

Designing Professional Development to Support Teachers' Learning in Complex Environments

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The new Queensland *Mathematics Years 1-10 Syllabus* differs from previous syllabuses in that it has an outcomes structure that describes how students think, reason, and work mathematically. The main challenge for secondary teachers implementing the new syllabus lies in taking a more investigative approach to “working mathematically”. This paper reports on a professional development project that supported a group of secondary mathematics teachers in planning and implementing mathematical investigations, consistent with the intent of the Queensland syllabus. The project was guided by a professional development model that applies Valsiner’s zone theory to teachers’ learning in complex environments. Participants were four pairs of mathematics teachers from four secondary schools in or near a Queensland regional city. Over five months the research team made three visits of two days each to work with the group of teachers. Follow up interviews were conducted nine months after the project’s conclusion to investigate issues concerning sustainability. Implementation of the professional development model is illustrated by two case studies demonstrating different configurations of personal and contextual factors that supported or hindered teachers’ learning. The outcomes of the project have implications for building a professional culture in schools, developing teacher leadership capacity, planning for sustainability, and scaling up.

Mathematics is one of eight nationally agreed key learning areas that form the basis of the curriculum for the compulsory years of schooling (Years 1-10) in Australia. Like all other key learning areas, Queensland’s new *Mathematics Years 1-10 Syllabus* (Queensland Studies Authority, 2004) has an outcomes focus that gives it a different structure from syllabuses developed in the past. Instead of specifying what should be learned in particular years or grades of school, outcomes-focused syllabuses are organised around a three-tiered hierarchy of learning outcomes. For the mathematics syllabus, this hierarchy comprises:

- overall learning outcomes that contain elements common to all key learning areas and collectively describe attributes of a lifelong learner;
- key learning area outcomes unique to mathematics that describe how students think, reason, and work mathematically; and
- core learning outcomes, sequenced in six levels indicating a progression of increasing sophistication and complexity in students’ understanding, that describe what students should know and do with what they know in the strands of Number, Patterns and Algebra, Measurement, Chance and Data, and Space.

The challenge for teachers implementing the new syllabus lies not only in using the new structure for curriculum planning, but also in designing learning experiences and assessment tasks that take an *investigative approach* to working mathematically (Queensland Studies Authority, 2005). Following Jaworski

(1986), Diezmann, Watters, and English (2001) describe mathematical investigations as “contextualised problem solving tasks through which students can speculate, test ideas and argue with others to defend their solutions” (p. 170). An investigative approach to the teaching and learning of mathematics aligns with long established curriculum reform movements in mathematics education (e.g., Australian Education Council, 1991; National Council of Teachers of Mathematics, 2000), and the notion of working mathematically is now represented in the mathematics curriculum documents of all Australian states and territories. However, recent Australian research has found little evidence of investigative, inquiry-based teaching practices in secondary mathematics classrooms. For example, the TIMSS Video Study (Hollingsworth, Lokan, & McCrae, 2003) revealed that in Australian Year 8 classrooms there was little emphasis on developing deep understanding of mathematical concepts or the connections between them. Instead, students experienced a diet of excessive repetition and problems of low complexity, with very few opportunities for mathematical reasoning — a cluster of features summed up by Stacey (2003) as constituting a syndrome of shallow teaching.

Several Australian studies have investigated how secondary mathematics teachers interpret curriculum reforms calling for more investigative teaching approaches, and these studies also provide insights into reasons why secondary teachers seem so resistant to change. Norton, McRobbie, and Cooper (2002) presented case studies of nine secondary mathematics teachers in relation to a new Queensland senior syllabus that emphasised problem solving and life-related applications of mathematics. Teaching practices typically involved “show and tell”, although some teachers used “explain” or “investigative” strategies when teaching new conceptual work to more able students. Teachers in this study said they found investigative strategies too difficult to implement because of the time pressures in covering prescribed content and preparing students for their examinations. Norton et al. (2002) also suggested that even the most experienced teachers in their study lacked a varied repertoire of alternative teaching strategies.

Similar findings were reported by Cavanagh (2006) in research carried out with secondary mathematics teachers in New South Wales to examine how teachers interpret the *Years 7-10 Mathematics Syllabus* (Board of Studies NSW, 2003) aims of working mathematically. Interviews with 39 teachers revealed that most had not embraced reform; they also had a poor understanding of the meaning of ‘working mathematically’ and what this might look like in the classroom. Barriers to change included: lack of time to prepare investigative lessons; a belief that students need to learn the basics first; concerns about student behaviour; and the over-riding need to prepare students for examinations. However, the main reason for resisting change seemed to arise from teachers’ satisfaction with their current teaching practices based on ‘chalk and talk’ and individual seatwork. Cavanagh proposed that professional development should provide examples of working mathematically tasks as well as practical advice on classroom implementation, a conclusion echoed by

Anderson (2005) in her research on the professional development needs of NSW secondary mathematics teachers in relation to implementing problem solving approaches.

Evidence from a multitude of research studies has emphasised that what teachers do in classrooms influences students' learning of mathematical content and their opportunities to understand the epistemology of mathematics as a discipline (see Anthony & Walshaw, 2007; Mewborn, 2003, for reviews of this research). Yet research also informs us about the challenges for teachers in changing their practice to enact the vision of curriculum reform (Remillard & Bryans, 2004). In this paper we argue that efforts to bring about teacher change via professional development are insufficient; careful attention also needs to be given to discovering teachers' epistemological and pedagogical beliefs, understanding teachers' institutional contexts and the complex, demanding nature of teaching in situ, and identifying how all these factors interact to influence teacher learning and development. In doing so, we report on a research and development project that supported secondary school teachers in planning and implementing mathematical investigations, consistent with the intent of the new Queensland syllabus. The aims of the paper are:

- to describe the professional development model and its implementation;
- to identify personal and contextual factors that support or hinder teachers' learning; and
- to consider broader implications of the project for policy and practice.

The Professional Development Model

The design of the professional development model was informed by previous research that has taken both theoretical and practice-based orientations.

Theorising Teacher Learning and Development. Previous research by Goos (2005a, 2005b) investigated how teachers learn from experience in complex environments, using a theoretical model originally proposed by Valsiner (1997) to conceptualise child development. This model re-interprets and extends Vygotsky's concept of the Zone of Proximal Development (ZPD) to incorporate the social setting and the goals and actions of participants. Valsiner regards the ZPD as a set of possibilities for development that are in the process of becoming actualised as individuals negotiate their relationship with the learning environment and the people in it. To explain how the actual emerges from the possible, Valsiner proposes two additional zones, the Zone of Free Movement (ZFM) and the Zone of Promoted Action (ZPA). The ZFM structures an individual's access to different areas of the environment, the availability of different objects within an accessible area, and the ways the individual is permitted or enabled to act with accessible objects in accessible areas. The ZPA comprises activities, objects, or areas in the environment in respect of which the person's actions are promoted. For learning to occur, the ZPA must engage with the individual's possibilities for development (ZPD) and must promote actions that the individual believes to be feasible within a given ZFM.

Applying Valsiner's theory to teacher learning and development, Goos (2005a, 2005b) argues that the ZPD represents *teacher knowledge and beliefs*, and includes teachers' disciplinary knowledge, pedagogical content knowledge, and beliefs about their discipline and how it is best taught and learned. The ZFM represents constraints and affordances within the *professional context*, which may include teacher perceptions of student background, ability and motivation, curriculum and assessment requirements, access to resources, organisational structures and cultures, and parental and community attitudes to curriculum and pedagogical change. The ZPA can be interpreted as *professional development strategies* that define which teaching actions are specifically promoted. Professional development may be provided informally by colleagues and mentors in a school or more formally through organised activities such as workshops and conferences. For teacher learning to occur, professional development strategies must engage with teachers' knowledge and beliefs and promote teaching approaches that the individual believes to be feasible within their professional context.

A Practice-Based Perspective on Professional Development. Mewborn's (2003) review of research on effective professional development for teachers of mathematics identified three key themes. First, change is a long term, evolutionary process that can be supported by giving teachers opportunities to engage with mathematical concepts and focus on their own students' thinking as they struggle to understand these concepts. Second, professional development is most effective when it occurs in school-based contexts so teachers can try out and validate ideas in their own classrooms. Third, teachers need time and opportunities to discuss pedagogical and curricular issues with supportive colleagues as they attempt to implement new practices. These themes are incorporated into a framework for designing professional development created by Loucks-Horsley, Love, Stiles, Mundry, and Hewson (2003) to capture the decision making processes that are ideally involved in planning and implementing programs (shown in Figure 1). Loucks-Horsley et al. (2003) explained that the shaded rectangular boxes represent a generic planning sequence, and that the "bubbles" above and below these boxes show inputs into the planning process and when it is most important to consider them. In Figure 1, we have re-interpreted these planning inputs in terms of Valsiner's zone theory. Loucks-Horsley et al. (2003) point out that the model is aspirational in the sense that it represents an ideal towards which to strive rather than an accurate picture of what may actually happen. Because providers of professional development often have limited resources, especially time, it is not always possible to attend to all inputs, or to do so at the optimal times.

Loucks-Horsley et al. (2003) describe the cyclic planning sequence as follows. Ideally, planning begins with teachers making a commitment to enhance teaching and learning, thus acknowledging that a tension exists between the current reality and the vision of mathematics teaching offered by new curriculum documents. In practice, however, it is not always feasible to delay the start of the professional development program until a whole school or group of teachers has established a shared commitment; instead the process of developing this

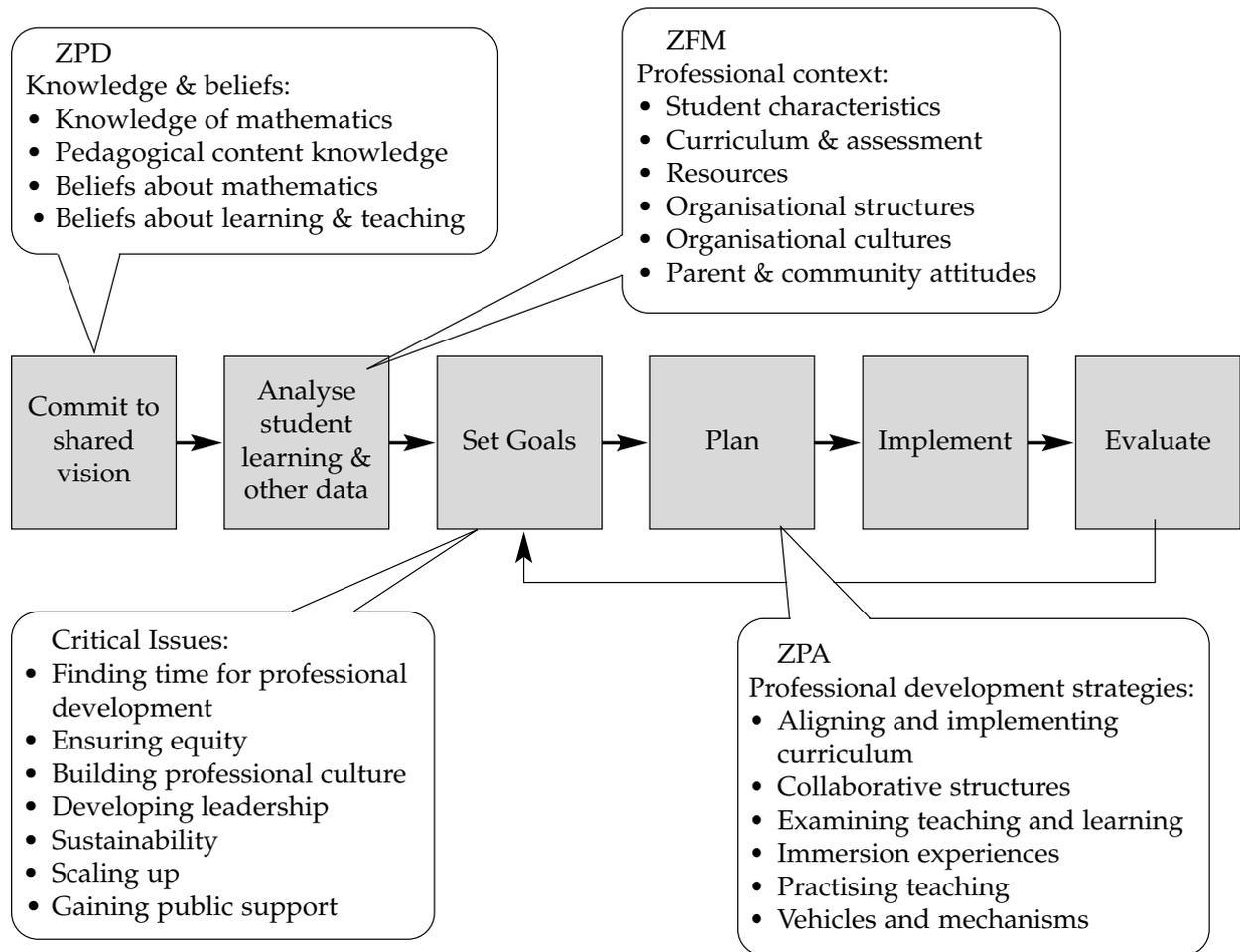


Figure 1. Design framework for professional development (adapted from Loucks-Horsley et al., 2003)

commitment and vision can continue throughout the program and is iterative with other phases of the design. Loucks-Horsley et al. (2003) note that teacher knowledge and beliefs are an important input into this phase (cf the ZPD discussed above). Analysis of student learning data sharpens the focus on setting targets for improvement and establishing goals for teacher learning and development. It is important here to study the context in order to know who the students are, to identify significant features of the learning environment, and to understand the school's organisational structures and cultures, the local curriculum context, and the views of parents and the community members (cf the ZFM discussed above). The framework suggests anticipating critical issues at the goal setting phase because each of these issues can influence the effectiveness of the program at some point. Planning for professional development can then draw on a wide range of strategies to achieve desired goals (cf the ZPA discussed above).

Design of the Project

A key element to our design was to align the project goals with the needs of the teachers. In particular, we recognised the importance of providing teacher participants with authentic, practice-based learning opportunities that included

examples of mathematical investigations, opportunities to experience these investigations as learners themselves, and opportunities to share their ideas and experiences with colleagues, including the challenges encountered and their insights into the process. In addition, previous work has informed us of the importance of validating the teachers' experiences, providing them with support and time for reflection, and utilising iterative cycles that allow teachers to reflect on the initial experience teaching their investigative unit and apply those experiences and learning to subsequent iterations (Makar, 2007). Implementing these key elements into the design of the project therefore required multiple visits, and a format that would provide time for learning as well as time for sharing of experiences. In addition, we considered it critical to schedule an on-site visit to each teacher's classroom to gain insight into their classroom and school context, to observe and videotape a lesson, and to discuss with each teacher the goals and implementation of their lesson.

Our choice of professional development strategies did not arise solely from consideration of contexts and goals, but also took into account findings from the literature that describes effective professional development as being long term, school based, and involving collaboration, experimentation, and reflection (e.g., Mewborn, 2003). These considerations led us to select three of the six key professional development strategies from the Loucks-Horsley et al. (2003) framework to support the teachers' learning: Immersion Experiences, Aligning and Implementing Curriculum, and Collaborative Structures (university-school partnerships and teacher networks). As these strategies do not work in isolation, they were integrated to allow each to support and inform the others.

Professional Development Strategies

Immersion experiences positioned teachers as learners in a mathematical investigation by providing them with curriculum, pedagogy, and assessment exemplars that align with the new syllabus. Unlike other forms of professional learning, this approach works to strengthen teachers' mathematical content knowledge in addition to emphasising mathematical processes that are often neglected in traditional classrooms.

A *curriculum alignment and implementation* strategy ensured that the investigations were directly applicable to the teachers' classrooms with support and time for reflection on the experience. Most short-term workshops that demonstrate innovative materials do not provide support, or even the expectation, for teachers to trial the materials in their own classrooms, and hence the ideas provided are rarely put to use (Cohen & Hill, 2001). By completing two cycles of curriculum implementation, teachers were able to put into practice elements learned and refined after the first attempt at implementation.

Collaborative structures provided opportunities for professional learning around topics negotiated by the group, ensuring common goals. The emphasis on collegiality and communication opened up a forum for the teachers to discuss specific issues related to their classrooms in an environment where the discussion was valued by their colleagues. We emphasised the need for the

teachers to share the challenges they experienced as well as their successes. We hoped that our emphasis on the authenticity of teachers' practice, rather than some idealised version of practice that denies the challenges of implementation, would relieve any fears they may have held that the researchers were there either to "fix" them or to intrude on their practice.

Project Participants

Four pairs of teachers from four schools located in or near a regional city in Queensland volunteered to participate in the project. Inviting *pairs* of teachers from each school was a deliberate strategy to encourage participants to connect with a colleague from their own school while extending their experience beyond this familiar context. Two schools were in the regional city (Cunningham and Churchill State High Schools), one was in a small rural town nearby (Sugartown State High School), while the fourth school (Seaside State High School) was located approximately 125 km away. The teachers in the project were a fairly experienced group with a mean 12.8 years (sd = 8.2 years) experience. They all had qualifications in either mathematics or science, so it is likely that their content knowledge in mathematics was stronger than that of many other lower secondary teachers who do not hold such qualifications.

Project Implementation

The main project activities took place over five months from October, 2005 to February, 2006. Two iterations of the research cycle were undertaken during the project (Figure 2). The design included three visits by the research team to work with the group of teachers for two consecutive days on each visit. The sequencing and goals for each visit are summarised below.

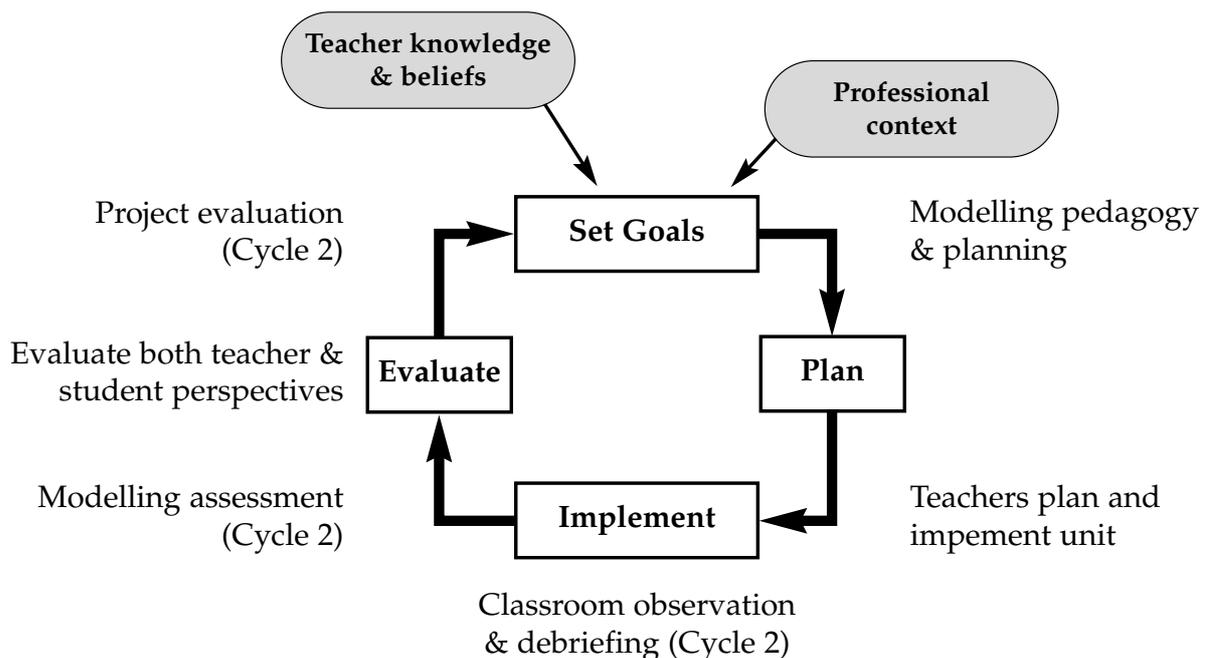


Figure 2. Research cycles (two iterations)

Visit 1: October 2005 (Cycle 1). The main purposes of the first visit were to seek information about the teachers' *knowledge and beliefs* and their *professional contexts*, to help them identify personal goals for the project, and to begin planning units of work with a focus on mathematical investigations. The researchers and teachers shared their "mathematical autobiographies" (audio taped discussion), including information about their own experiences of learning mathematics at school, the teachers who made a difference to them, the decisions and events that led to them becoming mathematics teachers, the significant moments that shaped their practice. This process also allowed the teachers to describe in some detail their professional contexts: the kind of students they taught, the learning environment they tried to create in their mathematics classrooms, their pedagogical approaches, their schools' organisational structures and cultures, and the influence of parents and the local community.

Further information about the teachers' beliefs was obtained via a Mathematical Beliefs Questionnaire (based on Frid, 2000). The questionnaire consists of 40 statements to which teachers respond using a Likert-type scale based on scores from 1 (Strongly Disagree) to 5 (Strongly Agree). Section 1 includes 14 statements about the nature of mathematics, Section 2 has 12 statements about mathematics teaching in secondary schools, and Section 3 has 14 statements on mathematics learning in secondary schools. Within each section, statements are paired to create positive and negative poles of a particular idea; for example, items 4 and 11 represent extreme positive and negative positions with respect to solution methods for mathematics problems:

4. There are often many different ways to solve a mathematics problem.
11. Mathematics problems can be solved in only one way.

In the literature, these poles have been variously referred to as representing beliefs about mathematics being rule-based versus non-rule-based (Tharp, Fitzsimons, & Ayers, 1997) or closed versus open (Boaler, 1998), and beliefs about mathematics teaching being transmissive versus child-centred (Perry, Howard, & Tracey, 1999). In our project we associated section 1 responses with open or closed views of mathematics as a discipline, and section 2 and 3 responses with teacher-centred versus student-centred pedagogical approaches. (A copy of the questionnaire, with items grouped in opposite pole pairs as described here, is provided in the Appendix.)

The first visit thus provided an opportunity to map teachers' Zones of Proximal Development and Free Movement onto the professional development design framework (Figure 1). An important focus of this visit was also to provide immersion experiences by engaging the teachers in three mathematical investigations suitable for use with lower secondary classes (Zone of Promoted Action). Teachers produced rough outlines of units they planned to implement before the next visit, and were asked to collect student data such as work samples, photographs, and attitude surveys (e.g., the IMPACT survey developed by Clarke, 1988) to bring to the next meeting.

Visit 2: November 2005 (Cycle 1-2). The focus of the second visit was on assessing student learning and *evaluating implementation* of the initial investigative unit. One activity involved developing assessment criteria for mathematical investigations. The researchers provided samples of student self-assessment rubrics and the teachers collectively modified these to suit their own students. In evaluating their first attempt at a mathematical investigation, the teachers discussed successes and problems they experienced during practical implementation of their new curriculum units. As a result of *evaluating* this completed unit the teachers were to set goals for *planning* a subsequent investigative unit of work with a new class at the start of 2006. (This represents the start of a second research cycle.) The teachers were given time to work on planning the next unit using an agreed template which also provided space for them to write an account of the actual implementation and student response.

Visit 3: February 2006 (Cycle 2). By the time of the third visit, sufficient familiarity and trust had been established for the researchers to visit the pairs of teachers in their schools and gain further insights into *implementation* of the investigative units by observing a lesson. The lessons were videotaped; the researcher and teacher(s) then discussed the lesson while watching the video together. This discussion was audiotaped for later review and analysis. The *evaluation* component of the second research cycle comprised debriefing interviews after the lessons that were observed and an informal evaluation of the project as a whole. A session was also devoted to preparing for a forum the following month in which all project participants would present their work to an audience of curriculum officers from each education administrative district in Queensland.

Follow up visit: November 2006. Nine months after the project ended the researchers returned to the regional centre and visited three of the pairs of teachers in their schools. (We did not visit Seaside State High School because one of the participating teachers had been transferred to another school at the start of the year.) The teachers were interviewed to discover their perceptions of progress towards the goals they had identified at the start of the project and the extent to which they seemed to be influencing other teachers to try investigative approaches.

Data Analysis

Data collected during the project included questionnaires, interview records, videotapes and field notes of lessons, student work samples and attitude surveys, and teacher planning documents. Table 1 demonstrates how these data provided evidence relating to elements of the professional development model shown in Figure 1.

Table 1.
Relationship between Data Sources and Elements of Professional Development Model

Data Source	Element of Professional Development Model			
	Knowledge & beliefs (ZPD)	Professional context (ZFM)	Goals	Plan, Implement, Evaluate
Mathematical Beliefs Q'aire				
Mathematical autobiographies				
Teacher planning documents				
Student attitude surveys				
Student work samples				
Lesson video tapes & field notes				
Teacher interviews				

Teachers' responses to the Mathematical Beliefs Questionnaire were analysed by comparing their individual scores on item pairs representing negative and positive versions of the same construct (see Appendix). Evidence from the full range of data sources was organised and selected to "fill in" the abstract analytical categories represented by elements of the professional development model (e.g., Zone of Proximal Development = teacher knowledge and beliefs; Zone of Free Movement = professional context; Zone of Promoted Action = professional development strategies) in order to construct case study accounts for each pair of teachers.

Personal and Contextual Factors Influencing Teachers' Learning

Potential supportive and inhibitory conditions influencing teachers' learning may be either personal (knowledge and beliefs) or contextual (students, curriculum, etc). Using selected elements of the professional development model, we examined how the project teachers negotiated these conditions in pursuing investigative approaches to mathematics teaching and assessment. This information is summarised in Table 2.

Table 2.
Teacher Knowledge and Beliefs, Contexts, Goals

Element of PD Model	Schools and Teachers			
	<i>Sugartown</i> Skye & Chris	<i>Seaside</i> Val & Shanti	<i>Cunningham</i> Peter & Ron	<i>Churchill</i> Tony & Ralph
Knowledge & beliefs (ZPD)	Qualified in mathematics & mathematics education	Mixed qualifications in mathematics & mathematics education	Qualified in mathematics & mathematics education	Mixed qualifications in mathematics & mathematics education
	Open/student-centred beliefs	Open/student-centred beliefs	Open/student-centred beliefs	Closed/teacher-centred beliefs
Professional context (ZFM)	Low achieving students	Test & textbook dominated practices	High achieving students take Project Mathematics as extension subject	School has strong academic reputation
	Poorly resourced	Poorly resourced	Other subjects use traditional teaching methods	Lecture approach
	Streamed classes	Classes streamed via frequent tests	Well resourced	Streamed classes
	HOD supportive of change	Organisational culture resistant to reform approaches	Flexible timetable	Other teachers resistant to change
	Low socio-economic status families		Teachers plan & teach together	Tony as HOD of Middle Years seeks curriculum reform
	Little parental support for school			
Professional development strategies (ZPA)	Immersion experiences, curriculum alignment & implementation, collaborative structures	Immersion experiences, curriculum alignment & implementation, collaborative structures	Immersion experiences, curriculum alignment & implementation, collaborative structures	Immersion experiences, curriculum alignment & implementation, collaborative structures
Goals	Engaging learners in meaningful mathematics	Making assessment more authentic and practical	Making Project Mathematics mainstream	Integrating mathematics with other key learning areas

Each pair of teachers came to the project with a common desire to align their teaching and assessment practices with the new mathematics syllabus, but because of the unique context of each teaching situation these desires were expressed as personalised goals. Skye and Chris (Sugartown) wanted to take a new approach to mathematics specifically for engaging and supporting a Year 8 class of low achieving students. Skye explained:

The kids see an exam and just freak. Even though they know it, they can't do it ... whereas if you put it into a context where it's not pen and paper they tend to cope better.

Val and Shanti (Seaside) felt disillusioned and marginalised by the traditional teaching and assessment approaches expected at their school and wanted further reassurance to continue exploring authentic assessment tasks that met with disapproval from fellow mathematics teachers. When Val had her class do group work or work on units that took her class outside, she said the other teachers disapproved:

I get kind of the eyebrows up as if to say, "You should be in there, you know, doing textbooks".

Shanti expressed the same problem:

I used to do as Val did and take the kids out ... but I'd get complaints from the other teachers.

Peter and Ron (Cunningham), who were already implementing a Project Mathematics extension program with a strong investigative flavour, aimed to integrate investigations into mainstream mathematics classes. The teachers at this school, according to Ron and Peter, felt a certain amount of pressure to "cover the content" required to meet the school mathematics work program for each year level and this precluded teachers from undertaking extended investigations or problem solving tasks. Unlike the other pairs of teachers, Tony and Ralph (Churchill) had quite separate reasons for volunteering for the project despite coming from the same school. Tony, a science teacher who had just been appointed Head of Middle Years, wanted to support his colleagues in taking a more integrated approach to curriculum design; while Ralph was teaching a low achieving Year 9 mathematics class and hoped to develop more effective approaches for engaging these particular learners.

The brief summary provided in Table 2 is useful for highlighting the diversity of personal and contextual factors featuring potentially helpful and unhelpful influences on the teachers participating in the project; however, it does not show how these influences interacted to either support or hinder teacher learning. Abbreviated versions of two contrasting case studies provide further insight for this purpose.

Case Study of Teacher Learning: Skye and Chris

Responses to the Mathematical Beliefs Questionnaire revealed that both Skye and Chris held similar open beliefs about the nature of mathematics, and generally student-centred beliefs about mathematics teaching and learning. For example, they agreed that there are many ways of interpreting and solving a problem, and that it is important to encourage students to build their own mathematical ideas. However, other responses showed they were uncertain about the benefits of more traditional approaches such as memorisation and

practice. This suggests that Skye and Chris were interested in moving towards more student-centred, investigative teaching practices, but that they needed to try out these practices with their own classes to find out whether this would lead to improved learning.

At the first project meeting, Skye and Chris explained that the most frustrating obstacle in their professional context was the students themselves and their apparent lack of interest in learning. This was evident in disruptive and uncooperative behaviour and students' frequently stated belief that they were "dumb" and simply couldn't do mathematics. Skye indicated that these students did indeed have tremendous gaps in their mathematical knowledge:

We had a lot of learners that just weren't coping in the [regular] classes ... The majority of them were below the state benchmark in numeracy tests.

Both teachers also referred to the students' family and community context. Many students in this small rural community came from low socio-economic status, single parent families, and the teachers felt that, on the whole, parents did not care much whether their children were learning anything at school. In these circumstances, they struggled to interest students in academic work. The experience of teaching unmotivated students led to the goal of *engaging learners*, or, as Skye explained, "for them to learn maths without being terrified of it". Both saw investigations as a way of presenting mathematics differently that would allow them to make mathematics more interesting for students by engaging them in purposeful tasks with real world relevance. Skye observed that:

We had to kind of make the decision: do we want them in the classes having failure all the time, never feeling success, or do we want these kids to actually come out and engage in learning and have some success at maths?

With support from their Head of Department, Skye and Chris decided to team teach the Practical Mathematics class and their teaching timetable was altered to enable this to occur. Skye and Chris's first unit of work asked students to investigate whether it is more economical to buy groceries in Sugartown or drive to the larger regional centre nearby. In their report on the investigation, students were required to include a family shopping list of at least 20 grocery items, a price list for these items if purchased from the local supermarket in Sugartown and a larger supermarket in the regional town, a price comparison between the two supermarkets, a calculation of the cost of fuel in driving to the more distant town, and a comparison of the advantages and disadvantages of shopping at the two locations. Although about half of the class was much more engaged in learning mathematics than previously, the teachers were not completely satisfied with this investigation because they realised that the topic of 'grocery shopping' was not sufficiently interesting to all students.

Skye and Chris planned a second investigation that they hoped would more closely connected to students' lives. This 'School Rage' investigation asked students to create a Top 20 song list for the school radio station, based on a survey of a sample of students attending the school. To make the task more

realistic, a letter from the 'radio station manager' (one of the other mathematics teachers) was given to the students asking for assistance in designing a new radio program similar to the Rage Top 20 program broadcast on television. The group submitting the best quality report would have their Top 20 songs played on the radio station during a designated lunchtime. Thus the task had an authentic purpose and a real audience comprising the entire school community. Core learning outcomes embedded in this task related to designing and carrying out data collections, using data record templates, organising data and creating suitable displays, making comparisons about data, and working with whole numbers, fractions and percentages.

Our observation of one lesson supported the teachers' judgment that students were engaged in the 'School Rage' investigation. Student comments suggested that they welcomed this new approach:

Student: Why are we doing this? This isn't maths!

Chris: What do you mean?

Student: It's like we're doing SOSE (Study of Society and Environment) — this isn't like a normal maths classroom.

Chris: Is that a negative thing?

Student: No, I normally hate maths but I don't mind doing this.

In the post-lesson interview, Skye and Chris not only identified the benefits for the students (engagement, confidence, alternative opportunities to demonstrate their learning) but also the challenges the new approach presented to them. They were now spending more class time responding to unanticipated ways students tackled investigations, often by asking questions to scaffold students' thinking, such as "What does it mean if you include the same person twice in your survey?" and "What if this person votes for two different songs?" Skye pointed out that she welcomed such unexpected responses as she regarded it as a sign of growth of sophistication in students' thinking.

In their presentation to the education forum at the conclusion of the project, Skye and Chris identified several reasons why they had been successful in implementing an investigative approach. They emphasised the importance of taking into account the students' prior experience and interests, and the local context of the school and community. Access to sample investigations was critical, as were human resources in the form of a supportive school administration team, a network of like-minded mathematics teachers across the schools participating in the project, and their teaching partner. Planning and teaching as a team was a significant benefit for both teachers because this reduced their workload, expanded their repertoire of teaching strategies, and provided opportunities for mutual observation and feedback. Skye and Chris's professional learning experience is summarised by the relationships between their knowledge and beliefs (ZPD), professional context (ZFM), and professional development strategies (ZPA), shown in Figure 3. Although they experienced

Support for development of new teaching approaches was consistent with teacher knowledge & beliefs

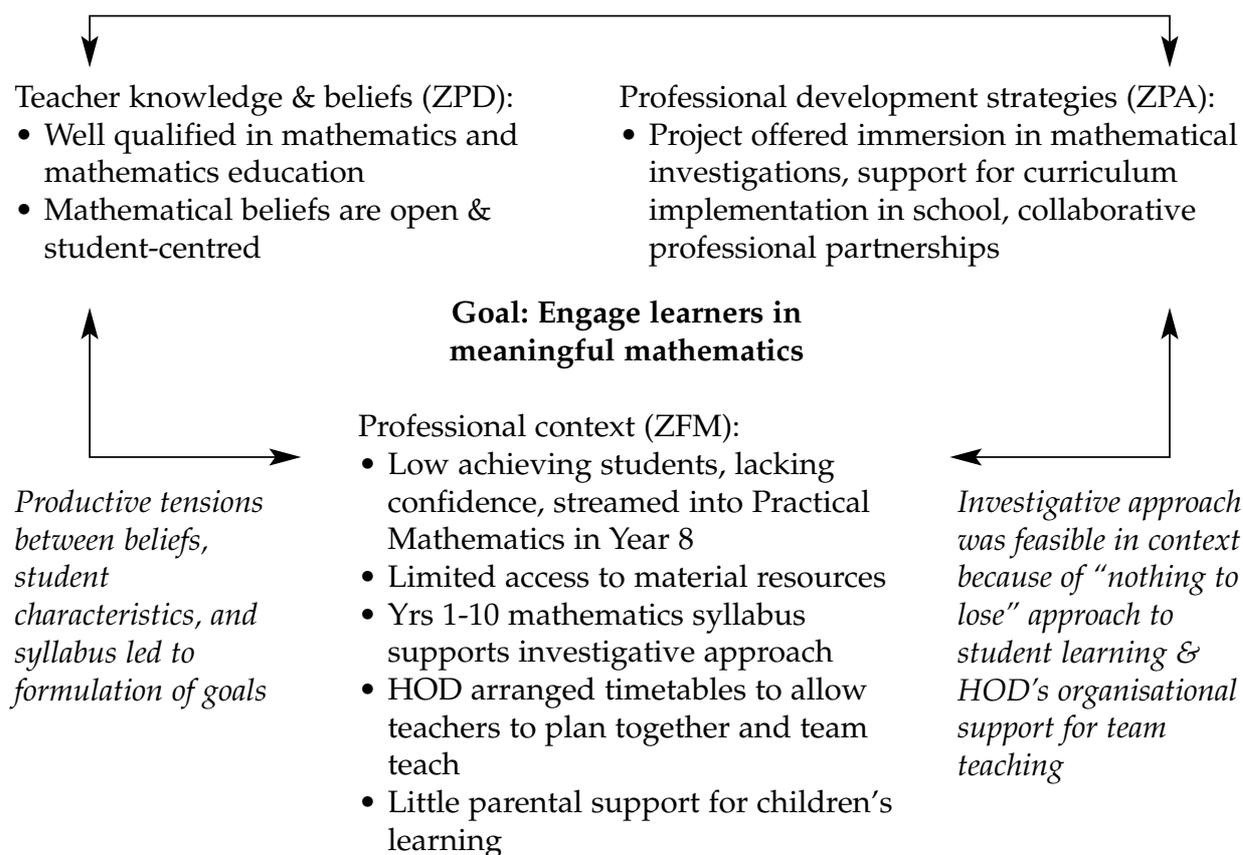


Figure 3. Relationships between professional learning factors for Skye and Chris.

hindrances within their professional context, productive tensions between aspects of the context and their pedagogical beliefs led them to formulate and pursue the goal of engaging learners.

Case Study of Teacher Learning: Tony and Ralph

As explained earlier, Tony and Ralph came to the project while undertaking different roles as teachers of mathematics within their school, and their individual goals led them to follow somewhat different pathways throughout the project. Because of Tony's potential influence in a new leadership role as Head of Middle Years we have chosen to focus on his activities in reporting on the Churchill case study.

Tony's responses to the Mathematical Beliefs Questionnaire were quite different from those of the other teachers in the project. For example, Tony agreed that in mathematics something is either right or wrong and that all important mathematics is already known by mathematicians; he did not agree that there are different ways to solve a mathematics problem. He also strongly agreed that solving a mathematics problem usually involves finding a rule or formulae that applies, and that mathematics learning is about getting right answers. These responses suggest a rigid and narrow view of mathematics, not uncommon from

those who have had little opportunity to reflect upon mathematics reform literature or to consider the nature and purpose of mathematics. Tony even admitted to the group that he did not consider himself as a teacher of mathematics.

Churchill State High School has a strong academic tradition and produces more high achieving Year 12 graduates than all other schools in the district. The school timetable is organised along traditional secondary school lines, except for Year 8 (the first year of high school). All Year 8 students are allocated to one of three groups, each comprising approximately 90 students, which stay together for the whole year with teaching teams allocated to each group. Mathematics classes are conducted in the Year 8 Centre — a large, open plan building where a group of 90 students sits in lecture-style chairs facing a single whiteboard. During our visit to the school, we observed how one of the three teachers assigned to this large group wrote notes on the whiteboard for students to copy, while the other two teachers patrolled the room to ensure that all students were paying attention.

Tony's goal in joining the project was to find out more about the model of middle years that operated in his school and to introduce a more investigative approach to teaching and learning mathematics. He organised formal meetings with teachers to discuss their perceptions of the middle years and found they had many concerns. Teachers felt that students moving from Year 8 into Year 9 were not coping well, and this was evidenced by an increased number of suspensions in the year. One teacher said:

The middle years students are basically nuts and we have to do something to improve the situation and our stress levels.

Tony felt that the time was right to consider change in middle years' structure and culture at this school, and he was hoping that the project might provide an avenue of support. Tony wanted not only to trial some new approaches to teaching mathematics to his Year 8 class, but also to develop some units of work that would integrate mathematics with other key learning areas such as English, Science, and Studies of Society and Environment (SOSE).

Tony trialled his first investigation, *Reading the News* (adapted from Erickson, 2001), with a small group of Year 8 students during class time at the end of the school year. Students were to conduct an experiment to investigate the claim that listening to a 10 minute news broadcast on the radio was equivalent to reading one whole page of the newspaper. Although students' reactions were less positive than he had hoped, Tony was pleased with the level of student engagement and the way he could revisit the concepts of gradient and speed through this task and informally assess students' understanding. Tony then decided to gather feedback from all Year 8 students about their experiences in learning mathematics by developing and administering a modified version of Clarke's (1988) IMPACT survey. The survey asked students to describe how they felt in mathematics classes at the moment, state the biggest worry affecting their work in mathematics, and suggest ways to improve mathematics classes. Results indicated that 60% of students claimed to be "bored"; 33% were "relaxed"; 40%

wanted mathematics to be “more fun, interesting and worthwhile”; 40% stated that they were “most worried about the end of year exam”; and 30% expressed concern about the crowded nature of the Year 8 room and their dislike of the lecture style presentation of mathematics classes. Many students wrote several paragraphs suggesting ways to improve mathematics classes, voicing concerns such as the following:

- their questions were not answered in class;
- the teachers talked too much and then students had to copy their notes from the board;
- the homework was too demanding and often students didn't know what was required;
- the teachers didn't know all the students' names;
- the teachers only answered students who sat at the front of the room; and
- the class sizes should be smaller.

This feedback provided Tony with further evidence to support his initial concern that things were not right with the existing middle years' model, and strengthened his resolve to do something about it.

Tony developed a second unit of work, entitled *Fire! Fire!*, with links between Mathematics, English, Science, and SOSE. Students were to explore the means to put out a fire that had started in a paddock adjoining the school (e.g., start a bucket brigade from the school swimming pool; make a water pump or hose to deliver water over distances). He hoped that the Year 8 teachers of these subjects would plan together to incorporate the investigation into their existing program and collaboratively develop assessment tasks and criteria sheets, using the guidelines he had developed. Because this was an ambitious venture, Tony had not yet implemented the *Fire! Fire!* investigation by the conclusion of the project, but he had nevertheless secured agreement from all the Year 8 teachers that they would try this integrated unit.

At the beginning of the project, we were uncertain about Tony's commitment and reasons for volunteering to participate. However, his design and implementation of the *Reading the News* investigation represented a major change to his usual style of mathematics teaching, and this experience, together with data from the student IMPACT survey, seemed to encourage him to create more investigative units across four key learning areas. As the project unfolded, we watched Tony interact with his colleagues in a more lively and collegial way as he asked questions about their planning and practices and shared his own ideas. At each project meeting, his vision for a new middle years' model gathered more substance and he actively sought opinions on his ideas from the rest of the group. In the group discussion evaluating the project as a whole, Tony noted that his involvement provided the impetus for trialling an investigative task with Year 8 students, and from this experience he learned how he could design a more integrated curriculum that would be accepted by the rest of the Year 8 staff.

Tony's professional learning is represented by the relationships between knowledge and beliefs (ZPD), professional context (ZFM), and professional

development strategies (ZPA) shown in Figure 4. His participation in the project exposed him to pedagogical approaches (professional development) and curriculum documents (new syllabus) that supported mathematical investigations but were inconsistent with his entering beliefs and the existing organisational structures of his school. It seems that these tensions may have led him to formulate and pursue his goal of reforming the middle years' curriculum.

Support for development of new teaching approaches may have led to shifts in teacher knowledge & beliefs

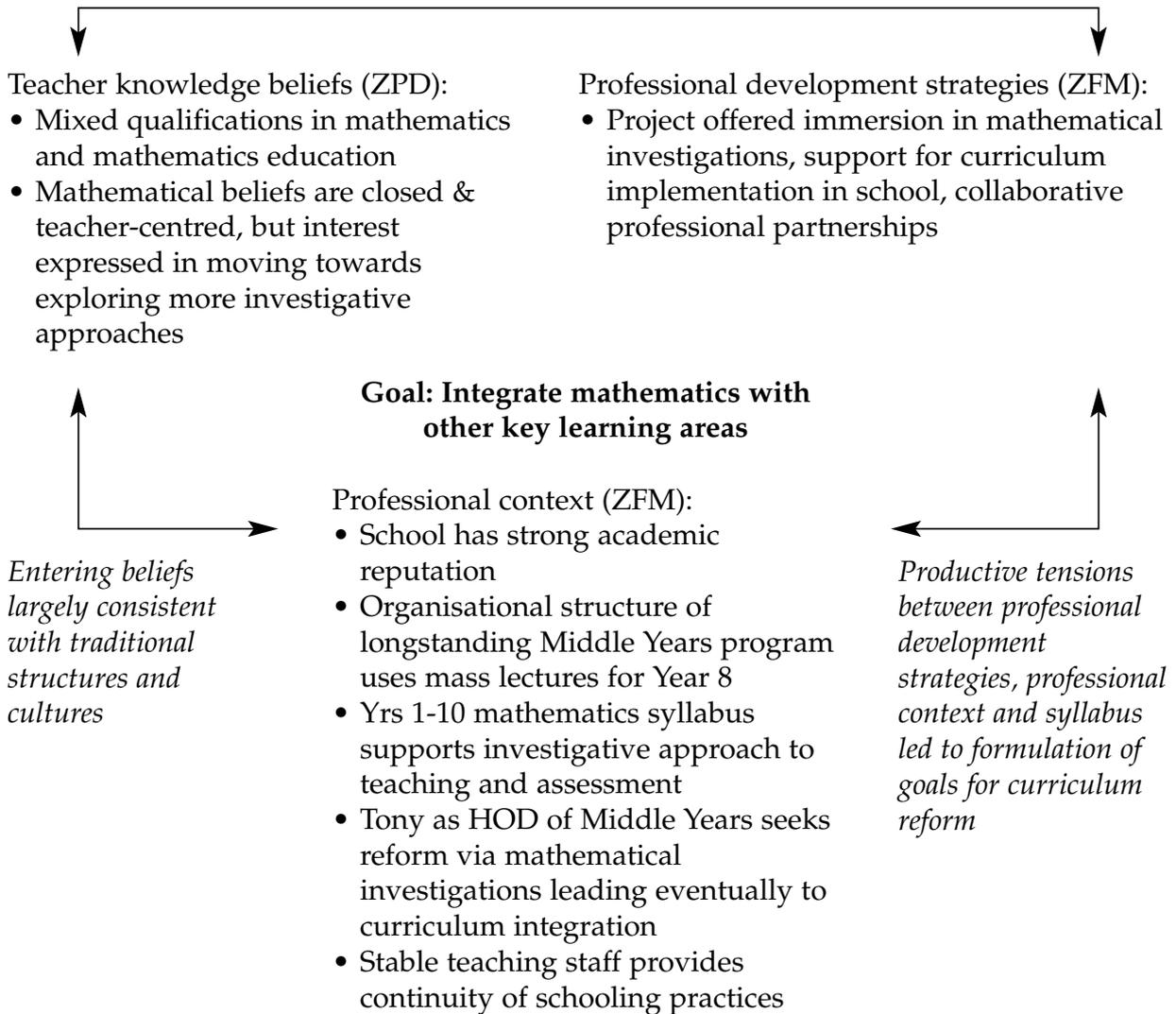


Figure 4. Relationships between professional growth factors for Tony

Significance of the Findings

Using the professional development model we were able to identify a different configuration of teacher knowledge and beliefs (Zone of Proximal Development), professional contexts (Zone of Free Movement), and professional development strategies (Zone of Promoted Action), and how these factors came together to shape opportunities for teacher learning during the life of the project.

Although there were some differences in the teachers' espoused beliefs about mathematics and how it is best learned and taught, all of them came to the project with a sense of dissatisfaction with their current teaching practices (a factor promoting change identified by Cavanagh, 2006), and some were already experimenting with investigative approaches to mathematics teaching. Nevertheless, when discussing its overall impact the teachers commented that it was unlikely significant change would have occurred without the impetus provided by this project, because the opportunity to participate provided them with realistic models of tasks and teaching and assessment approaches (as recommended by Anderson, 2005, and Cavanagh, 2006), while validating the changes that they wanted to achieve. In these circumstances, the other barriers to change identified in previous research — such as lack of time to cover the curriculum, student behaviour, the need to prepare students for standardised assessment tasks (Cavanagh, 2006; Norton et al., 2002) — seemed less important in influencing their actions. The credibility and authority the teachers gained from participation were also vital for helping them deal with relatively inflexible organisational structures and resistance from more traditionally minded mathematics teachers in their schools. Although these teachers worked in diverse professional contexts that offered both supports for, and hindrances to, innovation, all were able to draw on their knowledge and beliefs and the professional development opportunities available to them to plan and implement teaching approaches consistent with the intent of the new syllabus.

Implications for Policy and Practice

Loucks-Horsley et al. (2003) describe seven critical issues that must be taken into account when planning a professional development program, preferably in the goal setting phase (see Figure 1). Although we were conscious of these issues throughout the project, their influence is best analysed by looking to the future and asking how we might improve on the conduct of this project in the light of our experiences. We have selected four issues to illustrate implications for extending similar professional development opportunities to secondary mathematics teachers in other schools.

Building Professional Culture

Supportive school cultures are vital in building professional learning communities characterised by a strong vision of learning, shared responsibility for making learning happen, and de-privatisation of practice through collegial interactions. One strategy we used to begin building such communities in the participating schools was to request that pairs of teachers, rather than individuals, become involved in the project, and then to bring all the teachers together so they could share ideas and problems and seek solutions amongst colleagues from their own and other schools. This worked particularly well when some of the teachers had more experience in using investigations than others; for example, the teachers at Cunningham State High School were able to

offer advice to the other teachers on planning and assessment because of their experience with Project Mathematics. The two pairs of teachers in another two schools (Churchill and Cunningham) also joined the researchers to visit each other's classrooms, and this provided an opportunity that would not otherwise have been available for mutual observation of colleagues working in very different school environments.

Developing Leadership

Beyond those in designated leadership positions, it is important to develop teacher leaders who have the capacity to improve the quality of teaching and learning in their schools. Often the most powerful leadership exercised by teachers is simply in modelling new practices for colleagues to demonstrate that they actually work with students. Some of the teacher participants have also been identified as leading teachers at a district level, and have presented workshops for teachers in other schools where they model their approach to planning and implementing investigations in mathematics. While this kind of recognition is a fitting reward for participants in the project, there are disadvantages in burdening teachers with too many responsibilities and expectations that take them away from their classrooms. Consideration needs to be given to the kind of teacher leader roles that could be developed as a result of this project.

Building Capacity for Sustainability

The structure of our professional development model was designed to build sustainability because, in contrast to the common "one shot workshop" model, it was based on at least two cycles of goal setting, planning, implementing, and evaluating. However, it takes much more than this to ensure that any changes achieved within the life of a professional development project are sustained after it ends. In our follow up visit, nine months after the project ended, interviews with the teachers identified structural features in three of the participating schools that seemed to create a space for them to continue growing:

- At Cunningham State High School, Ron and Peter are extending Project Mathematics pedagogies into mainstream mathematics classes taught by other teachers.
- At Sugartown State High School, Skye and Chris have taken charge of writing the mathematics programs for Years 8, 9 and 10, thus extending their own investigations approach throughout the lower secondary school.
- At Churchill State High School, Tony is taking advantage of his leadership position as Head of Middle Schooling to enlist the support of teachers in other Key Learning Areas for extending an investigative approach across the curriculum.

Only at Seaside State High School are circumstances unfavourable to sustainability, since Val has left the school and Shanti lacks support from other teachers in the mathematics department.

Scaling Up

Scaling up is a vital concern for an education system as teachers and school districts implement new teaching and learning approaches. Loucks-Horsley et al. (2003) point out that programs are often initiated with volunteer teachers who are “early adopters” of innovations, and that it is essential to have a plan for bringing on board other teachers in the school as well as a plan for district and state wide change. The challenge here is one of designing professional development programs that reach a significant number of teachers throughout the system. Scaling up factors that contribute to success include:

- Ensuring that the innovation is clearly defined and based on a sound foundation (as in the case of implementation of a new syllabus);
- Having strategies for providing professional development opportunities to large numbers of people, using either online technology or a multiplier effect (training teacher leaders);
- Providing each teacher with sufficient support to change his or her practice;
- Establishing mechanisms for quality control of the professional development for all, especially when a multiplier strategy is used; and
- Developing a plan at each unit of implementation (school, district, state) for ongoing use, support, and implementation.

Conclusion

This project has shed some light on how researchers and education systems can work together to support teachers' professional learning in times of curriculum reform. We see a need to work with education system personnel at three levels:

- *Teachers*: to identify spaces within their school contexts where they can exert influence (such as in the case studies above);
- *Heads of Department/Principals*: to ensure they understand the goals of pedagogical reform and the organisational structures and cultures necessary to achieve these goals; and
- *District Officers*: to develop strategies for creating a “ripple effect” to other schools.

To these we may add a fourth layer — *parents and community members* — since gaining public support for mathematics education is necessary for building consensus around reform efforts, thus leading to a more informed public understanding of effective methods for teaching mathematics and of the role of mathematics in preparing young people for productive work, leisure, and citizenship.

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Appendix: Mathematical Beliefs Questionnaire

Scores ¹		Section 1. Nature of mathematics		Scores	
T1 ²	T2	Item (positive pole)	Item (negative pole)	T1	T2
		1. The ideas of mathematics can be explained in everyday words that anyone could understand.	8. Technical mathematical language and special terms are needed to explain mathematics.		
		9. In mathematics there are often several different ways to interpret something.	2. In mathematics something is either right or it is wrong.		
		10. Mathematics is an evolving, creative human endeavour in which there is much yet to be known.	3. Everything important about mathematics is already known by mathematicians		
		4. There are often many different ways to solve a mathematics problem.	11. Mathematics problems can be solved in only one way.		
		5. As computers increase in sophistication, mathematicians become more important to society.	12. Mathematicians are hired mainly to make precise measurements and calculations for scientists and other people.		
		13. In different cultures around the world there are different forms of mathematics.	6. Mathematics is essentially the same everywhere in the world.		
		7. Doing mathematics involves creativity, thinking, and trial-and-error.	14. Solving a mathematics problem usually involves finding a rule or formula that applies.		

Scores		Section 2. Mathematics teaching in secondary schools		Scores	
T1	T2	Item (positive pole)	Item (negative pole)	T1	T2
		21. Good mathematics teachers show students lots of different ways to look at the same question.	15. Good mathematics teachers show students the proper procedures to answer mathematics questions.		
		16. Cooperative group work and class discussions are important aspects of good mathematics teaching.	22. Good mathematics teachers plan so that students regularly spend time working individually to practise doing mathematics.		
		17. The role of the mathematics teacher is to provide students with activities that encourage them to wonder about and explore mathematics.	23. Good mathematics teachers only teach what is important for mathematics tests.		
		24. Teachers should let students determine if their methods and answers are right or wrong.	18. Teachers should provide problem solutions and answers when they are not in the back of the textbook.		

Scores		Section 2. Mathematics teaching in secondary schools		Scores	
T1	T2	Item (positive pole)	Item (negative pole)	T1	T2
		19. Teachers should regularly devote time to allow students to find their own methods for solving problems.	25. The teacher should provide examples of problem solutions and help students learn to replicate them when doing problems.		
		20. Good mathematics teaching involves class discussion in which students share ideas and negotiate meanings.	26. Good mathematics lessons progress step-by-step in a planned sequence towards the lesson objectives.		

Scores		Section 3. Mathematics learning in secondary schools		Scores	
T1	T2	Item (positive pole)	Item (negative pole)	T1	T2
		34. Understanding ideas and procedures is essential in mathematics learning.	27. Mathematics learning is about learning to get the right answers.		
		28. Students should be encouraged to build their own mathematical ideas, even if their attempts contain much trial and error.	35. Students learn mathematics by being shown the correct ways to interpret mathematical symbols, situations and procedures.		
		36. Mathematics learning is enhanced if students are encouraged to use their own interpretations of ideas and their own procedures.	29. Being able to memorise mathematical facts and procedures is important for mathematics learning.		
		30. Calculators can assist mathematics learning by serving as tools for exploration and consolidation of ideas.	37. Students who have access to calculators learn to depend on them and do not learn computational skills properly.		
		31. Students' mathematics errors often reflect their current understandings of ideas or procedures.	38. Students' mathematical mistakes are usually caused by a lack of practice.		
		39. Teachers should value periods of uncertainty, conflict, confusion or surprise when students are learning mathematics.	32. Students learn mathematics best if they are shown clear, procedures for doing things. precise		
		33. Use of physical objects and real life examples to introduce mathematics ideas is an essential component of learning mathematics.	40. Doing lots of problems is the best way for students to learn mathematics.		

¹ Strongly Agree = 5, Agree = 4, Undecided = 3, Disagree = 2, Strongly Disagree = 1

² Responses for each pair of teachers were recorded in the columns labelled T1 and T2.

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