Using SPREADSHEETS to make algebra more accessible

Part 2: Solutions to equations

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In the first article in this series (Green, 2008), we quoted Robert H. Lewis's analogy (2000) in his paper titled "Mathematics — The most misunderstood subject." He said that the rules, formulae and algorithms were the scaffolding, which was not of much use unless one built the building. The building is the true mathematical understanding, involving ability to think, perceive, and analyse mathematically.

This article is the second in a series of two papers which suggest some practical, spreadsheet-based ideas for helping students to make appropriate connections between particular algebraic concepts.

Solutions to equations

Solving equations has traditionally been taught as a pen-and paper process. Spreadsheets, such as that of Excel, provide a contemporary, and powerful context for the teaching of processes for the solution of equations. The following sections give some examples, then discuss pen-and-paper methods for their solution, and subsequently revisits these examples using spreadsheets.

Example 1

When working with percentage and trigonometry problems we use what we know to reduce the problem to a simple equation of the form $aX = b$. For example, finding 15% of 125 can be found by solving the equation

$$\frac{X}{125} = \frac{15}{100}$$

When cross-multiplied it becomes equivalent to solving $100X = 1875$.

Example 2

The hypotenuse of a right-angled triangle can be found given an angle of, say, 30° included by a side of two units and the hypotenuse by solving the equation

$$\frac{\cos 30^\circ}{1} = \frac{2}{X}$$

which when cross-multiplied is equivalent to solving $0.866X = 2$.

Example 3

Converting 50 degrees Fahrenheit to Celsius requires the solution of the equation $50 = 1.8C + 32$, which is, in the general form of $a = bX + c$.

Example 4

When working with area the problem can be reduced to an equation of the form $a = bX$. For example, the perpendicular height (H) of a trapezium given parallel side lengths of 10 and 20 units and an area of 100 square units can be found by solving the equation, $100 = 0.5(10 + 20)H$.

Pen-and paper methods

Being able to solve equations with a single unknown variable is important. Unfortunately, it is a process that is often learned by rote, resulting in misconceptions such as the following: a student was absolutely certain that even his teacher would agree that the equation $2X - 5 = 20$ reduces to $X - 5 = 10$ because, he said, "The multiplication has to be undone before the subtraction." When teaching about methods for solving equations, students must be given a context and taught in such a way so as to promote deep understanding of the concepts that underpin the procedures being used. By their very nature, statements such as, "Undo the multiplication and division before the addition and the subtraction," or, "Use the BODMAS rule in reverse," promote a rote-learning approach (sometimes incorrectly remembered) over approaches that encourage inference-making, connections and understanding.

When it comes to the teaching of equation-solving there are numerous approaches, such as back-tracking, balancing, flowcharts and function-machines. This article focusses only on the procedure of back-tracking. It is a conceptually powerful procedure, and can be applied to more advanced equations.

The first point that needs to be made is that back-tracking as a procedure will make better sense if students are first introduced to the (perhaps less well-known) procedure of forward-tracking. Students might be given a number, and asked to double it and finally add five. This essentially requires a process of forward-tracking. Spreadsheets are an excellent tool for facilitating this procedure, but the ideas need to be introduced by students using what can be loosely referred to as number chains (Figure 1). The spreadsheet conventions for the multiplication (*) and division (/) symbols have been used. Twelve, in the first circle, when multiplied by three is 36. This is recorded in the second circle. When four is then added to the 36, 40 is recorded in the next circle, and so on until we get to the last circle. This activity is establishing the ideas of forward-tracking.

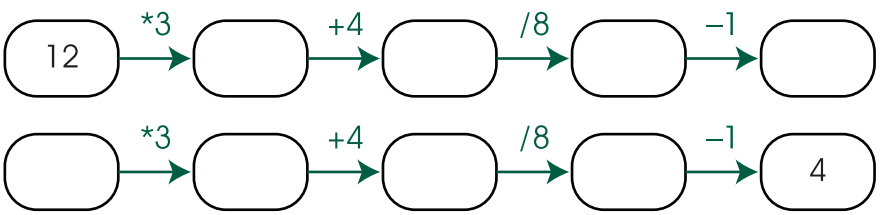

Figure 1

In the second example in Figure 1, the result of a series of computational steps is recorded in the last circle as four. It has become four after one is subtracted from the previous result, so before this, it must have been five. Five is recorded in the second-last circle. Five is the result after the previous number was divided by eight. Before this, the number must have been eight times more, that is, 40. The 40 is then recorded. Forty is the result after four was added to the previous number, which must, therefore, have been 36. The 36 is recorded. Thirty-six is the result of multiplying the previous number by three, which must, therefore, have been 12. The 12 is recorded in the first circle. This activity is establishing the ideas of back-tracking.

Using spreadsheets to solve equations

Another way to conceptualise forward-tracking is through the generation of paired values of data that fit a particular mathematical relationship. This was discussed in detail in a previous article. For example, a starting number might be 10 which when doubled becomes 20 which finally becomes 25 after five is added.

This number could now be conceptualised as an empty cell in a spreadsheet. For example, the number is A1 which when doubled becomes 2A1 which finally becomes 2A1+5 after the five is added. The final result will occupy cell B1. This leads to the relationship B1=2A1+5. This is entered into a spreadsheet as shown in Figure 2. (In the spreadsheet formula 2A1 has been reversed to read A1*2. This represents the order in which the operations are actually taking place.)

Cell A1 and B1 are next highlighted and the fill-down feature is used to create a data set. (The tiny light-blue square on the bottom right of the 25 is used to click and drag down.) The values in column A are increased by one. The same formula is copied into each cell in column B. The result is a set of paired values that describes the mathematical relationship represented by B=A*2+5.

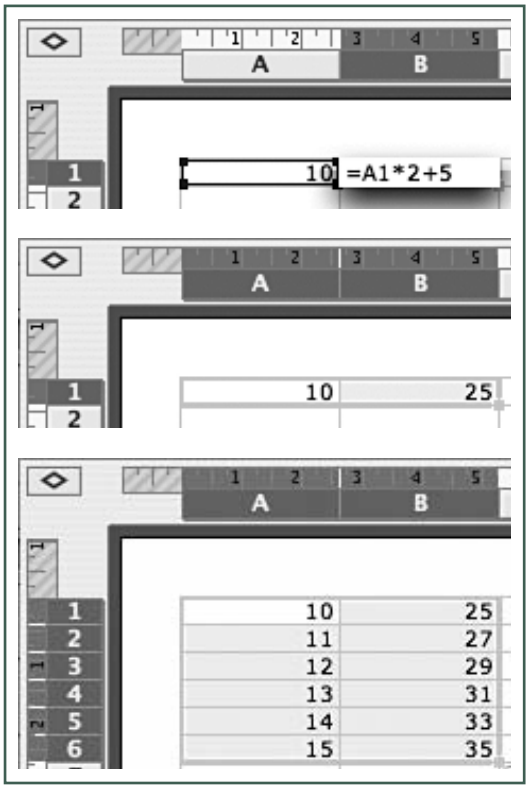

Figure 2

To show how the relationship is expressed in each particular cell the formula view mode of Excel is used. (This is activated by going to <preferences> and checking <formulas> in <view window options>.) When this is done the spreadsheet appears as in Figure 3. While the relationship stated in general terms is B=A*2+5, in specific terms this means that B1=2A1+5, B2=2A2+5, B3=2A3+5 and so on. At this early stage the variables X and Y are not used. One of the strengths in using a spreadsheet to introduce the idea of an

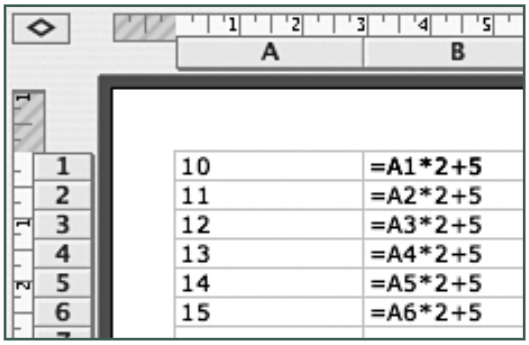

Figure 3

algebraic variable is that it gives the clear message that alphanumeric indicators are little more than a label for a box, the box being the cell in the spreadsheet. Final calculations cannot be performed unless a value is first assigned to a previously empty box. In other words, the variable has to be assigned a numerical value.

Now the procedure of back-tracking can be introduced. Ask students to back-track from, for example, 25 until they obtain the number 10. The following thinking should be encouraged: "After the five was added, the result was 25; so before the five was added, the number must be 25 take five, which is 20. After the number was doubled it became 20, so before it was doubled it must be half of 20, which is 10."

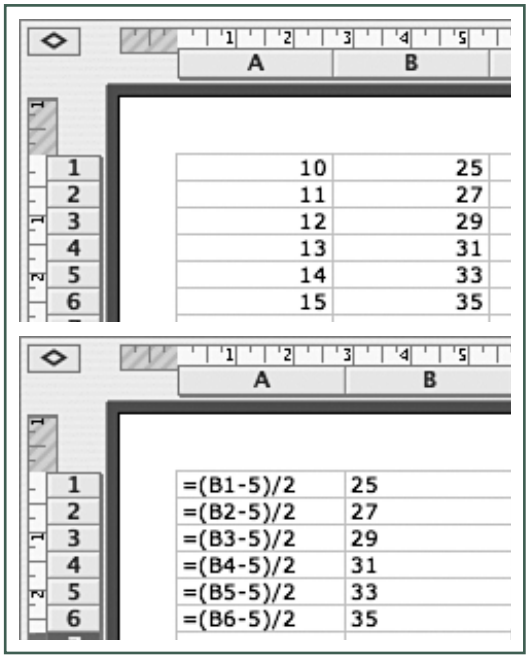

Figure 4

While the value 10 is forward-tracked using the relationship $B = A \times 2 + 5$, it is back-tracked using the relationship

$$A = \frac{(B-5)}{2}$$

Figure 4 illustrates the thinking in spreadsheet form.

The above process is essential to understand equation-solving. The forward-tracking process results in the equation $B = A \times 2 + 5$ while the back-tracking process results in the equation

$$A = \frac{(B-5)}{2}$$

Equation-solving can be contextualised through number-solving puzzles as follows: if you were to think of a number, double it and add five, then I would be able to use your answer to back-track, and from my result I would be able to tell you the number with which you started. For example: think of a number A; double it — which is $2 \times A$ — and add 5 — which is $2 \times A + 5$ — and you get 10. The equation to be solved is $25 = 2 \times A + 5$. Back-tracking produces $25 - 5 = 2 \times A$, which can then be rewritten as

$$\frac{25-5}{2} = A.$$

Once this process is clearly understood, it makes the topic of equation-solving much more accessible to students. This is partly because the spreadsheet can be used by students to check the correctness of their answers. Students can now be challenged to set up more complex forward-tracking scenarios as a way of challenging each other in back-tracking.

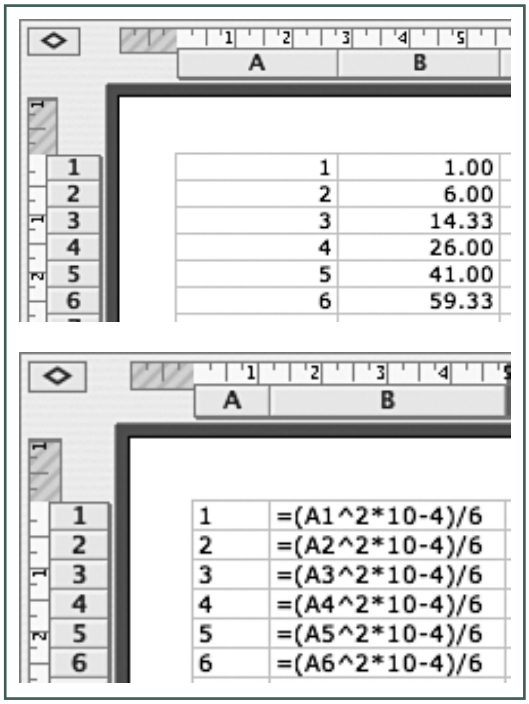

Figure 5

Figure 5 illustrates the forward-tracking process when a student squares a number, multiplies it by 10, subtracts 4 and divides the answer by 6. When this is done to the number 2 (cell A2), in the spreadsheet the result is 6 (cell B2). When it is done to the

number 3 (cell A3), the result is 14.33. When it is done to the number 4 (cell A4), the result is 26, and so on. Back-tracking this last result, for example, can be written as the equation

$$26 = \frac{10A^2 - 4}{6}.$$

Before dividing by 6, the equation would be $156 = 10A^2 - 4$. Before subtracting 4, the equation would be $160 = 10A^2$. Before multiplying by 10, the equation would be $16 = A^2$. Before squaring, the equation would be $4 = A$ and this is confirmed by the spreadsheet.

The previous article described how a spreadsheet might be used to generate a formula for converting temperatures from Celsius to Fahrenheit. The result was the generation of the equation $F = C \times 1.8 + 32$. By back-tracking, a formula for converting Farenheit to Celsius is obtained by the following steps: the initial formula is $F = C \times 1.8 + 32$. Before 32 was added, the equation was $F - 32 = C \times 1.8$. Before multiplying by 1.8, we had

$$\frac{F - 32}{1.8} = C$$

which is the required conversion formula.

It is often helpful to present equation-solving as a process that might be used to find one half of a set of paired values when the other half is missing. Consider the following formula where neither variable is the subject of the formula: $2A - 5 = 3B + 4$. To find the missing half in the paired value $(1, B)$ back-tracking can be used to arrive at

$$B = \frac{2A - 9}{3}.$$

To find the missing half in the paired value $(A, 8)$ back-tracking results in the equation

$$A = \frac{3B + 9}{2}.$$

Conclusion

The two papers in this series have described ideas for helping students to gain a contemporary view of the processes through which algebra, equations and equation-solving develop. In doing so, students will have a better understanding of and appreciation for the discipline of algebra. Successfully achieving this necessitates a less compartmentalised view of mathematics. It requires an integrated approach to the teaching of mathematics using examples from the various strands of mathematics. The challenge of doing this is made easier with access to the appropriate computer technology.

Developing algebraic understandings such as equation construction and equation solving can be woven into the very fabric of the mathematics curriculum. The approaches described in this paper (and the previous paper) promote inference making, connections and understanding over rote and procedural repetition.

References

Lewis, R. H. (2000). *Mathematics: The most misunderstood subject*. Accessed 4 February 2009 at http://www.fordham.edu/mathematics/whatmath.html.

Green, J. (2008). Using spreadsheets to make algebra more accessible — Part 1: Equations and functions. *The Australian Mathematics Teacher, 64*(4), 7–11.