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You Asked Open-Ended Questions, Now What? Understanding the Nature of Stumbling Blocks in Teaching Inquiry Lessons

Noriyuki Inoue & Sandy Buczynski

Undergraduate preservice teachers face many challenges implementing inquiry pedagogy in mathematics lessons. This study provides a step-by-step case analysis of an undergraduate preservice teacher's actions and responses while teaching an inquiry lesson during a summer math camp for grade 3-6 students conducted at a university. Stumbling blocks that hindered achievement of the overall goals of the inquiry lesson emerged when the preservice teacher asked open-ended questions and learners gave diverse, unexpected responses. Because no prior thought was given to possible student answers, the preservice teacher was not equipped to give pedagogically meaningful responses to her students. Often, the preservice teacher simply ignored the unanticipated responses, impeding the students' meaning-making attempts. Based on emergent stumbling blocks observed, this study recommends that teacher educators focus novice teacher preparation in the areas of a) anticipating possibilities in students' diverse responses, b) giving pedagogically meaningful explanations that bridge mathematical content to students' thinking, and c) in-depth, structured reflection of teacher performance and teacher response to students' thinking.

The things we have to learn before we do them, we learn by doing them.

-Aristotle

Many school reform efforts confirm the importance of inquiry-based learning activities in which students serve as active agents of learning, capable of constructing meaning from information, rather than as passive recipients of content matter (Gephard, 2006; Green & Gredler, 2002; National Council of Teachers of Mathematics, 1989, 2000; National Research Council [NRC], 2000). In inquiry-based mathematics lessons, students are guided to engage in socially and personally meaningful constructions of knowledge as they solve mathematically rich, open-ended problems.

Van de Walle (2004) emphasizes that conjecturing, inventing, and problem solving are at the heart of inquiry-based mathematics instruction. In inquiry-based lessons, students develop, carry out, and reflect on their own multiple solution strategies to arrive at a correct answer that makes sense to them, rather than following the teacher's prescribed series of steps to arrive at the correct answer (Davis, Maher, & Noddings, 1990; Foss & Kleinsasser, 1996; Klein, 1997). Inquiry-based lessons can be structured on a continuum from guided inquiry, with more direction from the teacher and a small amount of learner self-direction, to open inquiry, where sole responsibility for problem solving lies with learner.

In order to deliver an effective inquiry lesson, a set of general principles typically suggested in pedagogy textbooks are (a) to start the lesson from a meaningful formulation of a problem or question that is relevant to students' interests and everyday experiences; (b) to ask open-ended questions, thus providing students with an opportunity to blend new knowledge with their prior knowledge; (c) to guide students to decide what answers are best by giving priority to evidence in responding to their questions; (d) to promote exchanges of different perspectives while encouraging students to formulate explanations from evidence; and (e) to provide opportunities for learners to connect explanations to conceptual understanding (e.g., NRC, 2000; Ormrod, 2003; Parsons, Hinson, & Sardo-Brown, 2000; Woolfolk, 2006). In effective mathematics inquiry lessons, students are supported in reflecting on what they encounter in the environment and relating this thinking to their personal understanding of the world (Clements, 1997).

Noriyuki Inoue is an Associate Professor of Educational Psychology and Mathematics Education at the University of San Diego. His recent work focuses on inquiry pedagogy, Japanese lesson study, action research methodology, and cultural epistemology and learning.

Sandy Buczynski is an Associate Professor in the Math, Science and Technology Education Program at the University of San Diego. She is the co-author of recently published: Story starters and science notebooking: Developing children's thinking through literacy and inquiry. Her research interests include professional development, inquiry pedagogy, and international education.

Preservice Teachers' Difficulties with Inquiry-Based Lessons

Though research indicates the importance of students' construction of knowledge, multiple research reports show that preservice teachers are poor facilitators of knowledge construction in inquiry-based lessons, and that this persists even when they have gone through teacher-training programs focused on inquiry-centered pedagogy (Foss & Kleinsasser, 1996; Tillema & Knol, 1997). These research reports suggest that preservice teachers have a tendency to duplicate traditional methods, rather than implement the inquiry-based pedagogy they experienced in their teacher education programs. Traditional pedagogy is typically associated with a style of direct instruction that is teacher-centered and front-loaded with subject matter. It is characterized by the teacher reviewing previously learned material, stating objectives for the lesson, presenting new content with minimal input from students, and modeling procedures for students to imitate. Throughout the lesson, the teacher periodically checks for learners' understanding by assessing answers to closed-ended tasks and providing corrective feedback. In contrast, inquiry pedagogy is student-centered and allows time for metacognitive development. In an inquiry classroom, the teacher presents an open-ended problem, and the learners explore solutions by defining a process, gathering data, analyzing the data and the process, and developing an evidence-supported claim or conclusion.

Preservice teachers' tendency to duplicate traditional methods has been attributed to a lack of a sound understanding of the mathematics content that they teach (Kinach, 2002a; Knuth, 2002), an inability to consider various ways students construct mathematical knowledge during instruction (Inoue, 2009), and a failure to consider how the content, curriculum map, and classroom situations contribute to students' understanding (Davis & Simmt, 2006). Other researchers report that preservice teachers' reluctance to stray from traditional methods is originates in the difficulty that they feel in conceptualizing their teaching in terms of the classroom culture and its social dynamics (Cobb, Stephan, McCain, & Gravemeijer, 2001; Cobb & Bausersfeld, 1995). These researchers suggest that preparing a non-traditional lesson requires the teacher to predict the possibilities of classroom interactions and carefully consider ways to shape the social norms of the classroom to facilitate student-centered thinking. However, many preservice teachers go into teaching believing that knowledge transmission and teacher authority take precedence over students constructing ideas (Klein, 2004). Even if preservice teachers learn about inquiry lessons in their teacher-training programs and believe students' construction of ideas should take priority, they struggle to consider the multiple issues that are key for a successful inquiry lesson, limiting their ability to implement effective inquiry lessons.

Current literature on inquiry learning focuses on identifying and theorizing various psycho-social factors that contribute to teachers' ability to deliver an effective mathematics inquiry lesson in the classroom. Some researchers stress the importance of transforming teachers' perceptions and understanding of inquiry teaching (Bramwell-Rejskind, Halliday, & McBride, 2008; Manconi, Aulls, & Shore, 2008; Stonewater, 2005) and transforming teachers' beliefs (Robinson & Hall, 2008; Wallace & Kang, 2004). Others examine teachers' personally constructed pedagogical content knowledge (PCK) that stems from their experiences as learners and their perceptions of students' needs (Chen & Ennis, 1995). Wang and Lin (2008) add that students' conception and understanding of inquiry lessons needs attention as well. Though some of these research findings are based on studies of inservice teachers' struggles with implementing inquiry lessons, we believe that a majority of these research findings are applicable to preservice teachers as well.

Rationale for Study

Though the literature provides many insights on preservice teachers' struggles in implementing inquiry-based lessons, it is also essential to obtain a practice-linked understanding of why and how preservice teachers, particularly those who are motivated to teach mathematical inquiry lessons, encounter difficulty in authentic teaching contexts. This approach, taken together with the theoretical knowledge the literature provides, strengthens our understanding of how preservice teacher training should be improved. In this paper we address this identified need by presenting the results of one representative case study in which we analyzed a preservice teacher's inquiry-based lesson taught in a mathematics classroom. Obtaining a practice-linked understanding of the nature of the difficulties that a preservice teacher might encounter in an inquiry lesson provides detailed insight into how specific contexts affect inquiry pedagogy.

Research Questions

In the process of implementing inquiry lessons, many interactions can serve as stumbling blocks to the inquiry process. Here, a *stumbling block* refers to instances where a teacher poses an open-ended question, the students respond (or fail to respond), and the teacher does not know how to reply to students' comments or questions and, therefore, fails to guide the learning activity towards the rich inquiry investigation initially envisioned. With this in mind, the questions guiding this investigation are: 1) What instances serve as stumbling blocks for preservice teachers motivated to teach inquiry lessons? 2) How do preservice teachers respond to stumbling blocks and how do those responses influence the direction of the lesson?

Any preservice teacher who crafts an inquiry lesson could encounter these types of stumbling blocks. Therefore, the knowledge gained from this study can inform preservice teacher education in two ways: It can increase teacher educators' awareness of preservice teachers' issues in implementing inquiry-based lessons, and it can guide teacher educators in helping preservice teachers deliver effective mathematics lessons that are characterized by meaningful construction of knowledge through mathematics inquiry activities.

Methodology

Context

University faculty from the Mathematics Department in the School of Arts and Science were joined by faculty from the Learning and Teaching Department in the School of Leadership and Education Sciences to conduct a summer mathematics camp for third- through sixth-grade students. This cross-campus collaboration provided an opportunity for the faculty to mentor undergraduate preservice teachers to help them bridge mathematical content with pedagogical practice and knowledge of context. Preservice teachers were offered the opportunity to serve as camp instructors in order to gain experience teaching inquiry lessons. We then observed their inquiry-based lessons in order to answer our research questions.

The summer mathematics camp served as an ideal environment for this investigation since the camp's novice teachers could practice implementing inquiry lessons free from the pressure of supervisor evaluation and externally imposed state standards or tests. The camp also created an environment where learners were given time to be curious and to develop positive attitudes toward learning mathematics. The mission of math camp was two-fold: to provide mathematical enrichment for a diverse group of children and to support the mathematical and pedagogical development of preservice elementary school teachers.

The summer math camp had unique contextual constraints that distinguished it from a traditional classroom. The mathematics instruction was embedded in a thematic context of Greek mathematicians. Each class included combined grade levels; one for rising second through fourth graders and one for rising fifth through sixth graders. Students from across the city attended the camp. While this context diverged from a typical classroom, some features of the camp provided a context similar to a typical mathematics class: both classes had a heterogeneous mix of diverse students and class periods lasting 90 minutes. We believe that the educational context also highlighted opportunities for a preservice teacher to implement a quality inquiry-based lesson because the students attended voluntarily and were not pressured to perform on tests or homework. Similarly, there was little pressure on the instructors to cover certain material or deliver inquiry lessons with the goal of students' performing well on tests.

Camp instructors (preservice teachers)

University mathematics professors recruited camp instructors from an undergraduate elementary mathematics methods course. The professors informed preservice teachers enrolled in the course about the opportunity to practice inquiry-based lessons in this summer camp, and a number of them applied to be camp instructors. As part of the recruitment process, the candidates were informally interviewed about their interests and goals in mathematics teaching. Eight preservice teachers were selected to serve as camp instructors based on their enthusiasm and willingness to work in the team. All the eight camp instructors were female undergraduates working towards a bachelor's degree in liberal studies combined with an elementary teacher credential. During the interview, all of the camp instructors professed an interest in developing their teaching skills and math content knowledge in an activity-rich environment and were willing to commit to one week of camp preparation mentoring and one week of classroom teaching during camp. Each camp instructor's experience working with children varied, as did their time in the teacher education program. Two were sophomores, three were juniors, and three were seniors. Though they were at different points in the program, half of the camp instructors had completed foundation courses in education, and all had completed the mathematics teaching methods course.

Camp students

Because the camp was advertised in the local newspaper, children from across the city, as well as faculty and university-neighborhood children, applied and were accepted on a first-come, first-served basis. The price of the camp for each child was approximately $300. The university helped cover the operational cost of the camp with a $7,490 academic strategic priority fund award which applied to the camp instructors' salaries, classroom resources, and tuition reduction for eligible children. Each of the two classes enrolled 30 students with approximately ten each of rising second, third, and fourth graders in the lower grade class and approximately 15 each of rising fifth and sixth graders in the upper grade class. Caucasian, Latino, and Asian students made up approximately 60%, 30%, and 10% of the student campers respectively. Because of the age range in each class, a wide range of skill levels was observed.

Undergraduate preservice teacher preparation

For entering the undergraduate elementary teacher education program preservice teachers must be in the university's Bachelor's degree program in a content area of their choice. To become a licensed elementary teacher they must then complete the 33-credit hour multiple-subject education program and pass a standardized state content exam. Most of the students who enroll in the undergraduate credential program are liberal studies majors with a concentration in one of the content areas. The credential program includes coursework in educational psychology, content pedagogy (including elementary mathematics teaching methods taught by mathematics faculty with expertise in pedagogy), educational theory, and courses on children's learning. Through this coursework, the students gain field experience through a series of practicum placements in K-6 schools. In these placements they observe classroom instruction and teach inquiry lessons under the guidance of a school-based and a university-based supervisor.

Camp instructor preparation

Before the math camp program began, the camp instructors attended a required week-long preparation program focused on deepening their mathematics content knowledge, as well as mathematics pedagogy. Camp instructors learned about key developmental and learning theories and were exposed to current research on K-12 learners' social and personal construction of meaning. They also learned how to develop lesson plans using a wide variety of instructional approaches that focused on helping students construct knowledge. Because exposure to inquiry-based lesson development differed across camp instructors, faculty mentors provided both group and one-on-one instruction and mentorship in this pedagogy.

Four faculty mentors led seminars on the general principles of inquiry lessons. These faculty members also taught in the university's regular preservice credential program, therefore, the seminars were highly comparable to the university's regular preservice program. Constructivist philosophy influenced the design of the seminars. Preservice teachers were taught to encourage children to actively make sense of mathematics instead of teachers presenting and modeling procedures for solving problems. In other words, giving authoritarian feedback to students was not a pedagogical strategy valued by the math camp faculty mentors.

The camp instructors were also taught lesson planning based on detailed task analyses of instructional goals called "backward design" (Wiggins & McTighe, 2005). In backward design, the teacher begins with the end in mind, deciding how learners will provide evidence of their understanding, and then designs instructional activities to help students learn what is needed to meet the goals of the lesson. Based on this model, the camp instructors started designing a camp lesson with an initial mathematical idea and then discussed with their peers how students' understanding of this idea could be gauged. During the process, camp instructors were introduced to strategies including cooperative learning, active learning, mathematical modeling, and the use of graphic organizers. The instruction in these strategies emphasized inquiry pedagogy with the goal of learners developing understanding beyond rote knowledge.

Faculty members also guided camp instructors in how to navigate the disequilibrium between what children *want to do* versus what they *can do*. Though the camp preparation lasted only one week, students instructors reviewed the basic principles of learning and designed a camp lesson based on pragmatic instructional fundamentals. They learned what to include in a lesson plan, how to pace activities within the 90-minute class period, how to pose appropriate questions, how to make use of wait time, how to manage the classroom, and what to consider in a thoughtful reflection on teaching experience. Camp instructors' lessons were required to (a) provide a mathematically rich problem allowing for open-ended inquires of mathematical ideas, (b) ask open-ended questions, (c) encourage students to determine answers with rationales in their responses for problem solving activities, (d) and elicit exchanges of different ideas.

Faculty mentorship

Though the camp instructors had a theoretical understanding of how students make sense of mathematical ideas and lesson planning, they did not have any practical experience in planning appropriate inquiry-based mathematics lessons for students. To guide and support them through this process, faculty mentors were available to provide generous assistance and offer advice. Two mathematics professors and two education professors, one specializing in educational psychology and the other in curriculum design and STEM education, served as mentors. During the pre-camp training session, the eight student instructors were paired into four teams of two instructors each. All four mentor professors worked with each team. Mentors met individually with each team to discuss their proposed lesson activities in terms of developmental appropriateness, mathematics content, and pedagogy. At the end of the preparation week, a survey developed by the education faculty members (see Appendix A) was administered to get a sense of teachers' beliefs and attitudes toward inquiry learning after the camp instructor training program. According to this survey, all eight camp instructors had positive views about inquiry-based lessons and were motivated to deliver effective inquiry-based, activity-rich lessons in the camp.

Each camp instructor team member designed one inquiry lesson for the lower grade class and then one for the upper grade class, or vice versa. These two lessons focused on the same content, but were modified to be appropriate for each age range. For instance, one camp instructor of each team-taught her lesson for the lower grade class during the morning session and the other taught her lesson for this class in the afternoon session. The teams then presented the upper grade lessons in the same manner later in the week. The camp instructors were completely responsible for classroom instruction, however, mentor professors were present in the classroom for additional support as needed. When camp instructors were not teaching, they were observing their peer camp instructors' lessons. At the end of each day, all camp instructors met as whole group with all of the faculty mentors. These whole group meetings included discussions of how the day went and what aspects of the lesson were effective or ineffective, what revisions could be made, and what concepts should be revisited. Following this schedule, the camp instructors taught each lesson variation during the camp week and had a chance for individual feedback and advice from a faculty member after each presentation of their lesson. A large part of the camp instructors' experiential learning arose from their reflection on their daily teaching experience and the mentors' input about their classroom performance.

Data collection and analysis

During the camp session, the authors observed a total of 12 of the camp instructors' inquiry lessons: three randomly chosen pairs of lower and upper grade lessons and six other randomly chosen lessons. These observations allowed the researchers to gain a conceptual understanding of the inquiry process that these novice teachers enacted from their lesson plans. Researchers made field notes and video-taped lessons as video cameras and audio-visual staff were available. Camp instructors also completed a post-lesson questionnaire (Appendix B) that probed their perceptions of their effectiveness as math teachers and their success with inquiry pedagogy.

The 12 observed lessons offered a wide range of information about the camp instructors' approach to inquiry learning in elementary mathematics. The cross-case analyses of observed lessons led us to believe that the camp instructors followed the design principles of an inquiry lesson. However, camp instructors had moments of difficulty that we have termed stumbling blocks. As described earlier, in these moments, the camp teacher responded to an instructional situation in such a way that derailed the inquiry-based goals of the lesson and created moments that significantly undermined the quality of the inquiry lesson.

There were many different kinds of stumbling blocks. When we looked into the cases more closely, we found that the nature of the stumbling blocks was highly contextual and content specific. In each case, stumbling blocks emerged in math camp lessons, one after another, in ways that were nested. By nested we mean that once one stumbling block appeared in the lesson, it had the potential to contribute to the emergence of a subsequent stumbling block. For example, when a preservice teacher was faced with no student response to a question she posed, she resorted to guiding students with leading questions without giving ample opportunity for students to make sense of the concept. In this case, the initial problem that was created from the first stumbling block (i.e. not knowing how to respond when students have no input) served as a foundation for another stumbling block to emerge (i.e., guiding students with leading questions). These in-depth case study analyses revealed that each inquiry component of the lesson depended on other components of that lesson that developed from previous actions and interactions in the lesson. The only way to evaluate the inquiry process and conduct meaningful analyses of the stumbling blocks in inquiry pedagogy appeared to be step-by-step deconstructions of the camp instructors' actions and utterances within each lesson.

We reasoned that presenting a representative individual camp instructor as a case study was the most effective way to capture the nature of stumbling blocks that the camp instructors encountered during the presentation of their lessons. An analysis of one camp instructor's performance provided the best insight into strengths and weaknesses of the inquiry teaching process. The following section describes the findings of this study based on this methodological framework.

Findings

The case analyses of the observed lessons indicate that all the teachers were not successful in giving mathematically and pedagogically meaningful explanations, ignored creative responses from the students, or switched the nature of instruction to the direct transmission model where the teacher simply gave answers to students as an authority with little attention to students' thinking about mathematics. A variety of kinds of stumbling blocks were identified, and each type of stumbling block was found in multiple cases. The type of stumbling blocks depended on the mathematical content covered in the lessons, the students, and the particular dynamics of the interactions in the classroom. We analyzed and identified different stumbling blocks that the camp instructors encountered when teaching a mathematics inquiry-based lesson. Based on the cross-case analyses of the observed lessons, we identified a total of thirteen stumbling blocks, summarized in Table 1.

To exemplify these stumbling blocks, the following section describes an in-depth case study that illustrates the ways a preservice teacher actually encountered the stumbling blocks during the

Table 1
Stumbling Blocks

Location of Stumbling Block	Type of Stumbling Block	Teacher Response
Planning the Inquiry Lesson	1. Problematic problem design	The teacher uses a poor or developmentally inappropriate set up of an inquiry problem or question for the lesson.
	2. Insufficient time allocation	In the interest of time, the teacher moves on to the next planned activity scheduled in the lesson plan in spite of students' confusion or teaching opportunities created by students' responses.
Teacher Response to Student Input	3. Unanticipated student response	The teacher fails to anticipate students' input and cannot give a pedagogically and mathematically meaningful response to the students.
	4. No student response	The teacher fails to give a meaningful response to students' silence or lack of input in reply to the teacher's question.
	5. Disconnection from prior knowledge	The teacher's response severs connections between the lesson and students' prior knowledge or their attempt to make sense of the concept using their experiential knowledge.
	6. Lack of attention to student input	The teacher ignores the students' input in reply to the teacher's open-ended questions.
	7. Devaluing of student input	The teacher diminishes student input by rejecting their suggestions and shuts down their attempts at making sense of a problem.
	8. Mishandling of diverse responses	The teacher does not know how to effectively manage or give meaningful traffic controls to diverse responses that the students gave for open-ended questions.
Teacher Delivery of Inquiry Lesson	9. Leading questions	The teacher's questions directly guide learners to the answer without creating enough opportunities for learners to make sense of the concept.
	10. Premature introduction of material	The teacher introduces a new concept or symbol without giving enough opportunity for students to make sense of previous content.
	11. Failure to build bridges	The teacher misses important opportunities to effectively connect his or her question to the problem solving activity or the ideas that the students formulated during problem solving.
	12. Use of teacher authority	The teacher uses his or her authority to impose the answer or strategy or judge the students' answer or strategy as right or wrong.
	13. Pre-empting of student discovery	The teacher provides the main conclusion that students were supposed to discover.

presentation of an upper grade lesson. This descriptive case study (Yin, 2003) illustrates a thick description of some of the issues faced in mathematics inquiry pedagogy. We chose this particular case among all the observed cases since it most vividly informs us of the nature of stumbling blocks that the camp instructors typically encountered in the inquiry lessons observed in the study. We labeled each stumbling block that the preservice teacher encountered at various points of the lesson in reference to the above table.

Case study

Jessica (pseudonym) was a university senior majoring in liberal studies and enrolled in the university's elementary school teaching credential program. She had successfully completed an educational psychology class and other credential courses, but did not have any formal mathematics teaching experience. In the pre-survey Jessica described effective teaching as, "The teacher needs to prepare the students for what they will learn by getting them interested and providing a foundation to build on (pre-teach if necessary). Also the lesson/activity must be engaging (hands-on, collaborative)." This comment is representative of all the camp instructors' responses to this survey item; many indicated their belief in the importance of using activities meaningful to children, eliciting children's interest, and scaffolding students' personal construction of knowledge that is grounded in their prior experiences. Even though camp instructors' comments did not encompass the entirety of inquiry-based learning principles, they did show understanding of the key ideas. Jessica, in particular, showed an understanding of her intention and plan to deliver an inquiry lesson in the summer camp.

Jessica's instruction contained a wide variety of stumbling blocks and can inform us of the nature of the difficulties that preservice teachers can encounter in teaching inquiry lessons. As discussed before, Jessica prepared her lesson plan in the pre-camp session with guidance from the faculty mentors. The objectives of Jessica's lesson were to help children (a) understand the concept of ratio and (b) understand π as a constant ratio for any circle. As was true with the other camp instructors, Jessica was friendly and made personal contact with children very well. In the upper grade classroom, the children were divided into six groups sitting at different tables.

First, with a picture of trail mix containing M&Ms projected, Jessica asked her students if they liked M&Ms. After hearing a positive response from most of the children, she indicated that she had three brands of trail mix, each containing M&Ms, nuts, and raisins. She said, "We need to find out which brand we should buy if we would like to get the most M&Ms." With this problem statement, she has started with an interesting story and formulated an open-ended question relevant to students' everyday experiences, a key component of an inquiry-based lesson.

Jessica then explained that each brand of trail mix advertised that it contained two scoops of M&Ms. She showed ladles of varying sizes and said that she was not sure which ladle each brand used to measure their two scoops. She asked the children how they might determine which brand of trail mix to purchase to maximize the amount of M&Ms. The children were listening to her attentively and appeared to be thinking about this question. Then one child answered, "What about finding how much sugar that they have on the box?" This child knew that the package should indicate its amount of sugar on the nutrition label and that this would vary directly with the amount of M&Ms. She had not anticipated the direction of this response that overall sugar content would indicate quantity of M&Ms nor had she anticipated this particular question from one of the children. Jessica did not know how to respond. If she simply said no, her inquiry lesson would have lost its real life meaningfulness and stumble just as it was starting. After a pause, Jessica responded, "But the raisins also have sugar, so we cannot compare trail mixes based on sugar [to determine amount of M&Ms in each brand]." With this clever response, the child who asked the question seemed convinced and began to consider other approaches. In responding to the child's unexpected answer, Jessica managed to avoid using her authority as a teacher to silence the child. This child came up with a creative solution which she responded to by acknowledging his creativity while re-directing his thinking.

While the children were still considering solutions, Jessica suggested using actual trail mixes as stimuli and distributed three plastic bags that contained different brands of trail mix along with a worksheet to each group of students. She asked the children to collaborate at each table to record 1) the number of M&Ms, 2) the number of nuts, 3) and number of raisins. First through her failure to elicit additional solution strategies from students to connect their thinking to the problem and second through her imposing a particular strategy to count M&Ms for problem solving, two stumbling blocks (SB11: Failure to build bridges & SB12: Use of teacher authority) emerged. In other words, this strategy of counting pieces of trail mix did not come from the students, and

Jessica did not help the children make sense of what they were asked to do. One thing that needs to be pointed out here is that these stumbling blocks emerged even though a) she was trying to follow some aspects of the inquiry teaching principles by having students gather evidence and by giving priority to this evidence in responding to questions (NRC, 2000) and, b) the students were given the opportunity to connect the process of problem solving with the concrete experience of counting M&Ms and comparing their results for the different brands.

After receiving the bags, the children immediately started collaborating and using various strategies to count the pieces in the trail mixes. When they finished, Jessica recorded and displayed their results to discuss with the class (Figure 1).

Brand of Trail Mix	M&Ms	Nuts	Raisins	Total pieces in trail mix
Crunch Beans	66, 67	110, 117, 126, 111	32, 35, 36, 34	220
Sweet & Salty	30	69, 70	11	110
Snick Snack	71	167	91	329

Figure 1. Results of each group's counting

Note: Each cell displays the counting results from the groups. If the groups' counting results are the same, the same number was not added to the table to avoid repetition.

It was not until this point in the lesson that we realized that each group's bag of a particular brand of trail mix had the same number of M&Ms, nuts, and raisins; Jessica had set up the brands to have no counting variations among groups. Of course, the children made minor counting mistakes and this resulted in the variations shown in Figure 1. After the completing the chart, she suggested the correct number of pieces for each brand and totaled them in the table for the children. In other words, she told them the right answers as an authority (SB12: Use of teacher authority).

After the counting activity, she asked the class, "Which one [brand] has more M&Ms compared to the whole package?" When no child responded to the question (SB4: No student response), Jessica pointed out the numbers in the table (Crunch Beans brand: 67 M&Ms in 220 pieces and Snick Snack brand: 71 M&Ms in 329 pieces). Again, she asked the question, "Which brand had more M&Ms compared to the total number of trail mix pieces in the package?" Jessica attempted to assist children in finding the answers to her close-ended question by directing them to relevant evidence. However, the children remained confused because her explanation did not clarify that she was asking about the proportion of M&Ms compared to the total amount of trail mix. Still, with no child answering, she then asked "67 over 220 or 71 over 329?" (SB9: Leading questions). A child asked, "You mean, if the price of the packages is the same?" Again, Jessica clearly did not anticipate this question (SB3: Unanticipated student response), and responded by saying, "It's a good question," but went on to say that price was not important here since the price of three packages of one brand could be the same as one package of another brand; she pointed out that price comparison can be very complicated, and is not what they should consider in the problem solving. Jessica's reply indicated she did not understand the issue the student raised. The student was questioning a tacit assumption that Jessica did not address: if the prices were different then the comparison was invalid (SB5: Disconnect from prior knowledge). Jessica's response confused this student and many students began interjecting comments about the price and taste of various trail mixes they liked. Finding out which trail mix to buy by holding the price constant is a meaningful assumption for the children since it is what shoppers (and parents) do in choosing a brand of trail mix in everyday life. However, this line of thinking was different from how Jessica's problem set up: Her assumption was to hold the number of pieces constant, not a very meaningful set-up in everyday life. This discrepancy in interpretation of the problem served as another stumbling block for the inquiry process (SB1: Problematic problem design). She responded, "Let's not think about the price; let's explore this problem" (SB7: Devaluing student input). No one resisted this suggestion or asked why they needed to make such an assumption. Jessica began to subordinate children's meaning construction with her response loaded with authority (SB12: Use of teacher authority).

Then she asked the children if they knew what a ratio was, and wrote on the board, "Ratio = The relationship between quantities" (SB10: Premature introduction of material). At this point, the children began to be increasingly quiet. Without explaining why she was introducing the concept of ratio here, Jessica indicated that the children could use calculators to divide numbers and compare the ratios. She asked, "Does anyone know why divide?" No one answered the question, but some of the children were silently taking a note of the formula on their notebooks. Here, Jessica did not follow up on her question or support learners' meaning-making in the lesson (SB4: No student response and SB5: Disconnect from prior knowledge). This created another stumbling block that seems to have led the children to gradually shut down their personal construction of meaning in the following lesson segments. She pointed out to the children, "67/220 is like dividing a pizza. If you divide, you can compare, right?" (SB5: Disconnect from prior knowledge and SB9: Leading questions)

Then she told the children that she could use her calculator to execute the division. She input two numbers to the calculator and wrote on the board $(67/220) \approx .30\overline{54}$.[1] Here, she did not take the time to explain what she was doing or why she was doing this procedure prematurely assuming that students knew the meaning of the mathematical symbols (SB5: Disconnect from prior knowledge and SB10: Premature introduction of material). The children became increasingly confused because she failed to give an effective explanation of the meaningfulness of the assumption (i.e., holding the number of pieces constant for ratio comparison), why they needed to divide the numbers, or why this relates to the action of dividing a pizza. It was apparent that her failure to provide a meaningful rationale for the new mathematical idea created another stumbling block in the lesson. It was no surprise that, at this point, most of the children became quiet and watched her actions rather than participating in a discussion about the mathematics, which created the atmosphere of a traditional mathematics classroom.

Once she introduced the concept of ratio she began moving forward in her lesson plan despite student confusion (SB 2: Insufficient time allocation). Jessica hesitated for a while, but, in the interest of completing her planned lesson, she proceeded to introduce a new concept, a constant. She wrote *Constant* on the board and said, "Let's think about a constant. What is the quantity that does not change?" (SB10: Premature introduction of material) Jessica did not relate this question about constants to the M&M problem (SB11: Failure to build bridges). However, many of the children suddenly became engaged and raised their hands. They actively responded, "speed of light", "fingers", "gravity." The sudden increase in participation was possibly because they knew that they could answer the question and project personal meaning in the activity. Jessica smiled and nodded in response to each of the children's responses, but did not give any other reply (SB8: Mishandling diverse responses).

Next, Jessica suddenly introduced a story where Romans killed Archimedes while he was thinking about a circle he drew on beach sands. Without providing a rationale for the story (SB11: Failure to build bridges), she asked the children, "So… what's so interesting about circles? Again, given this opportunity to participate in the open-ended question-and-answer activity, children presented many different responses: "The circle is round," and "It looks like a hole." She responded with nodding and smiling (SB8: Mishandling diverse responses). Then one child answered, "Unlimited angle, no end, no beginning." Jessica looked a little puzzled by this child's answer. Clearly, she did not expect this response, and did not know how to react (SB3: Unanticipated student response). She missed this educational opportunity to discuss central angles of a circle (SB5: Disconnect from prior knowledge). The child's creative, yet unexpected, response served as another stumbling block in the lesson. She told the child that it was an interesting idea, and asked other children for more ideas.

Without clarifying the link with her original activity (SB11: Failure to build bridges), she then distributed objects that contained circles (cans, lids, duct tape, etc.). She explained what circumference and diameter of a circle are, and asked each group to measure them on their object and explore the relationship. However, she did not give any instruction about how to measure these accurately (SB1: Problematic problem design). After some exploration, most of the groups could reason that the ratio is a little more than three (though some children already knew the ratio to be 3.14 from school mathematics classes). Then Jessica wrote on the board the symbol π and mentioned that this ratio is a constant for any circle, pointing out that the relationship values calculated were almost the same across the various groups (SB13: Pre-empting student discovery).

At this point, a child raised her hand to say that their ratio was "a little less than three" in her group. Jessica approached this group and, while other groups were waiting, realized they were saying that three times diameter is a little less than circumference and therefore the ratio of circumference to diameter is a little less than three. She needed to spend a significant amount of the time for this particular group since she could not understand the logic underlying their claim, which served as another stumbling block in the lessons (SB3: Unanticipated student response). Essentially the group claimed that $3d < C$ implied that $(C/d) < 3$. Jessica missed another opportunity to compare these different ideas, address the misconception, and help the children construct their own meaning of ratio. Without sharing this group's information with the entire class, Jessica began to explain the next activity (SB2: Insufficient time allocation). Then she was stopped when one child suddenly asked how to find π of an oval. Again, Jessica did not expect this question and did not know how to respond. This moment served as another stumbling block in the lesson (SB3: Unanticipated student response). She simply told the child that she would think about it. She continued her lesson by drawing examples of inscribed and circumscribed triangles on the board and asked if the circumference of the circle or perimeter of the triangles are bigger. At this point, she did not have enough time to finish the lesson (SB 2: Insufficient time allocation). She thanked all the children for their interesting ideas, and the lesson ended within the class's allotted time.

As shown in the above case study, the ways the stumbling blocks appeared and influenced the lessons and student instructor's responses were complex. No simple descriptions seem to be able to capture the complexity and dynamics of these factors that were intertwined with each other. Please note that the camp instructors chose to teach in the inquiry-centered math camp and were highly motivated to teach inquiry lessons. This makes this group an unlikely representative of preservice teachers across the nation. However, we do not believe that this weakens our argument, but strengthens it. It highlights one of the key points of the study: Even if pre-teachers are motivated to teach inquiry lessons, they encounter stumbling blocks and often do not know how to overcome them. The following examples illustrate stumbling blocks that the camp instructors encountered in other lessons observed in the study.

1. When students explored how to expand a 2' x 3' picture of a face into a larger dimension without distorting the image, one of the students responded 5' x 6' since 2 + 3 = 5 and 3 + 3 = 6. The teacher simply responded, "That's not quite right," in front of all the students without explaining or examining this. (SB6: Lack of Attention to Student Input and SB7: Devaluing Student Input)
2. In a lesson to understand the effects of volume and mass on water displacement, the teacher started the lesson by asking very broad questions: "Have you ever heard the term volume?", "How does it relate to math?", and "What are some ways to find volume?" When the students gave diverse responses to these broad questions, the camp teacher merely listened to them without giving any sort of meaningful response and proceeded on to the planned water displacement activity. This lack of validation or even acknowledgement of students' responses quieted their eagerness to answer, as after that the students spoke up much less in the lesson. (SB8: Mishandling Diverse Responses and SB6: Lack of Attention to Student Input)
3. In a lesson to find the height of a pyramid, the camp teacher asked students to measure their shadows and compare the measures with their actual heights. Though the children made the connection between this activity and finding the height of a pyramid, the teacher did not make the relationship between the two activities explicit. (SB5: Disconnect from prior knowledge)

Most of the camp instructors expressed a sense of failure in their first round of teaching, but did not clearly know the reasons why their inquiry lessons did not work very well. After the first lesson, each teacher had an individual meeting with the faculty mentor(s) who observed the lesson to go over their reflection and receive suggestions for improvement. This opportunity to discuss the lesson presentation with faculty helped the camp instructors determine reasons why these *stumbling blocks* were encountered and provided them expert advice on what to do next.

Discussion

Teachers are known to possess personalized understanding of how to support children's construction of knowledge based on their own learning experiences (Chen & Ennis, 1995; Segall, 2004). The weaknesses observed in the student instructors' inquiry lessons could be seen to stem from a novice teacher's immature understanding of how elementary school students think and understand mathematics. Ample literature supports this point that teachers' beliefs and understanding about how children learn significantly impact the effectiveness of teaching (e.g., Kinach, 2002b; Warfied, Wood, & Lehman, 2005).

The camp instructors knew that helping children make connections between abstract concepts and the material representations of those concepts is critical to a meaningful inquiry-based lesson. However, there were many instances in our observations where such connections were not made, in spite of the camp instructors knowledge about inquiry lesson principles and their willingness to deliver an inquiry lesson. Through the study, we found that asking preservice teachers to take command of a full classroom of students with only a crash preparation course was insufficient to circumvent problems. In a way, it is not a surprise that the camp instructors had so much room for improvement in lesson planning, time management, and transitioning from one activity to the next. However, we believe that Jessica's lesson was informative for any teacher, novice or veteran, who attempts to deliver an inquiry lesson because it addresses several important stumbling blocks that anyone could encounter in delivering an inquiry lesson, yet not recognize in the moment of teaching.

There are several lessons that could be learned from this case study. First, Jessica often asked open-ended questions to help the children personally construct meaning. But it was often the case that she had not considered possible learner responses, and caught off guard, did not know how to respond, as she admitted in the post-teaching interview. As a result, in the face of these rich educational opportunities provided by the diverse learner responses, Jessica was unprepared, inflexible, and unable to make use her knowledge of content and children's thinking to improvise within the parameters of the lesson. Consequently, the children who contributed these thought-provoking answers did not receive any meaningful response or validation of their ideas from the teacher or their peers. We observed many such instances throughout the math camp and suspect this is true in many classrooms where teachers attempt to deliver an inquiry lesson.

To be fair, in preparing these camp instructors to teach a math concept, anticipating student response was not a part of their lesson plan template. Inoue (2011) points out that this should be a key component of lesson design; in his cross-cultural lesson study research, "failure to anticipate students' diverse responses" was one of the reasons that an inquiry lesson was ineffective and deviated from the initially planned instructional goal. One solution for this could be adding a section to the lesson plan template that includes thinking through possible student answers to questions, as Japanese educators are known to include in their lesson plans (Fernandez & Yoshida, 2004). This would help them prepare for conceptual conversations in the classroom and help them evaluate the lesson by envisioning students' diverse perspectives.

Furthermore, we discovered that preservice teachers were more focused on their own performance than on their students' performance in these classroom experiences. Berliner (1994) reports similar finding from his research that inexperienced teachers had a tendency to focus on teachers' actions, rather than students' actions, and lacked the ability to identify meaningful sub-activities integrated within a larger lesson. The camp instructors' tendency to focus on their own performance could work in favor of their learning from their pedagogical mistakes, strengthening their content delivery, and gaining insight into the inquiry process. However, it could do little to help them learn to consider each action in the lesson in reference to the goals of student learning, a necessity for successful inquiry instruction.

Passing over or ignoring a response that has merit in the conceptual framework of the lesson could not only lower the learner's inclination to participate in the lesson but also invalidate or devalue the learner's prior knowledge (Cooper, 1994, 1998). If a learner's response falls outside the realm of anticipated responses, yet presents an opportunity to expose the class to a different facet of understanding of a mathematical concept, the teacher needs to first validate a student's legitimate response and then use that response to navigate to the instructional goals. In addition, the teacher needs to have the flexibility and confidence in content matter to build a consensus among the students and achieve the instructional objective within the allocated time. More importantly, expecting diverse and high-quality responses and knowing how to incorporate a learner's prior knowledge in the lesson is an important skill for teachers to have when delivering an inquiry lesson. What holds the key seems to be a deep understanding of how children think and might react to concepts. For example, Lubienski (2007) points out that lower socioeconomic students are more likely to use "solid common sense" (p.54) than they are to use a sophisticated mathematical concept.

Researchers point out that mathematical word problems are often written without accurately reflecting the experiences described in the problems (Greer, 1997; Inoue, 2005; Verschaffel, Greer, & De Corte, 2000). In Jessica's episode, this was evident when students tried to compare prices of brands of trail mix rather than use ratios of ingredients. Being aware that students often become engaged with the real-world aspects of math problems rather than focusing on the mathematical concept intended by the problem would help teachers anticipate students' responses and prepare a means to incorporate that line of thinking into the math concept being studied.

Second, in spite of the camp instructors' attempt to explain abstract concepts in ways that were grounded in students' prior experiences and concrete models, they often failed to explain mathematical concepts in pedagogically meaningful ways. Jessica simply did not know how to explain the concept of ratios effectively, and gave irrelevant, misleading, and disconnected instructions to the students. This issue could be due to her lack of deep knowledge on how to deliberately unpack a mathematical concept, as seen with her treatment of ratio, constant, and π. As Jessica herself pointed out in the post-teaching interview, it is important for teachers to possess multi-layered content knowledge in order to utilize it in a pedagogical meaningful way to make connections among concepts and to a learner's prior knowledge. In this sense, teaching inquiry lessons effectively requires going beyond merely following the principles of inquiry lessons to developing a deep pedagogical understanding of how one could construct each mathematical concept in a meaningful way (Ball, Hill, & Bass, 2005; Ma, 1999; Mapolelo, 1998).

This point is emphasized by Shulman (1987) who claimed "the key to distinguishing the knowledge base of teaching lies at the intersection of content and pedagogy" (p. 15). Shulman (1986, 1987) described the construct of pedagogical content knowledge as an integrated synthesis of subject matter content knowledge and pedagogical knowledge that is specific to education and separates teachers from mere content experts. For instance, when the student says a circle is interesting because it has "unlimited angles, no end, no beginning," the teacher needs to be able to confidently respond to this mathematical statement in a pedagogical meaningful way without losing the scope of the planned lesson. For instance, the teacher could have responded by first pointing out that central angles are an important concept to explore in understanding a circle. She could have instructed the students on drawing central angles and challenged them to draw a 180° central angle, the diameter, before starting the activity to discover π, the ratio of diameter to the circumference. Preservice teachers must have meaningful criteria for suitable open-ended questions that are supported by deep pedagogical content knowledge. This will enable them to anticipate probable responses and have sufficient confidence in their content knowledge to determine which avenues are worth exploring and how best to follow up on diverse student input.

Finally, we learned that the evaluation of an inquiry lesson for teacher training requires step-by-step analyses of the preservice teacher's actions and utterances linked with prior actions, appropriateness of content, and students' understanding. Instructional dialogues that teachers engage in to support students' understanding are highly complex and do not allow linear, simplistic formulation (Inoue, 2009; Leinhardt 1989, 2001). We found this to be the case with the inquiry lessons that we observed. It is also true that we cannot expect epistemological enlightenment to arise spontaneously through two weeks of mentoring, no matter how strong the mentoring or the mentees. However, we infer that experience, combined with consistent, constructive step-by-step analysis of teacher performance, curricular materials, and learner interaction with both, are needed to support the teachers in order to build an effective teaching practice. Likewise, what is helpful to any teacher is to plan a lesson, deliver the lesson, and reflect on their step-by-step actions in the classroom. From this careful scrutiny of the meaningfulness of their every action and reaction, the teacher can become aware of possible stumbling blocks in their instructional path and use this awareness to strengthen future performances.

We do not deny the importance of learning the guidelines for delivering effective inquiry lessons. However, we also learned that actually teaching an inquiry lesson based on the guidelines had many possible pitfalls for teachers. We learned the importance of improving teachers' ability to explain content, anticipate children's responses, respond appropriately to children's answers, and link new content to appropriate models and experiences. For meaningful knowledge construction to occur, implementing an inquiry lesson is not enough. It is more important to effectively negotiate the topic's meaning as different perspectives and interpretations emerge at each moment in the classroom's instructional dialogue (Cobb & Yackel, 1998; Voigt, 1996). Without such micro-level support for students' thinking, any attempt to deliver inquiry lessons will encounter many serious stumbling blocks.

Implications for Teacher Training

These stumbling blocks of inquiry-based lessons are not bumps to be ignored. In designing professional development for teachers or coursework for preservice teachers, highlighting the role of teacher awareness on teacher actions and re-actions to learners is critical to developing practice (Buczynski & Hansen, 2010). For example, a teacher may not spend enough time acknowledging or validating students' responses. If, from careful examination of a teaching event, the teacher is made aware of this behavior, then this awareness creates a heightened sensitivity to the issue and a potential for change in the teacher's future behavior. Teacher practice then, goes beyond principled pedagogy to conscious responsiveness.

We found that teachers asking open-ended questions instead of giving answers provided learners with an opportunity to blend new knowledge with prior knowledge. However, this approach also presented a stumbling block. The teacher opens herself up to the unexpected nuances of the mathematical concept. By being aware that posing open-ended questions can lead to uncharted territory and take extra instructional time, teachers can design a lesson plan that includes consideration of strategies for anticipating responses and allowing contingency time for the subsequent discussion that might arise. A planned approach to the student comment would allow validation of the student's ideas and integration of student's prior knowledge with the topic at hand, two essential components of an inquiry learning activity.

Conclusion

This close examination of a preservice teacher's performance in math camp resulted in valuable information about potential stumbling blocks that stand in the way of effectively executing a well designed inquiry lesson. This study points teacher educators to focus teacher preparation in the areas of (a) anticipating possibilities in children's diverse responses, (b) developing deep pedagogical content knowledge that allows them to give pedagogically meaningful responses and explanations of the content, and (c) step-by-step analysis of a teacher's actions and responses in the classroom. Although the case study described in this article provides only a snapshot of one novice teacher's practice, we believe that uncovering these stumbling blocks across all camp instructors overcomes this limitation. To truly transform traditional teaching toward the inquiry model, we need to make every effort to help teachers become aware of potential missteps so that they may avoid these stumbling blocks in future inquiry lessons.

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[1] Though "≈" was incorrectly used, and likely unfamiliar to the students, we do not classify this as a significant stumbling block given the context of the on-going issues in the lesson.