Teaching Mathematics: Issues and solutions.

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Abstract

The ability to compute, problem solve, and apply concepts and skills in mathematics influences multiple decisions in our lives. The National Research Council (1989) reported that mathematics is especially evident in our technology-rich society, where number sense and problem solving skills have increased the importance and demands of advanced levels of proficiency. However, mathematics is often challenging for students with and without disabilities to master. Comparison studies have focused on student results which show US students not performing as well in math as students in many other developed countries (USDOE, 2000). This manuscript describes the changing context and expectations of math standards and curriculum, given the specific characteristics of students with disabilities. Various research-based instructional methods and strategies are described to address the revised standards in math for teachers to effectively meet the learning needs of students with and without disabilities to master mathematics.

Keywords
Mathematics, progress monitoring, standard
Mathematics is used throughout our lives - every day. The National Mathematics Advisory Panel (2008) reported that mathematics is the invisible culture of our age and emphasizes that mathematics is embedded in our lives in many ways: practical, civic, professional, recreational, and cultural. This is especially evident in our technology-rich society. Number sense and problem-solving skills have increasing importance, as technology (e.g., calculators, computers, software programs, etc.) enhances both the opportunities for, as well as the demands of, advanced levels of proficiency in mathematics.

Mathematics is often challenging for students with and without disabilities to master. Comparison studies from recent commissions and reports have focused on student results (NCES, 2004). Students in the United States are not performing as well in math as students in many other developed countries (USDOE, 2000). In both 1995 and 2003, U.S. fourth-graders showed no measurable gain in mathematics and twenty-three percent of grade four students and thirty-two percent of grade eight students scored below the “basic” level (NCES, 2004). Additionally, in the 2005 NAEP report, only two percent of U.S. students attained advanced levels of mathematics achievement by grade 12 (NCES, 2006). These data regarding mathematics suggest that math achievement of U.S. students was lower in 2003 than in 1995 relative to their peers in 14 other countries.

Concerns regarding the poor math performance of students with disabilities have also increased. Researchers have noted that math difficulties emerge in elementary school grades and continue as students progress through secondary school, typically performing over two grade levels behind their peers without disabilities (Cawley, Parmer, Yan, & Miller, 1998). Specifically, students fail to achieve a sufficient conceptual understanding of the core concepts that underlie operations and algorithms used to solve problems that involve whole and rational numbers (Fuchs & Fuchs, 2001).

Current Issues to Consider

Changing Policies

To address these issues, Congress passed the No Child Left Behind Act (NCLB), with the purpose of ensuring that all children have a fair, equal, and significant opportunity to obtain a high-quality education and reach, at a minimum, proficiency on challenging state academic achievement standards and state academic assessments (20 U.S.C. § 6301). NCLB focuses attention on the general education curriculum by requiring that states develop challenging academic standards for both content and student achievement for all children in mathematics, reading/language arts, and science (20 U.S.C. §§ 6311(b)(1)(A)-(C)). The development of new content standards was initiated to define and to raise the expectations for the general education curriculum.

Changing Standards in Mathematics

When beginning the revision of math standards, it was important to determine possible reasons for the decline of student performance in mathematics, as well as consider the new federal requirements and mandates.
related to increased rigor and accountability for results of all students. One explanation is that mathematics instruction includes too many superficially taught topics in a given year. More successful approaches, found particularly in Asian countries, tended to focus on few topics. The lessons are often devoted to the analysis of a few examples, and teachers encourage students to share different solutions to problems (Office of Educational Research and Improvement, 1998; Stigler & Hiebert, 1999).

When considering issues related to reported student results and recent revisions to federal legislation, the National Council of Teachers of Mathematics (NCTM) initiated reform efforts in math education, including a revision of the suggested math standards. As a result of published concerns about student achievement, NCTM recently revised their curriculum standards to include an increased process approach for a deeper understanding of a decreased amount of standards (NCTM, 2000). The Curriculum Focal Points outline comprehensive standards to ensure deeper, pedagogical content knowledge of conceptual understanding. One significant change in the mathematics standards is the shift in importance from memorizing computational facts to applying problem-solving to real life situations. NCTM highlights the importance of giving students opportunities to use and discuss multiple representations during problem-solving (NCTM, 2000).

The continued focus of the revised standards on high-level conceptual learning and problem-solving (Maccini & Gagnon, 2002) has been cited as being responsible for the instructional shift away from procedural practice for fluency of number facts (Goldsmith & Mark, 1999). Concerns regarding these new curriculum standards (www.nctm.org) as related to the successful inclusion of students with disabilities have been raised, as there is little mention of students with disabilities in the development of the standards (Woodward & Montague, 2002) and the process approach to teaching math may not meet the needs for explicit instruction needed by some students, especially students with disabilities (Jackson & Neel, 2006).

**Characteristics of Students with Disabilities**

Initially, students who demonstrate poor skills in numerical calculation abilities were described as students with dyscalculia (Johnson & Myklebust, 1967) and were eligible to receive special educational services if the instructional needs met the criteria (IDEA, 2004). Presently, it is estimated that between four and seven percent of the school-age population experience some form of mathematics-focused disability (Gross-Tsur, Monar, & Shalev, 1996). Approximately, one-fourth of the students identified with learning disabilities were identified because they underperformed in mathematics (Brian, Bay, Lopez-Reyna, & Donahue, 1991). It has been found that students with learning disabilities in mathematics perform several grade levels below their general education peers (Cawley, Parmer, Yan & Miller, 1998; Wagner, 1995), struggle in basic mathematics skills and have difficulty in problem-solving situations (Maccini & Hughes, 2000). Difficulties in mathematics are part of a larger educational concern. Students who exhibit deficits in mathematics skills also show evidence of social deficits such as deficiencies in self-help skills and poor organization (Rourke, 1993). In addition, students with learning disabilities are frequently characterized as having perceptual and neurological concerns that impact learning. Students with difficulties in math often have other related difficulties, such as in
memory, poor calculation skills, number reversals, and difficulty understanding conceptual and/or procedural processes, especially as represented through symbols and signs (Bryant, Hartman, & Kim, 2003; Bryant, Bryant, & Hammill, 2000).

There are several factors that may interfere with learning and subsequent mastery of concepts and skills in mathematics by students with disabilities (Ginsburg, 1997):

1. **Perceptual skills:** By definition, students with learning disabilities have difficulty with spatial relationships, distances, and sequencing. These difficulties may interfere with the acquisition of and demonstration of math concepts and skills, such as estimating size and distance, and problem-solving.

2. **Language:** Vocabulary and language of mathematical concepts is not only varied, but also abstract. Students with difficulties and/or disabilities in the area of language may also have difficulties with understanding such mathematical concepts as first, second, greater than, less than, as well as associated vocabulary terms such as vertex, complimentary, acute, etc. For students who have deficits in both reading and mathematics, the difficulty with word-problem solving is accentuated (Jitendra, DiPipi, & Perron-Jones, 2002).

3. **Reasoning:** Students with disabilities may not possess with abstract reasoning skills necessary for higher level math skills development. These skills in reasoning may also present difficulties if instruction in mathematics is at the conceptual, abstract level.

4. **Memory:** Many students with learning and behavioral problems have difficulties remembering information that was presented. This is especially evident with the abstract symbols used in mathematics (e.g., minus, greater than, less than, etc.).

**Considerations for Instruction in Mathematics**

Current legislation, reforms and revised curriculum standards in mathematics focus attention on research-based instruction for all students. Difficulties with learning mathematics occur in one or more domains and on a continuum of needs, from temporary to severe problems, which may manifest at different points in a child’s learning. Multiple instructional approaches and interventions may be necessary, since difficulties may be encountered at different ages and in different mathematical domains. Various research-based instructional approaches and metacognitive strategies both enhance and scaffold instruction for student mastery of abstract concepts (National Math Advisory Panel, 2008), especially within inclusive math classes (McLeskey, Hoppey, Williamson and Rentz, 2004; Miller and Hudson, 2007).

**Instructional Solutions**

Mathematics instruction for students with and without disabilities should include the recommended instructional practices:

1. differentiated instruction;
2. metacognitive strategies and instructional routines;
3. progress monitoring and formative assessment procedures; and
4. computer-assisted instruction and Universal Design for Learning (UDL).
Differentiated Instruction Using Levels of Learning

Differentiated instruction is an approach to planning and teaching based on the premise that teachers must consider who they are teaching as well as what they are teaching. The goal is student mastery of the curriculum. Development of differentiated instruction occurs along a continuum, beginning with units and lessons. Teachers start with the essential understandings and goals of the curriculum for the lesson or unit. Differentiating instruction includes clarity of the standards and learning goals of the curriculum, on-going assessment and adjustment, use of flexible grouping, and planning learning tasks that are respectful of each student’s needs (Tomlinson, 1999).

Differentiated instruction may be implemented in a variety of ways in mathematics classrooms. One way to differentiate instruction incorporates planning and teaching within flexible groups based upon students’ levels of learning. Levels of learning is a research-based instructional approach to teaching and differentiating mathematics that has been researched in mathematics education and special education. Levels of learning is most often described as an instructional sequence more commonly referred to as Concrete-Representational-Abstract (CRA). This instructional approach to differentiating instruction in math involves a sequence of instruction (Maccini, Mulcahy, & Wilson, 2007; Miller & Hudson, 2007) to master complex concepts and algorithms in mathematics. CRA involves utilizing manipulatives (concrete). Once the student has mastered the math concept using manipulatives, the objects are replaced with pictorial representations, such as a picture of the object (representational). This level is a critical bridge between the concrete manipulatives and the abstract symbols (e.g., equations, algorithm, etc.), as this step builds the mental schema bridging these two levels. It is critical to develop mathematics conceptual knowledge during the representational level of learning. Once the student is able to comprehend representational figures and designs, Arabic symbols and explanation of the algorithm (abstract) is taught. Successful performance at the abstract level is the goal, as mathematics is most often expressed and assessed at this level. The three levels of CRA are sequentially interrelated and interconnected. Each level prepares the student for the next level of learning (Witzel, 2005). The connections between the levels of learning are critical to foster student learning. For example, lessons and units must be designed to be easily represented pictorially and described abstractly. When lessons and units are planned to address the levels of learning, the teacher will be able to differentiate instruction to meet the students’ needs for content master.
Figure 1: Example of C-R-A using Base ten blocks with subtraction with borrowing

<table>
<thead>
<tr>
<th>Concrete</th>
<th>Representations</th>
<th>Abstract</th>
</tr>
</thead>
<tbody>
<tr>
<td>43</td>
<td>43</td>
<td>43</td>
</tr>
<tr>
<td>-26</td>
<td>-26</td>
<td>-26</td>
</tr>
<tr>
<td>4 longs + 3 shorts</td>
<td>4 longs + 3 shorts</td>
<td>40 + 3</td>
</tr>
<tr>
<td>2 longs + 6 shorts</td>
<td>2 longs + 6 shorts</td>
<td>-(20 + 6)</td>
</tr>
<tr>
<td>3 longs + 1 long and 3 shorts</td>
<td>3 long tallies + 1 long tally and 3 short tallies</td>
<td>30 + 10 + 3</td>
</tr>
<tr>
<td>2 longs + 6 shorts</td>
<td>2 long tallies + 6 short tallies</td>
<td>-(20 + 6)</td>
</tr>
<tr>
<td>3 longs + 13 shorts</td>
<td>3 long tallies + 13 short tallies</td>
<td>30 + 13</td>
</tr>
<tr>
<td>2 longs + 6 shorts</td>
<td>2 long tallies + 6 short tallies</td>
<td>-(20 + 6)</td>
</tr>
<tr>
<td>1 long + 7 shorts</td>
<td>1 long tally + 7 short tallies</td>
<td>10 + 7</td>
</tr>
<tr>
<td>17</td>
<td>17</td>
<td>17</td>
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</table>

Metacognitive Strategies and Instructional Routines

Metacognition refers to higher order thinking that involves active control over the cognitive processes engaged in learning (Montague et al, 2000). Metacognitive strategies teach students how to think about what they are doing and learning. Activities such as planning how to approach a given learning task, paraphrasing and monitoring for comprehension, analyzing all parts of problems, and evaluating progress toward the completion of a task are metacognitive in nature. Metacognitive strategies include mnemonic devices, problem-solving routines, self-monitoring skills, and the use of graphic organizers. Graphic organizers are designed to assist students in representing patterns, interpreting data, and analyzing information relevant to problem-solving. Other metacognitive strategies include prior knowledge prompts, advance organizers, mnemonics, and visual organizers (See Figure 2 for several research-based sample metacognitive strategies.)
<table>
<thead>
<tr>
<th>Definition</th>
<th>Classroom Use</th>
<th>Research-Base</th>
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<tbody>
<tr>
<td>Visual, organizational tool to increase engagement in active thinking in math by:</td>
<td></td>
<td></td>
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<tr>
<td>– (K) describing what is KNOWN about a topic</td>
<td></td>
<td></td>
</tr>
<tr>
<td>– (W) questioning about the potential learning</td>
<td></td>
<td></td>
</tr>
<tr>
<td>– (L) summarizing concepts learned after instruction</td>
<td></td>
<td></td>
</tr>
<tr>
<td>The Paraphrasing Strategy is designed to help students restate the math problem in their own words, therefore strengthening their comprehension of the problem.</td>
<td>2. Underline or highlight key terms.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3. Restate the problem in your own words.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>4. Write a numerical sentence</td>
<td></td>
</tr>
<tr>
<td><strong>Strategies that students and teachers can create to help students remember content. The verbal information promotes recall of unfamiliar information and content.</strong></td>
<td>Teacher predetermines critical information and key ideas from content and generates list of facts. Then, through the use of a memory devise from first letters, teacher creates a mnemonic. EX: PEMDAS and Please Excuse My Dear Aunt Sally or PEMDAS to remember the order of operations. Parenthesis, Exponent, Multiplication &amp; Division (left to right), Addition &amp; Subtraction (left to right).</td>
<td>Out of Memory (2008). Retrieved from the Web March 3, 2009. <a href="http://library.thinkquest.org">http://library.thinkquest.org</a></td>
</tr>
</tbody>
</table>
**Accommodations**

In addition, the use of accommodations by teachers may be necessary when considering the instructional needs of students within the context of specific math curricular goals and lesson objectives. The use of accommodations does not alter the standard in mathematics or curriculum goal taught for mastery by the students. Instead, accommodations involve a wide range of techniques and support systems to assure that all students participate and demonstrate mastery of that standard in mathematics or curriculum goal. Accommodations may be considered and implemented in five general areas: instructional methods and materials; assignments and classroom assessments; time demands and scheduling; learning environment; and use of special communication systems. When selecting accommodations, educators must analyze the learning needs of each student within the context of classroom and curriculum expectations. For example, a student’s disability in reading affects the student’s mathematics achievement when encountering word problems. Thus, if the student displays effective listening comprehension, then an appropriate accommodation would be to read aloud word problems (Johnson, 2000). In this example, reading the problems aloud enables the student to successfully participate and master the mathematics curriculum as a result of appropriate accommodations. It is important to examine the effectiveness of different accommodations in the classroom and keep a record of the student performance before using the accommodation in a high stakes environment. Consult state and school district policies and procedures regarding the use of accommodations within the mathematics classrooms and district and state assessments.

**Progress Monitoring and Formative Assessment**

Assessment is the process of gathering evidence about a student’s knowledge of, ability to use, and disposition toward mathematics and of making inferences from that evidence for a variety of purposes (National Mathematics Advisory Panel, 2008). Furthermore, assessment tasks should match student’s needs, the curriculum, and instructional strategies. In other words, the teacher should not teach one way and assess another way. Assessment should be viewed as a tool to assist the teacher design and revise instruction for the student. In order to determine the effectiveness of one’s instruction, a teacher administers various assessments on one or more skills to determine the growth rate of learning during the instruction and or intervention in mathematics.

Educational assessment is the systematic process whereby information about student learning is collected to make instructional decisions. Typically, assessment is equated with testing. Although testing is one way to answer educationally relevant questions, it is not the only way. Information about students can and should be obtained through other techniques, including observations, interviews, checklists, and rating scales (See Figure 3). Progress monitoring instruments provide valuable data of student performance for this systematic process of instructional decision-making by teachers (Allsopp et al, 2008). Stecker and Fuchs (2000) found that student performance increased when teachers made instructional adjustments based on individualized curriculum-based measurement data. Frequent assessment and linked instructional interventions are essential
to increasing student mathematics performance.

**Instructional Technology and Universal Design for Learning**

Universal Design for Learning (UDL) is a theoretical framework developed to guide the development of curricula that are flexible and supportive of all students (Hitchcock, C., Meyer, A., Rose, D., & Jackson, R., 2002). The concept of UDL was inspired by the universal design movement in architecture. This movement calls for the design of structures that anticipate the needs of individuals with disabilities and accommodate these needs from the outset. Universally designed structures are indeed more usable by individuals with disabilities, but in addition they offer unforeseen benefits for all users. Similarly, but uniquely, UDL calls for the design of curricula with the needs of all students in mind, so that methods, materials, and assessment are usable by all. Traditional curricula may present barriers that limit students' access to information and learning. In contrast, a UDL curriculum is designed to be innately flexible, enriched with multiple media, including assistive and augmentative technology, so that alternatives can be accessed whenever appropriate. A UDL curriculum takes on the burden of adaptation so that the student doesn't have to, minimizing barriers and maximizing access to both information and learning. Therefore, instructional needs of students can be offered in a wide variety of solutions in various curricular areas including mathematics. Specifically, the National Center on Accessing the General Curriculum (NCAGC) recommends eight curriculum enhancements that are effective in classrooms, which include:

1. **Anchor instruction**: Use authentic problem situations in conventional and digital environments. For example, have students conduct experiments comparing heights, distance, or temperature using some of the latest calculators.

2. **Modify text**: Change text to match the interests and reading level of students.

3. **Text-to-speech**: Record textbooks for students or have students record their work through digital pictures with verbal explanations.

4. **Manipulatives**: As mentioned previously, the use of concrete objects is important for conceptual understanding. Use concrete objects that match the purpose of the lesson at the level students should understand it.

5. **Simulations/virtual reality**: Interacting with media that shows the concept to student allows the student to see the social relevance of a standard and how they might use the information in their environment.

6. **Technology tools**: From calculators to the internet to simple concrete objects, work to increase student interactions with the mathematical skills and concepts.

7. **Concept maps**: Graphic organizers may be used to help students make connections between what mathematical concepts and skills (Edyburn et al, 2005; Rose, 2000). Both instructional and assistive technology provide resources within the educational environment (Edyburn, Higgins, and Boone, 2005). Instructional technology is essentially tools for enhancing the delivery of appropriately designed, research-based instructional strategies during mathematics instruction within the classroom setting. Typically, applications of instructional technology in classrooms include media such as DVDs, video, and more complicated forms of technology such as the internet and hypermedia. Instruction in mathematics is often enhanced through the use of technology. (See Figure 4 for examples).
<table>
<thead>
<tr>
<th>Student Characteristics</th>
<th>UDL Access Tools</th>
<th>Resources</th>
</tr>
</thead>
<tbody>
<tr>
<td>Antonio’s mind tends to wander in math class, but he can stay on task if he has a visual representation of the lesson’s concepts.</td>
<td>Antonio uses cognitive organizers and concept mapping software to visually depict lesson concept.</td>
<td>Inspiration and Kidspiration <a href="http://www.inspiration.com">http://www.inspiration.com</a></td>
</tr>
<tr>
<td>Steven is a bright student in understanding math concepts, however, has difficulty decoding and understanding the vocabulary contained in math problems.</td>
<td>Steven uses screen-reading software that translates text pages to spoken text by scanning the printed page using optical character recognition (OCP) software and listens to the text to audible speech.</td>
<td>OmniPage &amp; OmniForm <a href="http://www.scansoft.com">http://www.scansoft.com</a></td>
</tr>
<tr>
<td>Marcus understands complex math concepts at the concrete level, using manipulatives. However, his gross and fine motor skills, as well as his in class behaviors, limit his use of manipulatives.</td>
<td>Marcus uses virtual manipulatives, an extensive set of math manipulatives available on line in virtual classroom labs.</td>
<td>Gizmos <a href="http://www.explorelearning.com">www.explorelearning.com</a></td>
</tr>
<tr>
<td>Susan learns her math facts, but needs to develop increased accuracy and fluency with this skill.</td>
<td>Susan practices her math fact knowledge and recall to increase both her accuracy and speed with computer software that targets math fluency.</td>
<td>FASTT Math: <a href="http://tomsnyder.com">http://tomsnyder.com</a></td>
</tr>
<tr>
<td>Lashawn reads and understands the math word problems, but has difficulty sequencing the facts and details presented to create an equation to solve the problem.</td>
<td>Lashawn maps out the problem-solving process visually to use the information to solve the word problem.</td>
<td>CMAP: Cmap.ihmc.us</td>
</tr>
</tbody>
</table>

Numerous types of software programs include features specifically to support students in mathematics. Virtual manipulatives are very useful technology tools. They support complex thinking activities, enable students to experiment with various solutions to problems, and provide a visual way to look at data. In addition, students with gross or fine motor problems can often use virtual manipulatives easier than classroom sets of materials.

Two websites to consult when planning and using technology in conjunction with mathematics instruction include:

- **National Library of Virtual Manipulatives**: This site offers samples, resources, lesson plans using the levels of learning and mathematics curriculum frameworks.  
• National Council of Teachers of Mathematics: This site offers lesson plans and resources describing instructional technology in mathematics. http://nctm.org

Students with Disabilities and Mathematics Reforms

Comprehensive reform in mathematics involves numerous stakeholders in a process of continuous improvement (Little & Houston, 2003). An important initial step is to establish, articulate, and sustain the vision for high expectations for all students in mathematics within the school. Since an increased number of students with disabilities are participating in classes with students without disabilities (Nolet & McLaughlin, 2000), teachers need to be aware of and implement research-based instruction to meet the needs in mathematics for all students, with and without disabilities. In addition, the current reauthorization of IDEIA (2004) and the No Child Left Behind Act (2002) have placed a significant amount of pressure on teachers to demonstrate that all students are learning mathematics. Research-based instructional practices of differentiated instruction, levels of learning, metacognitive strategies, accommodations, and technology are often implemented in mathematics to meet the instructional needs of students with and without disabilities (McLeskey, Hoppey, Williamson & Rentz, 2004). Continuously monitoring student performance through various assessments and action research by teachers provides results of student learning to address instructional decisions and school accountability. Most importantly, however, actively engaging students through levels of learning, metacognitive strategies, accommodations, technology will have the greatest impact and use with those most directly affected-teachers and their students.
References


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