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Mathematics Assessment for Students With Mild Disabilities: Frameworks and Practices

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The overall purpose of this paper is to describe a practical and useful approach to mathematics assessment that is meaningful to teachers, students and families. The format is highly flexible, yet specifically designed to provide detailed information as to the performance characteristics of the student and a variety of curricula and instructional options for the teacher and school. The assessment focuses upon three key factors. They are: (1) the availability of an organized curriculum, (2) the utilization of multi-modal interactions between student and evaluator and (3) time for the teacher or other sources to actually conduct instruction in a manner indicated by the results of the assessment. The assessment we describe can be conducted in whole or in part in most every topic of mathematics and once the teacher has assured himself that he has mastered the curricula and instructional design, the teacher can ad-lib the instruction. As described, the assessment was an integral component of a four year project involving 23 teachers, about one-half from general education and about one-half from special education. The greatest obstacle encountered by the teachers was a lack of time to implement their instructional findings due to the rapid pace established by the district to cover the content of each grade in a year (Parmar, Cawley & Frazita, 1998).

The overall purpose of this paper is to describe a practical and useful approach to mathematics assessment that is meaningful to teachers, students and families. The format is highly flexible, yet specifically designed to provide detailed information as to the performance characteristics of the student and a variety of curricula and instructional options for the teacher and school. This will be presented in the form of frameworks or schemes that individual districts, schools or teachers can adapt to their own priorities.

Within the scope of these frameworks there is a commitment to content validity as it is represented in assessing student *knowledge, process,* and *skill.* In one sense, this paper represents a commitment to school or classroom developed forms of assessment with a lesser emphasis on norm-referenced practices that use technical forms of data analysis (e.g., item analysis) to select and sequence items.

For purposes of this paper, standardized assessments of three types will be described. The three types are: vocabulary, arithmetic computation and word problems of varying structures. An extensive data base for these components can be found in Cawley, et al (Cawley, Parmar, Foley, Salmon & Roy, 2001) These will be augmented with the systematic use of alternative representations, the use of alternative algorithms and a focus on how the student explains, proves or reasons with different types of mathematics.

One important element of assessment is its implication for program adaptation and utilization for the benefit of the student. In this regard, we propose a reasonably comprehensive view of assessment and an interpretation of it that will enable the user to detail specific needs of the student in mathematics. We will also outline how to employ the activities of mathematics to assist the student in areas often listed as reasons why the student does not learn mathematics. For example, a significant number of students with learning disabilities have difficulties in language comprehension. Teachers often describe the limitations in language comprehension as a reason why the student does not perform well in mathematics. What we propose is to use activities in mathematics to address the difficulties in language comprehension and enable the student to improve both language comprehension and mathematics simultaneously.

Standardized Assessments

Aside from the informal assessment conducted by the teacher, which generally is the basis for referral, the most commonly used forms of assessment are standardized models. The purpose of the standardized assessment is to provide data relative to the mathematics students *know* and *do* in relationship to themselves and to other students within a the framework of a logical sequence of mathematics. Three components of standardized assessment are reviewed. They are:

1. Mathematics Vocabulary
2. Mathematics Computation
3. Problems of Varying Structures and Dimensions

Mathematics Vocabulary

In its broadest sense, verbal development and proficiency are a key elements of success in school. Group tests commonly assess verbal development through the use of written multiple choice items. Individual tests frequently assess verbal development through the use of open-ended tasks in which the examiner states a term and the student provides a spoken definition for the term. The terms ultimately contained in the test are selected from a pool of test items that meet specified statistical criteria. To more fully address the mandates of legislation such as P.L. 94-142 and the criteria that assessments be equitable and fair, it is suggested that vocabulary assessment begin with a picture vocabulary format, similar to that of the *Peabody Picture Vocabulary* (Dunn & Dunn, 1965). This format provides relatively easy access and response capability in that the teacher states the term and the student marks or points to a picture that represents that term. The term itself can be presented in any language and is therefore, multi-lingual. The vocabulary chosen

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can be drawn from the teacher's guides of mathematics textbooks. One of the useful elements of this format is that selected picture sets remain the same while the stimulus word changes (*e.g., pennies, coins, dollars, currency, wealth, monetary).*

The use of a picture vocabulary assessment provides the teacher with information obtained from one mode of assessment. The emphasis is on listening comprehension, a process that is not common throughout psychoeducational assessment even though the change from reading to listening is common when modifying assessments. The teacher could utilize the same materials and explore formats in which the teacher selects terms known to the student to address the *content* (i.e., the word) versus *task* distinction (i.e., the format of the test item such as a picture vocabulary item).

Picture vocabulary assessments can also be developed for groups. In one instance, each student could be given a booklet and the teacher would pronounce the key term for each picture set. This is common in the assessment of primary grade vocabulary. In another instance, the word could be printed and each student would read the term and then mark the corresponding picture. The teacher might use formats such as:

1. Presenting a student with a picture and asking the student to name or describe the picture in mathematical terms
2. Stating a term and asking the student to state a term with the opposite meaning (e.g., addition/subtraction).
3. Presenting the student with a written word and asking the student to mark a picture that shows a meaning for the word.
4. Presenting the student with a written word and four choices of responses and requesting that the student mark the word that is most similar in meaning to the standard.
5. Stating or writing a term and asking the student to write a definition for the term.
6. Stating or writing a term and asking the student to write the term in a sentence that defines the meaning of the term.
7. Presenting a task for the student to perform in which the key word signals the correct action.

Students need to understand vocabulary and its direct and indirect importance for mathematics. To illustrate, examine the two problems below.

A	B
A zookeeper divided the reptiles so there were 8 reptiles in each of 3 habitats. How many reptiles in all were divided by the zookeeper?	A zookeeper divided the reptiles so there were 24 reptiles evenly placed in each of 4 habitats. How many reptiles were in each of the habitats?

Note that each problem began with the action, "divided," but division is not what is required in each problem. Students who are taught with an emphasis on a "cue" word often err when presented with problems of the type illustrated in A.

Mathematics Computation

Computation on whole numbers dominates the mathematics of special education and general education. Computation reigns supreme in nearly all intervention studies involving students with learning disabilities and in studies involving topics such as problem solving, there is the assumption that proficiency in computation has preceded the problem solving. As described in this paper, the assessment of computation focuses on two domains, the first being speed of response or automatization of single digit items and the second being proficiency in more robust computations ranging from two digit combinations to four digit combinations.

To conduct a meaningful assessment, it is important to specify the program goals and the elements of assessment that are assessed. Goals for multiplication might be:

- Student understands multiplication as a many-to-one relationship.
- Student understands that combining factors yields a product.
- Student knows about multiplication and how to determine products when they are unknown.
- Student understands that multiplication and division represent inverse operations.
- Student understands that finding a missing factor involves a quotient.
- Student understands how division is proof of multiplication and that multiplication is proof of division.
- Student understands basic principles such as commutativity, distributive property, identity element and zero.
- Student habituates fact combinations to enhance speed of response and does not habituate fact combinations as a means of learning to *do* multiplication.

Automatization

The general thesis underlying automatization of single digit combinations rests in the notion that "knowing the facts" is fundamental to success with more complex combinations. While that may be true, there is no guarantee that "knowing the facts" assures competence with more complex combinations for two reasons. Although "knowing the facts" is helpful, knowing the facts is only as good as understanding the process and the interrelationships within and among processes (Cawley, Smith, Shepard & Parmar, 1996). The present authors believe that (1) single digit multiplication and division should be taught simultaneously as factor by factor = product relationships, (2) that automatization of the facts should not take place until there is a complete understanding of the factor by factor = product relationship, (3) that automatization should not take place in the form of memorizing the tables, (4) extensions beyond single digit combinations should not take place until the student has a qualitative grasp of place value and expanded notation, (5) that the solving of word problems of varying structures should take place within a contextual setting sans specific word cues and (6) students are able to demonstrate what they *know* and *do* with multiplication through the use of alternative algorithms, alternative representations and explanation and proof. The aforementioned are important for a number of reasons. Some of these are:

- The teacher or the assessment specialist may fail to differentiate one error from the other and make an erroneous interpretation of the student.

A common example exists when the student utilizes an alternative algorithm correctly, but errs in computation and the teacher wants to change the algorithm.

- The manner in which students acquire "knowledge" of the basic facts is typically through drill and practice with materials such as flashcards, paper-pencil drills or responding to computer routines.

Exercises of these types fail to integrate number sense with response requirements and the students never learn any way to determine the solution to combinations they do not remember.

- The field does not have precise knowledge as to what constitutes "fast" or just how "fast" a student should be to have automatized his response to all combinations.

The materials used within the field do not provide common and needed numbers of replications such that students will have adequate practice with all of them. For example, 3 x 5 may occur 65 times across the spiral curriculum, but 9 x 7 only occurs 31 times. Yet, 9 x 7, the one with a fewer number of replications, is generally viewed as being more difficult.

Single Digit Assessment:

FACTOR-BY-FACTOR=PRODUCT

A beginning step in the assessment of factor-by-factor= product relationships is to determine student understanding for the one-to-many correspondence that represents multiplication. This provides information to the effect that the student senses multiplication as a process that determines the number of times one group of *many* (e.g., 4) are repeated or calculated (e.g., 2).

One assessment for the factor-by-factor=product relationship with children as young as first grade involves the student in cartesian relationships. Here the student is provided with combinations of items such as three shirts of different colors and 2 pair of slacks of different colors. The student is asked to determine the number of different combinations of clothes that can be worn (i.e., 3 x 2).

Young children also learn skip counting, first by two's and then by other combinations. Skip counting in the form of 2, 4, 6, 8, etc. is a representation of multiplication. The student can be asked to represent these on a number line or facsimile. Students can also be asked to complete number sequences such as 2, 4, _ , 8, 10, 12, _ , etc. or to extend number sequences such as 2, 4, 6, _, _ .

Students can be assigned to small groups and provided with materials that are grouped so that each member of the group will get the same number (e.g., 3). The student is to retrieve the needed number (3 times x) from a larger pile and distribute them.

Students can be assessed for understanding the distinction between multiplication as represented by a number of equal sized sets, Figure 1 and represented as an array, Figure 2 (Cawley, Fitzmaurice, Goodstein, Lepore, Sedlak & Althaus, 1974).

A symbolic form of assessment focuses on the factor-by-factor = product relationships using item combinations such as

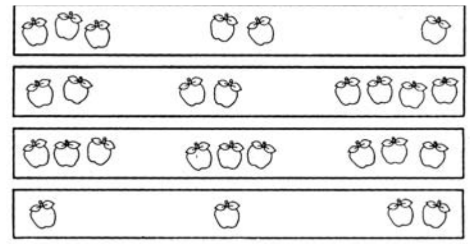
$3 \times 2 = \_$	$5 \times 9 = \_$
$\_ \times 2 = 6$	$\_ \times 9 = 45$
$3 \times \_ = 6$	$5 \times \_ = 45$

along with the commutative of each.

SPEED OF COMPUTATION

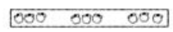
One method of the assessment of speed and accuracy of response consists of the following. Items were selected from a pool that consisted of all single digit combinations for each operation. These are commonly referred to as *the facts.* A total of 48 items were selected and the students were provided with one minute in which to complete as many as possible.

Figure 1 Set Depiction of Multiplication



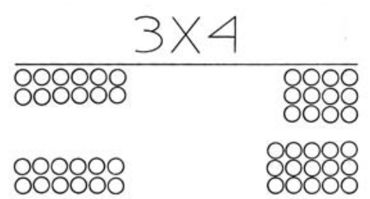
Look at each group of pictures on the page.
Which group could represent a multiplication expression?

Answer:



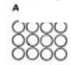
Strand -NUMBERS
Array - MULTIPLICATION
Concept -REPEATED ADDITION SETS

Figure 2 Array Depiction of Multiplication



Look at the multiplication expression at the top of the page.
Which picture shows what the expression says?

Answer:



Strand - NUMBERS
Area - MULTIPLICATION
Concept - ARRAYS

A second form of assessment (Figure 3) for single digit combinations tabulates both the number of items and number of strokes attempted and accurately completed for each of the four operations. For multiplication an item such as 1×5 requires only 1 stroke, whereas an item such as 9×5 requires two strokes. This assessment embedded the original set of single digit items into larger items where 5×1 and 5×9 are combined to make 5×91 and then to 5×911 where renaming is required in the 100s place and 5×119 where renaming is required in the 1s place. The data show that 88 percent of the items attempted are correct in set 1, but only 51 percent are correct for set 4. Yet, the items for set 4 are comprised of the items in set 1. The data tend to show that algorithmic complexity slows the student and also affects accuracy (Cawley, Smith, Shepard & Parmar, 1996).

Figure 3:

Composite Items for Addition and Multiplication

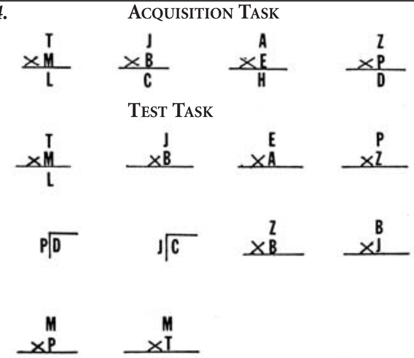
	Set 1	Set 2	Set 3	Set 4
Addition	2 2 6 +1 + 8 +1	22 +81	221 +816	122 +618
Multiplication	1 9 6 ×5 ×5 ×3	91 × 5	911 × 5	119 × 5

One philosophical issue relative to single digit automatization centers on the extent to which the student demonstrates knowledge of the combinations before speed or automatization is introduced. If the student does not know the item, speed of response is diminished. Further, if the students do not know the combination, do they have an efficient means to generate the response independent of memory and how much time is taken to generate the response. Possibly, separate measures could be taken. First the student should be assessed for knowledge of F × F=P combinations. Those that are known could be put in one pile and those that are unknown put in another. Speed of response could be examined with the pile of "knowns." Both the ability to generate correct responses and the speed with which they are generated are measured with the "unknown." Qualitatively, the assessment ought to include an inquiry relative to the procedures used by the student to generate the "unknowns."

Associate Learning

A fourth form is displayed in Figure 4. This format assumes a degree of student knowledge relative to the process of multiplication and it focuses on determining the process-by-effort-by-mass requirements of individual students in the acquisition of specific paired associate combinations. The student is shown the card with the four items. The student is then told that he is to remember each of the four items and that he/she can use any means to help remember (e.g., student can look at the combinations and say them to herself; student can write each combination as many items as needed, etc). Each item is then shown without the answer and the student completes the item. If corrections are needed, the student may practice some more.

Figure 4. ACQUISITION TASK



When all are recalled correctly, the card is turned over and the student is requested to complete the items as shown on the card. The items assess the sense of the student for factor-by-factor relationships and also determines the conceptual level of the student. The student is instructed to leave blank any items that can not be completed. To begin, the student examines the sample item and then goes on to complete the remaining items as follows:

(1) J × B is a direct recall item.
(2) E × A, B × J, M × T, and P × Z invoke the commutative property.
(3) $P\overline{)D}$ invokes the F × F=P relationship in that if Z can be multiplied by P to get D, either can be divided into D, as is so with J, B and C.
(4) Neither Z × B nor M × P can be completed and the correct student response is to leave them blank. This highlights the need to approach "fact" acquisition from a meanings perspective.

A first step in this form of assessment is to determine the process selected by the student to *acquire* the items (e.g., Does the student repeatedly write each one. If so, are they written one at a time or are all four written repeatedly at a single time). This provides information relative to a preferred technique of the student. The *effort* is the amount of time or repetitions needed by the student. One student may use a specific technique and master the items in 4 repetitions whereas another student may require 10 repetitions with the same technique. The third set of information relates to *mass* or the number of items the student can manage at a time. Although the assessment begins with four items, four items may be too many for some students whereas other students could manage more than four in a single setting.

Briefly, the instructional implications from the above may show: (1) that a change in acquisition process may enhance performance, (2) the student may need a greater or fewer number of repetitions and the student could participate in determining these and (3) the number of items provided to each student may differ. The need for meaning is evident. Students who *know* and *do* multiplication ought to be able to determine the correct responses for items such as M × T never having had them taught to them if they possess number sense through relationships with cardinality.

Queries relative to student reasoning can be undertaken with assessments such as:

- Give 3 × 4 = 12 and 2 × 6 = 12 and have the student explain why two different combinations produce the same product.
- Give the student 3 × _ = 12 and 6 × _ = 24 and explain why the missing factors are the same.
- Give student 2 × 3 × 4 = 24 and then give 2 × _ × 4 = 24 and have the student solve for the missing factor.
- Give the student (3 × 5) + (2 × 5) and ask the student to explain their comparability to 5 × 5.
- Give $3\overline{)12}$ and $2\overline{)12}$ and have the student explain why there are two different quotients.
  Also, have the student explain why the quotient is actually a factor.
- Give student $3\overline{)12}$ and $4\overline{)16}$ and have student explain why the quotients are the same.

There exists a multitude of formats for assessing proficiency with single digit combinations. All should be considered in the process of assessment and instruction.

COMPLEX MULTIPLICATION

For purposes of this paper, complex multiplication is defined as items composed of two-digits by a single-digit through items composed of 4 digits by 4 digits.

Two qualitative aspects of number sense must accompany or precede complex multiplication. These are that the student have a clear and meaningful knowledge of place value and proficiency with expanded notation. Assume a student is confronted with an item comparable to

$$\begin{array}{r} 6\,0\,0\,6 \\ \underline{\times \quad\ \ 6} \end{array}$$

and the assessment requests the student to complete the item as shown. The student is then presented with the item in the expanded notation form of

$$\begin{array}{r} 6000 + 0 + 0 + 6 \\ \underline{\times \qquad\qquad\quad 6} \end{array}$$

A student with an appropriate understanding of place value and expanded notation should be able to complete each item and, if requested, explain their similarity.

Adhering to the emphasis on curriculum validity (Jitendra, Parker & Kameenui, 1997), assessment of multiplication proficiency can be constructed within a curriculum framework. The items are selected and sequenced based upon their original occurrence in text book series. The procedure involved the sequencing of items from two digit combinations through four digit combinations in a curriculum format as shown in Figure 5. The items in bold are test items.

Figure 5: PARTIAL MULTIPLICATION SEQUENCE

	Example		Descriptor
M1	2	2	Single digit by single digit
	× 2	× 3	No renaming
M2	6	5	Single digit by single digit
	× 4	× 3	Renaming
M3	23	43	Two digit by single digit
	× 2	× 2	No renaming
M4	25	17	Two digit by single digit
	× 3	× 2	Renaming 1's
M5	43	32	Two digit by two digit
	× 12	×21	No renaming
M6	26	17	Two digit by two digit
	×13	×14	Renaming 1's
M7	48	27	Two digit by two digit
	×14	×16	Renaming 1's/10's
M8	53	74	Two digit by two digit
	×64	×75	Renaming all
M9	60	40	Two digit by two digit
	×45	×76	Renaming 10's//zero 1's
M10	36	74	Two digit by two digit
	×40	×60	Renaming 1's/10's
M11	243	321	Three digit by one digit
	× 2	× 3	No renaming
M12	863	389	Three digit by single digit
	× 8	× 7	Renaming 1's/10's/100's

Item selection from within a curriculum framework enables the examiner to describe student performance within the context of proximity analysis. Proximity analysis (Cawley Parmar, Yan & Miller, 1996) calls for the identification of those items in which the student is either competent or experiencing difficulty in relation to the full sequence of items within that topic. The performance of the student on the test items is examined relative to the last item passed and the first item failed. Proximity analysis focuses on the identification of the sequence of items between the last item passed and the first item failed. This is followed by a process in which the types of errors made on the first item failed are examined in relationship to the items between first item failed and last item passed. The point of any breakdown can be readily determined.

Note that all items are open-ended and it is our belief that items of these types must not only consider "percent correct," they must also consider response variability or the number and types of responses that are provided. It is suggested that open-ended items require the development of "exact response" files and their inclusion in standardized and clinical response analyses. One procedure for these types of analyses involves the division of the number of different incorrect answers by the total number of incorrect answers to determine error patterns. When the quotient is high (e.g. about 75%) the error pattern is due to calculation errors. When the quotient is low (e.g., 25%), the error pattern is due to algorithmic errors. This method of pattern analysis for errors ought to be included within the assessment data (Miller & Carr, 1997; Cawley, Parmar, Miller & Yan, 1996).

There are numerous formats for the assessment of complex multiplication in symbolic form. Two illustrations are provided. First is the importance of an in-depth examination of the inclusion of number sense with a single item as illustrated below:

$$\begin{array}{rrr} A & B & C \\ 3 & 2 & 4 \\ \hline \times & & 2 \end{array}$$

The following guidelines are provided:

Start with A, go to C and then B.
Start with B, go to A and then C.
Start with C, go to B and then A.
Start with B, go to C and then A.
Start with A. go to B and then C.
Start with C, go to A and then B.

The above illustrates the assessment of the intricacies of place value and multiplication by completing the *same* item six different ways. This is in contrast to the assessment of single procedure routines where 6 *different* items would all be done in the same way.

A second procedure is to systematically vary item composition so as to provide insights into the capabilities of the students using items such as

$$\begin{array}{r} 325 \\ \hline \times \quad 3 \end{array} \qquad \begin{array}{r} 352 \\ \hline \times \quad 3 \end{array} \qquad \begin{array}{r} 532 \\ \hline \times \quad 3 \end{array}$$

where renaming for each of the columns is located differently. A second format could be

$$\begin{array}{c} A \\ \begin{array}{r} 325 \\ \hline \times \quad 2 \end{array} \end{array} \qquad \begin{array}{c} B \\ \begin{array}{r} 325 \\ \hline \times \quad 3 \end{array} \end{array}$$

$$\begin{array}{c} C \\ \begin{array}{r} 325 \\ \hline \times \; 32 \end{array} \end{array} \qquad \begin{array}{c} D \\ \begin{array}{r} 325 \\ \hline \times \; 23 \end{array} \end{array}$$

where it is first determined that the student can respond correctly to each of the single-digit factors and then, assuming the student is correct on the single-digit factor items, how the student uses the two-digit algorithm. What might the assessment specialist do if the student responded to 32 ×325 with

$$\begin{array}{rr} 325 & \\ \hline \times \quad 32 & \\ 9000 & (300 \times 30) \\ 600 & (300 \times 2) \\ 600 & (20 \times 30) \\ 120 & (5 \times 30) \\ 40 & (20 \times 2) \\ \hline 10 & (5 \times 2) \\ 10370 & \end{array}$$

where the only mistake is in the partial product for 5 × 30 where the student obtained 120 instead of 150. Is there some way the scoring for this item would provide a higher indicator for the use of the alternative algorithm with an incorrect product than it would for a correct product done with the traditional routine.

Another format, the horizontal format, also assesses student performance in multiplication using the same 32 x 325 as an example.

$$\begin{array}{lrr} 325 \times 32 & & \\ & 9000 & 300 \times 30 \\ & 600 & 20 \times 30 \\ & 150 & 5 \times 30 \end{array}$$

6 0 0	300 × 2
4 0	0 × 2
1 0	5 × 2
1 0 4 0 0	

Here the student multiplied 300 x 30, 20 x 30, 5 x 30, and so forth.

Calculations beyond the complexity illustrated above should be done with calculators simply because there are too many opportunities for a careless error to occur. When large items are included, they should be used solely for the purpose of detecting algorithmic errors.

Word Problems of Varying Structures

Throughout the history of research on the solving of word problems by students with mild disabilities, there is overriding evidence of significant and chronic deficits. The degree of deficit is well established (Parmar, Cawley & Frazita, 1996). More disconcerting is the chronicity of the deficit in that students with mild disabilities make the same types of errors in the 1990s (e.g., Leon, 1993; Smith, 1994) as they did in the 1940s (e.g., Cruickshank, 1948). The great majority of problem solving intervention research shows merely that the students increase their performance solving specific types of traditional word problems following a set routine. The literature does not informs us that the students have effectively increased their capabilities as "problem solvers" for word problems of varying structures. Consistent with the intent that one of the primary uses for assessment is to influence curriculum and instructional decisions, we propose an alternative framework of assessment for word problems. We suggest the focus should be on curriculum, that is the selection of the types of problems that are the focus of the assessment. This can be accomplished with the development of a taxonomy that details the characteristics of each problem as illustrated below:

Direct	
EXT	NO-EXT
+ - x /	+ - x /
1 2 3 4	5 6 7 8

Direct = Direct Problem type
EXT = Contains Extraneous Information
NO-EXT = No Extraneous Information

Thus, a problem meeting the criteria as a 1 would be:

1. A naturalist gave 3 lizards to the girls.
   Another naturalist gave 4 lizards to the girls.
   Another naturalist gave 5 alligators to the girls.
   How many lizards did the naturalists give to the girls?

Alternative forms of word problem assessment include: Provide the student with two or more word problems and inquire into the student's comprehension of the characteristics of each problem. For example:

- A boy has 6 apples.
  A girl has 3 times as many apples as the boy.
  How many apples does the girl have?
- A boy has 12 apples. This is 3 times as many apples as the girl.
  How many apples does the girl have?

Note that each of the problems has the same question, one that is neutral in that it does not provide any "clues" to the student.

Have the students complete each problem. Verify their answers. Select combinations of students and have the various combinations explain the factors that influenced their choice of operation. Discuss the similarities and differences in the wording of each and ask the students to elaborate upon them.

Standardization would consist of the frequency of correct and incorrect responses and detailed descriptors of student responses. Comparisons can also be made between problems such as the following where each, although worded differently, involves the same operation.

- A boy has 6 apples.
  A girl has 3 times as many apples as the boy.
  How many apples does the girl have?
- A boy divided his apples among his 3 friends so that each friend got 4 apples.
  How many apples did the boy start with?

Alternative Representations

The development of alternative representations can be guided by the Interactive Unit (IU). The IU is a system of 16 teacher- student, teacher-student-materials interactions and it can be utilized across nearly any elementary science Cawley, Foley & Miller, 2003) or math content (Cawley & Reines, 1995). Each cell of the IU includes many different tasks. We have constructed some 400 different tasks for the write/write cell alone, a factor that allows for extensive diversity for item development. The items or subtests on most tests with which we are familiar can be cross-coded to the cells of the IU and the task patterns of assessment be identified and described. The IU is illustrated below:

	Input			
	Manipulate	Display	Say	Write
Output				
Manipulate	×	×	×	×
Identify	×	×	×	×
Say	×	×	×	×
Write	×	×	×	×

The following illustrates one input, **manipulate,** across four output combinations. The actual interactions may contain multiple activities.

INPUT	OUTPUT
Manipulate: Teacher demonstrates 3 × 4 in form of an array	**Manipulate:** Student moves chips to make an array
	Identify: Student selects picture representing 3 x 4 in form of an array **Say:** Student states description of 3 x 4 array **Write:** Student writes a number sentence.

The items that comprise this component of the assessment query the student as to the use of alternative representations, alternative algorithms and alertness/error detection. There is considerable interaction between the student and the assessment specialist and an opportunity for the latter to inquire into the unique qualities of the student.

Connected Assessments

A natural follow up to the use of alternative representations is to extend them to connected assessments such as illustrated with the **Parking Lot**.

The Parking Lot

Materials: Parking lot with different areas and set of vehicles.

Show the student the model of the parking lot.

Ask the student to point to section showing cars in 3×4 array.

Ask the student to point to section showing cars in 4×5 array.

Correct any errors.

Note that the screening takes place in the **Say/Identify** interaction, neither of which require reading or writing.

The assessment continues as follows:

Task 1: Instruct the student to use the 3×4 array and rearrange the cars in another area to show a 2×6 array.

Task 2: Instruct the student to use the 4×5 array and rearrange the cars to show a 4×2 array and a 4×3 array (i.e., representing the distributive property of multiplication).

Note that the tasks of the performance assessment take place in the **Say/Manipulate** interaction. Neither of these require reading or writing, nor are they memory-driven. What the student actually performs is the basis for the assessment.

In reality, the focus on connections is one of the more essential facets of subject-matter programming. Many of the errors made by students in the variety of word problems presented in Algebra and in other content courses such as science and carpentry are the results of a lack of content knowledge of the subject, not a limitation in the factor-by-factor = product relationship of arithmetic.

Connected assessments present an opportunity to explore the mathematical development of the student. To date, this paper has focused on multiplication in the form of FxF=P relationships. These relationships can be extended to other relationships such as *ratio* and *proportions,* where the ratio of one quantity to another is their quotient and proportions are the relationships between two ratios. The students can be engaged in determining the amount of work done in carrying 10 trees weighing 100 pounds a distance of 60 feet for planting (classroom objects and distances can be used). Here, the FxF=P relationship is expressed as *F*orce x *D*istance = *W*ork. The principle of ratio can be expressed as the relationship between the number of trees that grow and those that do not (e.g., There a 10 trees in the row. Four of the trees do not grow. What is the ratio (percentage) of trees that grow to the number of trees in the row?). Proportion can be assessed with items that compare the growth of trees in two or more rows (e.g., One row of trees is 40 feet long and has four trees. Another row of is 60 feet long. If the farmer wants to space the plants at the same distance apart, how many trees will go in the second row?). In a meaningful connected activity, the actual planting, measurement and data gathering and calculations would take place (e.g., planting radishes; moving objects).

Summary

This paper has described a number of alternative forms of assessment for use in the assessment of mathematics performance of students with mild disabilities. The focus is on curriculum-referenced assessment that stresses content validity. Given that a a major theme of contemporary assessment is to assist in the making of curriculum and instructional decisions, this paper has described a number of components of assessment that would substantively influence both curriculum and instructional decisions for students with mild disabilities. In effect, through the inclusion of alternative forms of assessment and the resultant curriculum and instructional developments, it is our contention that students with mild disabilities provide indicators that they know and can do mathematics and that they comprehend there is more than one way of *knowing* and *doing* mathematics.

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