Informally Multiplying the World of Jillian Jiggs

PAUL BETTS and AMANDA CRAMPTON describe how children develop rich understandings of multiplication by experiencing various representations, including repeated addition, equal grouping, and combinatorial situations.

Introduction

Mathematics education organisations such as the Australian Education Council (1991) and the National Council of Teachers of Mathematics (2000) advocate for reform of mathematics teaching, grounded in a social constructivist view of teaching, learning and knowledge. In this paper, we describe a reform-based activity concerning multiplication, developed within the context of the children's story *The Wonderful Pigs of Jillian Jiggs* by Phoebe Gilman (1988). We also provide vignettes of informal multiplicative thinking by Grade 2/3 children that occur during these activities. The informal multiplicative experiences of these children suggest to us that children can begin to construct informal understandings of multiplication, which provide a foundation for later formal experiences of multiplication.

Reform-based mathematics instruction is built on two principles. Children are mathematicians who, given the opportunity, actively construct mathematical meanings; and this activity is social. Vygotsky (1978) distinguishes between children's informal understandings, which are grounded in context and are intuitive; and formal understandings, which are socially accepted scientific and/or abstract representations of knowledge. Schooling, then, is the process by which learners, in social interaction, move from their informal observations of social reality to legitimate constructions of socially accepted formalisations of mathematical knowledge.

In our work as teachers, we have been attending to this shift from informal to formal thinking for the specific case of multiplication. Greer (1994) described various representations of multiplication that children should experience within K–12 mathematics. These include repeated addition, equal grouping, combinations (branching), folding, layering, area-producing, array-making, scaling/slope, proportioning, and stretching/compressing. Combinations, for example, is a formalised representation of multiplication because it is equivalent to equal groups. If we roll a die and flip a coin, for example, the possible outcomes are indicated in Figure 1.

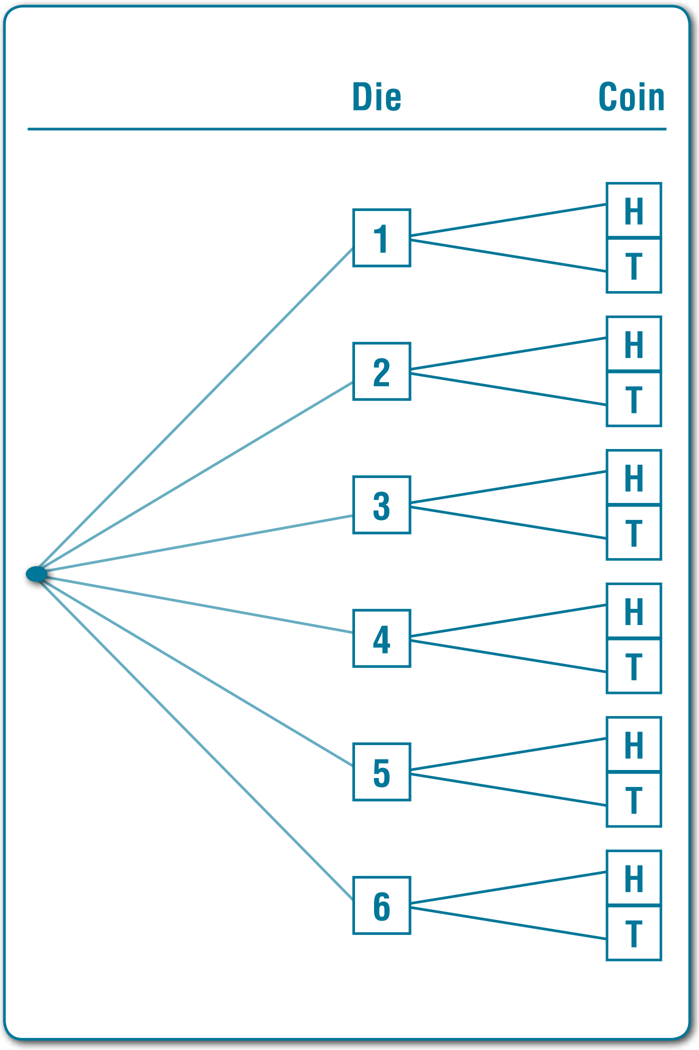

Figure 1. Branching diagram for rolling a die and flipping a coin.

There are 6 × 2 = 12 possible outcomes (e.g., 1-H, 1-T, 2-H, 2-T, etc.). The branching diagram in Figure 1 organises these 12 outcomes in six groups of two.

Children's literature provides a context for mathematical thinking, investigation and inquiry (Whitin & Whitin, 2004). During the story *The Wonderful Pigs of Jillian Jiggs*, the main character, Jillian, makes a pig from craft supplies. All of her friends want a pig too, so Jillian goes into the business of making and selling pigs; but each pig is unique and special to Jillian, so in the end she decides not to sell any of her pigs. There are numerous opportunities with the story to occasion mathematical thinking, such as counting, number operations and patterning. In this paper, we focus on attending to informal thinking of children as they engage with a combinatorial context for multiplication motivated by this story. We describe an activity and the responses of children in a Grade 2/3 class to this activity.

Using Jillian Jiggs to motivate combinations

During the story, Jillian considers the clothing that her pigs will wear, which provides an opportunity to think about combinations of outfits. We posed the following question to our students:

> Jillian's mother bought her blue cloth, yellow cloth and purple cloth so that she could make winter scarves and hats for her pigs. How many different winter outfits [scarf and hat] can Jillian make for her pigs?

We provided the students with six strips of coloured paper, each with 10 pictures of one item of clothing (i.e., blue scarves, blue hats, yellow scarves, yellow hats, purple scarves, purple hats). We also provided the students a page with pictures of 15 pigs. We emphasised that these supplies might or might not be enough to make all possible outfits. We instructed students to cut out the pigs and make outfits, and then glue the outfits onto a separate recording sheet.

We observed several types of responses to the situation. Many students started by randomly making outfits, while some started with all the same colour outfits. Many students quickly responded that they were done, even though they had only three or four outfits. We encouraged students to think further by asking if they were sure they had all possible outfits (later in the activity, we also asked if they had repeated the same outfit more than once). Although we observed various approaches, all converged on one of three possibilities, two of which illustrated evidence that the students informally experienced a combination representation of multiplication while solving the problem. In what follows, we describe examples of student work for each of these three possibilities.

The first possibility involved the use of an organised procedure for making every outfit exactly once. One student, Alice (all names of children are pseudonyms), was the only one who started with an organised approach. She reasoned that an outfit with a blue hat could have either a blue, yellow or purple scarf. On her recording sheet, Alice glued these three outfits in a row. Her second row showed all three outfits with a yellow hat (starting with a yellow scarf), and her third row showed all three outfits with a purple hat (starting with a purple scarf). We were surprised that a student generated this organised thinking so quickly. Other students, with some guidance also produced this organised approach, but their recording sheets looked random and their thinking was only evident through verbal explanations. These students informally experienced a 3 × 3 multiplicative representation of outfit combinations, some of whom explicitly recognised such a structure.

The second possibility involved focussing on one-colour outfits versus two-colour outfits. Beatrice, for example, started with all same-colour outfits and claimed she was done. We encouraged her by asking,

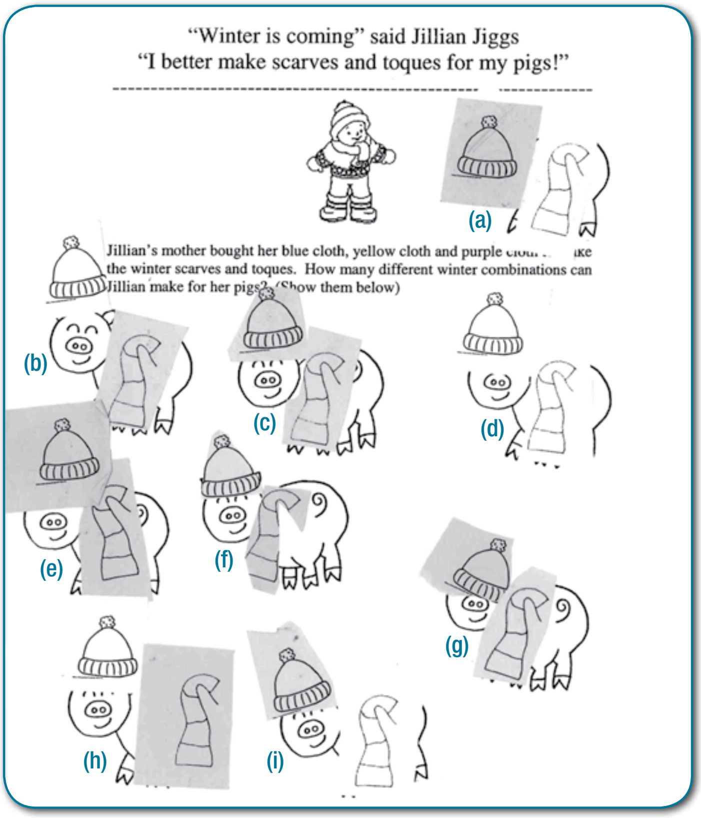

Figure 2. Beatrice's recording sheet: (a) pink hat, yellow scarf (b) yellow hat, blue scarf (c) blue hat, blue scarf (d) yellow hat, yellow scarf (e) pink hat, pink scarf (f) blue hat, pink scarf (g) pink hat, blue scarf (h) yellow hat, pink scarf (i) blue hat, yellow scarf.

"What about an outfit with a yellow hat and purple scarf?" Later, we needed to encourage further thinking by Beatrice by suggesting she had repeated an outfit. When Beatrice found the repeated output, she realised that she needed to organise her collection of outfits. When asked to explain her finished work, Beatrice said, "I put them in groups of two." While she pointed to her work (see Figure 2), she explained how the first pair had a blue scarf, the second pair had a yellow scarf (top right of recording sheet), and the third pair had a purple scarf (second row). Each pair had a one-colour outfit and a two-colours outfit, and both outfits in a pair had the same colour scarf. When asked about the final three outfits (bottom row) Beatrice explained that she checked to make sure she had all the outfits. After finding the final three, she was convinced she had all possible outfits (it may have been that she was convinced because she knew from other students that there were nine outfits and she was sure that she had no repeat outfits).

Beatrice did not generate a systematic list like Alice's, but her thinking is organised so that informal experience with a combinations representation of multiplication is still evident. In recognising that she needed a one-colour and two-colour outfit for each colour of scarf, Beatrice represented an incomplete collection of outfits with an informal experience of 2×3 (two outfits in each of three groups). In searching for missing outfits, Beatrice informally constructed a $2 \times 3 + 3$ representation of the situation.

Several students produced organised lists in ways that mapped onto a blended additive and multiplicative structure, similar to Beatrice. Another approach we observed involved separately listing all same coloured and all different coloured outfits. This list is an informal experience of $3 \times 1 + 3 \times 2$ (i.e., 3 colours for scarf $\times$ 1 possible same colours for hat + 3 colours for scarf $\times$ 2 possible different colours for hat). These students usually reasoned that they had all outfits because the different coloured outfits were listed by considering the two possibilities needed to make an outfit when one article of clothing was chosen. In particular, they have shifted their attention from counting singles or adding to making multiple combinations. When these students tried to organise their list of possible outfits, they informally represented multiplication as a combination.

The final possibility involved those students who did not seem to experience a multiplicative structure when they explored the problem. These students' recording sheets appeared random (see Figure 3), and their explanations did not move beyond, "I just keep going and going," which suggested that they did not recognise a need to organise their list of outfits. It may be that these students are not developmentally ready to notice combinations, or perhaps we were unable to provide the kinds of scaffolds for these students to begin organising their thinking, which is a necessary step toward informally experiencing the multiplicative structure of this problem. At the very least, these students successful created and counted a list of distinct outfits.

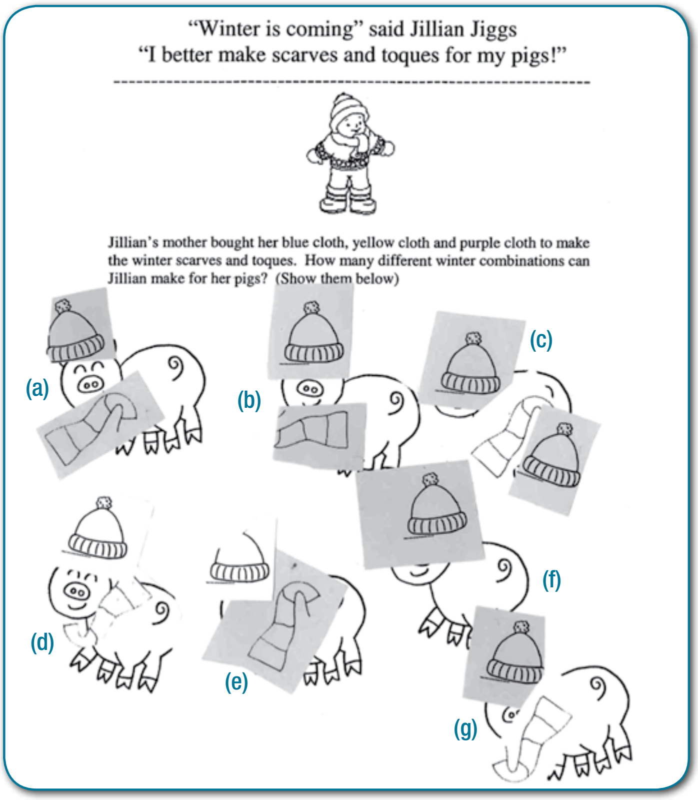

Figure 3. Random outfits: (a) pink hat, blue scarf (b) blue hat, pink scarf (c) blue hat, yellow scarf, blue hat (d) yellow hat, yellow scarf (e) yellow hat, blue scarf (f) pink hat (g) pink hat, yellow scarf.

Conclusion

The above activity occasioned an opportunity for many students to informally experience multiplication. The key feature of this activity, we believe, was that our questioning fostered a shift in student thinking from making a random list to generating an organised list of outfits. By organising their outfits, the students were able to justify when they had a completed list. It is in organising and justifying that the students informally experienced a combinations representation of multiplication. Subsequent activities could continue to build an informal base of experiences with multiplication, which is a necessary foundation for shifting from informal to formal understandings of multiplication.

The informal understandings we are describing, based on Vygotsky's distinction between informal and formal, are distinct from the formal computational fluency described by others. Bobis (2007), for example, describes a pathway for shifting strategy use by students from inefficient to more efficient multi-digit multiplication processes, by building on conceptual and skill-based knowledge of single-digit multiplication. All of the processes, understandings and skills described by Bobis are formal experiences with multiplication. We are suggesting that these formal experiences should be built on various, rich and repeated informal experiences with multiplication, of which the activity described in this article is one example.

Attending to informal mathematical experiences has potentially shifted our perceptions of planning. We have always assumed that the informal understandings of children developed from out-of-school ad hoc experiences, and that teachers needed to discover these experiences in order to build on them. However, we have realised another possibility: that teachers could design in-school opportunities for children's informal experiences. The activity above occasioned the use of organised lists and justification, both of which are mathematical processes fundamental to mathematics instruction. We wonder if contextualised mathematics activities, where mathematical processes emerge, might naturally occasion opportunities for students to informally experience a mathematical concept. Planning should be anchored in children's prior experience, which teachers still seek to draw on; but perhaps we can deliberately design various activities intended to enrich the informal experiences of children, which would be an intermediate step toward activities designed around formalising student understandings. An enriched informal experience base would provide a stronger foundation for helping students shift from informal to formal constructions of mathematics concepts. We are learning that reform-based mathematics is more than just a sequence of rich mathematical activities; teaching a sequence of specific outcomes is replaced by experiences with doing mathematics informally, which can provide a foundation for subsequent formal mathematical experiences.

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