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Cognitive Processes That Account for Mental Addition Fluency Differences Between Children Typically Achieving in Arithmetic and Children At-Risk for Failure in Arithmetic

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This study investigated whether processing speed, short-term memory, and working memory accounted for the differential mental addition fluency between children typically achieving in arithmetic (TA) and children at-risk for failure in arithmetic (AR). Further, we drew attention to fluency differences in simple (e.g., 5 + 3) and complex (e.g., 16 + 7) mental addition. Results suggested two important findings. First, working memory completely accounted for simple mental addition fluency differences between TA and AR children. Second, while working memory had the strongest effect on reducing differences in complex mental addition fluency between TA and AR children, group differences remained after accounting for the contributions of processing speed, short-term memory, and working memory. Results are discussed in terms of directions for future research on the cognitive processes that contribute to mental addition fluency.

Key Words: Arithmetic, Mental Addition, Working Memory, Processing Speed

One of the most consistent findings across studies examining the calculation performance of children with mathematical learning disabilities and those at-risk for failure in mathematics (both subsequently referred to as MLD) is their difficulty developing fluency in basic number facts such as mental arithmetic (e.g., Geary, 1993). Fluency in mental arithmetic is the ability to produce the solution to computational problems accurately and quickly without the aid of assistive devices such as calculators (Siegler, 1986). For instance, when performing mental addition (e.g., 4 + 5), children with MLD are less accurate (e.g., Ostad, 1998) and provide answers more slowly (e.g., Jordan & Montani, 1997) compared to their typically achieving peers. Examination of the strategies used by children with MLD to produce solutions to mental addition problems indicates that they tend to rely upon inefficient manual calculation strategies (e.g., finger counting) well beyond the stage when they would be expected to move to more efficient strategies such as direct retrieval from long-term memory (e.g., Geary & Brown, 1991). Although recent studies have indicated that number sense and working memory are significant contributors to written addition fluency (Locuniak & Jordan, 2008), investigation into the cognitive underpinnings that account for poor mental addition fluency is in need of attention. One of the most prominent hypotheses implicates a difficulty encoding basic addition facts into long-term memory (Geary, 1993). Extant research with children typically

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achieving in mathematics and children with MLD suggests that processing speed, short-term memory, and working memory are important factors to consider when examining this hypothesis. The purpose of this study was to investigate whether these cognitive processes account for differential mental addition fluency between children with and without arithmetic difficulties. Specific attention was also directed at differences between simple and complex mental addition. We reviewed research germane to the cognitive processes that are associated with both levels of problem difficulty in turn.

Predictors

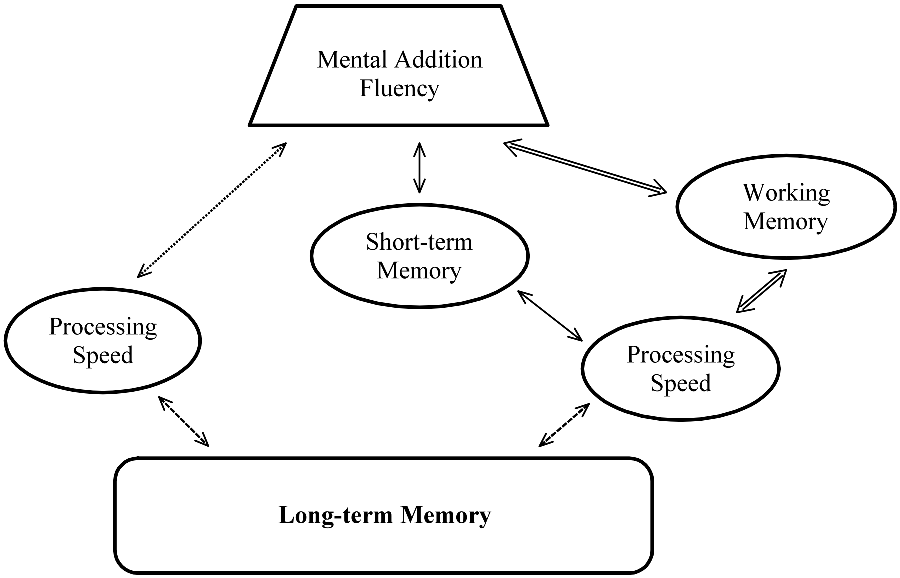
Simple Mental Addition: Processing Speed

By definition, processing speed is implicated in simple mental addition fluency. Fluency is conceptualized and operationalized as reflecting quick access to long-term memory representations of basic number facts. While processing speed has been associated with general mathematical achievement in Kindergarten (Mazzocco & Myers, 2003) and written calculation in late elementary grades (Berg, 2008b), its role in mental addition is less well established.

Processing speed, referring to direct access to long-term memory representations (see Figure 1), has been proposed as an explanation for the mental addition difficulty of children with MLD (Geary, 1990, 1993). Typically achieving children might first access the numerical representation of the addends from long-term memory (Dehaene, 1992). Subsequently, they would combine these representations by initiating a calculation strategy (e.g., verbal counting) (Logie & Baddeley, 1987). As children's fluency develops over time, these combined representations become stored in long-term memory as basic number facts. At this stage, processing speed functions as a resource to access these basic facts directly from long-term memory, rather than a resource utilized during problem solving strategies (e.g., counting). In essence, processing speed mediates fluency early in simple mental addition development through its connection to problem-solving strategies and mediates fluency later in mental addition development through its connection to stored number facts in long-term memory. For children with MLD, because there is an impairment in their ability to encode into or to retrieve numerical-based representations quickly from long-term memory, then it is plausible that these children would have difficulty encoding and retrieving associations between simple addition problems and these problems' respective answers.

Further implicating processing speed in mental addition fluency is research which reports that children with MLD are characterized by processing speed impairments related to accessing information quickly from long-term memory. McLean and Hitch (1999) reported that children with MLD were characterized by a marked deficit in long-term memory. Using a missing items task designed to measure a child's ability to retrieve information from long-term memory, these researchers found that children with MLD performed more poorly than did their typically achieving peers. A processing speed impairment, however, has not been found consistently within the literature. Berg (2008a) reported no significant difference between these groups on three measures: counting dots, articulation speed, and rapid automatized naming.

***Figure 1*. Cognitive models for mental addition fluency.**



Note. Theoretical path of linking processing speed to simple mental addition fluency (dotted line) is located on the left side of diagram. Theoretical paths linking short-term memory to complex mental addition fluency (single solid line) and working memory to complex mental addition fluency (double solid lines) are located on the right side of diagram

Predictors of Complex Mental Addition: Short-Term Memory and Working Memory

The role of memory in mental addition has often included foci upon systems other than long-term memory, in particular short-term memory and working memory (see Figure 1). The influence of these latter memory systems in mental addition has been implicated in the increased storage and processing demands associated with performing complex addition problems. Short-term memory's role has been related to the mental storage space required to hold problem addends or partial solutions when performing complex mental addition problems. More specifically, its importance has been suggested to be related to interactions with processing speed Geary (1990, 1993). Case (1985) argued that faster processing speed would increase short-term storage capacity by stabilizing the amount information that can be held in conscious attention. That is, fast processing speed counters a potential decay effect whereby information will be lost if not rehearsed quickly. Alternatively, Case argued that slow processing speed would increase the amount of time-rehearsed information, thereby decreasing the amount of available storage space. In essence, more efficient use of short-term storage space provides more cognitive resources available for combining a set of information (Case, 1985). In relation to the mental addition difficulties experienced by children with MLD, an increase in the time between paired associations increases the potential for decay of one or more of the pieces of information to be remembered. Thus, a paired association between a problem and its answer might not be established, which decreases the likelihood for encoding the paired association into a long-term memory representation. Impairments in short-term memory (remembering sequences of digits and words) have been found to be characteristic of children with MLD (e.g., Berg, 2008a). These explanations accord with findings that children with MLD experience prolonged difficulty shifting from procedural-based problem solving (e.g., counting) to memory-based problem solving (i.e., direct retrieval) (Ostad, 1998). Rather, these children tend to rely upon less efficient and more manual problem-solving strategies, such as finger counting.

Increasingly, the field of mathematics has been placing emphasis upon the importance of working memory in the development of arithmetic calculation. Working memory is a limited-capacity information processing resource that has two principal processes: the preservation of information and the concurrent processing of the same or other information (Baddeley & Hitch, 1974). Two central reasons have emerged that underscore working memory's relationship to mental addition. First, numerous studies have reported that children with MLD are impaired in several memory-based processes (e.g., Berg, 2008a; Passolunghi & Siegel, 2001). Related evidence has been found in the examination of the differential mental addition performance between children with poor and normal working memory functioning. Barrouillet and Lépine (2005) found that children's working memory capacity was related to the use of direct retrieval and to solution speed in solving simple mental addition problems. Compared to children with poor working memory, children with normal working memory functioning used direct retrieval more frequently and provided answers more quickly.

Second, the processes of mental addition—counting procedures and memory-based strategies—parallel the operational characteristics of working memory (Case, 1985). An important distinction between the roles of short-term memory and working memory is that working memory is a more complex cognitive system that involves a wider range and more specialized processing and storage capabilities. For example, to solve the problem 6 + 7 one must concurrently retain the two addends, select a problem-solving strategy, and then employ one or more procedures (e.g., counting) to combine the numbers and produce an answer. Geary, Hoard, Byrd-Craven, and DeSoto (2004) contended that the importance of working memory in the development of mental addition fluency is more strongly associated with the process of strategy development than with the retrieval process. For example, with the introduction of complex mental addition problems, the utilization of sophisticated strategies has also been related to working memory (Adams & Hitch, 1998). With such problems, children are taught to use column re-alignment and regrouping strategies that involve combining partial sums.

Regrouping involves maintaining recently processed information while conducting a related operation; to add 16 and 7, the 3 from adding numbers in the ones column (6 + 7 = 13) must be retained while the newly processed 1 is added to the 1 provided in the original problem. These processes involve transformation and manipulation of problem representations and have been linked to working memory in adults (Heathcote, 1994).

Over time, as children develop expertise using a variety of problem solving strategies to perform mental addition, these strategy-problem associations become solidified. Increased proficiency in calculation strategies leads to a decrease in the amount of working memory resources required for solving these problems and supports transitions to increasingly complex arithmetic calculation abilities. Indeed, the relationship between working memory and mental addition is more profound during childhood and decreases over time; however, the relationship does not seem to disappear entirely. Little and Widaman (1995) examined the relationship between working memory and mental addition in school children in second grade to eighth grade and in college students. They found that, while working memory was more significantly related to the mental addition performance of school children than of university students, working memory remained important to mental addition performance in university students.

Importance of the Study

To date, theoretical and empirical research in the field of mathematical cognition implicates processing speed, short-term memory, and working memory in adult mental addition and children's arithmetic calculation. Less clear is the relative importance of these cognitive processes specific to children's mental addition fluency. By extension, the importance of these cognitive processes in understanding the mental addition difficulties of children who experience difficulty developing proficiency in arithmetic has received comparably little attention. The central purpose of this study was to investigate whether processing speed, short-term memory, and working memory account for the differential performance in mental addition fluency between children typically achieving in arithmetic (TA) and children at-risk for failure in arithmetic (AR). Two questions each associated with a particular hypothesis informed our general purpose. First, is processing speed the strongest predictor of differences in simple mental addition between children TA and children AR? We hypothesized that in light of AR children's difficulty accessing numerical-based representations from long-term memory, their poor fluency in simple mental addition would be more strongly related to processing speed than to storage ability (i.e., short-term and working memory). Said differently, processing speed would have the greatest effect in reducing the simple mental addition fluency differences between children TA and children AR. Our second question focussed upon whether difficulty performing complex mental addition was related to engagement of more complex cognitive processes beyond the use of processing speed? We hypothesized that—in light of the memory impairments characteristic of children AR and the particular demands complex mental addition places upon the cognitive system (e.g., executing more sophisticated strategies such as regrouping)—storage abilities (i.e., short-term and working memory) would have the greatest influence in accounting for fluency differences between TA children and AR children on complex mental addition.

Parallel with developing a more complete understanding of children's mental addition fluency, the present study has implications for curricular and instructional development. There is a developmental trajectory within the field of arithmetic calculation that underscores the progression from simple to complex calculation, in term of both mental and written calculation (Geary & Brown, 1991). Similarly, a progression is noted in relation to different arithmetic operations, where proficiency in addition often precedes the development of multiplication skills (Cooney, Swanson, & Ladd, 1988). A better understanding of the cognitive processes involved in developing mental addition fluency will assist educators in modelling pedagogical approaches that help students transition across curricular strands and toward more advanced areas of mathematics.

Definition of At-risk for Failure in Arithmetic

In examining the mental addition fluency of children with MLD, the field is challenged by a lack of clarity in the identification of children with MLD. Across the literature, a range of terms has been used to describe children who experience significant challenges developing mathematical abilities: *mathematics difficulties* (Hanich, Jordan, Kaplan, & Dick, 2001), *mathematics disabilities* (Geary, 1993, 2004), and *poor math achievement* (Mazzocco & Myers, 2003). These terms are often used interchangeably within the literature. While the term *MLD* was used when reviewing literature germane to the present study, the term *at-risk for failure in arithmetic (AR)* was adopted to describe the participants in the current study. Adoption of AR was based upon a two-part rationale. First, the term *disability* suggests an underlying condition marked by persistent cognitive impairment(s). To date, research has yet to reach consensus on cognitive processes that represent the core deficit or deficits in individuals with MLD (Geary, 2004, 2005). Rather, the term *at-risk* was used to classify those children within the lowest quartile on the arithmetic achievement continuum; representing approximately a two-year difference in typical arithmetic achievement. Second, while many studies use standardized achievement tests to identify difficulties in arithmetic, the term *mathematics* is often applied to describe participant groups. *Mathematics* is a more general term representing many areas including problem solving, algebra, and geometry, whereas arithmetic more accurately reflects computational competency. Thus, given that a test assessing arithmetic computation was administered in the present study, the term *arithmetic* was used rather than the more general term *mathematics.*

Method

Participants

Forty-eight elementary school children served as participants for this study. Children were classified as being at-risk for failure in arithmetic (AR, $n = 24$, 10 boys and 14 girls) if they scored at or below the 25th percentile in arithmetic and above the 25th percentile in reading. This percentile score cut-off has been used by other researchers to identify disparate groups of children typically achieving in arithmetic and children experiencing severe arithmetic difficulty (e.g., Siegel & Ryan, 1989; Swanson & Sachse-Lee, 2001). The typically achieving group (TA, $n = 24$, 11 boys and 13 girls) represented children who were similar in chronological age to the AR children and whose percentile scores were above the 25th percentile in arithmetic and in reading.

In accordance with school requests, the socioeconomic status of individual students was not assessed; however, each of the schools that participated in the study was located in a predominately middle-class neighbourhood. All children spoke English as their first language. No child had been identified as having a neurological disorder (e.g., Attention Deficit Hyperactivity Disorder) or identified as having English language difficulties that would have made it difficult for them to complete any part of the study.

Instruments

Academic achievement. The Wide Range Achievement Test-Third Revision (WRAT3) (Jastak & Jastak, 1993) was administered to measure children's arithmetic achievement and reading achievement. The arithmetic subtest focuses upon reading numbers, counting, mental arithmetic, and written calculation. The reading subtest focuses upon recognizing and naming letters, and pronouncing words. The WRAT3 has been used extensively to assess children's achievement and to identify children with learning difficulty in arithmetic (e.g., Mabbott & Bisanz, 2003; Wilson & Swanson, 2001). Raw scores and standard scores ($M = 100$, $SD = 15$) based on age-appropriate norms were calculated for each child. Cronbach alpha for the arithmetic and the reading subtests measured .87 and .86, respectively.

Mental addition. The mental addition battery included 20 mental addition problems. Problems were divided equally into two sets, corresponding to two levels of difficulty: 10 simple (e.g., 5 + 3) and 10 complex (e.g., 15 + 8) problems. Problems were categorized as either simple or complex based upon literature underscoring the problem-size effect (Adams & Hitch, 1998; Groen & Parkman, 1972). Children were asked to solve each problem as quickly and as accurately as possible and to use any strategy necessary to solve each problem. It was emphasized to children that the most important part of the task was to get the correct answer, but to get the correct answer as quickly as possible. Administration of each problem set was initiated by the researcher by turning a card containing the problems to face the child. Children were asked to answer problems verbally as soon as the card was turned to face them. Using a stopwatch, timing began when the card was turned over. Time was stopped when the child began to articulate an answer. Scores were calculated for each child based upon response times for correct answers for each level of difficulty set. Cronbach alphas for the simple mental addition and complex mental addition problems sets were .93 and .87, respectively.

Cognitive processing. Eight tasks were administered to measure the three cognitive processing domains under investigation. Two tasks assessed processing speed: digit naming and number articulation. Two tasks assessed short-term memory: digit span forward and word span forward. And four tasks assessed working memory: auditory digit sequence, semantic categorization, visual matrix, and Corsi blocks.

Processing speed. The digit naming task was administered to assess children's speed to identify numerical representations in long-term memory. This task was a modified version of a similar task used by Compton (2003). In the present study, children were required to read aloud sets of 9 randomly ordered Arabic digits as accurately and quickly as possible. Digits, 1 through 9, were arranged across three rows within three columns. Two trials were administered, with each trial containing a different arrangement of digits. A stopwatch was used to measure each child's naming times. A child's score for this task was their digit naming rate, calculated by dividing the number of digits read per trial (9 digits with 3 digits in each of 3 rows) by the mean time for the two trials. Cronbach alpha for the digit naming task measured .79.

The number articulation task assessed children's speed of speech for numbers. This task was adapted from a similar task used by Kail (1997). Children were asked to repeat a pair of single syllable numbers as quickly as possible 5 times. Four trials were administered using the number pairs: 1-4, 5-8, 3-6, and 2-9. Each number pair was presented orally by the researcher to the child. A stopwatch was used to measure the time to articulate each number pair five times. A child's score for this task was their articulation rate, calculated by dividing the sum of the number pairs articulated per trial (10 digits with 2 numbers repeated 5 times) by the mean time for the four trials. Cronbach alpha for the articulation task was .86.

Short-term memory. In the digit span forward task, the child was asked to listen to a series of single-digit numbers articulated by the researcher. Next, the child was asked to repeat the number sequence in the order presented by the researcher. If the child correctly stated the number sequence, another trial was administered. Successive trials increased by one-digit until the child failed two attempts within the same trial. The maximum possible span was nine digits. A child's score for this task was the highest number of digits correctly recalled in sequence.

Word span forward task was similar to the forward digit span task. Words were one-syllable frequently used words (Carroll & White, 1973). The maximum possible span was nine words. A child's score for this task was the highest number of words correctly recalled in sequence. Cronbach alphas for digit span and word span forward measured .75 and .79, respectively.

Working memory. The auditory digit sequence task (Swanson, 1995) assessed a child's ability to recall numerical information contained within a short sentence. A sentence containing a street address was read aloud to the child, a process question was presented, the child was asked to select a strategy depicted on a display card that would help them to remember the address, and then asked to recall part of the address. If the child answered the process question correctly, the child was asked to identify the strategy he or she used to remember the numbers and then asked to recall the number embedded within the address. If the child answered the process question incorrectly or recalled the address number incorrectly, the test was stopped. If the child answered the process question correctly and recalled the address number correctly, the next address was presented. Sets ranged from 2 to 9 sentences. A child's score was the number of sets recalled correctly. Cronbach alpha for the auditory digit sequence task measured .66.

The semantic categorization task (Swanson, 1995) assessed a child's ability to recall related words within prearranged groups. A set of words was read aloud to the child with a 2- second interval between words. Next the child was presented with a process question, asked to choose a strategy depicted on a display card that would help them remember the groups and words, and then asked to recall each group name and each word within its respective group. If the child answered incorrectly, the task was stopped. If the child answered correctly, the child was asked to state the strategy he or she would use to remember the group and the words within the group. Next the student was asked to recall the group and the words within that group. If the child responded correctly, the next word set was administered. Item-set difficulty ranged from one group with two words to eight groups with three words in each group. A child's score was the number of sets recalled correctly. Cronbach alpha for semantic categorization measured .60.

The visual matrix task (Swanson, 1995) assessed a child's ability to recall dots arranged within a matrix. The child was presented with a matrix containing a series of dots, given 5 seconds to study the matrix. The matrix was withdraw from sight, and then the child was asked a process question. If the child answered incorrectly, the task was stopped. If the child answered correctly, the child was then asked to reproduce the dot arrangement onto a blank matrix of the same size. If the child correctly reproduced the original matrix, the next matrix was administered. The items ranged in difficulty from a matrix of 4 squares with 2 dots to a matrix of 45 squares and 12 dots. A child's score was the number of matrices recalled correctly. Cronbach alpha for the visual matrix task measured .67.

The Corsi blocks task consists of nine blocks arranged randomly on a wooden board (Milner, 1971). The researcher pointed to a sequence of blocks at a rate of one per second. After the researcher completed tapping the sequence, the child was asked to replicate the sequence. If the child correctly recalled the sequence of blocks, another trial was administered. Successive trials increased by one block until the child failed two attempts within the same trial. No feedback was given to the child throughout the task. The maximum possible span was nine blocks. The score of this task was the highest number of blocks correctly recalled in sequence. Cronbach alpha for the Corsi blocks task measured .70.

Procedure

All children were assessed individually by the principal researcher in two sessions each corresponding to a specific test battery, with each session lasting approximately 30 minutes. The WRAT3 and the mental addition tasks were administered in the first session. The WRAT3 was administered first and the mental addition tasks were administered second. The order of presentation for the mental addition problems began with the simple level and progressed to the complex level. The cognitive processing battery was administered in the second session.

Results

Descriptive Statistics

Classification measures and mental addition. Means and standard deviations for chronological age, achievement measures, and mental addition tasks are presented in Table 1. Independent sample t-tests with alpha set at $p < .05$ were used to identify significant group differences on all measures. Effect sizes were calculated using Cohen's d (1988). Interpretation of effect sizes was based upon Cohen's estimates of strength ranges: small effect, approximately $d = .20$; medium effect, approximately $d = .50$; large effect, greater than $d = .80$. AR children and TA children did not differ in age $t(46) = -.59$, $p = .557$ or in reading $t(46) = -1.79$, $p = .139$, on the WRAT3-R standard score. However, as expected, the two groups differed in arithmetic calculation, with AR children's standard score on the WRAT3-A significantly lower than TA children, $t(46) = -8.71$, $p < .001$, $d = 2.59$. AR children and TA children were equally accurate on simple mental addition $t(46) = -1.19$, $p = .241$, and complex mental addition $t(46) = -.891$, $p = .377$. However, the two groups differed significantly on mental addition fluency, with AR children performing poorer than TA children on simple problems $t(46) = 3.07$, $p = .005$ and on complex problems $t(46) = 4.05$, $p < .001$. Effect sizes indicated that the magnitude of differences between the groups were large; simple mental addition $d = 1.04$ and complex mental addition $d = 1.29$.

Cognitive processing. Means and standard deviations for all cognitive processing measures are reported in Table 2. No significant differences were found on processing speed: digit naming $t(46) = -1.43$, $p = .158$, $d = .42$, or in number articulation $t(46) = -1.78$, $p = .082$, $d = .52$. Significant differences appeared for both short-term memory tasks: digit span $t(46) = -2.30$, $p = .026$, $d = .67$, and word span $t(46) = -2.24$, $p = .030$, $d = .64$. TA children scored higher than AR children on both measures. Significant group differences were found on three working memory measures. AR children performed poorer than TA children on auditory digit sequence $t(46) = -2.06$, $p = .046$, $d = .59$, visual matrix $t(46) = -3.61$, $p = .001$, $d = 1.04$, and Corsi blocks $t(46) = -2.92$, $p = .005$, $d = .85$. AR children and TA children were comparable on semantic categorization $t(46) = -1.89$, $p = .065$, $d = .54$. Effect sizes for all significant differences in cognitive processing were within the medium to large range (Cohen, 1988).

Table 1
Descriptive Statistics for Chronological Age, Academic Achievement, and Mental Addition

Measures	AR (n = 24)		TA (n = 24)	
	M	*SD*	*M*	*SD*
Age (months)	121.21	13.85	123.42	11.92
Arithmetic				
Raw	25.25	3.17	31.38	3.99
Standard	84.42	5.23	102.42	8.67
Percentile	16.13	6.46	54.96	9.13
Reading				
Raw	35.58	3.89	37.54	3.70
Standard	104.25	8.83	108.08	8.82
Percentile	59.42	10.55	68.17	9.53
Simple mental addition				
Accuracy	9.63	.77	9.83	.38
Fluency	6.06	4.52	3.14	1.08
Complex mental addition				
Accuracy	8.46	1.56	8.88	1.68
Fluency	15.12	7.44	8.57	2.71

Note. AR = at-risk for arithmetic failure; TA = typically achieving.

Table 2

Descriptive Statistics for Cognitive Processing Measures

Tasks	AR (n = 24)		TA (n = 24)	
	M	*SD*	*M*	*SD*
Processing speed				
Digit naming	2.64	.51	2.84	.44
Number articulation	3.68	.51	3.96	.56
Short-term memory				
Digit span forward	5.15	.92	5.69	.70
Word span forward	4.63	.97	5.21	.83
Working memory				
Semantic categorization	1.67	.92	2.17	.92
Auditory digit sequence	1.92	.78	2.38	.77
Visual matrix	3.38	1.14	4.50	1.02
Corsi blocks	5.17	.87	5.79	.59

Note. AR = at-risk for arithmetic failure; TA = typically achieving.

Predictive Models of Mental Addition Fluency

In correspondence with procedures used in similar studies (e.g., Swanson & Sachse-lee, 2001), raw scores of cognitive measures were aggregated into composite scores along theoretical lines by summing *z*-scores for each appropriate task based on the total sample. Empirical support for this approach was established in a previous study with a larger group of children. A confirmatory factor analysis conducted on the processing speed, short-term memory, and working memory measures used in the present study indicated that these measures formed different though related scales (Berg, 2008b). Intercorrelations among mean composite scores are reported in Table 3. Prior to conducting regression analyses to address the posited hypotheses, we examined intra-item characteristics that might influence emergent models. Correlations between the cognitive processing domains and the simple mental addition tasks indicated all predictors were strongly associated with both simple and complex mental addition (see Table 3). Correlations among the predictor variables were moderate to strong; however, they were not at levels that would be problematic for multiple regression analyses. Both tolerance (range .48-.93) and variance inflation factor (range 1.07-2.09) values for all predictors were within acceptable ranges indicating that collinearity was not a problem (Hair, Anderson, Tatham, & Black, 1998). In preview, across all regression models the Durbin-Watson statistic ranged from 1.26 to 1.55 indicating that the assumption of independence of errors was satisfied.

Table 3
ntercorrelations Among Cognitive Processing Tasks, Mental Addition, and Achievement Contrast

	1	2	3	4	5	6
1. Processing speed	-					
2. Short-term memory	.54**	-				
3. Working memory	.46**	.62**	-			
4. Simple mental addition	-.64**	-.44**	-.45**	-		
5. Complex mental addition	-.65**	-.56**	-.50**	.86**	-	
6. AR vs TA	-.26	-.35*	-.55**	.41**	.51**	-

Note. AR = at-risk for arithmetic failure; TA = typically achieving.
* $p < .05$. ** $p < .01$.

Table 4
Hierarchical Regression Analyses: Predictive Models of Simple Mental Addition

Order of entry in equation	R^2	R^2 Change	*df*	*F* Change
		Model 1a		
1. AR vs TA	.17	-	46	9.47**
		Model 2a		
1. Processing speed	.41	-	46	32.40***
2. AR vs TA	.48	.07	45	5.63*
		Model 3a		
1. Short-term memory	.19	-	46	10.90**
2. AR vs TA	.27	.08	45	4.72*
		Model 4a		
1. Working memory	.20	-	46	11.47**
2. AR vs TA	.24	.04	45	2.40

Note. AR = at-risk for arithmetic failure; TA = typically achieving.
* $p < .05$. ** $p < .01$. *** $p < .001$.

Using the composite *z*-scores for each cognitive construct, a series of fixed-order multiple regression analyses were conducted to examine the individual contributions of each cognitive domain in reducing differences in mental addition fluency between children with AR and TA children. To measure fluency variability between AR and TA children, a contrast variable was entered into the regression equations. Similar to other researchers (Swanson & Sachse-Lee, 2001), we reasoned that any reduction in the contribution of this contrast variable to fluency after entering a cognitive domain was attributable to the influence of that cognitive domain. Further, if the contrast variable contributed variance to fluency after accounting for the variance of one or more cognitive domains, we reasoned that any remaining variance could be ascribed to an unexamined variable. Two sets of models were constructed, each corresponding to one of the two levels of mental addition difficulty under examination. Within the regression models, we assessed the effect sizes for the variance accounted for by individual and combined cognitive domains. Effect sizes for all significant predictors were estimated using Cohen's (1988) f^2, where values of .02 represent a small effect, values of .15 represent a medium effect, and values of .35 represent a large effect.

Hypothesis 1: Processing speed and simple mental addition. The first objective of our study centred upon examining whether the difficulty children with AR experience accessing numerical-based representations from long-term memory is primarily responsible for their poor simple mental addition fluency. We hypothesized that compared to short-term memory and working memory, processing speed would have the greatest effect in reducing the simple mental addition fluency differences between children TA and children AR. Table 4 shows the individual variance accounted for in simple mental addition fluency attributable to differences between TA children and AR children and to the three cognitive domains under examination. Model 1a revealed that the contrast between children with AR and TA children accounted for 17% of the variance in fluency. Cohen's f^2 value for the contribution of the AR vs. TA contrast variable was .20, which represented a medium-to-large effect. Three subsequent regressions, Models 2a-4a, captured the individual variance in fluency related to processing speed (41%), short-term memory (19%), and working memory (20%). Effect sizes for short-term memory (f^2 = .23) and working memory (f^2 = .25) reflected medium-to-large effects. Processing speed, however, with a Cohen's f^2 value of .69, represented a large effect.

Further examination of Models 2a-4a indicated that working memory had the greatest effect in reducing fluency differences between children with AR children and TA children. When working memory (Model 4a) was entered into the regression equation, the contrast variable contributed no significant variance to fluency. That is, working memory accounted for all variance in simple mental addition fluency between children with AR and TA children. Short-term memory (Model 3a) had the least effect in reducing fluency differences between children with AR and TA children, 52.9%, from 17% to 8% (i.e., [.17 - .08] / .17]) while processing speed reduced group differences by 58.8%, from 17% to 7% (i.e., [.17 - .07] / .17]). The effect size for the remaining group difference remaining after accounting for processing speed (f^2 = .13) and short-term memory (f^2 = .11) represented small-to-medium effects (Cohen, 1988).

Hypothesis 2: Memory and complex mental addition. Our second objective focussed upon investigating whether the memory-related impairments of children with AR are associated with their poor complex mental addition fluency. More specifically, we hypothesized that in light of the working memory impairments characteristic of children AR and the particular demands complex mental addition places upon the cognitive system, storage abilities would have the greatest influence in accounting for fluency differences between TA children and AR children on complex mental addition. Table 5 illustrates the individual variance accounted for in complex mental addition fluency attributable to differences between TA children and AR children and to the three cognitive domains under examination. Model 1b indicated that the contrast between children with AR and TA children accounted for 26% of the variance in fluency. Cohen's f^2 value for the contribution of the AR vs. TA contrast variable was .20, which represented a medium-to-large effect.

Table 5
Hierarchical Regression Analyses: Predictive Models of Complex Mental Addition

Order of entry in equation	R^2	R^2 Change	*df*	*F* Change
		Model 1b		
1. AR vs TA	.26	-	46	16.42***
		Model 2b		
1. Processing speed	.42	-	46	35.18***
2. AR vs TA	.56	.14	45	12.82**
		Model 3b		
1. Short-term memory	.31	-	46	20.97***
2. AR vs TA	.43	.12	45	8.96**
		Model 4b		
1. Working memory	.25	-	46	15.31***
2. AR vs TA	.33	.08	45	5.51*
		Model 5b		
1. Processing speed	.42	-	46	35.18***
2. Short-term memory	.49	.07	45	5.22*
3. AR vs TA	.58	.09	44	9.61**
		Model 6b		
1. Processing speed	.42	-	46	35.18***
2. Working memory	.48	.06	45	4.24*
3. AR vs TA	.56	.08	44	7.97**
		Model 7b		
1. Processing speed	.42	-	46	35.18***
2. Short-term memory	.49	.07	45	5.22*
3. Working memory	.51	.02	44	1.19
4. AR vs TA	.58	.07	43	8.06**

Note. AR = at-risk for arithmetic failure; TA = typically achieving.
* $p < .05$. ** $p < .01$. *** $p < .001$.

Three subsequent regressions, Models 2b-4b, captured the individual variance in fluency related to processing speed (42%), short-term memory (31%), and working memory (25%). The effect size for working memory ($f^2 = .33$) represented a medium-to-large effect. Cohen's f^2 values for processing speed (.72) and for short-term memory (.45) reflected large effects. Further examination of Models 2b-4b indicated that working memory (Model 4b) had the greatest effect in reducing the fluency differences. Specifically, variance attributable to group differences was reduced by 69.2%, from 26% to 8% (i.e., [.26 - .08] / .26]), with a Cohen's f^2 value of .12 representing a small-to-medium effect. Processing speed (Model 2b), had the least effect reducing fluency differences; the contrast variable was reduced by 46.2%, from 26% to 14% (i.e., [.26 - .14] / .26]), while short-term memory reduced group differences by 53.8%, from 26% to 12% (i.e., [.26 - .12] / .26]). Effect sizes for the remaining group differences in mental addition represented values within a small-to-medium range, with a f^2 value of .32 after accounting for processing speed and a f^2 value of .21 after accounting for short-term memory.

Models 1b to 4b supported our initial hypothesis that working memory would have the greatest effect in reducing group differences in complex mental addition fluency. However, we were interested in the observation that variance attributable to group differences remained after accounting for working memory (8%). We constructed three subsequent models to examine the cumulative influence of the cognitive domains on reducing group differences in fluency. Specifically, models were constructed to examine whether short-term memory (Model 5b) and working memory (Model 6b) decreased group differences in fluency beyond the influence of processing speed. Taken together, these two models revealed that short-term memory and working memory each contributed unique variance beyond the influence of processing speed (7% and 6%, respectively). The additional variance attributable to short-term memory and working memory reflected effect sizes (.14 and .12, respectively) within the small-to-medium range.

Of note, each model failed to account for group differences in fluency (contrast variable contributed 9% and 8%, respectively). Effect sizes for the remaining group differences in complex mental addition represented values within the medium-to-large range, with a f^2 value of .21 after accounting for processing speed and short-term memory and a f^2 value of .18 after accounting for processing speed and working memory. A final model (Model 7b) was constructed to examine whether the cumulative influence of all cognitive domains would completely account for fluency differences between the groups. Inspection of this model revealed two significant findings. First, the influence of working memory in reducing group differences was eliminated in the presence of processing speed and short-term memory. Second, significant group differences in fluency remained (9%, refer to Model 5b for a more parsimonious result) after accounting for the influence of all cognitive domains.

Discussion

While failure of AR children to develop mental addition fluency is one of the most salient findings in the field of learning disabilities (e.g., Geary, 1993; Jordan & Montani, 1997; Ostad, 1998), extant literature offers only a brief sketch of the cognitive processes that are associated with their difficulty. In light of a growing body of evidence indicating that AR children are characterized by processing speed, short-term memory, and working memory impairments (e.g., Berg, 2008a; Siegel & Ryan, 1989), we investigated whether these cognitive domains account for fluency differences between AR children and TA children. Further, we were interested in whether levels of mental addition problem difficulty would be associated with different cognitive processes. Our first hypothesis, that processing speed would account for simple mental addition fluency differences between children with AR and TA, was partially supported. Our rationale rested upon research (Dehaene, 1992) that implicates slow access to numerical representations in long-term memory as a constraint upon AR children's ability to access stored associations between simple mental addition problems and their respective answers. Although this premise was supported (processing speed did significantly reduce group differences by 58.8%), variance attributable to group differences remained (7%). Working memory, however, completely accounted for the variance associated with group differences in fluency.

An explanation of our results is provided by Geary (1990, 1993) who suggested that the mental arithmetic performance of AR children is related to interactions between working memory and long-term memory (Geary, 1990, 1993). Geary (1993) argued that the relationship between working memory and mental addition was related to an individual's ability to maintain an association between the component parts of a problem (e.g., partial results) while performing an additional step (e.g., borrowing or regrouping). For instance, poor working memory would disrupt establishing an association between a combined set of addends and an answer. A disruption in the development of paired associations would decrease the probability for encoding basic facts into long-term memory. That neither processing speed nor short-term memory totally accounted for group differences in fluency underscores the likelihood that the nature of AR children's poor fluency is related to complex cognitive processes beyond simply accessing long-term memory or the temporary storage of information.

Consideration of the involvement of a complex of cognitive processes in mental addition was also evidenced in view of those cognitive domains that contributed to reducing group differences in fluency on complex problems. Our hypothesis related to complex mental addition fluency stated that given the working memory impairments characteristic of AR children and the particular demands complex mental addition places upon the cognitive system, storage abilities would have the greatest influence in accounting for fluency differences between TA children and AR children. While this hypothesis was also partially supported, follow-up analyses provided a more complete picture of the cognitive processes related to complex mental addition fluency after processing speed and short-term memory were inserted into the regression equations (Model 5b to 7b). That the unique influence of working memory on complex mental addition was eliminated (after accounting for processing speed and short-term memory) is possibly related to the componential structure of the working memory system. Since working memory is considered a processing and storage system (Baddeley & Hitch, 1974), the cumulative influence of processing speed and short-term memory likely paralleled the influence of working memory. That is, the contribution of working memory would be comparable to the combined contribution of processing speed and short-term memory.

With group differences in complex mental addition fluency unaccounted for in the regression models, the question arises as to what cognitive domains(s) would account for differential performance between TA and AR children? A potential avenue for understanding these findings is the attentional aspects of executive functioning, such as switching attention. For instance, when solving 5 + 8 one must not only access the numerical representations from long-term memory and remember the numbers and operation sign (i.e., storage), but one must also actively direct attention among these elements of the problem when calculating a solution. One must switch, or redirect their focus, from one part of the problem (e.g., an addend) to another part of the problem (e.g., the operation), and switch to the strategy chosen to solve the problem. Recently, executive functioning has been receiving increased attention as playing a role in mathematical achievement and arithmetic performance (Bull &

Scerif, 2001). Moreover, research has highlighted executive functioning impairments of children with learning disabilities (Van der Sluis, de Jong, & Van der Leij, 2004).

Limitations and Future Research

An important direction for future research would be to examine relationships between specific problem-solving strategies and individual cognitive domains. For instance, does working memory mediate the relationship between finger counting or verbal counting proficiency and mental addition fluency in children with AR? Without assessing children's strategy use as a contributing factor in their mental addition performance (accuracy and fluency) any relationships between cognitive processing in mental addition calculation must be viewed as indirect and suggestive only of potential contributing factors. Additionally, research is needed that addresses the cognitive processes that predict the development of mental arithmetic fluency. While working memory has been viewed as central to mental addition performance (e.g., Geary et al., 2004), with additional support from the present study, does working memory hold similar importance in related areas of arithmetic such as subtraction and multiplication? Further, with increasing attention being directed at numerical-associated abilities (i.e., number sense) in the area of arithmetic (written calculation cognition, Locuniak & Jordan, 2008), where does cognitive processing fit within the complex of predictors of arithmetic fluency?

A key limitation of the present study is its potential for generalisation. While the sample size of the at-risk group in the present study (n = 24) is relatively small, it does correspond to other studies in the field (e.g., Jordan & Montani, 1997) and is moderately larger than some studies (n=15, Geary & Brown, 1991; n=12, McLean & Hitch, 1999). We contend, however, that the present study's results are informative to the field in general. First, while other studies have focused upon word problem solving (e.g., Swanson & Sachse-Lee, 2001) and written addition (e.g., Locuniak & Jordan, 2008), our research joins a smaller group of studies that focus specifically on mental addition in children at-risk for arithmetic difficulties and those with mathematical disabilities (e.g., Geary, et al., 2004). Second, the present study focused on a particular comparison between children typically achieving in arithmetic and children at-risk for arithmetic difficulty. Other studies in the field have employed methodologies that create sample groups based upon general mathematical performance, a practice that invariably creates heterogeneous groups each with a range of competencies within a variety of mathematics areas (e.g., calculation, algebra). Our methodology allowed us to focus upon a specific area of mathematics (mental addition) within a more homogeneous participant group.

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