CRITICAL NUMERACY AND ABSTRACTION

PERCENTAGES

PAUL WHITE, MIKE MITCHELMORE, SUE WILSON and **RHONDA FARAGHER** describe an approach to teaching percent that puts contexts and application before abstract concepts and procedures.

Have you ever taught a class a concept and then been disappointed that they seem unable to apply what they have learnt to a slightly different context? Many teachers have despaired over this issue which is at the heart of teaching for numeracy. Being numerate involves using mathematical ideas efficiently to make sense of the world, which is much more than just being able to calculate. What is needed is the accurate interpretation of mathematical information and the ability to draw sound conclusions based on mathematical reasoning. This skill may be called "critical numeracy", defined by Stoessier (2002, p. 19) as "being able to critique or make critical interpretations of mathematical information." There is a clear analogy with critical literacy, which involves the realisation that all texts represent different views of the world (Statkus, 2007) and requires students to go beyond acceptance to analysing and challenging (Luke & Freebody, 1999).

How should students be taught mathematics to develop critical numeracy? The traditional approach – sometimes called the "ABC method" because Abstract concepts and procedures are taught Before Concrete examples and applications (Mitchelmore & White, 2000) – certainly seems inadequate. In the ABC method, "knowledge acquired in 'context-free' circumstances is supposed to be available for general application in all contexts" (Lave, 1988, p. 9) – but research

consistently shows that, in practice, this intention is rarely fulfilled.

A teaching method which is more consistent with a critical numeracy standpoint is to start with real-world contexts and gradually draw out general mathematical principles; concepts become "the end-product of … an activity by which we become aware of similarities … among our experiences" (Skemp, 1986, p. 21). This approach, which we call "teaching for abstraction", is the reverse of the ABC method: teaching starts with contexts and examples and leads to abstract concepts.

The teaching for abstraction method was initially shown to be useful and successful for teaching angles (NSW DET, 2003; White & Mitchelmore, 2004). We have now extended the approach to teaching percentages, a topic with which students have many difficulties. A detailed analysis of percentages by Parker and Leinhardt (1995) highlighted the multiplicative–additive confusion with percentages, demonstrating that the concise, abstract language of percentages often uses misleading additive terminology with a multiplicative meaning (for example, there will be a pay rise of 15% paid in three instalments of 5%). Their review of research into teaching percentages showed that a method which linked the development of calculation procedures closely to percentage contexts was likely to be more effective than one in which procedures were taught abstractly and then applied. Our attempt to develop such a method is described below.

The teaching unit

A unit of work was developed in which Year 6 students investigated a variety of situations where it was or was not appropriate to use percentages. The skill of calculating percentages (limited to multiples of 10%) was also addressed.

Participants were students and teachers of five Year 6 classes in three regional primary schools. The unit contained the eight lessons shown in Table 1. The lesson descriptions used familiar terms, addressing the appropriate syllabus skills and outcomes. The recommended lesson structure would, however, have been less familiar to the teachers. Each lesson began with a problem being posed for whole class discussion. Students were then given similar tasks to investigate in small groups. The class then discussed and explained their findings and finally looked for generalisations.

Table 1. Topics for lessons on percentages.

1. Thinking per cent	Students interpret percentages in situations involving bar models. The focus is on per cent as a part of 100.
2. Calculating percentages	Students extend their previous experience of percentages to simple percentages (multiples of 10%) of 200, 300 and 50 objects.
3. Calculating more percentages	Students further extend their previous experience of percentages to simple percentages (multiples of 10%) of any number of objects.
4. Discounts	Students investigate discounts and compare percentage discounts with fixed discounts.
5. How do I choose?	Students compare the appropriateness of additive versus multiplicative strategies.
6. Taxes	Students compare different ways the GST could have been charged and decide on fair ways of doing so.
7. What is the best way?	Students investigate problems involving different comparisons and decide the best way to solve these problems.
8. Summary	Students bring together the main ideas and skills learnt in this unit.

Teaching examples

The following excerpts illustrate how teaching the unit proceeded in practice.

Known percentage contexts

In the first lesson, teachers began by bringing in a variety of grocery containers whose

labels included percentages. They discovered that students had an understanding of the difference between "per cent fat" and "per cent fat-free" and that for any product the two values added up to 100%. This discussion clearly assisted students to clarify their current understanding of percentage and gave direction for future teaching.

Sorting jelly beans

Students were presented with the following context: A jar holds a number of jelly beans, which are then sorted into 10 boxes as shown in Figure 1.

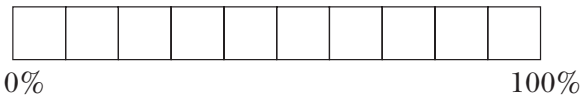

Figure 1

[The actual bar was about 14 cm long]. There were four tasks like this; the number of beans being 100, 200 300 and 50 respectively. In each case, students were told the percentage of red, blue and green jelly beans in the jar and asked to colour in the diagram accordingly. They were then asked the number of beans of each colour.

A final task asked students to colour in 10%, 50% and 90% of three undivided bars marked only with 0% and 100% at the beginning and end.

This task confirmed that students understood per cent to mean "out of 100." A common error early on was to calculate percentages of 200, 300 and 50 as if there were 100 jelly beans; in other words, treating a percentage as if it was always out of 100. Students had little difficulty colouring in 50% of the empty bar, but 10% and 90% gave more trouble. A common mistake was to colour in 1 cm for 10% and 13 cm [14 cm – 1 cm] for 90%.

Teachers' feedback indicated that these colouring activities and the discussion of errors helped students to think beyond 50% and to calculate percentages of numbers other than 100.

Fixed discounts viewed as percentages

Students were told about a fast food outlet where they could get $1 off meal deals for "math burgers" and asked whether it was better to buy two $5 deals (Nell) or one $10 deal (Grace). A typical answer was: "Nell, because she would get a $2 discount whereas Grace only gets a $1 discount." In one school, a student spontaneously came up with the idea that Nell gets 20% discount whereas Grace only gets 10%. The teacher was able to take up this idea and introduce it to the whole class.

It was clear that fixed discounts were familiar, but the discussion helped students relate them to percentage discounts.

Comparing a fixed tax with a percentage tax

Students were asked to compare the GST to a fixed tax method where $10 was charged on all transactions. Students' reactions were mixed:

"[Percentage is fairer because] otherwise you could buy a $1 lollypop and the tax would come in and it would cost you $11 which is a rip off."

"[Percentage is not fair] because if you get something that's expensive, you pay a lot of tax."

Some students thought that the GST was fair because the money comes back to you, but one student was adamant that the government should not take 10% "because they did not make anything."

Context played an important role in this task. Students had to argue their case within the context and discussion quickly moved beyond mere calculation.

Comparing differing discounts of different amounts

The problem shown in Figure 2 was posed.

After some debate, all classes came to the final conclusion that a bigger percentage reduction does not always mean a cheaper buy. Importantly, they observed that both the discount and the size of the whole, of which the percentage is calculated, were relevant.

Your team's football jersey is on sale at two different stores over two weeks. At Store A, the normal cost is $160. At Store B, the normal cost is $120.

(a) In Week 1, at Store A they reduce the normal price by 25%. At Store B, they reduce the normal price by only 10%. Which is the cheaper buy?

(b) In Week 2, at Store A they reduce the normal price by 40%. At Store B, they still reduce the normal price by only 10%. Which is the cheaper buy?

(c) Is the bigger percentage reduction always the cheaper buy? Explain your answer.

Figure 2. Problem comparing discounts.

However, the notion of "best" could still have different interpretations, with one student thinking the best deal occurred when the cost was lower, not the discount bigger, because they spent less money.

Making judgements about comparisons

To further help students distinguish contexts where percentages were appropriate from those where they were not, a number of problems were posed. Some are shown in Figures 3–6.

Two schools' results on a mathematics competition were as follows:

School	Enrolment	Number of certificates
Sunny Valley	500	60
Paradise Junction	800	80

Which school performed better? Explain your answer.

Figure 3

Most students correctly used percentages in the problem shown in Figure 3. They were able to calculate that 10% of the students in the second school were awarded certificates and to see that the percentage in the first school was greater (even though the calculation of 12% was beyond most of them).

Two basketball players compare their shooting from the free throw line. The first player has scored 20 goals from 40 shots. The second player has scored 25 goals from 50 shots. Which player is the better shooter? Why?

Figure 4

Responses to the question shown in Figure 4 included the expected assertion that each player scored 50%, but also arguments like this: "The 25 was better because they were the same but kept it up longer."

The government decides to give all public schools 10 extra computers.

(a) Is this a fair way to give out extra computers?

(b) If not, what would be a fairer method?

(c) Tell the Minister of Education exactly how to work out how many computers should go to each school.

Figure 5

As well as using proportional reasoning, some responses to the question shown in Figure 5 raised social justice issues of fairness with respect to how rich a school was.

Which is worse: losing 50% of $1, $10, or $100?

Figure 6

Value judgements also came into play in the question shown in Figure 6. Some students argued that 50% of $1 is not very much but 50% of $100 is. Others took the view that losing 50% of $100 still left you with $50, which is better than 50 cents.

Conclusions

The overwhelming response from teachers was that the extended discussion generated by the lesson materials was a great success and promoted student engagement and

learning. They were unanimous that the greatest development occurred in the students explaining how an answer was arrived at — in particular, in identifying percentage as a relative comparison and the need to identify "per cent of what." Both teachers and students indicated that the time spent talking about what a discount is, with examples from real life, was particularly valuable. One teacher described Lesson 6 as "the epiphany lesson" where the students realised why they needed to be able to calculate percentages. Another teacher comment was: "The high point of the whole thing was that they did have to nut things out, discuss."

"Nutting things out" is certainly one aspect of critical numeracy; dealing with value judgments and social justice issues present in a number of the teaching activities is another. Interestingly, teachers said they were not as comfortable with these types of activities as they were with others, because "right answers" were either unclear or non-existent.

Although the teachers agreed about the benefits of open discussion, time was often a limiting factor (especially when students got carried away with digressions). Teachers also faced the challenge of moving beyond their normal practice. There was the natural feeling, perhaps arising from traditional practice, that it is important for students to get class activity answers correct. This is not likely to happen when the focus is on long-term learning, and activities are challenging problems which encourage students to struggle and resolve problems for themselves. One result is that teachers give students too much "help" and thus reduce the struggle.

Our investigation shows that teaching for abstraction can lead to student engagement and empowerment when students understand they can make judgements about things that are part of their lives. Teaching for abstraction thus involves critical numeracy as well as computational skills and certainly requires a significant shift from the traditional approach to teaching mathematics.

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