Secondary Pre-Service Teachers’ Content knowledge for State Assessments: 
Implications for Mathematics Education Programs

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Abstract

Seventy secondary mathematics pre-service teachers from two universities were assessed on their content knowledge, vocabulary knowledge, and their perceived confidence in teaching the content addressed on the eleventh grade state assessment. The results indicate the pre-service teachers had significant content weakness in data analysis, algebra, and pre-calculus concepts. Correlation analyses revealed strong associations between their content knowledge and vocabulary knowledge, and between their content knowledge and their perceived comfort with teaching the content. The study supports the need for mathematics education programs to include a strong emphasis on making connections between the college-level content courses, secondary methods courses, and secondary mathematics content across the curriculum.

Key Words: Content Knowledge, Teacher Education, Mathematics Teacher, State Assessments

Introduction

Teachers’ content knowledge is a key factor to students’ academic success in today’s classrooms (Darling Hammond, 2000). To be certified for teaching, many states in the United States require prospective secondary mathematics teachers to pass the Praxis II: Mathematics Content Knowledge (10061) test. The test focuses on measuring their mathematical content knowledge, specifically their undergraduate mathematical knowledge. Unfortunately, this test does not assess their pedagogical content knowledge (Shulman, 1986), nor their understanding of the mathematics they will teach in a secondary classroom. One could question whether the completion of undergraduate mathematics courses adequately prepares prospective teachers for teaching secondary mathematics. Specifically, does the completion of courses such as linear algebra, multivariate calculus, differential equations, and advanced calculus prepare future teachers to be highly qualified to teach secondary content such as algebra II, trigonometry, precalculus, etc.?

Many mathematics educators, including Usiskin (2001), emphasize the need for pre-service teachers to acquire content knowledge different from the kind they normally receive in college level instruction. Policy documents such as A Call for Change, (Leitzel, 1991) and The Mathematical Education of Teachers (Conference Board of the Mathematical Sciences, 2001) recommend that the undergraduate preparation of secondary mathematics teachers involve courses that deepen and enhance prospective teachers’ knowledge and conceptual understanding of the mathematics they will teach.
**Secondary Pre-Service Teachers**

In reality, many pre-service teachers find they never had an opportunity to really study the middle or high school mathematics curriculum in depth, yet are expected to know the secondary mathematics content and be expected to teach it with meaning in their student teaching and beginning teaching experiences. As a result, many of them have gaps in their content knowledge or deficiencies in knowing how to apply and teach the mathematics addressed across the secondary school mathematics courses (Mansfield, 1985). If this is the case, how can prospective teachers help their students achieve mathematical proficiency if they have deficiencies in the same expected content knowledge? If pre-service teachers’ secondary mathematics content knowledge is weak, will they be able to explain the conceptual and procedural aspects of the mathematics, pose higher-level thinking questions, make the mathematical connections necessary for students to understand the content, or raise students’ performance on state level assessments? Also, if pre-service teachers do not feel competent with the secondary mathematics content, how will they feel about teaching this content to students? This paper will describe a study that was conducted to assess pre-service secondary mathematics teachers’ knowledge of the secondary content addressed on state level assessments, their confidence with the mathematical content, and their perceived confidence with teaching the mathematics content.

**Review of Literature**

There have been many debates on the underlying interpretation of what knowledge of mathematics is needed to teach mathematics effectively; subject-matter content knowledge or pedagogical content knowledge (Shulman, 1986; Wilson, Shulman & Richert, 1987; Ball, & Bass, 2000). Subject-matter content knowledge refers to general mathematics ability (U.S. Department of Education, 2002). Pedagogical content knowledge was defined by Shulman as the special nature of the subject matter knowledge required for teaching (1986). Ball’s studies (1990) showed effective mathematics teaching has been linked to both teachers’ subject matter content knowledge as well as their pedagogical content knowledge. Ma (1999) posits that teachers must have a profound understanding of the mathematics so they can make connections and explain the conceptual basis behind many of the operations and procedures. Heid et al. show secondary mathematics’ teachers content knowledge influences their instructional planning and classroom practice (Heid, Blume, Zbiek, & Edwards, 1991).

While many prospective secondary mathematics teachers complete a major in mathematics, their subject matter knowledge is generally lacking in depth (Bryan, 1999). In general, mathematics teachers’ subject matter knowledge is not dependent on the number of university level courses, their grade point average, or their scores on a standardized test (Even, 1993, Ball, 1990). In fact, Monk (1994) found that the number of undergraduate mathematics courses taken by secondary mathematics teachers did not correlate to their students’ achievement. In reality, few college level mathematics courses actually address the mathematics pre-service secondary teachers will teach (Conference Board of Mathematical Sciences [CBMS], 2001).

Studies, such as those reported by Ball and Wilson (1990), have noted weaknesses in the mathematical content knowledge among pre-service secondary teachers. They cite the inability of pre-service teachers to understand the meaning behind many mathematical concepts they will be teaching. Mathematics understanding is described as “the dynamic, constructed, and reconstructed process of sense making by the learner” and “learning to represent or communicate mathematical ideas or interpret mathematical representations through the use of language, diagrams, pictures, manipulatives, and other tools” (Heaton, 2000, p. 4). Thus, it is important for
pre-service teachers to have a firm grasp of the mathematics they will teach in order to facilitate their students’ explorations and investigations of mathematics concepts (Whittington, 2002). They must be able to teach in a challenging in-depth manner while showing practical applications and providing meaning to the mathematics. All mathematics teachers must, “actively involve students in meaningful mathematics problems that build upon their experiences, focus on broad mathematical themes, and build connections within branches of mathematics and between mathematics and other disciplines so that students will view mathematics as a connected whole relevant to their lives” (Association of Two Year Colleges [AMATYC], 1995). As a result, pre-service teachers need better preparation for this teaching approach (Furner & Robison, 2004). “To foster students’ conceptual understanding, teachers must have rich and flexible knowledge of the subjects they teach. They must understand the central facts and concepts of the discipline, how these ideas are connected, and the processes used to establish new knowledge and determine the validity of claims” (Putnam & Borko, 2000, p. 6).

What may be the strongest findings of the effect of teacher content knowledge are studies that show student achievement is higher when teachers have strong knowledge of the discipline they teach (Goldhaber & Brewer, 1999; Fetler, 1999; Monk, 1994; Hill, Rowan, & Ball, 2005). When teachers are comfortable with the content they are teaching they become more confident with designing effective lessons and teaching the content. “If American teachers dislike teaching a topic, they may take it upon themselves to eliminate topics altogether from the curriculum or be selective in what and how they teach” (Furner & Robison, 2004, p.4). They may also pass this weakness along to their students. The same may be true when teachers lack confidence in their ability to teach a topic. Teachers teach what they are most comfortable with.

“Students learn mathematics through the experiences that teachers provide. Thus, students' understanding of mathematics, their ability to use it to solve problems, and their confidence in, and disposition toward, mathematics are all shaped by the teaching they encounter in school. The improvement of mathematics for all students requires effective mathematics teaching in all classrooms” (National Council of Teachers of Mathematics [NCTM], 2000, p. 16-17).

As mathematics teacher educators aware of the need to have prospective teachers competent with the subject matter they will teach, we were concerned about their knowledge of the mathematics content secondary students are assessed on in state level standardized tests. How well would they perform on the same tests that their future students must achieve proficiency? These assessments, which are given annually to students in eleventh grade across the state, include questions related to numbers and operations, geometry and measurement, algebra and functions, and data analysis and probability. We were also curious as to the prospective teachers’ knowledge of pre-calculus content, since this content is fundamental for students’ understanding of higher level mathematics such as calculus. In addition, we wanted to determine how confident the prospective teachers were with their content knowledge and their perceived confidence with teaching the content.
Research Questions

This study considered the extent to which secondary mathematics pre-service teachers, who are in their final year of a four-year program, are knowledgeable about the secondary mathematics content and vocabulary assessed on an eleventh grade state assessment. Also, what level of perceived confidence do the pre-service teachers have regarding teaching this content to secondary mathematics students? Specifically:

To what extent are secondary mathematics pre-service teachers able to solve mathematics content questions expected of typical eleventh grade students from the National Association of Educational Progress (NAEP) and the state achievement test?

To what extent are secondary mathematics pre-service teachers able to define mathematics vocabulary identified on the eleventh grade state achievement test?

To what extent are secondary mathematics pre-service teachers confident about teaching the eleventh grade mathematics content to students in their student teaching or first year placements?

What is the relationship between secondary mathematics pre-service teachers’ mathematics content knowledge and their ability to define the mathematics vocabulary identified on the eleventh grade state achievement test?

What is the relationship between secondary mathematics pre-service teachers’ mathematics content knowledge and their perceived confidence with teaching this content?

Methodology

Participants

Data collection for this study took place during the Fall 2005-2008 semesters. Seventy pre-service secondary mathematics teachers participated in the study. The convenience sample consisted of sixty pre-service teachers from a Pennsylvania state university and ten pre-service teachers from a Pennsylvania land grant university. The pre-service teachers, all traditional students, were enrolled in the secondary mathematics methods course at each university and would student teach either in the following Spring or Fall semesters. There were 26 males and 44 females with three minority students in the sample. Their GPA’s ranged from 2.9 to 4.0 and they had completed similar mathematics courses during their undergraduate programs. The required mathematics courses included Calculus I, II, & III, Linear Algebra, Discrete Mathematics, Euclidean/Non-Euclidean Geometry, Applied Statistics, Differential Equations, Abstract Algebra, Mathematical Modeling, and the History of Mathematics.

Instrument

The assessment instrument was designed to assess the pre-service teachers’ (1) mathematics content knowledge (based on the eleventh grade state assessment items from the Pennsylvania achievement test), (2) vocabulary knowledge, and (3) perceived confidence in teaching the eleventh grade content in secondary classrooms.

The content assessment instrument used in this study consisted of 55 multiple choice questions derived from the 11th grade Pennsylvania mathematics assessment released items. The standards-based criterion-reference assessment is used to measure a student's attainment of the
mathematics academic standards while also determining the degree to which school programs
enable students to attain proficiency of the standards. The questions related to all areas of
secondary mathematics including: advanced number and operations (5 questions), geometry and
measurement (7 questions), algebra and functions (11 questions), and data analysis and
probability (12 questions). All items were tested for reliability and validity for use in the state
assessments. Because pre-calculus content items were not available, 23 questions similar to those
used on the Praxis II secondary mathematics content exam were included. The survey also
requires the pre-service teachers to identify the level of confidence (on a scale of 0 (none) to 3
(very)) they have with the correctness of each answer as well as their perceived confidence in
teaching the content addressed. A panel of four mathematics educators and two mathematicians
from the two universities reviewed the items on the instrument for content validity. Each
member of the panel reviewed the questions and completed the assessment to assure that correct
answers existed and the questions were stated clearly. The panel suggested adding a multiple-
choice response of “I do not know how to solve this problem” to provide a selection for a
response that would not require guessing. The panel also suggested eliminating some of the
example problems that were redundant. The instrument was then given to the secondary pre-
service teachers in early Fall of 2005, Fall 2006, Fall 2007, and Fall 2008.

In addition to the content questions, the instrument listed vocabulary terms from the
Pennsylvania Department of Education website that eleventh grade students are expected to
know. The pre-service teachers were asked to identify whether or not they could define the
different terms by selecting one of the following responses: “definitely know and can define the
term,” “might be able to define the term,” “have heard of the term but do not know how to define
the term,” or “have never heard the term used before.” For validity purposes, several placebo
terms were included.

While the released items are intended to be sample items eleventh grade students in
Pennsylvania are assessed on, their use in this study was to determine to what extent secondary
mathematics pre-service teachers are competent with this same subject-matter content
knowledge. The argument here is that pre-service teachers should be knowledgeable about the
content they may teach.

Data Analysis

Content Questions

We begin by describing the results of pre-service teachers’ performance on the content
questions according to the five strands: number and operations, geometry and measurement,
algebra and functions, data analysis and probability, and pre-calculus. A mean proportion of pre-
service teachers who answered the content questions correctly in each of the five strands was
found using the proportion of pre-service teachers who answered each question correctly in each
strand. The mean proportion of correct responses for number and operations, geometry and
measurement, algebra, and data analysis and probability strands were .86, .84, .79, and .78
respectively. The pre-calculus strand stands as an outlier at .37. The results are displayed in
Figure 1.

Figure 2 shows the results when an analysis was performed to determine the mean proportion
of pre-service teachers who answered the content questions in each strand “I do not know how to
solve the problem.” The mean proportion scores for number and operation, geometry and
measurement, algebra and functions, data analysis, and pre-calculus were: .01, .03, .05, .06, and
.36 respectively. Again, the pre-calculus strand stood out as an outlier.
Figure 1:
Mean proportion of correct answers to content questions by strand.

![Graph showing mean proportion of correct answers to content questions by strand.](image)

Figure 2:
Mean percent of content questions answered as “I do not know how to solve the problem.”

![Graph showing mean percent of content questions answered as “I do not know how to solve the problem.”](image)

**Vocabulary Questions**

Figure 3 shows the results when the vocabulary terms were divided into four strands: number and operations, geometry and measurement, algebra and functions, and data analysis and probability. Pre-service teachers were asked to select one of the following responses which was assigned a numeric value: “definitely know and can define the term” (4), “might be able to
define the term” (3), “have heard of the term but do not know how to define the term” (2), and “have never heard the term used before” (1). The mean scores were computed for each of the four strands by using the mean scores on each of the terms for the different content strands. The mean scores for number and operations, geometry and measurement, algebra, and data analysis and probability were 3.86, 3.44, 3.46, and 3.21 respectively.

**Figure 3:**
Mean scores of ability to define mathematics terms by strand.

![Mean Value of Vocab Comfort Level by Strand](image)

**Correlation Analysis**

Figure 4 shows a comparison of the mean proportion of content questions answered correctly in each of the four strands: number and operations, geometry and measurement, algebra and functions, and data analysis and probability (explanatory variable) with mean score for the vocabulary terms in the same strands (response variable) along with the strength of the linear association between the two variables. The correlation value 0.78 is also shown.

Figure 5 shows a comparison of the data with mean proportion of content questions answered correctly in each of the four strands: number and operations, geometry and measurement, algebra and functions, and data analysis and probability (explanatory variable) with the mean score for the comfort levels of teaching the mathematics in the content questions in each of the four strands (response variable). The correlation value is 0.96.
Figure 4:
Association of mean percent of correct answers by strand with mean score on vocabulary terms.

**Perceived Level of Vocabulary Understanding versus Proportion of Correct Answers by Strand**

![Graph showing the relationship between perceived level of vocabulary understanding and proportion of correct answers by strand. The equation is $y = 6.046x - 2.273$ with $R^2 = 0.778$.]

Figure 5:
Association of mean percent of content questions answered correctly with mean score for the comfort levels of teaching the mathematics in the content questions.

**Perceived Comfort Level Teaching Content of Question versus Correct Answers to the Question**

![Graph showing the relationship between perceived comfort level teaching content of question and proportion of correct answers. The equation is $y = 2.555x + 0.3119$ with $R^2 = 0.9622$.]
Two categorical variables: mean percent of content questions answered correctly in each of the four strands: number and operations, geometry and measurement, algebra and functions, and data analysis and probability and the mean score for the comfort levels of teaching the mathematics in the content questions in each of the four strands were analyzed using a Chi-square test. Mean proportion of content questions answered correctly in each of the four strands: number can be viewed as a categorical variable simply by looking at the mean proportion of questions answered correctly and incorrectly. Some modifications are needed for the comfort levels of teaching the mathematics in the content questions in each of the four strands. A comfort level score of zero or one was considered low and a comfort level of two or three was considered high. In all, four chi-square tests were run, one for each strand. The hypotheses used were: $H_0$: The two variables (accuracy in a strand and comfort teaching the content in that strand) mentioned above are independent and $H_a$: The two variables (accuracy in a strand and comfort teaching the content in that strand) mentioned above are not independent. The level of significance was set at $\alpha=.05$. The results of the chi-square are as follow:

Mean proportion of numbers & operation questions answered correctly and perception of comfort level teaching number and operation $\chi^2 (1, N = 70) = 6.59, p < .05$.
Mean proportion of geometry & measurement questions answered correctly and perception of comfort level teaching geometry & measurement $\chi^2 (1, N = 70) = 5.36, p < .05$.
Mean proportion of algebra & functions questions answered correctly and perception of comfort level teaching algebra & functions $\chi^2 (1, N = 70) = 8.06, p < .05$.
Mean proportion of data analysis & probability questions answered correctly and perception of comfort level teaching data analysis & probability $\chi^2 (1, N = 70) = 15.3, p < .05$.

**Discussion**

The results show slight decreases in the mean percent of questions answered correctly in the strands of number and operations, geometry and measurement, algebra, and data analysis and probability, in that order. The number and operations strand had the highest percent of questions answered correctly and the data analysis and probability strand had the lowest. The pre-calculus strand showed a significant, almost alarming, decrease in the mean percent of questions answered correctly. It is apparent that pre-service teachers need some form of pre-calculus review before the completion of their coursework, especially if pre-calculus will be part of their student teaching assignments.

Probably the most alarming piece of data shows the mean percent of pre-service teachers who answered, “I do not know how to solve the problem.” While the number and operations strand shows a rather low instance of this, that number increases slightly for geometry, measurement and algebra. The alarming piece of data comes with the pre-calculus strand where a mean of nearly 40% of the pre-service teachers answered the questions with the responses that they did not know how to solve the problem. This is content that the pre-service teachers might actually be teaching as student teachers or in their first teaching placements. More attention needs to be focused on this particular strand in the undergraduate programs to help improve pre-service teachers’ knowledge.

The vocabulary component of the study shows a rather consistent knowledge of the vocabulary terms in the four content strands. Again number and operations is the highest
category and data analysis and probability is the lowest, but there is no significant difference in the level of the vocabulary content knowledge.

The first comparison between mean percent of content questions answered correctly in each of the four strands: number and operations, geometry and measurement, algebra, and data analysis and probability, and the mean score for the vocabulary terms in the same strands confirms what would be predicted. There exists a strong association between those who answered the questions correctly and those with a high ability to define the mathematics vocabulary terms. The second comparison between mean percent of content questions answered correctly in each of the five strands: number and operations, geometry and measurement, algebra, data analysis and probability, and pre-calculus and the mean score for the comfort levels of teaching the mathematics in the content questions in each of the five strands also confirmed what would be predicted. This is in line with one of the original theses of this paper: those with strong mathematics content are more comfortable teaching mathematics content.

The results of the chi-square test are also in line with what would be predicted. The predictions involved the hypotheses: $H_0$: The two variables (accuracy in a strand and comfort) mentioned above are independent and $H_a$: The two variables (accuracy in a strand and comfort) mentioned above are not independent. It was not surprising to find that the two variables were not independent in any of the four strands. Again, this is in line with one of the original theses of this paper: those with strong mathematics content are more comfortable teaching mathematics content.

**Implications**

The results contribute to research that highlight the need for discussions on pre-service secondary mathematics teachers’ preparation. The results may also prompt secondary mathematics teacher programs to conduct similar studies using sample items from their state assessments, and if they find similar results, consider revising some of their undergraduate mathematics courses to connect more to the secondary mathematics content. Secondary mathematics methods courses may consider the infusion of content strands when content is needed to provide a context for modeling lessons that demonstrate pedagogical or assessment strategies. Assignments that require pre-service teachers to compare and analyze textbooks and curriculum materials can require them to explore how curriculum developers approach instruction about a particular mathematical topic related to one of the content strands. Incorporating inquiry-oriented mathematics curricula that address the content areas can help the pre-service teachers develop more complex understandings of the mathematics they will soon teach. Also, peer teaching or microteaching lessons could be framed around content areas such as pre-calculus and data analysis.

As is the case of this study, and possibly elsewhere, secondary mathematics pre-service teachers take rigorous mathematics courses, yet do not feel comfortable with the content they may be teaching. One approach is to examine the mathematics’ education curriculum to allow pre-service to take a precalculus course or provide opportunities for them to serve as a supplemental instructor or tutor for students in a pre-service course to reacquaint them with the content. Another option is to incorporate a course designed to review content that includes the use of graphing technology and emphasizes the content in an in-depth manner along with practical applications.
Conclusion

College level mathematics courses for prospective secondary mathematics teachers need to connect the mathematics being studied with the mathematics of the secondary curriculum (CBMS). They also need to focus on developing the pre-service teachers’ knowledge of mathematics for teaching and their ability to demonstrate and connect the mathematics. Specifically, the way mathematics educators need to know and use mathematics is different from the way pure mathematicians need and use mathematics (Ball & Bass, 2000).

With shortages in the number of secondary mathematics certified teachers, there is an increasing demand to have recent graduates be well prepared and knowledgeable with the mathematics they will be teaching to impact their students’ learning. As this study found, completing an undergraduate mathematics program may not mean the pre-service teachers are competent or comfortable with the content they will teach in the secondary mathematics classroom. Overall, the study suggests the need for secondary mathematics education programs to explore ways to engage their pre-service teachers in a more in-depth understanding of the secondary mathematics content they will teach.

Also, the study emphasizes the fact that achievement correlates to comfort with the content. The more competent pre-service teachers are with the content they may teach to their secondary students, the more comfortable they feel about the mathematics which may lead to their ability to craft more effective and challenging lessons. One would also expect that as these pre-service teachers gain experience in teaching mathematics, they will develop stronger content knowledge. The question is how long can their students wait to have a highly-qualified secondary mathematics teacher.

References


