Make your own PAINT CHART

A realistic context for developing proportional reasoning with ratios

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Proportional reasoning has been recognised as a crucial focus of mathematics in the middle years and also as a frequent source of difficulty for students (Lamon, 2007). Proportional reasoning concerns the equivalence of pairs of quantities that are related multiplicatively; that is, equivalent ratios including those expressed as fractions and percents. Students who do not learn to reason proportionally are unequipped to learn mathematics topics such as similarity, scaling, and trigonometry. Proportional reasoning is also essential to understanding rates and hence many science concepts such as speed, density and molarity.

Ratios express the proportions of components in a combination. They can relate the sizes of parts of a single whole (e.g., the number of boys and girls in a class), two wholes (e.g., the numbers of grade 5 students and grade 6 students), or one part to a whole (e.g., the number of girls in a class to the total number of students in the class). The last of these is the type of ratio most commonly represented as fractions but in fact each type can be represented in this way. A key part of teaching about ratio is helping students to connect the various representations. Because the proportional reasoning is difficult students need to encounter the ideas in many different contexts in order to build rich connected understanding.

This paper describes a series of activities related to mixing paint that was used with a group of middle school students learning about ratio. These activities were conducted after the students had done some introductory work with ratio using counters. The activities were open enough to cater for the diverse needs of students spanning Grades 5–8 classes in a rural Kindergarten to Grade 10 school. Although all of the students had been identified by their teachers and the school mathematics coordinator as capable students and likely to benefit from some extension mathematics activities, there was considerable variation in their experience with and understanding of ratio and their abilities to reason proportionally. Up to 14 students

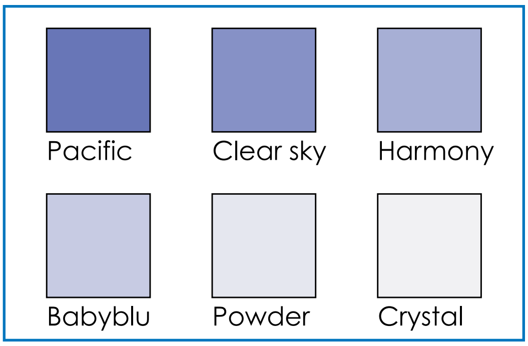

Figure 1. Part of a paint chart.

participated in weekly classes of approximately one and a half hours. Attendance varied due to other regular class activities. Two of the students were in Grade 5 and 12 were from composite Grade 7/8 classes.

Colour recipes

In the first paint mixing lesson, the students were shown paint colour charts from a local hardware store and some pages from an old paint colour "recipe" book that had until relatively recently been used to make the various colours for a certain brand of paint. An example of a paint colour chart is shown in Figure 1 and a page from the recipe book in Figure 2. The recipes use Ys and Ds (1Y = 64Ds) as units although the D is typically not written. For example in Figure 2, 10 litres of Dusty Plains requires 7Y + 32 of the tint denoted by E. That is, 7 × 64 + 32 = 480Ds of tint E are needed. The first activity, described below, gave students an opportunity to work with a recipe and become familiar with these units. Most of the students were already familiar with paint charts and had seen tints being added to base paint. The class discussed how computers had superseded the manual process but that the ratios described were still the basis of making the colours.

Harmony Blue

GLEAMING WHITE					
Tnt	250	500mL	1L	4L	10L
B	3	6	12	48	1Y+56
E	1	2	4	16	40

Floral Pink

DEEP DARK BASE					
Tnt	250	500mL	1L	4L	10L
M	24	48	1Y+32	6Y	15Y

Dusty Plains

OLD GOLD BASE					
Tnt	250	500mL	1L	4L	10L
E	12	24	48	3Y	7Y+32
G	4	8	16	1Y	2Y+32
M	4	8	16	1Y	2Y+32

Figure 2. Part of a page from a paint colour "recipe" book.

PAINT RECIPES

The following information will help you work out how to read the recipes.
The letters in the left hand column of each table stand for pigments (colours).

B = Blue
E = Yellow ochre
G = Red oxide
M = Black
J = Violet
V = Magenta

C = Green
D = Yellow
L = Red
KK = Bright yellow
F = Orange

Harmony Blue

This is the number of parts (called Ds) of blue pigment needed for 250 mL of Harmony Blue.

One Y is 64 Ds.

GLEAMING WHITE					
Tnt	250	500mL	1L	4L	10L
B	3	6	12	48	1Y+56
E	1	2	4	16	40

1. Answer the following using the table for Harmony Blue.
   a. What colours are mixed to make Harmony Blue?
   b. What colour is Harmony Blue?
   c. What is the ratio of B:E in Harmony Blue?
   d. Is the ratio the same for each volume of the paint? How do you know?
   e. What fraction of the tint in Harmony Blue is B?
   f. What fraction of the tint in Harmony Blue is E?
2. Choose another colour from the page and work out the ratio of the pigments used.

Figure 3. Worksheet on paint colour recipes.

The worksheet shown in Figure 3 helped students to explore the colour recipes and to notice the use of ratios to make the colour of different volumes of paint the same. For the 10 litre can, the amount of Blue is shown as 1Y + 56 meaning 1Y + 56Ds. Conversions among units such as between Ds and Ys and between millilitres and litres are themselves ratios. It was helpful for students to think about 250 mL as one quarter of a litre and 500 mL as half a litre. The questions on the worksheet were designed to reinforce links that had been made among different representations of ratios. For example the ratio of B:E is 3:1, which is the same as saying that three-quarters of the tint in Harmony Blue is B and one quarter is E.

Ordering shades of colour

Before making their own paint colour charts, the students were asked to choose a secondary colour (green, orange or purple) of which to make shades. They then had to come up with six different ratios of the two primary colours that made their chosen colour and order these—for example, from bluest green to yellowest green. Because the students chose their own ratios, the activity was able to accommodate the diversity of experience in the group, with some students, including one of the Grade 5 students, choosing unit ratios (1:1, 1:2, 1:3, 1:4, 1:5, 1:6) that were easy to order while others chose more challenging sets of ratios. Although it made ordering trivial it was pleasing that this student recognised that unit ratios would be easiest to order because it demonstrated important understanding of ratios. In the context of a whole class of a single grade it is likely that a similar diversity of understanding would exist, nevertheless, for some students it would be appropriate to limit the use of unit ratios to one or two or perhaps none in order to adjust the level of challenge.

An efficient way to order ratios, particularly if a calculator is available, is to change each ratio to a unit ratio but most of the students preferred to imagine the relative shades they would make and were largely accurate in their ordering. We were careful not to impose any particular method but rather encouraged students to reason about the problem in ways that were intuitive and meaningful for them. A common strategy was to compare the two sides of each ratio in terms of how close they were to the being the same or "balanced". In doing so the students drew upon or consolidated important understandings of fractions. For example, yellow and red mixed in the ratio 7:9 will be yellower than when these colours are mixed in the ratio 5:7 because, although in both cases there are two more red parts than yellow parts, the two extra red parts in the 7:9 mix are smaller in relation to the whole than the two extra red parts in the 5:7 mix.

Figure 4 shows Tess's (Grade 8) ordering of her ratios. Notice that she has included both 2:3 and 4:6. At his stage the error was not commented on because it was anticipated that the next activity, actually mixing the paint according to the planned ratios, would help students to realise such errors for themselves.

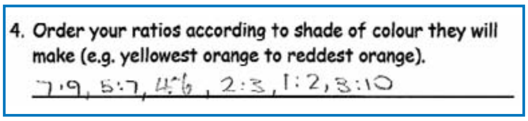
4. Order your ratios according to shade of colour they will make (e.g. yellowest orange to reddest orange).
7:9, 5:7, 4:6, 2:3, 1:2, 3:10

Figure 4. Tess's ordered ratios.

Making paint colour charts

For this lesson the group met in a specialist art room in the school. Before mixing the paint, practical issues such as what size a "part" would be and how these could be consistently measured needed to be thought through. The discussion around this issue helped students to focus on the key fact that in ratios the size of the parts does not matter but it is critical that all parts are the same. This got to heart of the proportional relationship that a ratio expresses in a way that simply working with pencil and paper exercises could not.

The best solution to measuring parts depends upon the form of the paint to be used and the size of the parts depends upon the supply of paint available. For example, powdered paint parts could be measured effectively using kitchen spoon measures. In this case we were using pre-mixed acrylic paints dispensed from tubes. Although not highly precise we decided to call a part a "blob" of paint approximately the size of a pea. This method proved adequate for the students to produce distinguishable shades that aligned with the ordering that their ratios suggested and was not too tedious or demanding in terms of the measuring implements required. The students made each shade and used each to paint an area of a page. With larger groups of students it could be useful to have students work in pairs with one member of each pair assigned the role of measuring the parts. This arrangement would assist achieving parts of the same size.

The students enjoyed mixing the paint and comparing the shades that were produced by the different ratios. Tess was quite annoyed when two of her shades looked the same but instantly recognised her error when she was pointed to her list of ratios: "Of course 2:3 and 4:6 are the same!" Tess changed her 4:6 ratio to 5:6 and ultimately produced the paint chart shown in Figure 5.

The painted pages were left to dry and brought to class for the next lesson in which the students cut out a section from each colour and arranged them on a chart. Most enjoyed coming up with names for their colours. For Ellen, in Grade 7, the activity was particularly helpful in reinforcing the connection between the ratio and the fraction of each paint colour and this was reflected in her labelling of her paint chart (see Figure 6). Rather than focussing on producing a neat chart or being concerned with names, Ellen emphasised the total number of parts involved.

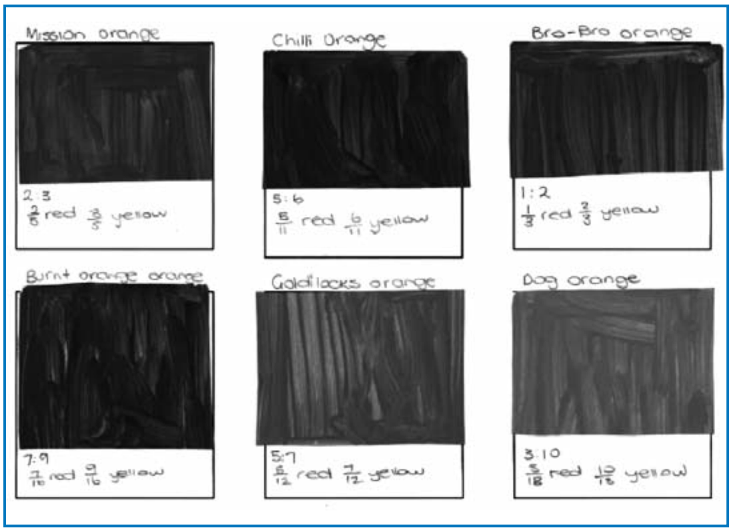

Figure 5. Tess's paint chart.

Reflections on the activities

The mathematics curriculum and linking ratios and fractions

The draft *Australian Curriculum: Mathematics* (Australian Curriculum, Assessment and Reporting Authority (ACARA), 2010) places great emphasis on fractions beginning with work with halves in Year 1. Teaching of the key idea of equivalence of fractions is mandated for Year 5 and links are to be made with decimals. The first mention of ratio is in Year 6 and limited to unit ratios. The only other mention of ratio is in Year 8 where it is stated that students will be taught to, "Solve problems involving use of percentages, rates and ratios, including percentage increase and decrease and the unitary method and judge the reasonableness of results." This describes a sophisticated ability to reason proportion-

Figure 6. Ellen's paint chart.

ally which must include the ability to connect ratios and fractions although there is no explicit mention of this. Similarly the National Council of Teachers of Mathematics [NCTM] (2000) includes flexible use of fractions, decimals and percents separately from understanding ratios and proportions in its Standards for Grades 6–8.

The intention may be to give explicit attention to ratio and rate and, in the case of the NCTM standards, to give prominence to ratio and proportional reasoning in the middle years but it is also important that links are made between ratio and fractions since both express quantities that are multiplicatively related. An important advantage of relating the two ideas is the opportunity it affords to highlight that ratios most often deal with part to part relationships whereas we usually use fractions for part to whole relationships. This is why the total number of parts is important when the fractions of each part involved in a ratio are calculated. In this activity, more could be done to link the ordering of ratios with ordering fractions.

Using realistic contexts

Using realistic contexts to teach mathematics is widely advocated and claimed to improve students' motivation and engagement, and attitudes to mathematics, as well as providing a means for them to connect mathematical concepts to familiar experiences, thus helping to build their understanding. Interesting problems of any kind presented in a supportive and safe classroom can achieve these aims so there is nothing inherently worthy about a task simply because it derives from a so called "real world" situation. Rather, their value depends upon the lesson's objectives: what mathematical ideas with which we want students to engage.

The paint mixing activities described here did seem to engage the students but so did other 'unrealistic' problems. The paint activity was designed and used not simply because it related to realistic context or because we suspected that the students would enjoy making their own paint charts, but because it provided an opportunity for students to "see" what different ratios looked like as mixtures. Previously the students had worked with counters and so they had images of ratios as distinct parts. It was hoped that seeing ratios as expressions of the relative proportions in a mixture would give them a richer appreciation of the meaning of proportion as reflected in ratios. It also provided an opportunity to make specific links between fractions and ratios. The success of the activity also relied on the fact that most of the students were familiar with paint charts and the idea of mixing paints to obtain desired shades.

As is the case with most "realistic" contexts it was necessary to simplify some aspects of the situation in order to focus on elements that served the purpose of the activities. Some of these aspects could, however, be usefully explored. One possibility is investigating the volume of the parts used in paint recipes. This offers opportunities to work with unit conversions. In fact, the units D and Y are not universal but vary with paint brand. Similarly the volumes of Ds and Ys and their equivalents also vary. Typically, approximately 33Ys make a litre so 1Y is approximately $1000 \div 33 = 30$ mL. This means that the volume of D would be $\frac{1}{64}$ of 30 or about 0.5 mL. Because different paint colours are produced by adding different amounts of tints to cans of base coats, the exact volumes of paint produced also vary and this needs to be allowed for when paint cans are manufactured.

There have also been changes over time to the tints that are used. For example, the tint Yellow Ochre that is represented by E in the recipe for Harmony Blue (see Figure 3) is now denoted by EE and is only half the strength of the original Yellow Ochre. In addition to discussing what is meant by "half as strong" there are opportunities for links to aspects of the curriculum beyond mathematics.

Although the paint colour recipes are now computerised, the people who work in paint shops develop tremendous skill in judging the effect on the final colour of small changes to the amounts of various tints that are used. This skill is not explicitly mathematical although it relies on an intuitive feel for proportions. It is not the same as understanding as described in the proficiency strand of that name in the *Australian Curriculum* (National Curriculum Board, 2009; ACARA, 2010). Rather than the specificity and requirement for accuracy within a narrow range of applications of much workplace mathematics, the curriculum demands the development of understanding "which includes building robust knowledge of adaptable and transferable mathematical concepts, the making of connections between related concepts, the confidence to use the familiar to develop new ideas, and the 'why' as well as the 'how' of mathematics." (NCB, 2009, p. 6).

Conclusion

The mixing paint activities provided an opportunity for students with widely varying experience and understanding of ratio and proportional reasoning to develop and consolidate some key ideas including the connections between ratios and fractions. Importantly students were able to experience the meaning of ratios in terms of a mixture and to see how changing proportions impacted the overall mixture. The activities also illustrate some important points about the value of realistic contexts in mathematics teaching.

It is hoped that the Australian curriculum is not interpreted as a list of discrete topics but that teachers actively seek out and capitalise on opportunities to connect different areas of the curriculum as they arise regardless of whether such connections are specified. Linking ratios and fractions presents such an opportunity and constitutes an essential part of a rich understanding of proportion.

Acknowledgements

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