# USING QUESTIONING TO STIMULATE MATHEMATICAL THINKING

**JENNI WAY** reminds us of the importance of questioning to stimulating higher order thinking.

Good questioning techniques have long been regarded as a fundamental tool of effective teachers and research has

questions were "lower order" knowledge-based questions focusing on recall of facts (Daines, 1986). Clearly this is not the right type of questioning to stimulate the mathematical thinking that can arise from engagement in problem solving and investigations. Unfortunately, research continues to show that teachers ask few questions that encourage children to use higher order thinking skills in mathematics (Sullivan & Clarke, 1990). Many primary teachers have already developed considerable skill in good questioning techniques in curriculum areas such as literacy and social studies, but do not transfer these skills to mathematics.

Teachers' instincts often tell them that they should use investigational mathematics more often in their teaching, but they are sometimes disappointed with the outcomes when they try it. There are two common reasons for this. One is that the children are inexperienced in this approach and find it difficult to accept responsibility for the decision-making required and need a lot of practice to develop organised or systematic approaches. The other reason is that the teachers have yet to develop a questioning style that guides, supports and stimulates the children without removing the responsibility for problem-solving from the children.

This article presents some approaches that can be used by teachers to scaffold children's mathematical investigations through utilising a hierarchy of questions.

## Four types of questions

Within the context of open-ended mathematical tasks, it is useful to group questions into four main categories (Badham, 1994). These questions can be used by the teacher to guide children through investigations and to stimulate their mathematical thinking. Another function of these questions is to gather information about the students' knowledge and strategies.

### 1. Starter questions

These take the form of open-ended questions that focus the children's thinking in a general direction and give them a starting point.
Examples:

- What does this remind you of?
- What are some examples of…?
- How could you sort these…?
- How many ways can you find to…?
- What happens when we…?
- What can be made from…?
- How many different… can be found?

### 2. Questions to stimulate mathematical thinking

These questions assist children to focus on particular strategies, connect to previous experiences and help them to see patterns and relationships. This aids the formation of a strong conceptual network. The questions can serve as a prompt when children become 'stuck.' Teachers are often tempted to turn these questions into instructions, which is far less likely to stimulate thinking and removes responsibility for the investigation from the child.

Examples:

- What is the same? What is different?
- How can you group these… in some way?
- What are some things you could try?
- Can you find more examples?
- Can you see a pattern? Can you explain it?
- How can this pattern help you find an answer?
- What do you think comes next? Why?
- Is there a way to record what you've found that might help us see more patterns?
- What would happen if…?

### 3. Assessment questions

Questions such as these ask children to explain what they are doing or how they arrived at a solution. They allow the teacher to see how the children are thinking, what they understand and at what level they are operating. Obviously they are best asked after the children have had time to make progress with the problem, record some findings and perhaps achieved at least one solution. These questions also prompt the children to reflect and self-assess and so help prepare them to contribute to class discussion.
Examples:

- What have you discovered?
- How did you find that out?
- Why do you think that?
- What made you decide to do it that way?
- What makes you confident it is correct?

### 4. Final discussion questions

These questions draw together the efforts of the class and prompt sharing and comparison of strategies and solutions. This is a vital phase in the mathematical thinking process. It provides further opportunity for reflection and realisation of mathematical ideas and relationships. It encourages children to evaluate their work and appreciate the thinking of others.
Examples:

- Who has the same answer/pattern/grouping as this?
- Who has a different solution?
- Are everybody's results the same? Why/why not?

- Have we found all the possibilities? How do we know?
- Have you thought of another way this could be done?
- Do you think we have found the best solution?
- What new questions/problems have you thought of?

## Levels of mathematical thinking

Another way to categorise questions is according to the level of thinking they are likely to stimulate, using a hierarchy such as Bloom's taxonomy (Bloom, 1956). Bloom classified thinking into six levels: Memory (the least rigorous), Comprehension, Application, Analysis, Synthesis and Evaluation (requiring the highest level of thinking). Sanders (1966) separated the Comprehension level into two categories, Translation and Interpretation, to create a seven level taxonomy, which is quite useful in mathematics. As you will see as you read through the summary below, this hierarchy is compatible with the four categories of questions already discussed.

1. ***Memory:*** The student recalls or memorises information.
2. ***Translation:*** The student changes information into a different symbolic form or language.
3. ***Interpretation:*** The student discovers relationships among facts, generalisations, definitions, values and skills.
4. ***Application:*** The student solves a life-like problem that requires identification of the issue and selection and use of appropriate generalisations and skills.
5. ***Analysis:*** The student solves a problem in the light of conscious knowledge of the parts of the form of thinking.
6. ***Synthesis:*** The student solves a problem that requires original, creative thinking.
7. ***Evaluation***: The student makes a judgement of good or bad, right or wrong, according to the standards he or she values.

## Combining the categories

The two ways of categorising types of questions overlap and support each other.

For example, the questions:

- Can you see a pattern?
- How can this pattern help you find an answer?

relate to ***Interpretation.***

The questions:

- What have you discovered?
- How did you find that out?
- Why do you think that?

require ***Analysis.***

The questions:

- Have we found all the possibilities?
- How do we know?
- Have you thought of another way this could be done?
- Do you think we have found the best solution?

encourage ***Evaluation.***

## Working with more able children

While all children benefit from developing higher order thinking skills, it has been suggested that gifted and talented children require a greater quantity of their learning time spent engaged in higher order thinking than 'regular' students (Davis & Rimm, 1989). The two triangles (Figure 1) illustrate the suggested difference in the proportion of time spent on the Bloom's six levels of thinking.

## Guide questions for mathematical investigations

In the process of working with some groups of teachers in England on using investigations in their mathematics teaching, Table 1 was developed. It provides examples of generic questions that can be used to guide children through a mathematical investigation, and at the same time prompt higher levels of thinking. Depending on the investi-

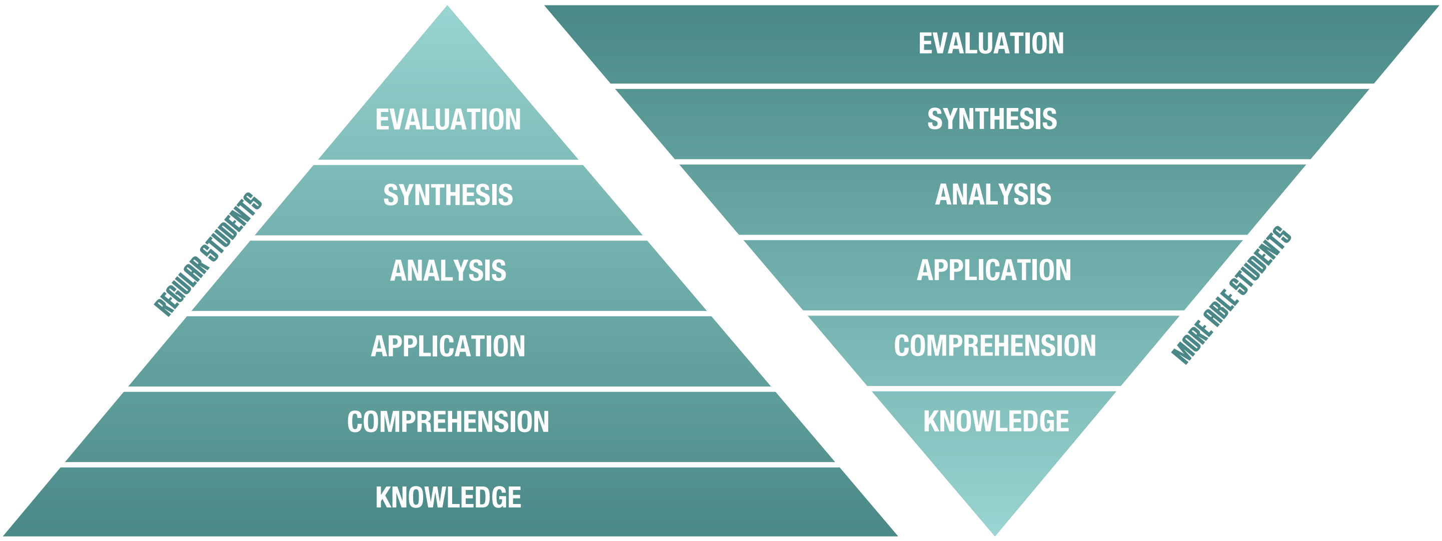


Figure 1. Comparison of learning-time spent on levels of thinking.

gation, the investigator might cycle through the hierarchy of questions several times. Adults and experienced investigators naturally ask these questions of themselves, but children, being inexperienced (formal) investigators, do not. Thus, the interaction with the teacher becomes a crucial factor in promoting mathematical achievement.

Table 1. Levels of thinking and guide questions.

| LEVELS OF THINKING | GUIDE QUESTIONS |
|---|---|
| Memory:<br>Recalls or memorises information | What have we been working on that might help with this problem?<br>Have you seen something like this before? |
| Translation:<br>Changes information into another form | How could you write/draw/model what you are doing?<br>Is there a way to record what you have found that might help us see more patterns? |
| Interpretation:<br>Discovers relationships | What is the same? What is different?<br>Can you group these in some way?<br>Could there be a connection between these…?<br>Can you see a pattern? |
| Application:<br>Works towards solving a problem — use of appropriate generalisations and skills | How can this pattern help you find an answer?<br>What do you think comes next? Why?<br>What could you do to explore this connection further?<br>Are there any ‘rules’ that can be followed? |
| Analysis:<br>Solves a problem — conscious knowledge of the thinking | What have you discovered?<br>How did you find that out?<br>Why do you think that?<br>What made you decide to do it that way? |
| Synthesis:<br>Solves a problem that requires original, creative thinking | Who has a different solution?<br>Are everybody’s results the same? Why/why not?<br>What would happen if this changed? |
| Evaluation:<br>Makes a value judgement | Have we found all the possibilities? How do we know?<br>Have you thought of another way this could be done?<br>Do you think we have found the best solution? |

## Example of scaffolding an investigation through levels of questioning

Table 2 shows an example of a lesson plan in which the teacher gradually transfers responsibility of the task to the students while continuing to scaffold the mathematical thinking through a hierarchy of questions.

*Task: Write 4 consecutive numbers. Insert an addition or subtraction sign between each number. Investigate.*

Table 2. Overview of lesson plan.

| PHASE | APPROACH | STRATEGIES AND QUESTIONS |
|---|---|---|
| Concept 'warm up' | Whole class Teacher directed | Show number line from –10 to +10 and discuss negative and positive numbers.<br>Demonstrate 2 + 4 – 8 + 2 on number line.<br>Ask class the answer to several more; e.g., 8–3–5+2 and demonstrate on number line. |
| Introduction of task | Teacher guided | Teacher presents task and gives an example of 4 consecutive numbers with + – signs inserted; e.g., 3 + 4 + 5 – 6 = 6<br>Ask for another way the signs could be inserted. |
| | Individual exploration | Invite students to find more ways. |
| | Teacher questions to whole class | How many ways can it be done? (Record on strips of paper to exploring facilitate various sequences)<br>How can we check if we have them all?<br>Which sequence of signs gives the smallest/largest answer?<br>What other ways can we arrange them?<br>That is just one example – what could we do now? |
| Development of investigation | Individual exploration | Students choose own consecutive numbers, write the set on paper strips and calculate the answers. |
| | Paired discussion | Ask pairs to look for similarities and differences in their sets and explore various arrangements of the strips, looking for patterns. |
| | Teacher questions to whole class | What have you noticed?<br>Are there any patterns?<br>What questions have you thought of?<br>What ideas do you have?<br>What numbers would you like to try next? |
| Continuation of investigation | Individual or small group exploration | Students decide what direction to explore and keep records.<br>What patterns can you find?<br>What have you discovered?<br>Can you make predictions about the answers?<br>Are there 'rules' that can be followed? |
| Closure | Class sharing | Individuals and groups share findings and further questions.<br>Who has a different solution?<br>Are everybody's results the same? Why/why not?<br>What would happen if this changed?<br>Have we found all the possibilities? How do we know?<br>Have you thought of another way this could be done?<br>Which way of explaining this pattern is best? Why?<br>What new questions have you thought of that we could investigate?<br>Review strategies that were most helpful in revealing patterns etc. |

Note: The content of this article is based on another by the same author, originally published February 1999 on the NRICH website (www.nrich.maths.org, University of Cambridge). This website is an excellent source of mathematical investigations and problems.

## References


Badham, V. (1994) What's the question? *PAMphlet 23*. Primary Association for Mathematics (Australia).

Bloom, B. (1956). *Taxonomy of Educational Objectives Handbook 1: Cognitive Domain*. New York: David Mackay.

Dains, D. (1986). Are teachers asking the right questions? *Education 1,* (4) 368–374.

Davis, G. & Rimm, S. (1989). *Education of the Gifted and Talented* (2nd Ed.). New Jersey: Prentice Hall.

Sanders, N. (1966). *Classroom Questions: What Kind?* New York: Harper & Row.

Sullivan, P. & Clarke, D. (1990). *Communication in the Classroom.* Geelong: Deakin University Press.

Wood, T. (2002). What does it mean to teach differently? In B. Barton, K. Irwin, M. Pfannkuch & M. Thomas (Eds), *Mathematics Education in the South Pacific: Proceedings of the Twenty-fifth Annual Conference of the Mathematics Education Research Group of Australasia,* Vol. 1 (pp. 61–67). Auckland: MERGA.



Jenni Way
University of Sydney
<j.way@edfac.usyd.edu.au>


APMC

# Resource Review


***Mathematics Assessment for Learning: Rich Tasks and Work Samples***

**Ann Downton, Rose Knight, Doug Clarke & Gerard Lewis**

Published by Mathematics Teaching & learning Centre (Australian Catholic University) & Catholic Education Office, Melbourne 2006, 152 pp., spiral-bound paperback, ISBN-13: 978-0-9756718-5-6

Year levels: K–8

Available from AAMT: $39.00 for members

This is a valuable addition to any primary/middle school library and has also been a real hit with student teachers. It presents a range of great tasks which are open-ended and have been "road tested" with many Australian students. Readers are provided with a scoring rubric for each task, along with the mathematical background for the task and work samples which illustrate student performance at each level of the rubric.

These three aspects of the book make it useful as an assessment tool and as a professional development resource for teachers. For those just starting out as mathematics teachers, the tasks themselves provide a model for designing rich and purposeful tasks linked to key mathematical ideas. For other teachers who wish to explore the idea of careful analysis of work samples to inform future teaching, the samples provided in the book can be used to promote discussion and build confidence in making assessment decisions.

Teachers are always searching for new or different tasks — ones which can be adapted to meet the needs of students. This book not only provides them with great tasks, but the inclusion of stories from the classroom and work samples makes it an even more valuable resource. It is not just another book of great activities but a professional learning resource suitable for junior primary to junior secondary teachers.

*Review by Denise Neal,*
*Department of Education, Tasmania.*