# The power of a single game to address a range of important ideas in fraction learning

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describe a fraction game used to develop key concepts including equivalence and addition of fractions.

As part of the Contemporary Teaching and Learning of Mathematics Project (CTLM[1]), the mathematics education team at Australian Catholic University has the privilege of working with principals, teachers, students, and parents in schools in the Melbourne Archdiocese.

A particular highlight is the opportunity to work alongside project teachers and their students in developing worthwhile and relevant classroom activities. In the last 3 years, we have had the chance to try out particular fraction and decimal lessons, making subtle changes each time, as we and our teacher colleagues learn from each classroom experience.

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In this article, we will describe one such activity, Colour in Fractions, including its evolution, the mathematics which it has the power to address, and some hints for teachers using it for the first time. We mentioned this activity briefly in Clarke, Roche and Mitchell (2007), but have gradually refined it, and in this article, we provide a detailed outline of the purpose and benefits of its use.

## Fractions: Difficult to teach and to learn

Fractions are widely recognised as a very important topic in the middle years, but one that is difficult to teach (Ma, 1999) and to learn (Behr, Lesh, Post & Silver, 1983). Among the factors which make rational numbers in general and fractions in particular difficult to understand are their many representations and interpretations (Kilpatrick, Swafford & Findell, 2001).

Many teachers complain, with justification, that we have a crowded mathematics curriculum in the middle years, leading to a strong tendency to treat topics at a "surface level", given the pressure to "get through it all." Our experience is that state/territory and school curricula and consequently teachers give inappropriate and premature attention to the four operations with fractions and decimals, while failing to give appropriate emphasis to more important foundational notions, such as, for example, fraction as division, fraction as operator, and fraction as measure (Clarke, 2006; Clarke et al., 2007).

Given the pressure of the crowded curriculum, it is exciting, therefore, to find games/lessons/activities which address a number of important ideas in a challenging but accessible and enjoyable way.

The data from 323 interviews with students at the end of Year 6 (Clarke, Roche, Mitchell & Sukenik, 2006) indicated that students need classroom experiences which assist them to understand more clearly the roles of the numerator and denominator in a fraction, the meaning of improper fractions, and the relative sizes of fractions. The game discussed in this article has been shown to provide assistance in all of these areas and others.

Teachers have used fraction walls of various kinds for many years in teaching fractions, and in many cases, played games of a similar kind to the one below. However, it is some of the refinements to the game which in our opinion have made the impact more substantial.

## Colour in Fractions: The rules

Students have dice that create fractions up to twelfths, and a fraction wall. They colour in sections of the wall that correspond to the fractions that they roll with the dice. They have:

- one die labelled 1, 2, 2, 3, 3, 4 in one colour
- another die labelled $\frac{*}{2}$, $\frac{*}{3}$, $\frac{*}{4}$, $\frac{*}{6}$, $\frac{*}{8}$, $\frac{*}{12}$ in another colour
- a fraction wall as shown in Figure 1.

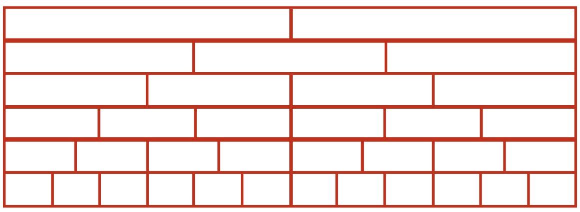

Figure 1. Fraction wall

Each horizontal strip is one whole. So, the first strip is made up of two halves, the next one three thirds, and so on. Players in turn throw both dice. They make a fraction, the first die being the numerator. They then colour the equivalent of the fraction shown. For example, if they throw 2 and $\frac{*}{4}$, then they can colour in $\frac{2}{4}$ of one line, or $\frac{4}{8}$ of one line, or $\frac{1}{4}$ of one line and $\frac{2}{8}$ of another, or any other combination that is the same as $\frac{2}{4}$.

If a player is unable to use their turn, they "pass." The first player who colours in their whole wall is the winner, but the other player is encouraged to keep going (with the support of the first player) to fill their fraction wall, if time permits.

## How we introduce the game to students

We believe that the game as described here is appropriate for an introduction to fractions from Grades 4 to 8, as different students will gain more from the game each time they play. In fact, we believe strongly that teachers should encourage students to play the game in full at least three times, in order for its potential to be realised.

In introducing the game, we make use of an A3 version of the fraction wall, stuck to the board, and gather the class around to demonstrate. We invite one student to roll the two dice and generate a fraction. Because we need a fraction that has a number of straightforward equivalences, we prefer something like $\frac{1}{4}$ or $\frac{4}{8}$, and if the student rolls $\frac{1}{12}$ or $\frac{4}{3}$, for example, we ask them to roll again. We choose not to demonstrate the consequences of rolling an improper fraction, as we prefer this to arise naturally during the game.

Assuming that $\frac{1}{4}$ has been created (say), we ask the student what they choose to shade in. The student usually shades in one of the quarters, but we ask the class to consider other possibilities, and usually $\frac{2}{8}$ is suggested. We point out that they can shade in any fraction equivalent to $\frac{1}{4}$, even part of one strip and part of another (e.g., $\frac{1}{6}$ + $\frac{1}{12}$). Of course, once they decide, they must only shade in one equivalence. We also mention that they do not have to start from the left of the fraction wall each time. Please note that the second player does not do anything with her/his board as yet. The second player's turn will be next.

We then show them the fraction wall and suggest the following:

Each roll, the student should use a *different colour* pencil or texta. This means that it is easy for them and for the teacher moving around to follow clearly the decisions made at each stage of the game. The latest version of the game sheet has "what I rolled" and "what I shaded" columns, so that these can be distinguished and considered at a later stage. For example, a student might have rolled "$\frac{3}{8}$", but shaded "$\frac{1}{4}$ + $\frac{1}{8}$". A sample game sheet is shown in Figure 2.

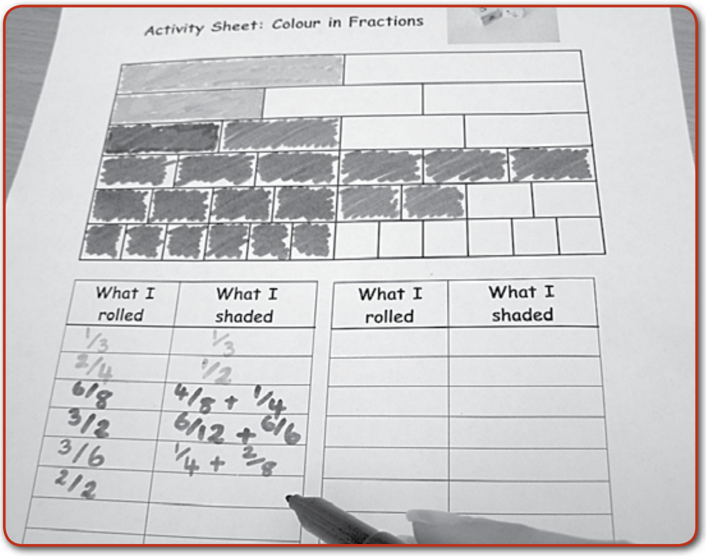


Figure 2. A sample game sheet.

The game then proceeds as follows:

1. They should take it in turns to roll and shade, and if the fraction rolled or its equivalence cannot be shaded, they miss a turn. This becomes more frequent later in the game.
2. They are not allowed to break up a "brick."
3. In finishing off the game, the student must roll exactly what they need. A larger fraction is not acceptable to finish. So, for example, if they need $\frac{1}{8}$ to finish and they roll $\frac{2}{4}$ (clearly more than is needed), they must miss a turn.

## Naming and representing improper fractions: A powerful opportunity provided by this game

As will be evident to the reader, no mention has been made to date of improper fractions, which are bound to arise at some stage in the game, because the dice are designed to make this happen. Although some teachers assume that a lesson on improper fractions should precede the game, our experience is that it is better to just let improper fractions arise naturally, and then look at how students choose to deal with them.

We have noticed that some students in the middle years who roll $\frac{4}{3}$ call it "four threes." This use of whole number rather than fractional language appears to be an indicator that the students do not yet understand which digit refers to the number of parts or the size of the parts. This provides the chance with the individual or the whole class to draw their attention to the meaning of the numerator and the denominator in a fraction. We see this as a key teaching point within the lesson.

When students are first trying to make sense of common fractions, teachers have typically defined them as follows:

"The denominator tells you how many parts the whole has been broken up into, and the numerator tells you how many of these parts to take, count or shade in."

Now, this works reasonably well for fractions between 0 and 1, but not well for improper fractions. We prefer this explanation for students:

"In the fraction $\frac{a}{b}$, $b$ is the name or size of the part (e.g., fifths have this name because 5 equal parts can fill a whole) and $a$ is the number of parts of that name or size."

So if we have $\frac{4}{3}$, the three tells the name or size of the parts (thirds), and the 4 tells us that we have 4 of those thirds (or $1\frac{1}{3}$).

With this explanation, students quickly see that rolling an improper fraction, early in the game, can work to their advantage

## Accessibility of the game to all students

One of the features of the game is its suitability for students with a range of levels of understanding of fractions. Some students will roll $\frac{3}{4}$ and shade in three of the quarters if the quarters are as yet unshaded, without considering any other options. If they have already "used" the quarters and they roll $\frac{3}{4}$, they will elect to miss a turn. However, students who have a good understanding of equivalence will of course look for a range of equivalences of $\frac{3}{4}$ and choose one which suits the situation. The advantage of playing the game in mixed ability pairs is that students will learn from each other as they play, gradually increasing their understanding of possibilities.

At the conclusion of the game, we often ask students to share with the class their most interesting turn and describe what they rolled and what they shaded. Reading straight from their game sheet they can recall a roll (say, $\frac{3}{4}$) and then describe the bricks that they chose to shade for that roll (e.g., $\frac{2}{8} + \frac{1}{4} + \frac{3}{12}$). This sharing of these choices will also contribute to the learning that occurs with peers.

Rather than waiting for one player to completely shade the fraction wall, teachers sometimes find that the game may be better concluded at a specified time, and the players can then consider who will be the winner—the player whose fraction wall has the least total yet to be shaded. Determining the winner in this case is an interesting mathematical task in itself.

At some stage, the teacher may wish to point out that if the fraction wall is completely shaded, then all of the shaded parts must add up to 6 (six wholes), or alternatively draw this from the students. Students can be invited to check this for their game.

## A discussion of strategies

In pulling the lesson together, we often ask students to consider one of two questions:

1. If you played the game tomorrow, what would you do differently?
2. If you were giving some hints to a younger brother or sister who was about to play the game, what would these hints be?

It is interesting that students' hints often relate to either shading the little bits first (e.g., the twelfths or the eighths) so that they are not "stuck" with them at the end, or to shading the large parts first, leaving them with more possibilities at the end.

## Where's the mathematics? A quick summary of the mathematical potential of the game

Given the crowded curriculum, it is important to realise the many aspects of fractions addressed during the game. Teachers identify the following, with the first four being the major points:

- Equivalent fractions—the physical representation of the fraction wall enables students to "line up" particularly difficult fractions to generate equivalent combinations (e.g., lining up $\frac{4}{12}$ as the same as $\frac{1}{6}$ and $\frac{2}{12}$ combined).
- This game encourages the use of "fractional language" (e.g., three-quarters instead of three out of four) which is helpful for understanding fractional parts within the part-whole construct.
- Understanding improper fractions—the game provides an excellent introduction to improper fractions, in a context where understanding is motivated by the need to use them to advantage in the game context.
- Addition of fractions—the game provides a very appropriate introduction to this notion, as students record their various sums in the "what I shaded" column, and notice some of the patterns involved.
- Problem solving—as there are many options possible at most stages of the game, the students have to weigh all of these up, before committing to a particular shading.
- Visualisation—sometimes the students are challenged to see how what they have rolled might be represented by a combination of some of the remaining parts—this can be quite a visual challenge on occasions.
- Probability—what is the chance that I will get what I need? As the teacher moves around the room, they can ask individuals or pairs, "What do you still need and how might you get this?" For example, a student might still need $\frac{2}{12}$ and could get this as either $\frac{1}{6}$ or $\frac{2}{12}$. If they only need $\frac{1}{12}$ to finish, it is a good chance to discuss the fact that they would only expect on average to form this fraction one in every 36 rolls (one chance in 6 on one die—the "1"; and one chance in 6 on the other die—the "$\frac{*}{12}$").

## Follow up investigation—each group gets a fraction

After a couple of lessons playing the game, each group could be given a particular fraction (e.g., $\frac{3}{4}$, $\frac{4}{6}$, $\frac{4}{3}$) and invited to find as many equivalences as they can, which are possible with the fraction wall in the Colour in Fractions game. A sample small group response is shown in Figure 2.

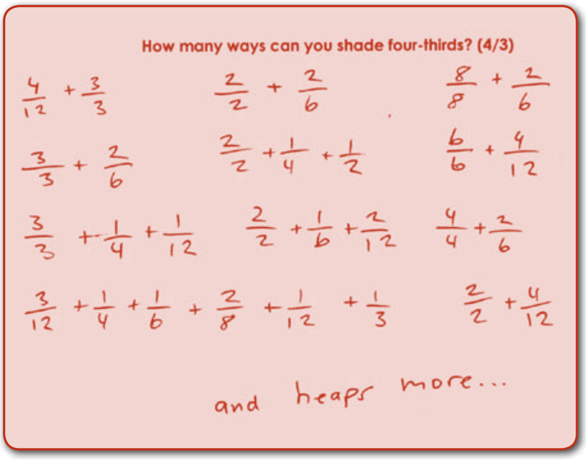


Figure 3. Expressions equivalent to $\frac{4}{3}$.

Another challenge is to present the students with a hypothetical game in progress, and invite them to suggest the way in which a particular roll might work out for the player.

For example, children could be shown the game in progress depicted in Figure 4 , where $\frac{1}{3}$, $\frac{2}{6}$, $\frac{2}{8}$, and $\frac{1}{12}$ remain unshaded. They are then told, "Imagine someone has just rolled the fraction $\frac{2}{3}$ (or $\frac{3}{8}$ or $\frac{4}{12}$ or $\frac{1}{2}$, respectively). What would you recommend they shade in, or do they have to miss a turn? In particular, is it possible to complete the fraction wall with one more roll of the dice?" (As the reader will determine, the answer is "Yes.")

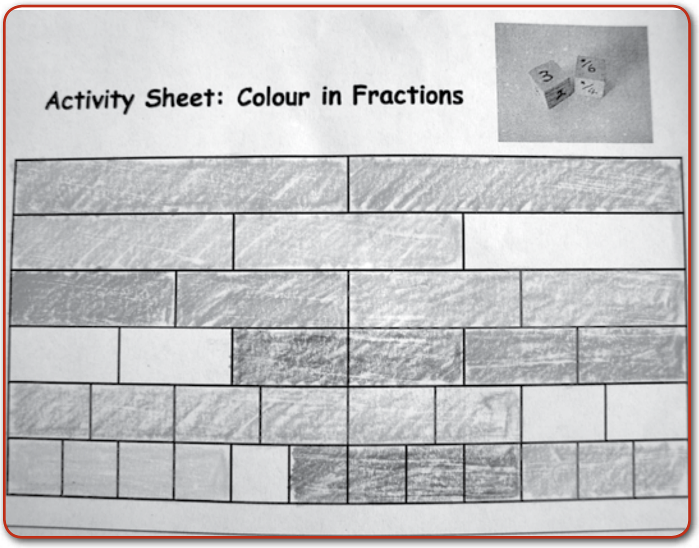


Figure 4. Example of a game in progress.

## An approach for students who find the creation of equivalences a challenge

One teacher told us about a way she makes the game accessible to students who find the equivalences a great challenge. She gives the students a clean fraction wall and asks them to label all the bricks (each one of the halves as $\frac{1}{2}$, each one of the thirds as $\frac{1}{3}$, etc.). They then cut out all the pieces and when they are playing the game and they roll $\frac{3}{4}$ for example, they physically manipulate some of the bricks to carefully match the $\frac{3}{4}$, and, having found one that works, they then complete their turn on the game sheet.

## In summary: A game with great potential

Hopefully, we have outlined the potential of a game in assisting students to develop key concepts of equivalence and addition of fractions. We invite the reader to try out the game with its various enhancements with a group of students at least three times, and let us know how it goes. The reader may also be interested in the *Decimat* game (Roche, 2010), which uses a similar model and game to assist students' developing understanding of decimal fractions.

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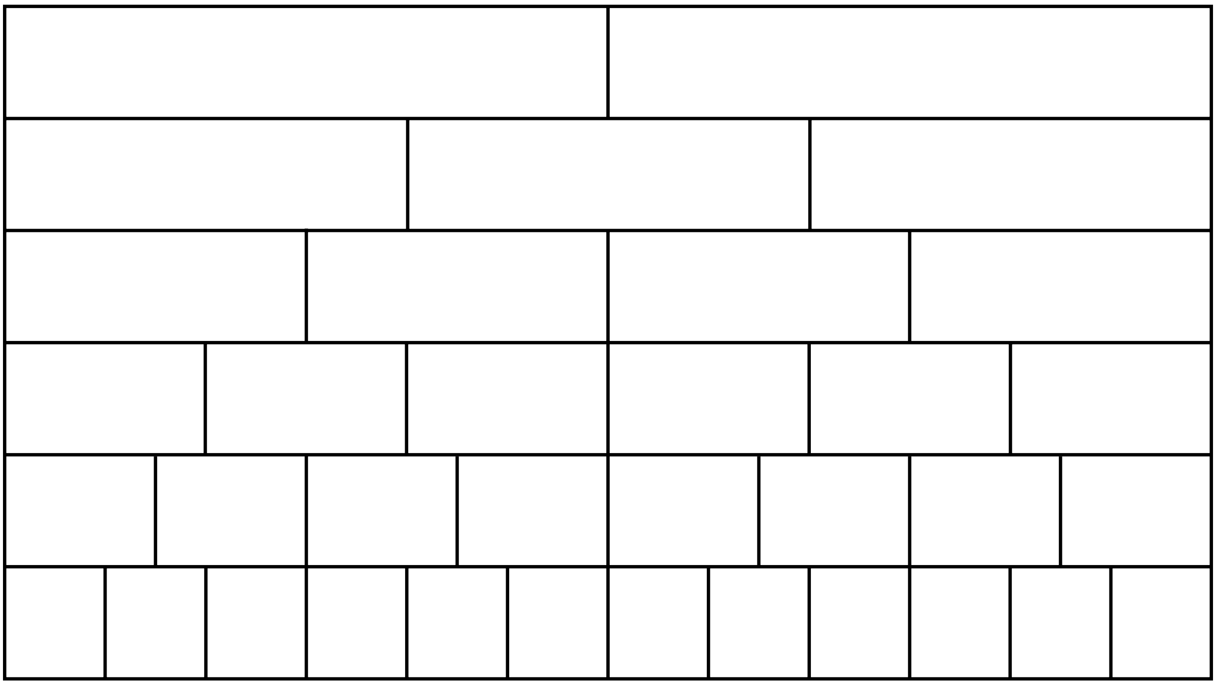
APMC

## Activity Sheet: Colour in Fractions



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