Mathematics Pedagogy and Content in a Blended Teacher Education Program

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There is a significant shortage of credentialed elementary school teachers in the state of California. In many urban school districts, between twenty-five and thirty-five percent of those teaching in grades K through 6 do not have a teaching credential (California Commission on Teacher Credentialing, 2002). Traditionally, students planning to become elementary school teachers in California complete an undergraduate bachelor's degree in a content area and then enroll in a post-baccalaureate credential program. The combination of these two programs usually requires six or more years before students receive their credential. As a result, the university systems in California have not been able to produce credentialed teachers fast enough to meet this ever-growing demand.

In response to this demand, the California Commission on Teacher Credentialing (CCTC) requested that universities offering multiple subject credential programs begin to develop alternative pathways for students who enter post-secondary education with the desire to become elementary school teachers. The goal of these alternative programs is to blend the baccalaureate and credential programs so that these "early deciders" can receive a teaching credential in a shorter period of time.

The California State University system has begun an initiative to create these undergraduate *blended* programs of teacher preparation. The programs are designed to blend subject-matter study with pedagogical training. While programs vary from campus to campus, they share a set of common features: giving students early field experiences in public schools, an early introduction to teaching as a career, concurrent enrollment in subject-matter and credential courses, and "blended courses."

These blended courses represent the most unique aspect of the alternative programs. Some of these courses integrate related subject matter, such as mathematics and science, within the same course. Others, however, blend subject matter and related pedagogy in the same course.

Developers of these courses argue that blending the study of teaching methodology with content encourages students to think pedagogically about the major themes in subject-matter courses, as well as make intellectual connections between disciplinary themes and key issues regarding teaching and learning. Moreover, it is believed that blending fosters greater collaboration between teacher education and subject-matter faculty in curriculum development and emphasizes the contributions of all university faculty in the preparation of future teachers.

Recently, the Department of Teacher Education and the Liberal Studies Program in the School of Education at California State University, Dominguez Hills (CSUDH) collaborated in developing a Blended Liberal Studies/Teacher Education Program (BLTEP) for upper division transfer students interested in completing an elementary credential. The BLTEP curriculum blends the regular Liberal Studies Program with the elementary credential program. Students take both subject matter and pedagogy courses concurrently, as well as a set of blended content and pedagogy courses in science, social science, and mathematics.

This article details the curriculum development process of the blended mathematics course, *LBS 360 Math Content and Methods*. The content in this course combines the study of real numbers and problem solving with pedagogy for teaching math content to elementary-aged students. The blending of content and pedagogy is described, along with a discussion of specific issues related to blending of disciplinary and professional studies in a teacher preparation program. The idea for the blended math course came about as a result of a collaborative effort of a professor in the Mathematics Department at CSUDH, a professor in the Teacher Education Multiple Subject Credential program, and the coordinator of the BLTEP. The mathematics professor taught the undergraduate course, *Real Numbers for Elementary School Teachers*, and the teacher education professor taught the

methods course, *Elementary School Mathematics Methods*, in the teacher credential program. The motivation for blending the mathematics content and methods courses came from the instructors' belief that learning mathematics content while at the same time learning how to teach it would deepen prospective teachers' understanding of content and alert prospective teachers to those aspects of mathematics that may be confusing to the elementary students they will eventually teach.

Balancing Content and Pedagogy

The creation of the blended mathematics course posed challenges. One of the first was logistical. The mathematics content class was a 3 unit class, while the methods class was 2 units. In order to streamline the BLTEP program and reduce redundancy, it was agreed that the blended course would be 4 units total. This meant giving up 50 minutes of instruction each week of a 15 week semester. Combining the content of these two courses into fewer hours of instruction was one of the first challenges.

During the discussion of how to fit 5 units of instruction into 4, it became apparent that the crux of the issue was how to balance the elements of content knowledge and knowledge of pedagogy. Obviously, elementary school teachers need a thorough understanding of mathematics in order to teach at the K-6 level; however, some teachers who have a deep understanding of mathematics are not able to transfer this knowledge to the students they teach. Likewise, some teachers who are well-versed in pedagogy have difficulty teaching elementary school mathematics effectively. This raises the question, "What knowledge does an elementary teacher need to teach mathematics effectively?"

In order to be an effective elementary school mathematics teacher, a teacher needs both a strong background in mathematics and a thorough understanding of pedagogy. Research in student learning (Darling-Hammond, 1999) indicates that there is a positive correlation between teachers' content knowledge and their students' success in learning mathematics. Other research has demonstrated a connection between teachers' pedagogical knowledge and students' performance (Rowan, Chaing & Miller, 1997). In their extensive survey of teacher education and learning research, Wilson, Floden, and Ferrini-Mundy (2002) point out that there are some indications that there is a third type of knowledge teachers need to be effective teachers. It is the knowledge that combines mathematics content and pedagogical skills. Elementary teachers need both an understanding of the central concepts and structures of mathematics and an ability to use that conceptual understanding to support their students' learning (INTASC, 2001; NCATE, 2001). Wilson, Shulman and Richert (1987) refer to this type of knowledge as *pedagogical content knowledge*.

Pedagogical content knowledge includes: knowing what topics are typically difficult for students and why, knowing different representations that are useful for teaching a specific idea, and knowing ways to develop students' understanding of

a particular idea (Ball, 2000). According to Ball, a teacher with good mathematical pedagogical content knowledge can break down mathematical knowledge into less polished and abstract forms, thus making it accessible to students who are at different cognitive levels. A teacher with good pedagogical content knowledge can unpack the mathematics into its discrete elements and can explain a concept or procedure at a level that includes the steps necessary for the students to make sense of the reasoning. Ball further indicates that elementary teachers with good mathematical pedagogical content knowledge understand where elementary students may have trouble learning the mathematics and can represent the mathematics in a way that their students can comprehend its structure and avoid these difficulties. In order to prepare effective elementary mathematics teachers, a teacher training program must focus on all three types of knowledge: content knowledge, pedagogical skills, and pedagogical content knowledge.

Along with developing prospective teachers' pedagogical content knowledge, the mathematical preparation of effective elementary school teachers must include experience using that pedagogical content knowledge. Lampert (1998) describes teaching as a thinking practice that integrates reasoning and knowing with action. The effective use of pedagogical content knowledge is the *action* to which Lampert is referring. According to Ball (1995), most programs that ask teachers to enroll in separate content and methods classes make the assumption that learning to effectively combine the knowledge acquired in these courses occurs after the teachers are in their own classrooms. Ball further claims that this transference is a difficult task, and for many, if not most, teachers it is something that never happens at all. In order to ensure that teachers develop this ability to use their pedagogical content knowledge, structured opportunities to apply their knowledge in a classroom setting, discuss the results, and get feedback on how to improve their techniques must be provided. These opportunities should be closely associated with teachers' learning of the pedagogical content knowledge in order to strengthen the connection between the pedagogical content knowledge, its application, and student learning.

Traditional Approaches

Finding the appropriate balance between content and pedagogy with the aim of helping teachers develop pedagogical content knowledge became the central goal in designing the new blended mathematics course. Exactly how to accomplish this required a review of traditional approaches to educating elementary school teachers in mathematics and an evaluation of the extent to which these approaches were useful or needed to be modified for the new course.

Traditional mathematics content classes for elementary school teachers are designed to add *depth* to the teachers' knowledge of how to *do* mathematics and increase their appreciation of the beauty of mathematics. These courses focus on increasing teachers' ability to *do* more complex and abstract mathematics and

usually not on the *why* behind the process. Mathematics methods courses traditionally focus on the tools of teaching: lesson plans, unit planning, classroom management, and assessment, and they are usually not designed to help teachers learn more mathematics or develop an understanding of its structure and logic. One desired outcome in traditional mathematics content classes is that teachers be able to solve problems individually and in an abstract format, without resorting to concrete objects or pictures. However, desired outcomes in methods classes often include the ability to use manipulatives and/or pictures to solve mathematical problems while working effectively in cooperative groups.

Many content specialists view elementary school mathematics teaching as a pattern of recitation and seatwork during which teachers introduce new topics and procedures to students, tell them how to do the problems, and then provide extensive periods of practice on written problems from the textbook. Methods specialists tend to view elementary school mathematics instruction from a constructivist classroom viewpoint, where teachers facilitate students' investigations of topics and where student learning occurs in collaborative group discussions. Thus, content courses tend to be structured around a teacher-centered lecture followed by individual assessment through homework, quizzes, and tests. Methods course instruction is more discussion-oriented using collaborative groups and a variety of assessments, including portfolios, journals, and writing assignments.

Given these different approaches to teaching content and methods, the authors sought to develop a course in which teachers acquired a thorough conceptual understanding of mathematics in addition to the ability to explain its processes in a way that elementary students can understand and assimilate. Before actually developing the course, however, it was necessary to determine what the objectives of the course would be, including the specific mathematics and methods content to include.

Blended Course Goals and Objectives

Since this course is designed for preservice teachers, it must emphasize those areas and strands which are included in state and national content and teaching standards. The *National Council of Teachers of Mathematics Principles and Standards for School Mathematics* (NCTM) and the *California Mathematics Academic Content Standards* are organized around five strands: number sense, algebra and functions, measurement and geometry, probability and statistics, and mathematical reasoning (NCTM, 2000; California Department of Education, 1999). Since the content class being blended was a numeration class, the strands of number sense, algebra and functions, and mathematical reasoning were incorporated into the development of the blended course. Another mathematics content class in the program focuses on the content of the other strands.

The California Standards for the Teaching Profession (CSTP) emphasize

organizing content for learning, assessing learning, engaging all students, planning effective instruction, maintaining effective learning environments, and professional development (California Commission on Teacher Credentialing and California Department of Education, 1997). All of the CSTP standards were incorporated into the development of the blended course.

Based on both selected content strands and the professional standards, the following general objectives for the blended course were developed:

1. Prospective teachers will understand the mathematics and be able to use representations, examples, and illustrations to explain it in ways that help their students understand the structure and logic of a topic.

2. Prospective teachers will be able to identify those aspects of a topic that may cause students difficulty and can plan learning experiences specifically designed to help students understand those aspects of the topic.

3. Prospective teachers will be able to assess their students' learning and identify those misconceptions and misunderstandings that are causing the students difficulty.

4. Prospective teachers will be able to understand their students' often inaccurate perspective on a topic and be able to engage and support the students in their development of a complete and accurate understanding.

In order to be effective in promoting student learning in mathematics, an elementary school teacher must have strong pedagogical content knowledge and know how to design instruction that motivates, encourages and supports student leaning. These four goals were developed to insure that a student who successfully completes this blended course would be that kind of teacher.

Having strong pedagogical content knowledge means that a prospective teacher not only understands the mathematics content but knows the most developmentally effective ways of representing and explaining various topics and concepts. Because of the often-abstract nature of even elementary school mathematics, there are aspects of various topics that elementary students have difficulty understanding. Having pedagogical content knowledge will prepare a prospective teacher to be able to identify these areas. Having once identified these troublesome aspects, the effective teacher knows several different representations and specifically designs developmentally appropriate instruction to help students understand those aspects.

All learning is built on prior knowledge. Sometimes that prior knowledge contains misconceptions that conflict with developing an understanding of a current topic. Sometimes, during the learning process, students develop misunderstandings that inhibit their learning. Pedagogical content knowledge includes a familiarity with these common misconceptions and when they may occur. Effective teachers use various forms of assessment to identify the misconceptions and

misunderstandings of their students and then plan instruction to help their students correct their learning.

Effective teachers encourage students to communicate and explain their mathematical understandings. Teachers with strong pedagogical content knowledge are able to recognize students' developing mathematical understanding and use appropriate representations, examples, and illustrations in designing future instruction to help fully develop students' understanding.

The Blended Math Course

The course that developed from these goals and standards uses a combination of instructional formats emphasizing the most appropriate format for the specific topic. It also incorporates a collaborative learning approach and provides prospective teachers the opportunity to work with students in actual classroom settings and reflect on that experience. Finally, it is designed to develop teacher's pedagogical content knowledge.

To promote this knowledge, the course emphasizes the mathematical development of ideas and topics through the use of hands-on manipulatives and problem solving oriented, investigative activities in cooperative group and individual settings. Prospective teachers are guided to discover the underlying conceptual and procedural structures of various mathematics topics and ideas.

In addition to helping prospective teachers learn the mathematics, these investigative-problem solving activities provide prospective teachers with opportunities to see how mathematical understanding develops in themselves and others. Instead of being given the solution in a polished abstract form, they discover the solutions in ways they and their group members understand. Through this process, the prospective teachers develop an understanding of the difficulties elementary students typically have in learning mathematics, and they learn strategies and techniques for dealing with those difficulties.

Cooperative-based activities (Kagen, 1992) provide a context in which prospective teachers can share, discuss, and negotiate meaning with regard to being both a learner and teacher of mathematics. These activities also help prospective teachers recognize and understand different perspectives on topics of mathematics, experience different problem solving strategies, and decide on their appropriateness in teaching elementary school students. As teachers work in collaborative groups, they become proficient at mathematical communication. For example, in their discussions with their group members, they develop abilities to describe and share their mathematical reasoning. This is reinforced as they write up the procedures and strategies they used in their investigations and report the solutions they reached in their problem solving activities. Having strong oral and written communication skills and being able to discuss a topic from multiple perspectives enables prospective teachers to explain mathematics in ways that their students can better understand.

In concert with this investigative approach to the mathematics, research-based discussions of how elementary students learn mathematics help teachers to further understand the difficulties students may have in their learning. These discussions, the investigative problem-solving activities, and the oral and written communication experiences all meld together to develop prospective teachers' mathematical pedagogical content knowledge. An example of this approach can be seen in the way problem solving as a topic of study is addressed in the blended mathematics course. The importance of this topic in elementary school mathematics curriculum is reflected in the fact that it is specifically addressed in both the state and national mathematics content standards for grades K-6. Elementary school teachers need to develop their own problem solving abilities, as well as their pedagogical content knowledge of this topic. In order to facilitate this development, prospective teachers in the blended course study problem solving techniques, such as those developed by Georg Polya (1945) and learn strategies for solving problems, such as making a chart, solving a simpler problem, looking for a pattern, or working backward. Working in cooperative groups, the prospective teachers work on a variety of problems, discussing them and applying Polya's techniques to their solution.

To further the development of pedagogical content knowledge, a technique called *Reciprocal Teaching* is also used in the blended course. This technique was developed by Brown and Palincsar (1988) and adapted by Chamot and O'Malley (1994). Reciprocal teaching as applied to mathematical problem solving consists of five steps:

1. Summarizing: Each student reads the math word problem aloud and states what the problem is about.

2. Generating Questions: Each student makes up a comprehension question about the math word problem.

3. Identifying Difficulties & Clarifying: Group discusses comprehension difficulties in understanding the word problem and clarifies vocabulary, mathematical terms, and the problem situation.

4. Representing the Problem: Students draw, write, make graphs, and use numbers and symbols (mathematical and non-mathematical) to illustrate the math word problem.

5. Discussing Possible Solutions: Group discusses possible solutions of the math word problem.

The use of this Reciprocal Teaching technique focuses the prospective teachers' attention on the areas of problem solving that traditionally cause students difficulty. The first three steps combine to emphasize students' differing and often misinterpreted views of the various problems. The last two steps emphasize effective problem solving approaches.

To give students in the blended course an opportunity to practice applying their pedagogical content knowledge, the course is designed to give students structured teaching experiences. As part of their on-going assignments, students regularly plan and design lessons and units for teaching the mathematics topics currently being investigated. Groups also present mini lessons to the class. As Lampert (1998) states, however, prospective teachers need to actually experience the use of their pedagogical content knowledge in actual classroom settings to develop their expertise.

In order to meet these needs, the BLTEP program requires prospective teachers in each of its blended courses to spend a minimum of 15 hours in a K-6 classroom while they are enrolled in the course. During this time, they observe mathematics teaching in the K-6 classroom and are assigned to develop and teach various topics. This combination of observation and participation provides students the opportunity to experience different instructional strategies and decide on their effectiveness in teaching various topics of K-6 mathematics. Students also think about their experiences by keeping a reflective journal in which they answer questions about their knowledge of mathematics curriculum for the grade level they are teaching and report the learning outcomes they observe in their students. Journals are shared among students and feedback is provided to encourage thinking about ways to improve mathematics instruction. This field experience and reflection, together with the in-class lesson and unit plans and the mini lessons, develops prospective teachers' understanding of how to apply their pedagogical content knowledge to enhance students' learning of mathematics.

Implementation and Outcomes

The blended course was implemented during fall semester, 2000. It was taught by a single instructor with expertise in both subject matter and pedagogy (one of the authors) in order to reinforce the blended nature of the course. Students worked in cooperative groups using manipulatives, doing problem-solving investigations, and discussing the mathematics. Various pedagogical techniques were modeled and learning theories discussed in the context of elementary school mathematics. In addition to the time spent in class, the prospective teachers were observing and participating in actual elementary school mathematics classrooms through their required fieldwork component.

In the beginning, the prospective teachers were unsure and tentative like most students who enroll in an elementary school mathematics class. By the end of the first month, students were beginning to make comments in their weekly reflections (done via email) and in class itself like, "Doing multiplication this way is so easy to understand. Why wasn't I taught like this?" and, "I never understood fractions until now! I am definitely going to teach my students this way!" These comments refer to techniques that break down the mathematics in ways that elementary

students can understand, thus demonstrating the prospective teachers' developing mathematical pedagogical content knowledge.

As the prospective teachers began participating in their fieldwork, comments began to appear in the weekly reflections concerning the teaching strategies used by the teachers they were observing and how these strategies seemed to leave the elementary students confused and frustrated over not understanding how to *do* the mathematics just as the prospective teachers themselves recalled feeling about mathematics when they were in elementary school. Part of the requirement of the fieldwork is to participate in the mathematics instruction through working with small groups, tutoring, or teaching some actual lessons. This aspect of the fieldwork produced very excited class discussions, as the prospective teachers reported trying some of the techniques discussed in class and being very successful. One prospective teacher talked about working with a small group in a second grade class on multiplication basic facts.

> This group of students was having a lot of difficulty memorizing their three's. So the teacher asked me to work with them and see if I could help them. We were practicing the facts using flashcards, and the students were getting more and more frustrated. Finally, one girl said, 'I wish these were as easy as the two's!' 'Why are the two's easy?' I asked. 'I can count them!' she replied. Then, I remembered in class that we had talked about skip counting for all of the multiplication facts. So I tried it. We practiced skip counting by three to thirty. Although they were slow, the students picked up the pattern fairly quickly. Then we applied the skip counting to the three's facts. The first 'hard' fact that came up was 3 x 6. The group skip counted '3,6, 9, 12, 15, 18' in unison. When I said, 'That's right!' they actually clapped they were so excited. After that, they really worked, and by the end of the math time, they were successfully skip counting almost all of the three's facts.

It should be noted that math was taught in a blocked 90-minute schedule at this school. Through the in-class activities and the fieldwork experiences, by the end of the semester the prospective teachers had developed good mathematical and pedagogical skills and a strong pedagogical content knowledge.

The one aspect of the blended course that did not meet expectations during the first semester was the development of prospective teachers' problem solving abilities. Even though they participated in problem solving oriented activities in class and did the reciprocal teaching, when given a new or different type of problem to solve, the prospective teachers would complain that it was too difficult or that the problem had not been covered in class. If these prospective teachers did not develop confidence in their ability to solve problems, they would not emphasize problem solving in their classrooms and, as a result, their students would not become problem solvers. To address this issue, the developers added problems-of-the-week (POW's) to the course. The POW's were weekly problems the prospective teachers worked on outside of class and then wrote up in their journals. The POW's seemed to help develop the prospective teachers' problem solving skills.

The real impact of the POW's became apparent during the second semester the course was offered. The POW's were used from the first week of the class. At first, students complained about the problem solving the same way the students had during the first semester, but half-way through the semester, the complaints ceased, to be replaced by questions about the POW's that were assigned for that week. Then the questions changed from the current POW to POW's that had yet to be assigned (all POW's were given to the prospective teachers at the beginning of the semester as part of the course packet). The prospective teachers were beginning to work ahead. When asked why they were working ahead on the POW's, some of the replies were "I want to get them done!" or "I like the challenge!" This behavior of working ahead clearly demonstrated the prospective teachers' development of confidence in their own problem solving abilities.

The blended course is now offered each semester. Each semester, the prospective teachers in the class ask, "This is so easy! Why wasn't I taught this way?" and "Why aren't all teachers teaching like this?" They have confidence in their mathematical abilities and are excited about teaching mathematics. Their comments in their reflections and in class show that this confidence and excitement can be directly attributed to their growing pedagogical content knowledge.

Conclusion

The challenge of blending mathematics pedagogy and subject matter in a single course has been a learning experience in itself. It has led the investigators to consider what Ball (2000) calls the *fundamental tension* in the training of teachers, that is, the proper relationship between theory (subject matter) and practice (pedagogy). The traditional approach in teacher education of introducing these components separate from one another is now being challenged in the new blended programs. Instead, the learning of content in the context of how it is taught is seen as important for the development of true teaching skill. The experience of implementation has produced a course that students actually look forward to taking because they feel like they are "really learning how to teach math."

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