An Investigation of Fifth Grade Students' Conceptual Development of Probability through Activity Based Instruction: A Quasi-Experimental Study

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Abstract

This study aims to compare the effects of activity-based instruction and traditional instruction on fifth grade primary school students' conceptual development of probability. The study was conducted through quasi-experimental method and carried out with 50 5th grade primary school students, 25 for experimental group and 25 for control group. The Conceptual Development Test consisting of 12 open-ended questions was administered to the students in the study before and after the treatment. Data were analyzed using independent samples t-test, and analysis of covariance (ANCOVA). It was determined that activity-based instruction was more effective in helping students develop the probability concepts than did traditional one.

Key Words

Mathematics Teaching, Probability, Activity-Based Instruction, Conceptual Development.

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Mathematics educators have used various activities and approaches so far to implement mathematics instruction effectively. Activities, designed to present abstract mathematical expressions in a concrete and visual way help students develop creative thinking and imagination (Thompson, 1992). Shaw (1999) agrees with a large number of educators who claim that students should not be passive during the learning process and states that the approaches which involve students in the process of constructing knowledge should be adopted. When mathematical concepts are taught only by verbal expressions or symbols to students who do not have enough mental maturity, they cannot understand these concepts which are abstract to them (Piaget, 1952). Piaget expresses that primary school students need ample experiences with many materials and drawings in order to understand mathematical concepts. Similarly, a large number of studies claimed that learning environments should be enriched with easily accessible activities so as to allow the comprehension of mathematical concepts (Baki, Gürbüz, Ünal, & Atasoy, 2009; Castro, 1998; Clements & McMillen, 1996; Durmuş & Karakırık, 2006; Gürbüz, 2006a, 2008; Moyer & Jones, 2004; Moyer, Bolyard, & Spikell, 2002; Sowell, 1989; Tatsis, Kafoussi, & Skoumpourdi, 2008; Thompson, 1992).

Literature Framework of the Probability

In the literature, it is possible to encounter with various studies about the impact of different strategies used in teaching probability subject on students development of probability concepts (Amir & Williams, 1999; Aspinwall & Shaw, 2000; Babai, Brecher, Stavy, & Tirosh, 2006; Baker & Chick, 2007; Castro, 1998; Gürbüz, 2006a, 2007; Gürbüz, Çatlıoğlu, Birgin, & Toprak, 2009; Jones, Langrall, Thornton, & Timothy Mogill, 1997; Memnun & Altun, 2007; Nilsson, 2007, 2009; Pijls, Dekker, & Van Hout-Wolters, 2007; Pratt, 2000; Quinn & Tomlinson, 1999; Tatsis et al., 2008; Watson & Kelly, 2004). For example, Nilsson (2009) conducted a study to determine how the interpretations of 12-13 year-old-Swiss students regarding probability concepts changed after they have seen the results of some experiments. For this purpose, he designed a game in which two different forms of dices were used for 8 primary students. In this game, students were required to identify the sum of two dices, sample space and probability distributions using dices designed as (222 444), (333 555), (111 333), (444 666), (222 555) and asymmetrical dices which were different from their traditional dice perception as (123 456). Group discussions were recorded and filmed. As a consequence, the students were found to make some mistakes due to their traditional dice perceptions and they were also found to remedy their mistakes after they noticed that the forms of the dices changed in the last two sessions. Baker and Chick (2007) studied to teach some probability concepts with the help of two spinners. In this research, two spinners were used which were divided into 9 identical parts on which numbers from 1 to 9 were written. After grouping their students, two teachers tried to teach probability concepts by letting students to do some experiments and play games with the spinners. At the end of the study, researchers observed that using these kinds of materials in mathematics class helped students develop their own methods and enabled students to learn practically rather than theoretically. Quinn and Tomlinson (1999) designed and implemented an activity based instruction so as to introduce students theoretical and experimental probability concepts. At the end of the applications, students stated that they have enjoyed the process, and that the process gave them the opportunity to make the closest estimations for each random variable.

Cognitive psychologists and mathematics educators examined students' misconceptions and misunderstanding of probability subject with studies made with different age groups (Batanero & Serrano, 1999; Çelik & Güneş, 2007; Dooren, Bock, Depaepe, Janssens, & Verschaffel, 2003; Fischbein & Schnarch, 1997; Fischbein, Nello, & Marino, 1991; Kahneman & Tversky, 1972; Konold, 1989; Konold, Pollatsek, Well, Lohmeier, & Lipson, 1993; Lecoutre, 1992; Polaki, 2002; Watson & Moritz, 2002). For example, Dooren et al. (2003) studied to determine and compare misconceptions of 10th and 12th graders. At the end of the research, no obvious difference was found despite the decrease in misconceptions with the increase in level of education. Polaki (2002) conducted a study with a group of six 9-year and six 10-year olds, total 12 students in order to determine how two different teaching practices affected the development of students' thinking levels. He made two groups from these 12 students and implemented teaching practices with them two times a week. These teaching practices continued for six weeks. In these implementations, first group of students were allowed to make 20 experiments and the second group was allowed to make 50, 100, 500, 1000, … of virtual experiments in addition to the former 20 experiments. A week after these implementations ended, first evaluation and after four weeks the second evaluation were made. As a result of the evaluations, some improvements were observed in the thinking levels of both groups. However, no significant difference was found among the development of the thinking levels of the groups.

When studies about teaching of probability concepts are examined, it is possible to encounter various studies in which different teaching materials were developed, applied and their reflections in the implementations were evaluated (Aspinwall & Shaw, 2000; Baker & Chick, 2007; Castro, 1998; Gürbüz, 2006b, 2007; Gürbüz et al., 2009; Memnun & Altun, 2007; Nilsson, 2007, 2009; Pijls et al., 2007; Polaki, 2002; Pratt, 2000; Quinn & Tomlinson, 1999; Tatsis et al., 2008; Thompson, 1992; Watson & Kelly, 2004). Most of these studies emphasized the necessity of using different teaching strategies to enhance students' development in probability concepts. In this context, one of foremost strategies used in enhancing conceptual development are activity-based instruction. But when studies about teaching of probability subject were analyzed, no studies were found in which the effect of different activities on conceptual development of probability concepts were researched in Turkey. For this aim, an instructional process on probability concept based on various activities was designed.

Difficulties in Teaching Probability

Probability concepts are widely used in decision-making processes related to uncertain situations we encounter in our daily lives. As in many other countries, concepts related to this area cannot be taught effectively in Turkey because of several reasons. This deficiency may be due to the common teacher-centered classroom environments, lack of appropriate instructional materials or the abstractness of prepared materials (Gürbüz, 2006a; Pijls et al., 2007), and students' incorrect theoretical knowledge or misconceptions (Barnes, 1998; Batanero & Serrano, 1999; Çelik & Güneş, 2007; Dooren et al., 2003; Fast, 1997; Fischbein et al., 1991; Fischbein & Schnarch, 1997; Konold et al., 1993; Lecoutre, 1992; Polaki, 2002; Pratt, 2000; Watson & Moritz, 2002). Furthermore, the other reasons why this subject is not effectively taught may stem from students' difficulty in probabilistic reasoning (Fischbein & Schnarch, 1997; Munisamy & Doraisamy, 1998; Polaki, 2002), students' incorrect relations or links between their daily life knowledge and scientific knowledge (Gürbüz, 2006a), students' negative attitudes towards the subject and low level of achievement in probability (Bulut, 2001), and teachers' lack of sufficient infrastructure in teaching probability (Bulut, 2001; Fast, 1997). These studies show that in resolving deficiencies in effectively teaching probability subject, traditional teaching strategies are inadequate.

Method

A quasi-experimental research design was used in the study. This study was conducted with fifty 5th grade primary students-25 for each group. All of the students were studying in a formal primary state school in South-East Anatolia Region in 2008/2009 academic year. By drawing a lot, one of the classes of the two teachers were determined as treatment group and the other as the control group. The study was initiated after the determination of groups.

Instruments

As data collection tool, a Conceptual Development Test-CDT was utilized that consists of 15 questions 9 of which were developed by the researcher and 6 of which were developed with the help of related literature (Baker & Chick, 2007; Fischbein et al., 1991; Gürbüz, 2007; Jones et al., 1997). In this test, the focus was on sample space (SS), probability of an event (PE), probability comparisons (PC) concepts and 5 questions were asked for each concept. The content and face validity of the test was ensured through two elementary teachers and two mathematics educators. Moreover, the pilot application of the test was conducted and the expressions that were hard to understand or that led to misunderstandings were corrected. In the final form of the test, the number of questions was decreased to 12 based on comments of the same experts.

Procedure

Before starting the applications within the scope of research, the CDT prepared for the probability subject, was applied to both groups as a preliminary test. Groups were encouraged to answer all questions. The applications were carried out in group E (experimental group) with 3-4 students. In group E, students were asked to do experiments and then they were asked to discuss their experiments and conclusions with each other. Moreover, since the students working in groups questioned each other with questions such as "why are you doing that?", "how did you get that?", "… oh no, it isn't right, because …," and "… but that's wrong, because …," it's believed that they constructed their knowledge more meaningfully and showed a better cognitive development. In this process, the teacher -instead of just lecturing, showing and evaluating- has become a helper, counselor, cooperator and supervisor. On the other hand, students have become more active, improved their knowledge, questioned the knowledge they got, and they were able to explain what they know instead of being passive during the class.

In group C (control group), the lessons were taught as teacher-centered and orally according to the book and the teacher noted down the necessary points on the board. During the process the students sat on their seats silently and listened to the teacher. Then the teacher gave them some time to note down from the board to their notebooks. The teacher also asked if they had any questions about the subject. Meanwhile, he walked around the class and answered their questions. And 70-75% of the probability lesson was composed of just the teacher's talk. At the end of the lesson, the teacher asked the students to answer the questions at the end of the unit. During this process the teaching was conducted by teachers, according to the 2008/2009 academic year instructional plan and it was applied in 4 teaching hours for each class. During the applications the researcher attended to the groups as observer. After the applications were finished in both groups, a month later, CAT was applied as the post test. In this way, the study provided sufficient time for students to forget the test items.

Data Analysis

The effect of activity-based instruction and traditional teaching was investigated by the developed CDT. Students' answers have been classified according to the levels in Table 1 by two mathematics educators.

Table 1.
Developed and Used Criteria for Conceptual Development Test

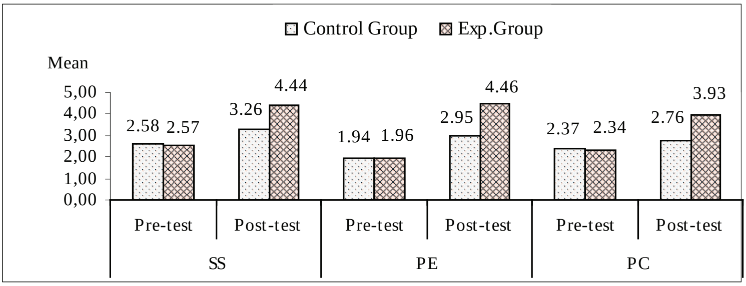
Levels	Score	Content	Sample response
Level A Completely Correct Answer	**5**	The explanations which are accepted as scientifically true, take place in this group	**PC 4:** Spinner 1: 1, 2, 3 Spinner 2 = 1: 1,1 1,2 1,3 Spinner 2 = 2: 2,1 2,2 2,3 Spinner 2 = 3: 3,1 3,2 3,3 The game is not fair because as can be seen on the table, the probability of obtaining an odd outcome is 4/9 and an even outcome is 5/9. So my friend wins.
Level B Partially Correct Answer	**4**	Explanations are true but when compared to the correct answers some parts are missing, so it takes place in this group.	**PC 2:** The probability of stopping at grape is the highest because the number of grapes is largest. The probability is $= \frac{3}{8}$ **SS 2:** I can write five different numbers as 312, 321, 213, 231, 123.
Level C Wrong Answer Type (1)	**3**	The explanations which contain partially correct statements but are connected to the right reasons or don't give reasons, take place in this group.	**SS1:**  We can paint it in six different ways. **PE 1:** The probability to stop at red is highest. **PE 2:** $\frac{2}{6} = \frac{1}{3}$

Level D Wrong Answer Type (2)	**2**	Expressions that contain wholly wrong or irrelevant explanations are in this group.	**PC 4:** Game is a matter of luck. Whoever is lucky, wins the game. **PC 3:** The probability of targeting at red is the lowest. Since red balls is in bottom. **PC 1:** I would choose spinner A because since the reds are given mixed on this spinner, the probability of red is higher.
Level E Uncodeable	**1**	Incomprehensible explanations or explanations that have no connection to the question are in this group.	**SS 1:** The wall should be painted with a single color. **SS 3**: It depends on what I want at that time. **SS 4:** Same numbers would be obtained because all numbers have pairs. 1=1, 2=2, 3=3. **PC 4:** I think it's not fair. The one who turns the spinner first has the highest chance.
Level F Unanswered	**0**	Those that made no explanations and those who wrote question itself in the explanation part are in this group.	**PC 2:** When the spinner is turned, which fruit has the highest probability to stop at? **PE 2:** Chosen geometric shape from the board

According to the scores given according to the criteria in Table 1, statistical comparisons of conceptual development levels of groups were made. To achieve this end, the mean scores gathered from four questions related to each three concepts in CDT were taken. Scores gathered were analyzed through SPSS statistical packet program. Data were analyzed using independent samples *t*-test, and analysis of covariance (ANCOVA).

Results

Pretest and posttest results of groups are given in Graph 1 and Table 2. When Graph 1 and Table 2 are analyzed, no significant difference SS [*t* (48)=-.077, *p*>.05], PE [*t* (48)=.100, *p*>.05] and PC [*t* (48)=-.258, *p*>.05] was found among pretest scores of groups related to three concepts in CDT. Here we can say that all groups were at the same level prior to implementation. However, when Graph 1 is analyzed, activity-based instruction and traditional teaching were seen to have different effects on students' development of probability concepts.



Graph 1.
Pretest and Posttest Scores of the Groups on SS, PE and PC Concepts

Table 2.
Pretest Results of Groups on SS, PE, and PC Concepts

Variable	Group	N	*M*	*SD*	*df*	*t*
SS	Experimental Group	25	2.57	.38	48	-.07
	Control Group	25	2.58	.52		
PE	Experimental Group	25	1.96	.71	48	.10
	Control Group	25	1.94	.70		
PC	Experimental Group	25	2.34	.45	48	-.26
	Control Group	25	2.37	.36		

As a result of the ANCOVA in Table 3, a significant difference was found between the adjusted total posttest scores of groups on the concepts of SS [*F* (1,46)=55.974, *p*<.01], PE [*F* (1,46)=71.154, *p*<.01], and PC [*F* (1,46)=48.614, *p*<.01]. This shows that students' achievement is related with the group they belong. When the adjusted total posttest scores of the groups are considered, the mean scores of the treatment group (For SS, *M*=4.44; for PE, *M*=4.45 and for PC, *M*=3.92) were found to be higher than the mean scores of the control group (For SS, *M*=3.25; for PE, *M*=2.94 and for PC, *M*=2.75). This indicates that the conceptual level of the students in the treatment group for the concepts of SS, PE and PC is higher than that of the students in the control group. On the other hand, when eta squared effect size values (For SS, $\eta 2$=.549; for PE, $\eta 2$=.607 and for PC, $\eta 2$=.514) are considered, it can be stated that the instruction made in the treatment group had a large effect on the development of the students on the concepts of SS, PE and PC.

Table 3.
Comparison of the Posttest Scores of Groups on Conceptual Development of the Concepts of SS, PE, and PC Using ANCOVA

Variable	Group	N	*M**	Std. Error	Mean dif.	*df*	*F*	Partial eta squared
SS	Exp.	25	4.44	.11	1.19	1-46	55.97**	.55
	Control	25	3.25					
PE	Exp.	25	4.45	.12	1.51	1-46	71.15**	.60
	Control	25	2.94					
PC	Exp.	25	3.92	.11	1.17	1-46	48.61**	.51
	Control	25	2.75					

p<.01 *M: Estimated marginal means for post-test

Discussion

When the concepts of SS, PE and PC concepts are examined respectively, while the students did not have difficulty in answering the questions SS 1, SS 2 and SS 3, but had difficulties in question SS 4 related to the concepts of SS in the pretest. Students made significant mistakes in questions SS 1, SS 2 and SS 3. For example, the reason why they answered question SS 1 as "*we can color in 2 different ways*" (Figure 1) and as "*we can color in 2 different ways*" (Figure 2), in the same way the reason why they answer question SS 2 as "*I can write as many numbers as I want*" and with a similar approach why they answered the question SS 3 as "*eating these fruits one after disturbs me so I don't eat them*" is thought to be related with their language development. This verifies Ford and Kuhs (1991), Gibbs and Orton (1994), Kazıma (2006) and Tatsis et al. (2008) who asserted that language development is important in understanding probability concepts.

Figure 1


Figure 2

Students showed improvement in posttest compared to the pretest. Because while students did not show any systematic technique in creating outcomes related to sample universe in the pretest which is parallel to what English (1993) found out, they developed a systematic technique in creating outcomes in posttest and they could apply this technique to questions SS 1, SS 2 and SS 3 in an effective way. In pretest related to question SS 4, some students commented that, as the numbers on both spinners were identical, the probability of getting the same numbers was higher. For example some students answered this question: "*we cannot say anything unless we see how these tiny spinners that are turned by someone else move*" or they replied "*nobody know what will be happen*". It is clear that students' pretest answers were generally based on their intuitions rather than formal knowledge.

In understanding the probability subject, the knowledge about the concept of SS is found to be very important. Therefore, in teaching of probability topic, various types of experiments and sample space of these experiments have to be emphasized so as to enable students to understand sample space concept better. In line with this, Vidakovic, Berenson, & Brandsma (1998) noted in their studies that in order to enable students to understand sample space concept better, there has to be a stress on various types of experiments and sample spaces of these experiments.

When pretest scores of the students related with the concept of PE are examined, the most frequent mistake made by the students were found to be using their visual intuitions and using the expression of "%" in numerical representation. The students made significant mistakes in questions PE 1, PE 2 and PE 3. For example, they interpreted the question PE 1 as "*100:2=50 so 50%*", "*20%*" and question PE 2 as *"6/8", "2%", "100:8=12 and 2.12=24 so 24%"* and with a similar approach the question PE 3 as *"3%"*. That students had difficulties in numerical displays could be correlated to their lack of knowledge on sample space concept or fractions. This finding is parallel to findings gathered in studies conducted by Carpenter Corbitt, Kepner, Lindquist, & Reys (1981) and Jones et al. (1997). The students had difficulty in questions PE mostly in question PE 4. When Graph 1 related to questions PE 1, PE 2 and PE 3 is examined, the students in the treatment group were found to have considerable improvements whereas those in the control group showed partial improvements. When Graph 1 about the question PE 4 is examined, it can be said that the students had difficulty in this question. The reason why the students have difficulty in this question was their custom dice perception as (123 456). For example, based on their classical dice perceptions, some students thought that the outcomes of an experiment when a dice is tossed are equally probable and numerically equal to each other. In other words, they thought that a random event depends on chance and therefore has equal probabilities. Amir and Williams (1999), Baker and Chick (2007), Batanero and Serrano (1999), Fischbein et al. (1991), Lecoutre (1992), Nilsson (2007) and Pratt (2000) reached similar conclusions in their studies. Although this kind of mistakes decreased in group E, they remained quite the same in group C in the posttest.

It is understood that students' knowledge on SS concept is highly important in their development on PE concept. Parallel to this, Keren (1984), Fischbein et al. (1991), Speiser and Walter (1998), Polaki (2002), Baker and Chick (2007), Nilsson (2007, 2009), pointed out to the same issue in their studies.

It was seen that in questions related to PC concept most students tried to give answers by using their intuitions and informal solution strategies in the pretest. It is possible to encounter with studies that are in line with this finding (Fischbein et al., 1991; Piaget & Inhelder, 1975). In question PC 1 related to PC concept, most students realized the importance of size, but focused on only one dimension of size. For example, they used expressions such as "*I would choose spinner A because colors are given in a mixed way on spinner A*" or "*I would choose B because colors are given together on spinner B*". A group of students attributed to the skill of the person turning the spinner and chance. For example, they made interpretations such as "*it depends on who turns the spinner, I can't say anything unless I see who's turning the spinner*". Amir & Williams (1999), Fischbein et al. (1991) and Greer (2001) reached similar conclusions in their studies. In the pretest, most of the students realized the importance of size in question PC 1 as in PC 2, however there were still some who attributed to skill of the player and chance. For example they used expressions such as "*it depends on chance but it may be orange or* *grape*" or "*if I turned it, I would stop it at any fruit I want*". In numerical representation though, the majority of students used percentage expressions. This finding is parallel to findings gathered in studies conducted by Carpenter et al. (1981) and Jones et al. (1997).

In question PC 3, students made mistakes because of concentrating on the location of the balls. For example "Since red balls are below, red can't be picked" or "Red balls are at the bottom but since they will come up when the spinner is turned, red will be picked". These answers show that students could not understand the questions well enough and focused on the locations of the balls. This finding is in line with findings gathered from studies carried out by Gürbüz (2007) and Jones et al. (1997). When Graph 1 about questions PC 1, PC 2 and PC 3 is examined, the students in group E showed a considerable improvement whereas those in group E showed partial improvement.

Regarding question PC 4 in the pretest, most students gave answers using their intuitions or subjective views. For example, they made interpretations such as "Since there are more odd numbers on the spinners, I win" or "I have no chance. So I lose the game". When Graph 1 about question PC 4 is examined, it can be stated that students had difficulty in this question.

When students' answers related with the concepts of SS, PE, and PC are examined, it can be claimed that students' knowledge about sample space concept plays an important role in answering the questions about probability.

In this study, it is observed that activity-based instruction is more effective in improving probability concepts compared to traditional instruction. Because activity-based instruction gives students opportunities to do experiment, to see results derived from experiments and to discuss the process. Moreover, it could be articulated that activity-based instruction have positive effects in view of the fact that student can get involved in more concrete experiments This finding is in line with studies carried out by Aspinwall and Shaw, (2000), Castro (1998), Gates (1981), Gürbüz (2006b;2007), Nilsson (2007;2009), Polaki (2002), Pratt (2000), Quinn and Tomlinson, (1999) and Tatsis et al. (2008) who stated that concrete experiments made on probability topic increased students' achievement and helped learning to take place at conceptual level.

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