An Investigation of the Pre-Services Teachers' Ability of Using Multiple Representations in Problem-solving Success: The Case of Definite Integral

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Abstract

Using different representations and having flexibility of transition between representations throughout the definite integral problem-solving process are thought to be effective on the development of conceptual understanding and students' performance. The study aims at considering the effects of representations used in the definite integral on pre-services teachers' problem-solving achievement. The research is a case study having qualitative paradigm. Within the context, a case study with 45 pre-services teachers from a mathematics teaching department in a state university has been conducted. This study uses multi-method approach to collect the data which are qualitative. The data were analyzed and interpreted through classification method and descriptive statistical techniques. The findings indicate that the skills of pre-services teachers in using multiple representations within the process of solving certain definite integral problems were not sufficient as required. It was specified that the candidates trying to solve problems with the domination of merely one representation were weak in terms of the transition of representation skills and they were in low level in terms problem-solving. After discussing the findings of the study in the light of the literature some suggestions are given for increasing the problem-solving success.

Key Words

Definite Integral, Multiple Representations, Problem Solving Achievement.

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Mathematical knowledge and skills are the basis of research on natural sciences, engineering, and social sciences. Scientists, who describe mathematics as the language of nature, need to explain natural phenomena and their relationship with other sciences by using different impressions of mathematics (Kaput, 1998). All of these different languages and display types used in the field of mathematics education are called multiple representations. In most general sense, multiple representations can be described as the modeling process by concretation of abstract concepts and symbols in the real world (Kaput, 1998). Moreover, the representations can be used as a flexible tool for solving the same mathematical concepts or problems in the case of transitions in themselves or with each other (Monaghan, Sun & Tall, 1994). Especially, the effect of multiple representations of the mathematics education has begun to be felt more heavily after the analysis reform in America (Tucker & Leitzel, 1994). As the NCTM report, published in 2000, separated a section to multi-representation approach that stressed the importance of subject.

Keller and Hirsch (1998) indicated that multiple representations are advantageous for students in using different ways in problem-solving and facilitating understanding by establishing relationships with the concept and cognitive representations. Berdnarz and Belanger (1987) have pointed to the importance of using multiple representations for the concept development of students. In this context, multiple representations and variables which affect these representations have an influence on students' learning processes of the definite integral concept in which students expressed lack of the conceptual understanding. In a topic such as definite integral which permits the use of multiple representations, students' choice of representations and their abilities of transferring one representation to the other is the research focus of the current study.

Definite Integral Concept and the Multiple Representations

Multiple representations are also used in the teaching of the analysis course. Several studies exist on the use of different representations in many mathematical topics such as functions, limit, and derivatives but not in integrals. Some studies on definite integral focus on the concept definition and concept image (Rasslan & Tall, 2002; Thompson & Silverman, 2007), others concentrate around the skills of using different representations (Ghazali, Abdullah, İsmail & İdris, 2005; Czarnocha, Loch, Prabhu & Vidakovic, 2001), prior knowledge requirements (Ferrini-Mundy & Graham, 1994; Sealey, 2008), and the use of technology in its teaching (Carlson, Persson & Smith, 2003; Robutti, 2003). However, the role of using different representations is vital in understanding definite integrals (Ghazali et al., 2005).

The requirements for the full understanding of the concept include using different representations in solving problems and ability to transfer one representation to the other (Camacho & Depool, 2003). Some studies indicate that there is a relationship between the representations used by students in solving an integral problem and the meanings they attribute to the integral concept (Berry & Nyman, 2003). The graphical representation of definite integrals are generally used in calculations involving areas under a curve or volumes of revolving objects, whereas numerical representations are of use in Riemann's cumulative addition problems (Thompson, 1994; Sealey, 2008). Moreover, solving integrals using common integration techniques make use of needs the algebraic representations (Finney, Thomas, Demena & Waits, 1994). Research findings suggest that in different problems different representations are used and that the conceptual understanding levels of students that rely on only one representation or cannot transfer between representations do not sufficiently develop (Lesh & Doer, 2003). Definite integrals are known to be a structure which is used in volumes of revolution and area calculations. There are a variety of numerical and geometrical approaches in making such calculations (Ostebee & Zorn, 1997). Moreover, making use of different representations in the teaching of mathematics is strongly suggested by NCTM (2000).

The topic of definite integrals which allegedly cause many learning difficulties among the majority of students (Orton, 1983) is investigated on the basis of multiple representations. 'Representation' in the context of the study refers to expressing a mathematical concept or a relationship with the help of forms including tables, equations or graphics.

Purpose

In this study, representations used by students in definite integral problems and students' skills of transfer between these representations and the relationship between these representations and students' problem-solving performances are examined. Under the light of the existing literature, the problem of the study can be defined as "the investigation of mathematics teacher candidates' use of types of representations in solving problems." Sub-questions are determined as follows.

1. What kinds of representations are used by mathematics teacher candidates in the process of solving definite integral questions?
2. What is the nature of relationship between representations used by mathematics teacher candidates and their performance in solving questions on definite integrals?

Method

The study uses a non-positivist paradigm and interpretive approach. This study uses multi-method approach and mainly qualitative in terms of data. The research is based on the "explorative case study" model since the problem situation is described within its own context (Cohen, Manion & Morrison, 2000).

Participants

Purposive sampling, a category of non-probabilistic sampling technique, was used for the selection of the individuals (Patton, 1990). In this context, the participants of this study is a total of 45 second grade, 19-22 years old students who registered in the mathematics teacher training program offered by Marmara University, Faculty of Education in 2008-2009 academic year. There are 23 females and 22 males in the sample. Data are collected during Calculus II course.

Data Collection

The data collection instruments were participant observation, interviews, document analysis, and essay type examinations which were used according to the nature of the problem and to the expectations of the researchers. One data collection instrument is used to collect two different types of data, and an interview form was used for the process analysis.

Using expert opinions obtained from the interviews and results from the validity-reliability analyses, the Representation Preference and Transition Test (RPTT) was developed. The test consists of nine items each of which represent a different objective of the course. Each question contains input representations defining the givens of the problem and output representations which the solution of the problem includes. Textbooks, exam questions, school grades, and relevant literature were taken into account to determine 71 questions which then were reduced to nine according to the predetermined objectives. There are input and output characteristics in each of these questions. In abbreviations of characteristics, input and output representations are presented with capital and small letters respectively as *N or n* (numerical representations), *G or g* (graphical representations), and *A or a* (algebraic representations).

The test has two sub-categories: 'Transition within representations' sub-category contains question types in which input and expected output representations are the same. 'Transition between representations' sub-category contains question types in which input and expected output representations are different. The test was found to have construct and content validity after the analyses based on the five experts in the area.

Having administered to 35 higher grade mathematics teacher candidates, the test was made free from structural and language mistakes. The testing time was determined to be 45 minutes. Inter-rater reliability was taken as a measure for the reliability of the testing instrument for which randomly selected 12 answer sheets from the RPTT were evaluated by three experts who had PhDs in mathematics education. High correlations between the answers of assessors are interpreted as sufficient reliability. Expert opinions are accepted sufficient for the validity and the reliability of the test. After administering the test, semi-structured interviews were conducted with six respondents for deeper understanding of their solution processes and skills of using representations.

Data Analysis Process

To analyze the data, first, in order to determine the levels of achieving the answers were assessed using the labels including 'correct answer', 'partially correct answer', 'incorrect answer' and 'no attempt'. The cases in which the solution of the problem was completed with correct concept, process, and answer, are labeled as "correct answer" for which 2 points were given. The cases in which the solution was completed with correct concept, wrong process and/or wrong answer, are encoded as "partial answer" and 1 point is given. The cases in which the solution was completed or not completed with wrong concept, process and answer, are encoded as "incorrect answer" and no points were given. Total score of the RTPT ranges between 0 and 18, the scores were operationalized as the scores for the representation conversion skills for each teacher candidate. Second, the representations respondents used were coded as numerical, graphical, and algebraic and mixed. A mixed representation is said to exist when, more than one representation are used interrelated for the same question. The relation between students' representations and success in problem solving were investigated. Findings of interview were analyzed using common categorizations and statistical analyses were done using a statistical software.

Findings

The Relationship between Representation Types and Levels of Achievement in Problem-solving

Findings indicated that algebraic representation yielded the highest correct response rate in the problem solution process (18.7 %) and numerical representation yielded the lowest (2.8 %). The representations used in the solutions in the RPTT were sometimes the reason for the wrong answer. The representation type with the highest 'incorrect answer' rate was observed is algebraic (13.1 %) and lowest is the mixed type (1.6 %). The reason that the use of algebraic representation has the highest proportion can be attributed to its frequent use in general.

The use of mixed representation also yielded notable findings. Among all the incorrect answers of candidates, mixed representation has constituted 7 % of all of the answers. This rate is 32 % for the algebraic representation, 21,8 % for the graphical representation, and 13,3 % for the numerical representation. In addition, the question which has the lowest proportion of correct answers (13.3 %) is RPTT/6 question, a transition between representations type, in which a graphical representation is given with the question and whose solution requiring the use of algebraic representations. This is followed by a transition between representations type question RPTT/3 (15.5%) given with a numerical representation and whose solution requiring also numerical representations. Finally, the highest correct response rate (82.2 %) came from RPTT/2 question in which both the presentation and the solution of the question involves algebraic representations.

The influence of problem characteristics on the solution processes: Definite integral questions given with a graphical representation, as opposed to other representation types, seem to be found much more difficult by the teacher candidates. The correct response rate is very low (20 %) in such questions. In cases in which the input representations are numerical, teacher candidates' correct response rate becomes 29%. The highest correct response rates were reached in the questions that were given with algebraic representations (50 %).

Identification of the Representations Used in the Problem-solving Processes

Findings indicate that the highest proportion of use of representation (93.3 %) is observed in a problem with an Aa characteristics (RFTT/2) in which an algebraic representation was used. This is followed by an An problem (RFTT/4) in which an algebraic representation was used. Numerical and graphical representations were not used in problems having Gg, Ga, and Aa characteristics. The high proportion of use of algebraic representations in all of the RFTT problems is another noteworthy finding. In addition, contrary to the expectation of a high proportion of use of graphical representation in the solution of RFTT/5, only 15.6 % seemed to have used it. Another important finding is related to the types of problems which were left undone. RFTT/6, for which a Ga transition was expected, was the problem with the highest proportion of blank responses. Besides, it is seen that the problems for which either numerical or graphical representations were expected had the higher blank proportion than the other questions.

When all answers given to the RPTT are considered, the proportion of using algebraic representation is 46% and the graphical representation is 17 %. The difference between percentages of the most widely used two representations algebraic and graphical suggested a dominance of relying on a single representation. The least preferred type of representation is the numerical representation; which is preceded by mixed representations. The findings of the RPTT's subsections suggest the domination of algebra representation in the within and between transition questions.

Discussion

The Representation Preference and Transition Test findings showed that the algebra representation was used in solving all types of problems. This is an indication of the fact that teacher candidates see algebraic representation as a support tool. It has attracted attention the high proportion of the use of algebraic represents in the problems having An, Ag, Gg characteristics is especially noteworthy. This suggests that the teacher candidates develop a habit of using algebraic representation even if the objective of the question is different. This may also be related to a limited meaning loaded into definite integral as "a case that needs a calculation". Findings of the interviews suggested that the candidates' concept definition and images were on the algebraic level. Candidates asserted that such conceptualization of define integrals was mostly shaped by the teaching methods used during their preparations to the University Entrance Examination (OSS). It seems that the rules and solution techniques in university preparation courses are mostly taught in relation to solving specific 'problem types' and mostly results-oriented which might lead to a limited understanding of the concept

Findings from the analysis of lecture notes revealed that teaching is algebraic-oriented and that little or no emphasis was given to real world applications. This seems to be resulting of the classical *definition-theorem-proof-applications* format which is used in the course. This might be responsible for candidates' reliance on single representation.

The most commonly encountered problem type in definite integrals is the one that is given with equations and for which some sort of area calculation is needed to reach a solution. Candidates seem to have difficulties in dealing with such Ag type problems resulting from incorrect use of algebraic representations. In problems related to calculation of areas under a curve or bounded areas, the candidates do not seem to benefit from graphical representations. Problems that these candidates faced are generally result from inability to think algebraically and incorrect use and misinterpretation of information obtained from graphics. Candidates who thought algebraically without needing any drawing simply accepted the boundaries of integrals as the roots by equating two algebraic expressions. Some candidates did try to make use of graphical representations but because of incorrect or misuse of graphical data ended up with incorrect answers. Lowride and Dieazmann (2007) had similar findings.

The candidates have difficulties in questions involving transitions between representations. Similar to Camacho et al.'s (2009) findings, definite integral is dealt with on the basis of only one representation in the traditional classroom seems to be a responsible factor for candidates dependence of single representation, Kendal and Stacey's (2003) findings supported the finding that candidates had more difficulty on the problems requiring the use of numerical representations. Participants that are not accustomed to solving problems by the help of integral tables, had trouble in applying the method of cumulative totals as a problem-solving tool. Findings indicate that the representation type with the highest success proportion is the mixed representations. Candidates seem to reach a correct solution more easily when they were able to relate two representations or make transitions between two representations (Ghazali et al., 2005).

Results

The findings indicate that candidates prefer more algebra representations in their definite integral problem-solving processes. While solutions of some questions require transition between different representations, their insistence of using algebraic representations is found to be related to the nature of the teaching. Especially low achievement rate in the questions requiring the use of transitions between different representations showed that the candidates suffer from the lack of knowledge/skills of synthesizing different representations to reach a solution.

Although the most used representation is the algebraic, it was observed that there was a high achievement rates in questions whose solutions involve algebra to algebra (Aa) transitions within representations and geometric to numeric (between representations transitions. On the other hand, although the least used representation is numerical, there was a low achievement rate in questions whose solutions involve numerical to numerical (Nn) transitions within representations and geometric to algebraic (Ga) (between representations transitions. The reason that most incorrect answers come from the questions in which algebraic representations were used may be related to the candidates' tendency of using algebraic representations even if the solution necessitates either numerical or graphical ones. In addition, the participants have difficulties with the problems given by numerical representations and in interpreting information given by a table. Possible reasons for the tendency include the lecturers' reliance of a single representation in their teaching, Lecturers' reluctance to show textbook examples that enable to use of multiple representation and students' lack of knowledge of alternative definitions. It is believed that the skill of the use of representations develops if the lecturers use multiple representations in the classroom and ask questions involving the use of multiple representations in their assessments (Goerdt, 2007).

The present study investigated the knowledge and skills of the students through the existing state, not looking into the effects of any software and material. Hence the influence of computer algebra systems on types of multiple representations is not covered in our literature review. We believe that study in this area would be beneficial to the researchers.

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