

Curricular Knowledge and the Work of Mathematics Teacher Educators

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Mathematics teacher educator-researcher-mentor is a mouthful, but an accurate title to describe the range of roles of mathematics teacher educators across the nation and around the world. There is limited research about the professional development of teacher educators (Cochran-Smith, 2003; Wilson, 2006), and even less research specific to mathematics teacher educators. This article draws from the existing studies about mathematics teacher educator development and additional mathematics education literature to examine Shulman's (1986) notion of curricular knowledge for mathematics teacher educators and how growth in curricular knowledge can be facilitated. The analysis builds upon a previous investigation (Chauvot, in press), a self-study that examined the knowledge growth and structure of a novice mathematics teacher educator in terms of all three of Shulman's (1986) categories of teacher content knowledge: subject matter content knowledge, pedagogical content knowledge and curricular knowledge. All three categories merit more in-depth attention for considering the work and professional development of mathematics teacher educators; the purpose of this report is to elaborate upon the notion of curricular knowledge. Such an elaboration has implications for discussions regarding mathematics education doctoral programs, support systems for novice mathematics

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teacher educators, and further research in the realm of mathematics teacher educator professional development.

This article is organized in the following way. First, I summarize Shulman's (1986) notion of curricular knowledge and illustrate how the ideas play out for mathematics teacher educators. I then summarize the existing research about mathematics teacher educator professional development and discuss how this research furthers a conceptualization of curricular knowledge. I conclude with insights related to further understanding the professional development of mathematics teacher educators.

A Definition and Three Hypothetical Cases

Shulman's (1986) description of *curricular knowledge* is summarized in Figure 1 in four components. The first component of *curricular knowledge* consists of knowledge of different programs and corresponding materials available for teaching the given content. *Content* can be interpreted in a broad sense where the content is say, teaching mathematics, or algebra, or in a more narrow sense, where the content may be a topic

Figure 1.

Four components summarizing *curricular knowledge* as described by Shulman (1986).

Component	Excerpts from p. 10 of Shulman (1986) <i>Curricular Knowledge is</i>
1 Programs & Materials	Knowledge of "the full range of programs designed for the teaching of particular subjects and topics at a given level [and] the variety of instructional materials available in relation to those programs."
2 Indications & Contradindications	Knowledge of: "the set of characteristics that serve as both the indications and contradindications for the use of particular curriculum or program materials in particular circumstances."
3 Lateral	Knowledge of: "curriculum materials under study by his or her students in other subjects they are studying at the time." (<i>Lateral curricular knowledge</i>)
4 Vertical	Knowledge of: "familiarity with the topics and issues that have been and will be taught in the same subject area during the preceding and later years in school, and the materials that embody them" (<i>Vertical curricular knowledge</i>)

such as cooperative learning or solving linear equations. Component two of *curricular knowledge* goes beyond an awareness of the different programs and materials to include knowledge of the effectiveness and implications of programs and materials for given contexts. Component three entails knowledge of content and corresponding materials in other subject areas of students (lateral curricular knowledge), and component four consists of knowledge of how topics are developed across a given program (vertical curricular knowledge).

Admittedly, the term *curriculum* and related terms have different meanings in different contexts (see e.g., Gehrke, Knapp, & Sirotnik, 1992; Remillard, 2005; Stein, Remillard, & Smith, 2007). Remillard (2005), for example, for the purposes of her review of research about teachers' curriculum uses, referred to *curriculum materials* and *curriculum* as "printed, often published resources designed for use by teachers and students during instruction" (p. 213). Shulman's (1986) description is notably broader. The purpose here is not to compare different definitions of terms. Rather, a specific goal is to explore what curricular knowledge, as Shulman (1986) describes it, might be in the context of mathematics education where the *teacher* is a *mathematics teacher educator*.

Teacher educators are a diverse group of professionals, housed in different, overlapping arenas, and serving multiple roles. Some are university-based; some are school-based; some are both. There are those who teach prospective teachers, who conduct in-service professional development, who conduct research in education, and who mentor future teacher educators and researchers (Koster & Dengerink, 2001). In mathematics education, mathematics teacher educators are faculty, adjunct instructors, and graduate students who teach community college and university courses for pre-service and in-service teachers, or fulfill supervision roles within student-teaching experiences. Mathematics teacher educators are also school and department leaders and mathematics curriculum supervisors providing support for practicing teachers within K-12 school systems. And mathematics teacher educators can be found as instructors within alternative certification programs.

To illustrate curricular knowledge for different contexts and roles, three hypothetical cases are discussed (see Figure 2). The first case considers a context in which the mathematics teacher educator is a university-based faculty member and instructor of an undergraduate secondary mathematics methods course. The second case considers a curriculum supervisor within a school district who provides professional development for practicing secondary mathematics teachers. The third case is a university-based faculty member who serves as a mentor for

Figure 2.

Three hypothetical cases of *curricular knowledge* for different contexts and roles of mathematics teacher educators.

	Case I University-based faculty and instructor of a secondary mathematics methods course for undergraduates	Case II School-based curriculum supervisor providing professional development for in-service secondary mathematics teachers	Case III University-based faculty and mentor of mathematics education doctoral candidates
1 Programs & Materials	<ul style="list-style-type: none"> Models of mathematics teacher preparation programs Textbooks/materials for secondary mathematics methods courses Materials for teaching about [cooperative learning, equity, manipulatives, technology, etc.] in a mathematics classroom 	<ul style="list-style-type: none"> Models of professional development for mathematics teachers Materials used with in-service mathematics teachers Materials for teaching about [cooperative learning, equity, manipulatives, technology, etc.] in a mathematics classroom 	<ul style="list-style-type: none"> Models of mathematics education doctoral programs Materials available for teaching about mathematics education research. Materials/activities for teaching about the [role of theory, writing a literature review, etc.] in mathematics education research
2 Indications & Contraindications	<ul style="list-style-type: none"> Pros and cons of different mathematics teacher preparation programs How pre-service teachers interact with and learn from mathematics methods curriculum materials Research knowledge 	<ul style="list-style-type: none"> Pros and cons of different models of professional development for mathematics teachers How in-service teachers interact with and learn from professional development materials Research knowledge 	<ul style="list-style-type: none"> Pros and cons of different mathematics education doctoral programs How doctoral candidates interact with and learn from materials and activities intended to teach about mathematics education research Research knowledge
3 Lateral Curricular Knowledge	<ul style="list-style-type: none"> The institution's mathematics teacher preparation program Other courses pre-service teachers are enrolled in at the time 	<ul style="list-style-type: none"> The school district's philosophy, procedures and policies Other professional development opportunities the district is providing at the time 	<ul style="list-style-type: none"> The institution's mathematics education doctoral program Other courses doctoral candidates are enrolled in at the time
4 Vertical Curricular Knowledge	<ul style="list-style-type: none"> The institution's mathematics teacher preparation program Other courses pre-service teachers are enrolled in before and after the methods course 	<ul style="list-style-type: none"> The school district's philosophy, procedures and policies Past and future professional development opportunities the district has provided 	<ul style="list-style-type: none"> The institution's mathematics education doctoral program Past and future coursework and activities of doctoral candidates

mathematics education doctoral candidates. For comparison, the components of curricular knowledge are discussed across cases.

Component One across Three Cases

For Cases I and II, component one of curricular knowledge would entail knowledge of different models of mathematics teacher preparation programs and knowledge of different models of professional development for mathematics teachers, respectively. It follows that knowledge of different mathematics education doctoral programs would be included in the first component of curricular knowledge for Case III (see e.g., Reyes & Kilpatrick, 2000). Essentially, this broad, "big picture" perspective provides an understanding of the wealth (or scarcity) of available programs and materials for the given content.

Wilson (2006), in discussing the preparation of future teacher educator-researchers, proposed a series of related questions:

How much of a big-picture view of teacher education do our doctoral students need?... Ought all future teacher educators know about differ-

ent ways to organize teacher education?... Do they need to know about alternative routes into teaching and about state policies concerning certification? To what extent do teacher educator-researchers need to understand the history of teacher education and arguments concerning whether there should be formal teacher education and/or where it should be located? ... In discussions of K-12 education, we often argue that teachers' practice is improved if they have a broad view of the curriculum. Why would that not also be true within teacher education programs? (pp. 317-318)

While Wilson's (2006) comments are specific to what doctoral candidates might need in terms of knowledge of teacher preparation programs, parallel questions might be asked regarding future teacher educators who will be providing professional development for practicing mathematics teachers and for teacher educators who serve as mentors of doctoral candidates. For example, Wu (1999) aptly points out that the in-service arena is very different from the pre-service arena. Borrowing from Wilson (2006), ought all future teacher educators know about different ways to organize professional development for practicing teachers? Ought all future teacher educators know about different ways to organize doctoral programs?

Narrowing the focus, component one of curricular knowledge for Case I also includes knowledge of available textbooks and materials specific to teaching secondary mathematics methods courses, as well as knowledge about materials for teaching specific topics of methods courses such as cooperative learning, or use of manipulatives in the mathematics classroom. For Case II, knowledge of materials generated for in-service contexts would be more useful, although specific topics would likely be similar to the pre-service context. Specific topics of curricular materials for Case III, however, would be very different. Here, knowledge of materials for teaching about mathematics education research such as the history of mathematics education as a disciplined field (see e.g., Kilpatrick, 1992), for example, might be the focus of interest.

Component Two across Three Cases

Moving on to component two (Figure 2), indications and contradictions of curricula, mathematics education examples for Cases I and II would include research about pre-service and in-service teachers' uses and interactions with curriculum materials (e.g., Ball & Feiman-Nemser, 1988; Lloyd, 1999, 2008; Remillard, 1999; Remillard & Bryans, 2004; Sherin & Drake, in press), and how curricula can be used as catalysts for mathematics teacher learning (e.g., Frykholm, 2005; Lloyd, 2006; Remillard, 2000; Tarr & Papick, 2004). This genre of research provides insight and evidence for why materials "work" in one context but not

another. To date, there exists virtually no parallel research regarding mathematics education doctoral candidates' interactions with curriculum materials or how curricula is used to facilitate doctoral student learning (Figure 2 - Component 2, Case III).

Broadening the perspective beyond the materials themselves, component two for the first two cases would also involve knowledge of the effectiveness of entire programs. For example, in the pre-service context, what kinds of field experiences are effective? How much, and when? Or how should coursework be sequenced? Or within the in-service context, are summer programs effective? By what measures? How long should a summer in-service program be? What kinds of experiences are useful during the school year? Indications and contraindications for Case III might consist of knowledge of the kinds and how many research courses are most appropriate, what residency requirements should entail, and the kinds of internship experiences that contribute to doctoral candidate learning.

It is worth mentioning that an ability to answer such questions, if answers are to be based in research, requires an ability to interpret and assess educational research and evaluation studies. In other words knowledge of indications and contraindications seemingly includes an ability to assess the chosen research design as well as the credibility of the theoretical perspective that grounds the study, methodologies used, analysis procedures and reliability and validity measures.

Components Three and Four across Three Cases

Finally, components three and four, lateral and vertical curricular knowledge, would require knowledge specific to the institution's secondary mathematics teacher preparation program, the district's professional development program, and the institution's doctoral program, for Cases I, II, and III, respectively. For example, in Case I, relevant lateral curricular knowledge would be knowledge of courses that the pre-service teachers are enrolled in at the time. Are the students of the methods course concurrently enrolled in another course that is addressing cooperative learning as an instructional model for teaching? Or, considering vertical curricular knowledge, have the methods students already studied cooperative learning, and to what extent?

For Case II, the curriculum supervisor should be knowledgeable about other professional development opportunities the district is providing at the time (lateral) as well as past and future professional development opportunities (vertical). And for Case III, the mathematics teacher educator needs to be knowledgeable of coursework and activities the candidate is, has been, or will be engaged in.

The previous analysis across the three cases ostensibly oversimplifies the work of mathematics teacher educators. However, if one considers the multiple roles that mathematics teacher educators simultaneously hold, one can see that even just in terms of one construct, *curricular knowledge*, the work of mathematics teacher educators is quite complex. Consider for example, the complex nature of the work of a university-based mathematics teacher educator who teaches a secondary mathematics methods course for undergraduates *and* directs a professional development grant for local practicing mathematics teachers while *also* mentoring mathematics doctoral candidates. The previous analysis also does not address all scenarios. For example, not all instructors of undergraduate methods courses are university faculty; it is worth considering what the four components of curricular knowledge might be if the instructor of the methods course was, say, a graduate assistant.

The existing studies about becoming a mathematics teacher educator, although scarce, provide further insight into describing curricular knowledge of mathematics teacher educators. I first provide a summary of the existing studies. The summary is followed by a discussion linking back to components of curricular knowledge.

Studies about the Professional Development of Mathematics Teacher Educators

Zaslavski and Leikin (2004) attended to the professional development of mathematics teacher educators who provide in-service for mathematics teachers. This 5-year project involved 20 mathematics teacher educators and 120 teachers. 14 of the 20 mathematics teacher educators were involved from the early stages of the project, and many of the 120 teachers participated for four consecutive years. All of the mathematics teacher educators were experienced secondary mathematics teachers, and none of them had had any formal training in teaching teachers.

Grounding their work in theories that consider teachers' knowledge as developing socially within communities of practice, and drawing from both Steinbring's (1998) model of teaching and learning and Jaworski's (1992) teaching triad, Zaslavski et al (2004) offered a three-layer model of growth through practice as a conceptual framework to think about becoming a mathematics teacher educator. First, the authors contend that Steinbring's (1998) model illustrated that teachers learn through practice due to the interdependence of two autonomous systems: students' subjective interpretations of mathematical tasks, and teachers' constructed understandings of students' learning processes.

From there, Zaslavski et al (2004) adapted Jaworski's (1992) triad of mathematics instruction (the management of learning, sensitivity to students, and the mathematical challenge) to consider a triad of mathematics teacher instruction (the management of mathematics teachers' learning, sensitivity to mathematics teachers, and the challenging content for mathematics teachers), where challenging content for mathematics teachers was the original teaching triad.

They then included a second layer, where the student is the teacher and the teacher is the teacher educator, and a third layer where the student is the teacher educator and the teacher is the teacher educator's educator. They reported that their model helped to explain ways in which all participants (the mathematics teachers (MT), the mathematics teacher educators (MTE), and the mathematics teacher educators' educators (MTEE)) may learn from their practice. They applied their model to the planning and implementation of one of the professional development workshops involving five mathematics teacher educators and concluded that the model helped to explain professional growth for not only the newcomers in the group of mathematics teacher educators but also for the more senior and experienced members of the community.

The planning phase of the workshop centered primarily on two of the mathematics teacher educators (MTE), Tami and Alex, and a mathematics teacher educator's educator (MTEE), Keren. Zaslavski et al (2004) reported that Tami and Alex had different teaching styles:

Tami, who had special expertise in developing and implementing cooperative learning approaches in mathematics, suggested managing the workshop in a cooperative learning setting; contrary to Tami's suggestion, Alex was inclined to organize the workshop in a more teacher educator centered fashion, where he would lead the teachers towards the consideration of the use of the domain and range of a function in solving equations and inequalities. (p.17)

Eventually, after mutually developing a set of 22 mathematical tasks, Tami and Alex began to design a structured cooperative learning workshop. However, they had alternative suggestions for how to present the tasks to the teachers: "[Tami] proposed to group the 22 equations and inequalities according to the families of functions ... [so that teacher could] infer the role of the domain (or range) when solving the equations and inequalities" (Zaslavski et al, 2004, pp. 19-20). Alternatively, "Alex insisted on grouping them according to the role of the domain and range of the functions in solving the conditional statements [as a way] to make sure that the different roles of the domain and range were made explicit" (Zaslavski et al, 2004, p. 20). After consulting with Keren, it

was decided to propose an open-ended prompt: teachers were asked to sort the 22 tasks in as many ways as they could.

In the implementation phase, both Tami and Alex expected the teachers to define two criteria for sorting the tasks; the teachers identified 11 criteria for the sort:

Tami was surprised to realize the numerous approaches of the MTs to the sorting task. By observing the teachers' work on the problems and reflecting in and on action, Tami became more sensitive to MTs' ways of thinking and became more aware of what may be expected of MTs. She also became convinced of the potential of sorting tasks as a vehicle for professional development. (Zaslavski et al, 2004, p. 27)

Tzur's (2001) self-study provided reflective analysis of what he called experience fragments from various times of his development from a learner of mathematics through his experiences as a mathematics teacher, a doctoral student, and finally as an assistant professor. A result of his work was a framework for conceptualizing mathematics teacher educator development in terms of four interconnected foci of reflection: (a) learning mathematics as a student, (b) learning to teach mathematics as a teacher, (c) learning to teach mathematics teachers as a teacher educator, and (d) learning to teach mathematics teacher educators as a mentor. He noted the recursive, non-linear nature of a teacher educator's development where advancement to a higher-level focus proceeds through reflecting on activity-effect relationships at the lower level(s):

A level is considered higher in that the reflective process engenders a conceptual reorganization of practices used at the lower level(s). Thus, each higher level focus embodies the lower level foci; it encompasses new, explicitly integrated ways of thinking of what at the lower level was used implicitly and/or locally. (p. 272)

For example, Tzur (2001) argued that as a teacher of mathematics, it was reflection on his students' mathematical thinking that deepened his own understanding of mathematical concepts that were previously understood superficially, as a learner of mathematics.

In one experience fragment, Tzur (2001) described his interactions with teachers within a curriculum development project:

The teachers' use of the curriculum often indicated misunderstandings on their part. ... I remember how I tried to convince Joe to let students solve problems before telling them procedures and rules of solution ... I also suggested specific activities, which I was using with my students ... Yet, Joe continued to first teach the procedures and rules as a means to obtain students' mastery, at which point he considered them ready to solve problems. (p. 269)

In analyzing this experience, Tzur (2001) realized that he had not understood the problematic nature of attempts to “give” ideas to Joe and that he had failed to consider Joe’s understanding of learning and teaching and how it influenced his ability to make sense of Tzur’s (2001) recommendations.

Chauvot’s (in press) self-study utilized all three of Shulman’s (1986) categories of teacher content knowledge to investigate her knowledge content, structure, and growth from her doctoral program into her third year of a tenure-track faculty position at a large southwestern United States university. Narrative inquiry was used to examine past artifacts such as artifacts from her doctoral program, past syllabi, documents generated from college and department committee work, and submitted narratives for promotion and tenure. Narrative inquiry also supported the collection of new data including a journal and several iterations of a knowledge map.

Chauvot (in press) reported that the analysis of her course-development activities and the resources she used revealed the significance of what she called “human resource knowledge,” or “knowledge of experts” in the field whose scholarly work reflected specific topics of interest such as proportional reasoning, meaning of variable, mathematics education history, and mathematics education research design and methodologies. Knowledge of the experts led to the desired curricular materials.

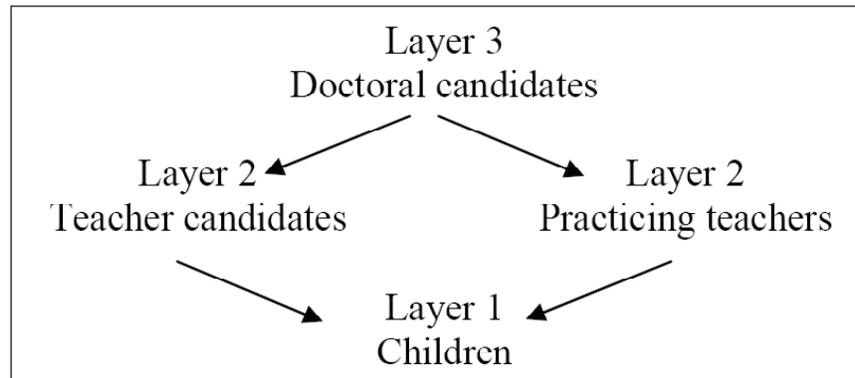
Activities within course development such as creating comprehensive syllabi and selecting textbooks/reading materials were examples of experiences that were very different from her classroom teaching experiences, mainly because of the different level of autonomy she experienced as a university instructor as compared to a classroom teacher. Accordingly, Chauvot (in press) reported that such activities supported growth in her curricular knowledge.

Consistent with Zaslavski et al’s (2004) three-layer model for growth through practice, and Tzur’s (2001) resulting layered conceptual framework, another pattern that emerged in the Chauvot (in press) study was also this notion of layers. For example, Shulman’s (1986) three categories of teacher content knowledge were identified as an initial layer of subject matter content knowledge for the teacher educator. The concept of layers was also applied in considering the students she served (see Figure 3):

Indirectly or directly, I encounter three layers of students: 1) *Children* in K-12 education, 2) *Teacher candidates/practicing teachers* in university courses, and 3) *Doctoral candidates* both in my university courses and as an advisor/mentor of future mathematics teacher educator-researchers. Once I separated the two contexts where I directly encounter students

Figure 3.

“Layers” of students of a university based mathematics teacher educator (adapted from Chauvot, in press).



(layers two and three), the creation of [my] knowledge map became much easier. (Chauvot, in press)

The different layers of students became an organizing tool for describing the content and structure of knowledge of the novice mathematics teacher educator. Each “layer” of students elicited different kinds of subject matter content, pedagogical content, and curricular knowledge, respectively. For example, her curricular knowledge for teaching prospective mathematics teachers was different from her curricular knowledge for mentoring doctoral students.

Van Zoest, Moore, and Stockero (2006) reported the collaborative experiences of three veteran mathematics teachers and a mathematics teacher educator-researcher as the veteran teachers transitioned to teacher educator within a doctoral clinical experience that involved a middle school mathematics methods course for preservice teachers. Final recommendations included the importance of engaging doctoral students in explicit conversations about what it means to be a teacher educator, the importance of emphasizing experiences that are different from K-12 classroom teaching, and the recommendation that collaborations with experienced teacher educators be a required component of doctoral programs. As an example, Van Zoest et al (2006) recommended breaking away from using doctoral students as co-designers of methods courses as internships because such activities focused doctoral students’ attention to *generating* preservice teacher thinking, an activity similar to K-12 classroom teaching, rather than *analyzing* preservice teacher thinking.

Taking a different approach, Sztajn, Ball, & McMahon (2006) recog-

nized the diverse arenas of mathematics teacher educators and examined how mathematical knowledge for teaching (e.g., Ball, Hill, & Bass, 2005) could serve as a “common intellectual space” within a summer institute for mathematics teacher educators. A goal was for the 65 participants to consider ways to plan, organize and implement instruction in mathematics content courses for prospective elementary teachers. Over a third of their participants were from mathematics departments while the others came from mathematics education departments, districts or other agencies. The institute activities included observations and analysis of activities of a laboratory class of 20 preservice elementary teachers who were studying mathematics for teaching. Sztajn et al (2006) reported several occasions in which mathematical knowledge for teaching served as a productive focus for their diverse group of individuals. In doing so, these researchers provide an example where a construct known for furthering the professional development of mathematics teachers (e.g., Ball, Hill, & Bass, 2005) was utilized for furthering the professional development of mathematics teacher educators.

Likewise, the previously cited studies illustrate that frameworks used to conceptualize mathematics teaching and mathematics teacher development (e.g., Jaworski’s (1992) teaching triad, Steinbring’s (1998) model of teaching and learning, Shulman’s (1986) categories of teacher knowledge) are effective frameworks for conceptualizing the development of mathematics teacher educators.

Connections

In what ways is *curricular knowledge* evident in the previously cited studies? There are several examples. One example comes from Tzur’s (2001) description of his interactions with Joe within the curriculum development project. In reflecting on this experience, Tzur (2001) realized that he had failed to consider Joe’s conceptions of learning and teaching and how it influenced Joe’s ability to make sense of the materials. This is an example of the second component of curricular knowledge, where mathematics teacher educators need to be knowledgeable about how teachers interact with and learn from curriculum materials (see Figure 2, Component 2, Case II). Similarly Tami and Alex (Zaslavski et al, 2004) experienced growth in the second component of curricular knowledge when they reflected on how differently the teachers sorted the tasks and how instrumental it was to provided the open-ended prompt.

An example of Component 2, Case III (Figure 2) emerges when one notes the incongruence between Van Zoest et al’s (2006) recommendation that doctoral candidates be less involved in course development and

Chauvot's (in press) claim that course development activities furthered her growth in curricular knowledge. An interpretation that acknowledges both claims might be that activities such as course development have the potential for facilitating growth in curricular knowledge whereas participating in activities that entail analyzing preservice teacher thinking, as recommended by Van Zoest et al (2006), afford opportunities to develop pedagogical content knowledge (Shulman, 1986). The doctoral students within the Van Zoest et al (2006) study may have already had significant experiences involving course development whereas Chauvot (in press) had not.

This distinction brings to the table that doctoral students and novice mathematics teacher educators, regardless of the arena in which they reside, come from varied backgrounds and programs. Mentors of doctoral students and administrators such as department chairpersons and others who provide support and professional development for mathematics teacher educators should be cognizant of assumptions made regarding past experiences of the candidate or novice mathematics teacher educator: "When looking at the complexity of the expertise of teacher educators, it is really quite remarkable that there is a common taken-for-granted assumption that a good teacher will also make a good teacher educator" (Korthagen, Loughran, & Lunenberg, 2005, p. 110). Although not illustrated in Figure 2, consider Components 1 & 2 of curricular knowledge for a hypothetical Case IV, a university-based administrator who has just hired a new mathematics teacher educator. What are different models of professional development for mathematics teacher educators? Which ones "work," and under what circumstances? Sztajn et al (2006) provides one model, a summer institute where mathematics knowledge for teaching was the focus of attention. Zaslavsky et al's (2004) work is grounded in collaborative efforts of communities of practice. What other models are there?

Extending the analysis further for Case IV, in what ways do mathematics teacher educators interact with curriculum materials? For example, through Tami and Alex's (Zaslavsky et al, 2004) initial suggestions and arguments for how to structure the workshop (cooperative learning vs. teacher-centered) and how to present the tasks to the teachers, we gain insight into how Tami and Alex's conceptions about teaching and learning interacted with their desired implementation of the materials. Just as studies have reported that mathematics teachers' conceptions influence curriculum implementation (e.g., Lloyd, 1999; Remillard, 1999), it is not surprising that similar conclusions can be made about mathematics teacher educators.

Concluding Remarks

This article draws from studies about the professional development of mathematics teacher educators to examine curricular knowledge (Shulman, 1986) for mathematics teacher educators. In doing so, what curricular knowledge is for a mathematics teacher educator within different contexts, and the kinds of experiences that may facilitate growth in curricular knowledge are highlighted. Furthermore, commonalities across studies about the professional development of mathematics teacher educators are illuminated, despite the full range of settings that are represented.

For example, it is agreed that collaboration and reflective analysis are key components in the professional growth of teacher educators. Zaslavski et al's (2004) work is grounded in theories that consider knowledge as developing socially within communities of practice; Van Zoest et al (2006) recommended that collaborations with experienced teacher educators be a required component of doctoral programs; Tzur identified a team-teaching experience within his doctoral program as significant in helping him make sense of his beginning role as a teacher educator; and the summer institute within the Sztajn et al (2006) study was purposely designed to engage participants in active exchanges of ideas related to mathematics knowledge for teaching. Foci of reflection vary: one may focus on kinds of knowledge (Chauvot, in press; Sztajn et al, 2006), significant experiences (Tzur, 2001), or interactions among members of a community of practice (Van Zoest et al, 2006; Zaslavsky et al, 2004). The complexity of the work of teacher educators requires multi-dimensional approaches for better understanding both what this work entails and how this expertise develops. What other dimensions might we consider? One possible line of inquiry which has already been alluded to could be examining the role that mathematics teacher educator beliefs play in their professional growth (see also Chauvot, Ice, Sanchez, Kastberg, Leatham, Lovin, & Norton, 2007).

It is also agreed that the work of mathematics teacher educators is conceptually different from the work of mathematics teachers. However, the two professions inform one another: If we adopt the notion of layers, there exists the potential that as we continue our research about the professional development of ourselves, as mathematics teacher educators, we can apply what we learn for constructing a better understanding of the professional development of mathematics teachers. In other words, as we study ourselves in transition, we have the potential for better understanding teachers in transition.

One might ask whether this report examined the work of mathematics

teacher educator-researchers through Shulman's (1986) construct of curricular knowledge or if this report examined Shulman's (1986) construct of curricular knowledge in terms of the work of mathematics teacher educator-researchers. Regardless of the direction, an understanding of both the construct of curricular knowledge and the work of mathematics teacher educators was enhanced by this analysis. One point being made here is that this article illustrates how past researchers are successfully utilizing existing frameworks within mathematics education in order to conceptualize the professional development of mathematics teacher educators. In turn, an understanding of the framework is enhanced. Kilpatrick (1995) states:

No one should expect to draw strong implications for practice from the results of a single research study. The results of a study may be its least important part. Research in mathematics education gains its relevance to practice or to further research by its power to cause us to stop and think. It equips us not with results we can apply but rather with tools for thinking about our work. (p. 25)

It is important to continue drawing from existing literature to further our understanding of what it means to become a mathematics teacher educator.

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