The Con Test

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The Con Test (FremantleMedia, 2007) is a recent Australian television game show which premiered on 7 February 2007 on Network Ten. The show followed the same general rules and game play as the UK show PokerFace. The format of the show is as follows:

- On each show, 6 contestants compete for a prize of $50 000.
- In Round 1, there are 8 multiple-choice questions worth $500 each. After the answer to each question is revealed, viewers at home can see which contestants got it right or wrong by a series of ticks and crosses appearing on the pictures of each contestant. At the end of each round, viewers at home see the leader board, and more importantly, who is in first and in last place.
- After the round has finished, the face-off begins. The contestants stand in front of red buttons that rise from the floor, and a ten second countdown starts. There are two possible outcomes:
  - Contestants fold by pressing the button. They lose their place in the game, but take home with them their winnings.
  - Nobody folds. The player with the least money will be removed from the game and they leave with nothing. If several players tie with the least amount of money lots are drawn to decide who is removed.
- There are five rounds in total. In each subsequent round, there are just five questions, but the prize money increases by $250 each round.

At the end of each round a contestant needs to decide whether or not he or she should fold. A contestant needs to know how likely it is that he or she is in last place. Suppose, for example, a contestant has answered two questions correctly in round 1. Is it likely that he is in last place? To answer this question one needs to know on average what proportion of questions is answered correctly. Over the first few weeks of the program, approximately 50% of questions were answered correctly. In order to solve the problem we will make the following modelling assumptions:

1. All the contestants are equally likely to answer a question correctly.

2. The probability a question is answered correctly is $\frac{1}{2}$.
3. The probability a question is answered correctly does not change throughout the program.

If a contestant has answered only two questions correctly in round 1, there are several scenarios in which he/she is in danger of being eliminated:

1. All the other contestants score more than two correct answers.
2. Four of the other contestants score more than two correct answers and one has two correct answers.
3. Three of the other contestants score more than two correct answers and two have two correct answers.
4. Two of the other contestants score more than two correct answers and three have two correct answers.
5. One of the other contestants scores more than two correct answers and four have two correct answers.
6. All the other five contestants have two correct answers.

(Note that if there are n contestants with the lowest number of correct answers there is a probability of $\frac{1}{n}$ of any one of them being eliminated.)

Scenario 1

The probability any one contestant scores more than two correct answers

$$=1-\left\{{}^{8}C_{0}\left(\frac{1}{2}\right)^{8}+{}^{8}C_{1}\left(\frac{1}{2}\right)^{1}\left(\frac{1}{2}\right)^{7}+{}^{8}C_{2}\left(\frac{1}{2}\right)^{2}\left(\frac{1}{2}\right)^{6}\right\} \text{ (using the binomial distribution)}$$

$$=219(0.5)^{8}$$

$$=0.855$$

Hence, the probability all five score higher than three correct answers is 0.855^5. Interestingly, it is likely that another contestant has scored as few as two correct answers!

Scenario 2

The probability a contestant scores exactly two correct answers (using the binomial distribution) is

$$^{8}C_{2}\left(\frac{1}{2}\right)^{2}\left(\frac{1}{2}\right)^{6}=28(0.5)^{8}=0.109$$

The probability four of the other contestants score higher than two correct answers and exactly one scores two correct answers is

$$^{5}C_{4}(0.855)^{4}(0.109) = 0.291.$$

Hence, the probability the contestant is eliminated if they do not fold is

$$\frac{1}{2}\times 0.291=0.146$$

The probabilities associated with the other scenarios can be calculated in the same way. Table 1 shows the probabilities of elimination in each case.

Table 1. Probililities of elimination scenarios.

Scenarios	*Probability of elimination (to 6 decimal places)*
1. All the other contestants score more than two correct answers	0.458164
2. Four of the other contestants score more than two correct answers and one has two correct answers	0.146445
3. Three of the other contestants score more than two correct answers and two have two correct answers	0.024965
4. Two of the other contestants score more than two correct answers and three have two correct answers	0.002394
5. One of the other contestants scores more than two correct answers and four have two correct answers	0.000122
6. All the other five contestants have two correct answers	0.000003

Hence the probability the contestant is in danger of elimination if two correct answers are given is 0.63 (2 significant figures.). In other words, there is a slightly greater than even chance that the contestant will be eliminated if no-one else folds. If the contestant folds only $1000 will be taken away, whereas not folding leaves the possibility of winning far more money in the next rounds. Clearly it is in the contestant's interest not to fold if two correct answers are given.

In practice most contestants who correctly answer only two questions decide to fold. They believe, fallaciously, that there is little chance of someone else doing as badly as them. The mathematics shows, however, that if a contestant does badly and only correctly answers two questions it is more than likely that someone else has done as badly.

A knowledge of probability theory is most useful for determining the optimal strategy to use. When the results obtained are counter-intuitive, as in a case like this, any contestant who understands probability theory is at a distinct advantage over the other contestants!

It is interesting to note that in later rounds contestants' intuition lets them down even more. Readers might like to try the following: calculate the probability that a player who answers two questions correctly in round one is in last place after round two.