

Hashing scheme based on the Chinese remainder theorem

Based on the Chinese remainder theorem, Chang and Lee (1986) proposed a letter-oriented minimal perfect hashing scheme for a set of words. For a finite word set $K=\{L_1, L_2, \dots, L_n\}$, it is heuristically assumed that there exist s_1 and s_2 such that the extracted letter pairs (L_{i1}, L_{i2}) are distinct, where L_{i1} and L_{i2} are the s_1 -th and s_2 -th characters of the word $L_i, i=1, 2, \dots, n$. Chang and Lee's hashing function is defined as $h(L_i) = H(L_{i1}, k_{i2}) = d(L_{i1}) + C(L_{i1}) \pmod{p(L_{i2})}$, where d and C are integer value functions, and p is a prime number function. Chang and Lee's applied the hashing scheme to 12 months and 9 major planets with 0.154 and 0.103 loading factors respectively.

When applying the Chinese remainder theorem to the design of letter-oriented minimal perfect hashing functions, we often encounter the intractable issue of extracting letters from the word sets to form distinct letter pairs, especially from large data sets. Chang and Shieh (1985) used a zero value rehash index to resolve the problem. They successfully applied the technique to rehash the 59 reserved words for data-flow language VAL,

the 65 Z-80 commands, and the 256 frequently used words. Furthermore, Chang and Wu (1988) utilized the characteristics of Mandarin phonetic symbols to cluster the word set and then produced 1303 distinct letter pairs. The hashing scheme is introduced in the next section.

Mandarin phonetic symbols hashing scheme (Chang & Wu, 1988)

Chinese characters are constructed by 37 Mandarin phonetic symbols accompanied by one of the five tones. There are a total of 1303 distinct Mandarin phonetic transcriptions of Chinese characters. The phonetic symbols are divided into three categories: (1) the consonant, (2) the first vowel, and (3) the second vowel. For each symbol x in the symbol set, we have its order $O(x)$. In the hashing scheme, Chang and Wu translate all the phonetic transcriptions to letter pairs of two phonetic symbols.

Chang and Wu (1988) then cluster all letter pairs according to the five tones. In each equal-tone cluster, letter pairs with the same leading character are further grouped together. We see that the maximum number of character pairs in one group might go up to 33. From the experiment, as applying the Chinese remainder theorem, this would make the constant C quite large. By dividing the character pairs into three sets, they thus can assign the least 11 prime numbers to the corresponding characters in each group of the three sets. The minimal perfect hashing function is defined as $H_j(L_{i1}, L_{i2}) = d_{jk}(L_{i1}) + C_{jk}(L_{i1}) \bmod p(L_{i2})$, where d_{jk} and C_{jk} are integer value functions of each L_{i1} in the k -th set of each j -tone cluster, and p is a prime number function of each L_{i2} . The total size of space used is $38 \cdot (3 \cdot (5 \cdot 2 + 1) + 3) + 1303 = 2671$, where 38 stands for 37 phonetic symbols and 1 dummy symbol; $3 \cdot (5 \cdot 2 + 1)$ stands for d_{jk} and C_{jk} of 5 clusters and index k in three sets. The number 3 is for the functions O , p , and W . Thus, the loading factor is about 0.4878. If only the contiguous space is considered, the size of the space that is used becomes $38 \cdot 3 \cdot 14 + 1303 = 2899$; the loading factor is about 0.45.

Taiwanese phonetic transcriptions hashing scheme (Shieh, 2003)

The Taiwanese phonetic transcription system, referred to a traditional but representative and authoritative Taiwanese dictionary (Shen, 2001), is composed of 7 tones (Table 1), 15 consonants (Table 2), and 45 vowels (Table 3). Each Taiwanese phonetic transcription consists of a vowel, a consonant, and a tone. Theoretically there are a total of 4725 transcriptions. However, only 3028 of the transcriptions are associated with Han characters. Shieh (2003) takes these 3028 transcriptions as study word set.

Table 1: Taiwanese Seven Tones

| Code k_{i1} | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---------------|-----|-----|-----|-----|-----|-----|-----|
| Tone | kun | kún | kùn | kut | kûn | kün | kuť |

Table 2: Taiwanese Fifteen Consonants

| Code k_{i2} | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
|----------------------------|-----|------|-----|-----|----|-----|-------|------|-----|----|-----|-----|----|-------|----|
| Consonant | liú | pinn | kiú | khì | tē | phó | thann | tsan | jíp | sí | ing | mng | gí | tshut | hí |
| Assigned Prime $P(k_{i2})$ | 2 | 3 | 5 | 7 | 11 | 13 | 17 | 19 | 23 | 29 | 31 | 37 | 41 | 43 | 47 |

Table 3: Taiwanese Forty-five Vowels

| Code k_{i3} | Vowel | Code k_{i3} | Vowel | Code k_{i3} | Vowel | Code k_{i3} | Vowel | Code k_{i3} | Vowel |
|---------------|-------|---------------|--------|---------------|-------|---------------|-------|---------------|--------|
| 1 | kun | 10 | kuan | 19 | kam | 28 | ka | 37 | kong |
| 2 | kian | 11 | ko | 20 | kue | 29 | ki | 38 | ń |
| 3 | kim | 12 | kiau | 21 | kang | 30 | kiu | 39 | muê |
| 4 | kui | 13 | ki | 22 | kiam | 31 | kenn | 40 | îng |
| 5 | ka | 14 | kiong | 23 | kau | 32 | kng | 41 | kiaunn |
| 6 | kan | 15 | kau | 24 | khia | 33 | kiô | 42 | tsim |
| 7 | kong | 16 | kai | 25 | kuè | 34 | kiunn | 43 | ngâu |
| 8 | kuai | 17 | kin | 26 | kam | 35 | kuan | 44 | kiann |
| 9 | king | 18 | khiong | 27 | ku | 36 | koo | 45 | kuan |

Shieh handled 3028 distinct letter pairs of (k_{i1}, k_{i2}, k_{i3}) 's, each with k_{i1} tones, k_{i2} consonants, and k_{i3} vowels. He sorted these letter pairs by their lexical orderings and then assigned each (k_{i1}, k_{i2}, k_{i3}) a unique address. According to k_{i1} , Shieh got seven groups and computed their starting addresses $d(k_{i1})$. For each group k_{i1} , based on 15 consonants, he produced 15 tone/consonant subgroups and computed their corresponding relative

subgroup starting addresses $d_{k_{i1}}(k_{i2})$. For each subgroup, there are at most 45 letter pairs. He clustered the subgroup into 5 bunches by $b(k_{i3})$ and also calculated each relative starting address $d_{k_{i1},k_{i2}}(b(k_{i3}))$, where each k_{i1} is associated with $b(k_{i3})$. Then he sequentially assigned the least 9 prime numbers $P(k_{i3})$'s to k_{i3} cyclically in each tone/consonant/vowel cluster. Finally, for every cluster, he applied the Chinese remainder theorem to compute constant $C_{k_{i1},k_{i2}}(b(k_{i3}))$ such that $C_{k_{i1},k_{i2}}(b(k_{i3})) \bmod P(k_{i3})$ equals the relative address of the cluster. The corresponding minimal perfect hashing function is defined as $H(k_{i1}, k_{i2}, k_{i3}) = d(k_{i1}) + d_{k_{i1},k_{i2}}(b(k_{i3})) + C_{k_{i1},k_{i2}}(b(k_{i3})) \bmod P(k_{i3})$. Totally, it takes 4235 spaces: 3028 for key words, 7 $d(k_{i1})$'s, $7*15 = 105$ $d_{k_{i1},k_{i2}}$'s, $7*15*5 = 525$ $d_{k_{i1},k_{i2}}(b(k_{i3}))$'s, $7*15*5 = 525$ $C_{k_{i1},k_{i2}}(b(k_{i3}))$'s and 45 $P(k_{i3})$'s. The loading factor is $3028/4235 = 0.715$.

Hakka phonetic transcriptions hashing scheme (Shieh & Hsu, 2007)

According to the selected Meixian Hakka dialect dictionary, the Hakka phonetic transcription system is composed of 6 tones (Table 4), 17 consonants (Table 5), and 72 vowels (Table 6). Each Hakka phonetic transcription consists of a tone, a consonant, and a vowel. However, only 1428 of the transcriptions are associated with Han characters. Shieh and Hsu took these 1428 transcriptions as study word set.

Table 4: Hakka Six Tones

| Tone | Yin Ping | Yang Ping | Shang | Qu | Yin Ru | Yang Ru |
|---------------|----------|-----------|-------|----|--------|---------|
| Code k_{i1} | 1 | 2 | 3 | 4 | 5 | 6 |

Table 5: Hakka Seventeen Consonants

| Consonant | p | p' | m | f | v | t | t' | n | l | ts | ts' | s | k | k' | ŋ | h | Ø |
|---------------|---|----|---|---|---|---|----|---|---|----|-----|----|----|----|----|----|----|
| Code k_{i2} | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 |

Table 6: Seventy-three Vowels

| Code k_{i3} | Vowel |
|---------------|-------|---------------|-------|---------------|-------|---------------|-------|---------------|-------|---------------|-------|
| 1 | i | 13 | eu | 25 | ok | 37 | e | 49 | ip | 61 | uo |
| 2 | au | 14 | uŋ | 26 | at | 38 | iap | 50 | əɱ | 62 | n |
| 3 | u | 15 | on | 27 | uk | 39 | iun | 51 | iai | 63 | uaŋ |
| 4 | oŋ | 16 | oi | 28 | ap | 40 | ot | 52 | ət | 64 | ep |
| 5 | a | 17 | en | 29 | it | 41 | iuk | 53 | ua | 65 | uat |
| 6 | o | 18 | iam | 30 | et | 42 | ɿ | 54 | uai | 66 | uet |
| 7 | ai | 19 | ui | 31 | ak | 43 | iet | 55 | uan | 67 | iut |
| 8 | un | 20 | aŋ | 32 | im | 44 | iak | 56 | ion | 68 | m |
| 9 | iau | 21 | iaŋ | 33 | iuŋ | 45 | iok | 57 | əp | 69 | iui |
| 10 | an | 22 | ien | 34 | ian | 46 | em | 58 | io | 70 | uen |
| 11 | in | 23 | ioŋ | 35 | ut | 47 | ən | 59 | uon | 71 | uak |
| 12 | am | 24 | iu | 36 | ia | 48 | iat | 60 | uoŋ | 72 | uok |

They handled 1428 distinct letter pairs of (k_{i1}, k_{i2}, k_{i3}) 's, each with k_{i1} tone, k_{i2} consonant, and k_{i3} vowel. Shieh and Hsu sorted these letter pairs by their lexical orderings and then assigned each (k_{i1}, k_{i2}, k_{i3}) a unique address. According to (k_{i1}, k_{i2}) , they had $6*17$ groups and compute their starting addresses $d(k_{i1}, k_{i2})$'s. Then, they assigned appropriate prime numbers $P(k_{i3})$'s for k_{i3} . Finally, for every group, they applied the Chinese remainder theorem to compute constant $C(k_{i1}, k_{i2})$ such that $C(k_{i1}, k_{i2}) \bmod P(k_{i3})$ equals the relative address of character pair (k_{i1}, k_{i2}, k_{i3}) in group headed with (k_{i1}, k_{i2}) . The corresponding minimal perfect hashing function is defined as $H(k_{i1}, k_{i2}, k_{i3}) = d(k_{i1}, k_{i2}) + C(k_{i1}, k_{i2}) \bmod P(k_{i3})$. It takes 1704 spaces: 1428 key words, $6*17$ $C(k_{i1}, k_{i2})$'s, $6*17$ $d(k_{i1}, k_{i2})$'s, and 72 $P(k_{i3})$'s. The loading factor is $1428/1704=0.838$.

The Unified Phonetic Transcription Design

Hashing Function Design

The unified Mandarin, Taiwanese and Hakka Romanized phonetic transcription is composed of 7 tones (Table 7), 29 consonants (Table 8), and 120 vowels (Table 9) associated with a prime number $P(k_{i3})$. Each phonetic transcription (k_{i1}, k_{i2}, k_{i3}) consists of a tone k_{i1} , a consonant k_{i2} , and a vowel k_{i3} . There are totally 24360 combinations of (k_{i1}, k_{i2}, k_{i3}) 's. According to our further analysis, we worked out that we can use exactly 4135 phonetic transcriptions to associate their corresponding Han characters.

Table 7: Tones

| Tone | 1 | 2 | 3 | 4 | 5 | 7 | 8 |
|------|---|---|---|---|---|---|---|
|------|---|---|---|---|---|---|---|

| | | | | | | | |
|---------------|---|---|---|---|---|---|---|
| Code k_{i1} | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---------------|---|---|---|---|---|---|---|

Table 8: 29 Consonants

| | | | | | | | | | | | | | | | |
|---------------|----|----|----|-----|----|----|-----|----|----|----|-----|----|----|------|----|
| Consonants | b | c | ch | chi | d | f | g | h | j | ji | k | kh | l | m | n |
| Code k_{i2} | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
| Consonants | ng | p | ph | r | s | sh | shi | t | th | ts | tsh | tz | v | null | |
| Code k_{i2} | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | |

Table 9: 120 Vowels

| Vowel | Code k_{i3} | $P(k_{i3})$ | Vowel | Code k_{i3} | $P(k_{i3})$ | Vowel | Code k_{i3} | $P(k_{i3})$ |
|-------|---------------|-------------|-------|---------------|-------------|-------|---------------|-------------|
| a | 1 | 2 | iat | 41 | 2 | mh | 81 | 2 |
| ah | 2 | 3 | iau | 42 | 3 | ng | 82 | 3 |
| ai | 3 | 5 | iauh | 43 | 5 | ngh | 83 | 5 |
| ainn | 4 | 7 | iaunn | 44 | 7 | o | 84 | 7 |
| ak | 5 | 11 | ie | 45 | 11 | oh | 85 | 11 |
| am | 6 | 13 | iem | 46 | 13 | oi | 86 | 13 |
| an | 7 | 17 | ien | 47 | 17 | ok | 87 | 17 |
| ang | 8 | 19 | iet | 48 | 19 | on | 88 | 19 |
| ann | 9 | 23 | ieu | 49 | 23 | ong | 89 | 23 |
| annh | 10 | 29 | ih | 50 | 29 | onn | 90 | 29 |
| ap | 11 | 31 | ii | 51 | 31 | onnh | 91 | 31 |
| at | 12 | 37 | iim | 52 | 37 | oo | 92 | 37 |
| au | 13 | 41 | iin | 53 | 41 | ot | 93 | 41 |
| auh | 14 | 43 | iip | 54 | 43 | ou | 94 | 43 |
| aunn | 15 | 47 | iit | 55 | 47 | u | 95 | 47 |
| aunnh | 16 | 53 | ik | 56 | 53 | ua | 96 | 53 |
| e | 17 | 59 | im | 57 | 59 | uah | 97 | 59 |
| eh | 18 | 61 | in | 58 | 61 | uai | 98 | 61 |
| ei | 19 | 67 | ing | 59 | 67 | uainn | 99 | 67 |
| em | 20 | 71 | inn | 60 | 71 | uan | 100 | 71 |
| en | 21 | 73 | innh | 61 | 73 | uang | 101 | 73 |
| eng | 22 | 79 | io | 62 | 79 | uann | 102 | 79 |
| enn | 23 | 83 | ioh | 63 | 83 | uat | 103 | 83 |
| ennh | 24 | 89 | iok | 64 | 89 | ue | 104 | 89 |
| ep | 25 | 97 | ion | 65 | 97 | ueh | 105 | 97 |
| er | 26 | 101 | iong | 66 | 101 | uei | 106 | 101 |
| et | 27 | 103 | iou | 67 | 103 | uen | 107 | 103 |
| eu | 28 | 107 | ip | 68 | 107 | uenn | 108 | 107 |
| i | 29 | 109 | it | 69 | 109 | uet | 109 | 109 |
| ia | 30 | 113 | iu | 70 | 113 | uh | 110 | 113 |
| iah | 31 | 127 | iuah | 71 | 127 | ui | 111 | 127 |
| iai | 32 | 131 | iue | 72 | 131 | uih | 112 | 131 |
| iak | 33 | 137 | iuah | 73 | 137 | uinn | 113 | 137 |
| iam | 34 | 139 | iui | 74 | 139 | uk | 114 | 139 |
| ian | 35 | 149 | iuk | 75 | 149 | un | 115 | 149 |
| iang | 36 | 151 | iun | 76 | 151 | ung | 116 | 151 |
| iann | 37 | 157 | iung | 77 | 157 | uo | 117 | 157 |
| iannh | 38 | 163 | iunn | 78 | 163 | ut | 118 | 163 |
| iaong | 39 | 167 | iut | 79 | 167 | yu | 119 | 167 |
| iap | 40 | 173 | m | 80 | 173 | Null | 120 | 173 |

We handle 4135 distinct letter pairs of (k_{i1}, k_{i2}, k_{i3}) 's, each with k_{i1} tone, k_{i2} consonant, and k_{i3} vowel. We sort these letter pairs by their lexical orderings and then assign each (k_{i1}, k_{i2}, k_{i3}) a unique address. According to (k_{i1}, k_{i2}) , we have $7*29$ groups and compute their starting addresses $d(k_{i1}, k_{i2})$'s. There are at most 120 characters k_{i3} in each group (k_{i1}, k_{i2}) . Then, we heuristically assign appropriate prime numbers $P(k_{i3})$'s for k_{i3} . Finally, for every group, we apply the Chinese remainder theorem to compute constant $C(k_{i1}, k_{i2})$ such that $C(k_{i1}, k_{i2}) \bmod$

$P(k_{i3})$ equals the relative address of character pair (k_{i1}, k_{i2}, k_{i3}) in group headed with (k_{i1}, k_{i2}) . The corresponding minimal perfect hashing function is defined as $H(k_{i1}, k_{i2}, k_{i3}) = d(k_{i1}, k_{i2}) + C(k_{i1}, k_{i2}) \bmod P(k_{i3})$.

The hashing function design of the unified phonetic transcription is summarized as follows:

Step 1: Using tone k_{i1} , consonant k_{i2} and vowel k_{i3} , we can have 4135 distinct letter pairs (k_{i1}, k_{i2}, k_{i3}) . We sort them in their lexical orders and assign each a unique address.

Step 2: We allocate each (k_{i1}, k_{i2}) group a $d(k_{i1}, k_{i2})$, the first address of the letter pairs headed with (k_{i1}, k_{i2}) 's.

Step 3: Associated with each group (k_{i1}, k_{i2}) , we assign each (k_{i1}, k_{i2}, k_{i3}) a relative address.

Step 4: We assign appropriately the prime numbers $P(k_{i3})$'s to k_{i3} .

Step 5: Consider the letter pairs (k_{i1}, k_{i2}, k_r) , $1 \leq r \leq n$, with the same (k_{i1}, k_{i2}) , that is they are in the same group (k_{i1}, k_{i2}) , and the corresponding assigned prime numbers of k_r 's are P_1, P_2, \dots, P_n , where $P_1 < P_2 < \dots < P_n$.

Applying the Chinese remainder theorem to find a constant $C(k_{i1}, k_{i2})$ such that $C(k_{i1}, k_{i2}) \equiv 1 \pmod{P_1}$, $C(k_{i1}, k_{i2}) \equiv 2 \pmod{P_2}$, ..., and $C(k_{i1}, k_{i2}) \equiv n \pmod{P_n}$. Our proposed minimal perfect hashing function is simply defined as $H(k_{i1}, k_{i2}, k_r) = d(k_{i1}, k_{i2}) + C(k_{i1}, k_{i2}) \bmod P(k_r)$. The values of all $C(k_{i1}, k_{i2})$'s are illustrated in the Appendix.

Loading Factor Comparisons

Loading factor is used to measure the efficiency of memory usage in hashing design. It is defined as a ration of the number of data and the total size of memory used. The loading factor of the hashing function designed in this paper is derived as follows: (1) Used spaces, 4135 key words, 7×29 $C(k_{i1}, k_{i2})$'s, 7×29 $d(k_{i1}, k_{i2})$'s, and 120 $P(k_{i3})$'s. We take 4661 spaces in total. (2) The loading factor is $4135/4661=0.887$. The following table (Table 10) shows the loading factors of various minimal perfect hashing functions designed for diverse word sets by the Chinese remainder theorem. Obviously, it can be shown that our hashing function is superior to others.

Table 10: Comparisons of Loading Factors

| Word Sets | Loading Factor | |
|---|----------------|---------------------|
| Names of 12 months | 0.154 | Chang & Lee, 1986 |
| 59 VAL reserved words | 0.312 | Chang & Shieh, 1985 |
| 65 Z-80 commands | 0.263 | Chang & Shieh, 1985 |
| 256 frequently used words | 0.472 | Chang & Shieh, 1985 |
| 1303 Mandarin phonetic transcriptions | 0.448 | Chang & Wu, 1988 |
| 3028 Taiwanese phonetic transcriptions | 0.715 | Shieh, 2003 |
| 1428 Hakka dialect phonetic transcriptions | 0.838 | Shieh & Hsu, 2007 |
| 4135 unified Mandarin, Taiwanese and Hakka phonetic transcription | 0.887 | This paper |

Number C Analysis

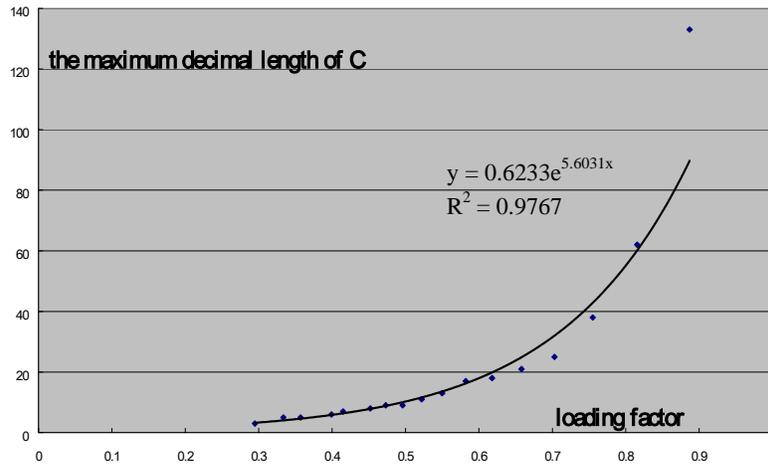
The numbers C 's are the most intractable ones as applying the Chinese remainder theorem to design hashing functions for data sets. On observing variations of C 's resulted by the experimental designs for the unified Romanized phonetic transcriptions, we have concluded that the size of C is dependent on the number of associated primes that we have used in each vowel group. In fact, during the hashing design, we can group k_{i3} 's vowels for each (k_{i1}, k_{i2}) in different sizes to have alternate C 's results. The smaller size each vowel group has, the smaller constant C we result. However, what we should pay for is loading factor. There is a tradeoff between the size of constant C and the loading factor. The following table (Table 11) shows the experiments.

Table 11: The C 's and Loading Factors of Different Groups

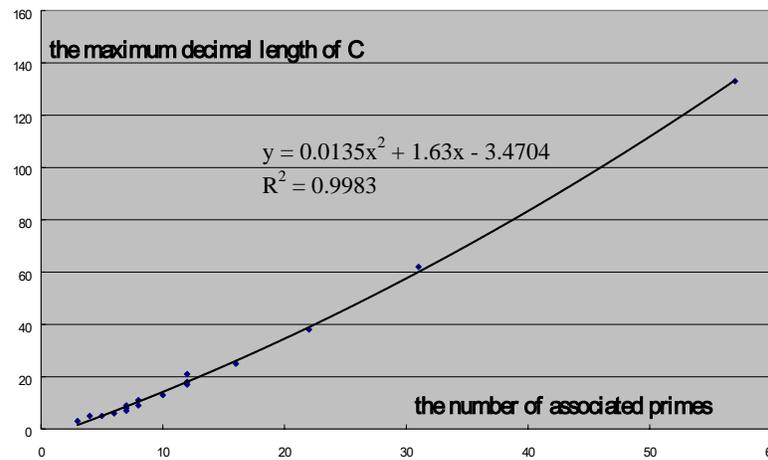
| Number of vowel groups | Number of associated primes | Loading factors | The maximum length of C 's |
|------------------------|-----------------------------|-----------------|------------------------------|
| 1 | 57 | 0.887 | 133 |
| 2 | 31 | 0.816 | 62 |
| 3 | 22 | 0.755 | 38 |
| 4 | 16 | 0.703 | 25 |
| 5 | 12 | 0.658 | 21 |
| 6 | 12 | 0.618 | 18 |
| 7 | 12 | 0.582 | 17 |
| 8 | 10 | 0.550 | 13 |
| 9 | 8 | 0.522 | 11 |
| 10 | 7 | 0.496 | 9 |
| 11 | 8 | 0.473 | 9 |
| 12 | 7 | 0.452 | 8 |
| 14 | 7 | 0.415 | 7 |
| 15 | 6 | 0.399 | 6 |

| | | | |
|----|---|-------|---|
| 18 | 4 | 0.357 | 5 |
| 20 | 5 | 0.334 | 5 |
| 24 | 3 | 0.295 | 3 |

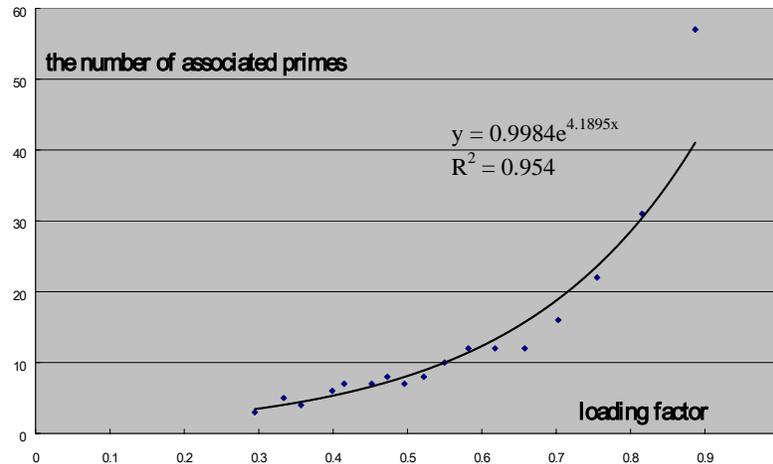
Next, we apply the statistical regression analysis to the experimental data to profile the correlations between the above parameters: loading factor vs. the decimal length of constant C, the number of associated primes vs. the decimal length of C and loading factor vs. the number of associated primes. The results are shown in the following Figure 3, where R^2 is the coefficient of determination and its value is between 0 and 1.



(a) Loading factor and the maximum decimal length of C



(b) The number of associated primes and the maximum decimal length of C



(c) Loading factor and the number of associated primes

Figure 3: The Regression Analysis of Effective Factors .

What the preferred situation is to have high loading factor with short decimal length of C. However, from the regression analysis, we have the fact that there is a tradeoff between two parameters. This will give us the concrete suggestion as we apply the unified phonetic transcription on diverse learning and teaching devices.

CONCLUSIONS

With the unified phonetic transcription system for mapping Mandarin, Taiwanese and Hakka mother tongue languages to their Han characters, people will be convenient to promote their learning and teaching activities, as well as to record and preserve their particular cultures. In this research, we have successfully applied the Chinese remainder theorem to design a novel minimal perfect hashing function for 4135 integrated Mandarin, Taiwanese and Hakka Romanized phonetic transcriptions of Han characters. We have achieved significant results in terms of loading factors. We further give experimental investigation and mathematical regression analysis for considered factors of hashing effectiveness. We get the conclusion that the size of number C is dependent on the number of associated primes. For the unified Romanized phonetic transcriptions case, we propose the grouping technique to promote the effective applications as concerning the practicability of accessing constants C's. However, we have explored that there is a tradeoff between the loading factor and the size of C. The unified phonetic transcription can be used to promote Chinese mother tongue languages applications and can be applied as a tool to popularize digital learning of the languages further.

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Appendix: The Values of C(k₁₁, k₁₂)'s

| Tone k_{i1} | Vowel k_{i2} | d Address | $C(k_{i1}, k_{i2})$ Value |
|---------------|----------------|-------------|--|
| 1 | 1 | 0 | 64944529802520314615071967316445377 |
| 1 | 3 | 18 | 22072260041420641168670456121911469647357 |
| 1 | 4 | 37 | 2341262191069822373794511463164075 |
| 1 | 5 | 51 | 17826075993273201357220661350819824012907 |
| 1 | 6 | 70 | 36750251768899802683830396935 |
| 1 | 7 | 84 | 853423971975501060369045480800064457 |
| 1 | 8 | 102 | 3230888234697313835737580133054940934078475906950 6405134158841920335470218544507965940504191556559 11774608617063627 |
| 1 | 9 | 153 | 2811216111161579226740265389366581840581217 |
| 1 | 10 | 173 | 2341262191069822373794511463164075 |
| 1 | 11 | 187 | 5580647162611650774150618419340213976176344914817 6938660322244270242937251421132853009341113616024 428807332733392463007 |
| 1 | 12 | 239 | 5992871137556135789475657067386415924456169544983 3894341282590670541979389250807654745457 |
| 1 | 13 | 279 | 2419721735365016321777114959630980944915355085821 32838475657383488099190217 |
| 1 | 14 | 313 | 3977418278170894117865883174693348709678451922127 67 |
| 1 | 15 | 339 | 696097658238809533265458610083531288767 |
| 1 | 16 | 359 | 837645492417954537087209440 |
| 1 | 17 | 371 | 9256816887250690574107803504585337791212251648534 025093024420752547647 |
| 1 | 18 | 403 | 5302917695889361096059961272251746978804318170782 2176722777 |
| 1 | 19 | 430 | 1 |
| 1 | 20 | 431 | 9139333575553435344217573681380336055117755848887 4096007881464715928500115628213403688313249448958 96180387845109617 |
| 1 | 21 | 482 | 295741854374958593532533406802157 |
| 1 | 22 | 498 | 2341262191069822373794511463164075 |
| 1 | 23 | 512 | 7107816320470116249432012767652915896330712723618 0232942362905093585975451695572232441855417 |
| 1 | 24 | 554 | 1015974035642889592010734726566928011766396482742 4965345545544520122099986242077 |
| 1 | 25 | 590 | 2568083120669429978818728146297905012787131406591 3680434272401072566577907876566001440722639060786 2742466451591047 |
| 1 | 26 | 640 | 4964913407385885314546459313197740006643945653388 43097797541044913863962802531791645790796217 |
| 1 | 27 | 682 | 71711593239345115255790093017 |
| 1 | 28 | 696 | 12678327602740390681 |
| 1 | 29 | 705 | 2544866659299794137538047059881254367721733177260 582332282721875158327900530386438590267244477609 41810631355723228823155204875566007 |
| 2 | 1 | 762 | 2659659253858485141086416953487398456564240347 |
| 2 | 3 | 784 | 4081499448220247076582253363034397 |

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|---|----|------|--|
| 2 | 4 | 800 | 22492123753135128914677172498929 |
| 2 | 5 | 813 | 18918548605449 |
| 2 | 6 | 820 | 15656398053976404884240051025293 |
| 2 | 7 | 835 | 32478880744933868806045498990713283027 |
| 2 | 8 | 852 | 4514287264331388834844767949128952386082923444244 662398453514708904398465937552711849399021517 |
| 2 | 9 | 893 | 5576652203238090897311189140037 |
| 2 | 10 | 908 | 46991006860557 |
| 2 | 11 | 914 | 7839378807204054516147988363233949331261449480452 1543383818411446409126296404252400573506901816689 336887 |
| 2 | 12 | 960 | 7712718818069213264262424523835158900799430218587 906050597966600767581607 |
| 2 | 13 | 993 | 1514316853780982837710041954454136582163357047154 489327522740884312427908550466792087 |
| 2 | 14 | 1031 | 4659130194219555724727590242215279123970263126843 386987416675537 |
| 2 | 15 | 1062 | 1597654559044358957796421802051547923604424138759 4606870299138871861 |
| 2 | 16 | 1094 | 188459278322167127153310528405995558315107 |
| 2 | 17 | 1113 | 7703351501819123908547394978381198438297936947366 177839853089887 |
| 2 | 18 | 1143 | 33551547097592679892970230932194841 |
| 2 | 19 | 1159 | 2966945680576791803941513 |
| 2 | 20 | 1170 | 9552262579405222272595537823100916958389536384672 0670887183208686293110648399890673471687 |
| 2 | 21 | 1210 | 80206105335638512818 |
| 2 | 22 | 1219 | 8100153285279498757088758568809 |
| 2 | 23 | 1232 | 3967019274860162899246388867812728434169084145156 22392718841084984610278516803037 |
| 2 | 24 | 1270 | 7291090621590647041439072207814174065327563872138 429998946 |
| 2 | 25 | 1297 | 2360804216443227435798667343668479052493606886105 2117344284790568803024813899597787348158445339764 18337 |
| 2 | 26 | 1344 | 1623532452270782186281332385066015582425401938240 4065514604339945261215768313481429641647 |
| 2 | 27 | 1383 | 879162072665 |
| 2 | 28 | 1390 | 1610599946459446681057 |
| 2 | 29 | 1400 | 1083206920889392469867123942521147945249568040525 7462415907115669137260295506321932051196389762439 74173060877007869777 |
| 3 | 1 | 1451 | 1749763638869608412023452687 |
| 3 | 3 | 1466 | 322241013048720509746348605415 |
| 3 | 4 | 1481 | 38996819222356748983769636 |
| 3 | 5 | 1492 | 1283654813299438293278779569397703907 |
| 3 | 6 | 1510 | 12944896164105491213843598330327 |
| 3 | 7 | 1526 | 115686602028041351850558986590963196427080136 |
| 3 | 8 | 1546 | 3357737135958534201527982731928588947077811758764 1764645865158324866160558099879928133577 |

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| 3 | 9 | 1585 | 112519155483150811495060023100337397 |
| 3 | 10 | 1602 | 7375532636858408518398308344372 |
| 3 | 11 | 1615 | 1684783329030632754731377940271226718883299528812 2796893082348548895450031281698617653759518975954 179726981687 |
| 3 | 12 | 1663 | 7452694336649028941633590459945878578444992496635 454164446302943434372609536040359022872257447637 |
| 3 | 13 | 1706 | 1107987255140470453405591416742240701173456169477 31252124664935737261232456137 |
| 3 | 14 | 1741 | 114671662452749816364636647442934425043646555139 7 |
| 3 | 15 | 1765 | 9378386571883512292646043084519325601841982717522 457 |
| 3 | 16 | 1790 | 129857631503867593810398312613400562127 |
| 3 | 17 | 1809 | 1324242754488800281714359122126851640079844770095 9730454708332410405727 |
| 3 | 18 | 1841 | 1300680649457591397293105543898150135112943781218 234973749353742267 |
| 3 | 19 | 1871 | 1938060110162566238 |
| 3 | 20 | 1880 | 1988671469739037638239403554225348077743556316160 6221940277390005968431648205240776763900096086435 69925366209153517 |
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| 3 | 22 | 1946 | 109377925939343038412220929 |
| 3 | 23 | 1957 | 5886821438505272931573958023368272745762577408572 173635907458823399172815340460914062560867 |
| 3 | 24 | 1998 | 1811038696058785063155490515767440481494605441286 52455346751735217536204765697 |
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| 3 | 26 | 2078 | 1215486198672120342411177158581553550434181396583 06461856626751777405797099461521832119009777 |
| 3 | 27 | 2119 | 37799600695088183854976164207 |
| 3 | 28 | 2133 | 16543624983193568638265 |
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| 4 | 1 | 2199 | 79913438279382459481489364265610711902572513 |
| 4 | 3 | 2222 | 724221717480452268740291021217 |
| 4 | 4 | 2237 | 6769896829431796783602782 |
| 4 | 5 | 2248 | 6980762466230080398861812412681587787 |
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| 4 | 9 | 2342 | 1794955116411917954709185512823943437 |
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| 4 | 11 | 2373 | 4780891390647215639980960268523434534887533915758 705362397056228729721990995951667915520218842 |

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| 4 | 12 | 2415 | 26795197191227870856298511674858202168681232360269914 |
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| 4 | 16 | 2532 | 28800703500190574616359876 |
| 4 | 17 | 2543 | 2610757393422535143644715869006825249716688643336441634608435164690063 |
| 4 | 18 | 2577 | 1255375958567247599798506332145332808 |
| 4 | 19 | 2594 | 55733162486284969083133 |
| 4 | 20 | 2604 | 43665913080555789728531326382132817474615494629781519356837546771964344981032202775906519620257 |
| 4 | 21 | 2647 | 155599977486564813010145580993355477 |
| 4 | 22 | 2664 | 2341262191069822373794511463164075 |
| 4 | 23 | 2678 | 56489372725693140581528568809536440159445884725284672118178475285043823045997390009233428753 |
| 4 | 24 | 2721 | 619401890263600802564735186852281568028143620 |
| 4 | 25 | 2742 | 15994428068795413828218144417715498169511362061528447687174623669850044300383819359940711176127 |
| 4 | 26 | 2784 | 117795152091889881210962711056161633914832844014897391 |
| 4 | 27 | 2808 | 124984874469926146257810472496 |
| 4 | 28 | 2822 | 5257164150129 |
| 4 | 29 | 2828 | 1623574972742378109983811417825200303692565072216639931617246225998695859273054766918434544702661591292579522920835441200893 |
| 5 | 1 | 2883 | 168878377699201632388054139240250981910241757 |
| 5 | 3 | 2904 | 1 |
| 5 | 4 | 2905 | 1 |
| 5 | 5 | 2906 | 1 |
| 5 | 6 | 2907 | 338826979773502888683565267 |
| 5 | 7 | 2921 | 546346856133854386461919403582789822821185162228595017 |
| 5 | 8 | 2946 | 2005397379617378022596671315414152052591005045578311624782663720962125478217362238156938751723125927 |
| 5 | 9 | 2991 | 2403837296262839863720324972 |
| 5 | 10 | 3003 | 20324972 |
| 5 | 11 | 3005 | 1526071016818703996330628204219975979083131452418117891912536510252620342305684332540851 |
| 5 | 12 | 3043 | 862323498696940745456525981650874688957556440144267190379151690016809838325631 |
| 5 | 13 | 3077 | 127673445075430236388007678348014265807775642697937591767821601530826359378197 |
| 5 | 14 | 3112 | 30539680348545329986649631833498718169991124565798288743276167 |
| 5 | 15 | 3141 | 9726901571118510230101687655349575127085799287 |

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| 5 | 16 | 3165 | 105872856608865224362358635091168325001400043541623189878881177 |
| 5 | 17 | 3194 | 1669521193801301929676310284362728611750183160207 |
| 5 | 18 | 3217 | 39456471054531983303920781002795535348650486549280944903607 |
| 5 | 20 | 3244 | 4952424038687225802218686195906740998248491334207651265315626967369909192024366546331790087 |
| 5 | 21 | 3285 | 31790087 |
| 5 | 22 | 3287 | 1 |
| 5 | 23 | 3288 | 179353023360131962892399509934221337210179621656370244686780813642822743121549977727 |
| 5 | 24 | 3326 | 1872459309563264049880522527678371796921339523547477369476731761841687 |
| 5 | 25 | 3358 | 62601391713058698035238897750198085861141820152628497996105936522158303747192330346427 |
| 5 | 26 | 3398 | 545554098200366892496794396950967555161884056444619587690053613027588611174921537280818084507 |
| 5 | 27 | 3440 | 1 |
| 5 | 28 | 3441 | 6284602471488624036687 |
| 5 | 29 | 3452 | 1063512091338202606821365361821105909555754133304111818326841617258522488818595962548625584587 |
| 6 | 1 | 3494 | 991291924043032694366940565068354227143 |
| 6 | 7 | 3512 | 189335613045177604054307401009276517 |
| 6 | 8 | 3529 | 152481775154523581819612252277908510306708217724519932343078540547 |
| 6 | 9 | 3559 | 77231424308070559 |
| 6 | 11 | 3566 | 59401758626585618088870537074886995150214030086871800571087 |
| 6 | 12 | 3592 | 35412161870767049414814 |
| 6 | 13 | 3602 | 1522525697674741080103176998268905023366602539287170193797867 |
| 6 | 14 | 3630 | 1032697793415830176466938 |
| 6 | 15 | 3641 | 289578503341955866322657 |
| 6 | 16 | 3653 | 3006263 |
| 6 | 17 | 3657 | 13859744833169121680225830534372541480386084855987 |
| 6 | 18 | 3680 | 98334985008736815942514736877516469074908177 |
| 6 | 20 | 3701 | 849461730393323420807603036505757830881548431298998996 |
| 6 | 23 | 3724 | 375340389653771341598367755063422370079071990209520661495865142500727 |
| 6 | 24 | 3756 | 6337471000405276586237115312859281 |
| 6 | 25 | 3772 | 96270146771858984062822000501356658417545186937843951171786971032177 |
| 6 | 26 | 3803 | 801128368398242695123417 |
| 6 | 29 | 3816 | 5189813134507463236347957040810627329486403882309551659472936601273 |
| 7 | 1 | 3846 | 214643791629891130652686735759237 |
| 7 | 6 | 3861 | 182287063653 |
| 7 | 7 | 3866 | 5068148460282900594363322574201 |

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| 7 | 8 | 3879 | 73682591563608850472748865300829284731733 |
| 7 | 9 | 3898 | 3452032766467 |
| 7 | 11 | 3903 | 124403403267415881506160389626 |
| 7 | 12 | 3916 | 4935409942089541429388313310860735781 |
| 7 | 13 | 3933 | 4482628447566173767329199931470555272675772565355 70497949 |
| 7 | 14 | 3959 | 2562991393797764131711391 |
| 7 | 15 | 3971 | 1661275737352881 |
| 7 | 16 | 3978 | 1456363052715719456 |
| 7 | 17 | 3986 | 12172994738807119694929487674 |
| 7 | 18 | 3999 | 16222681315494410513594737121487 |
| 7 | 20 | 4013 | 3718892506217348358334646674903246426744707814296 385 |
| 7 | 23 | 4036 | 7166523818239668976161736537297480181038766848 |
| 7 | 24 | 4057 | 200218239419106128490513795004 |
| 7 | 25 | 4071 | 1613452437872808691961211787657095315483678643035 81037 |
| 7 | 26 | 4095 | 33287520344187096554000336156494463862 |
| 7 | 28 | 4112 | 175858486 |
| 7 | 29 | 4116 | 10966326177647918930460307313434677092766934 |