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Enhancing Early Numeracy by Promoting the Abstract Thought Involved in the Oddity Principle, Seriation, and Conservation

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Today's climate of standards-based reforms and accountability holds teachers and schools responsible for the achievement of their students. Because of the No Child Left Behind Act (NCLB; 2001), states receiving federal funds are required to develop and administer assessments that enable them to report student progress on an annual basis. As a result of this emphasis, there appears to be a tendency for schools to introduce mathematical skills and concepts at much earlier ages, which may, in some cases, mean prematurely introducing mathematics skills and concepts that are beyond children's cognitive capabilities. The expectation is that children will achieve the standards once instruction has been provided. "The rhetoric of higher standards and achievement may be appealing, but the reality is not" (Neuman, 2003, p. 287). Neuman asserted that the law makes the erroneous assumption that all

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summary

Although many students who enter kindergarten are cognitively ready to meet the demands of the kindergarten mathematics curriculum, some students arrive without the early abstract reasoning abilities necessary to benefit from the instruction provided. Those who do not possess key cognitive abilities, including understandings of conservation, insertions into series, and the oddity principle, are at a disadvantage when attempting to master mathematical concepts and skills that require early abstract thought. Recognizing the need to address this gap, this study examined the effects of an intervention designed to teach children conservation, insertions into series, and the oddity principle. The study included 78 kindergartners enrolled in a culturally, linguistically, and socioeconomically diverse metropolitan school district. Students were randomly divided among one of three groups: cognitive intervention, numeracy instruction, and art instruction. Instruction for each group was matched in number, timing, and extent of sessions. The study found that kindergartners who received the cognitive intervention scored significantly higher on measures of cognitive ability than those in the comparison group who participated in the art instruction or those who received numeracy instruction. On the Woodcock-Johnson III Applied Problems scale, those in the cognitive intervention scored significantly higher than those who received art instruction. Those in the cognitive intervention and those in the numeracy intervention performed similarly. These results suggest that it is possible to provide instruction that enhances the cognitive abilities of kindergartners who do not possess key reasoning abilities. In addition, there is evidence that promoting early abstract thought can enhance kindergartners' mathematical abilities.

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children enter school "on a level playing field" (p. 287). In reality, disparity exists among students entering kindergarten.

Although many children enter kindergarten with key reasoning abilities that promote their academic success, some children do not. Children who are not as cognitively advanced may face serious difficulties as they try to navigate a curriculum filled with concepts and skills beyond their reach. Many mathematics concepts and skills require children to draw upon early abstract abilities, including the oddity principle, insertions into series, and number conservation. The oddity principle is the ability to identify the only item in a group that differs from all others on some dimension. Children who have not mastered the oddity principle may have difficulty learning basic kindergarten skills. For example, kindergartners are expected to differentiate among a penny, nickel, dime, and quarter; "sort and classify objects according to similar attributes (size, shape, and color)" (Virginia Board of Education, 1995, ¶ K.19); and "identify representations of plane geometric figures (circle, triangle, square, and rectangle), regardless of their position and orientation in space" (Virginia Board of Education, 1995, ¶ K.14). Children who have not developed the ability to identify an item that differs from the others on some dimension may struggle as they try to perform tasks related to these expectations.

The same is true of insertions into series, which is the ability to relate an item to others in an increasing or decreasing series and insert the item in its proper place in that series. This is an important cognitive ability that comes into play when kindergarten students compare the size (larger/smaller) of plane geometric figures (Virginia Board of Education, 1995). Number conservation also plays a role in children's success in mathematics. Number conservation is the understanding that the number of items in a group cannot change unless one or more is added or subtracted. This skill enables kindergartners to determine whether one set of objects has the same, fewer, or more objects than another set (Virginia Board of Education, 1995).

Fortunately, some aspects of cognitive functioning can be improved to enhance learning. Studies by Pasnak, McCutcheon, Campbell, and Holt (1991) and Pasnak, Hansbarger, Dodson, Hart, and Blaha (1996) found that when kindergartners were provided extensive instruction on the oddity principle, number conservation, and insertions into series, they scored higher in these reasoning abilities as well as measures of mathematics concepts and verbal comprehension, as measured by the Stanford Early School Achievement Test. Similar results were found with preschool children. When preschool children were taught oddity and insertion, their reasoning abilities in these areas improved (Ciancio, Sadovsky, Malabonga, Trueblood, & Pasnak, 1999). Ciancio, Rojas, McMahon, and Pasnak (2001) also found that preschool students could be taught the oddity principle and insertions into a series, and they showed subsequent gains in numeracy as measured by the McCarthy Scales of Children's Abilities.

Background

The Oddity Principle

Children who apply the oddity principle are able to identify the one object in a group that differs from all of the other objects in the group in one characteristic. The comprehension of relations involved in employing the oddity principle marks the transition from understanding events primarily in terms of perceptual thought to understanding based on early abstract thought. The ability to recognize similarities and differences, to sort reasonably well, and to categorize objects hierarchically into basic, subordinate, and superordinate classes are usually relatively well developed prior to age 4 (Gelman & Wellman, 1991; Mervis, Johnson, & Mervis, 1994; Waxman, 1994), but mastery of the oddity principle depends on more advanced relational responding. When confronted with a group of objects that are all identical except that one differs in size, preschool children try to solve the problem on the basis of some quality of an object, rather than on the relation between objects. For example, if shown one large and three small safety pins, a preschool child may correctly identify the large pin as different and not belonging with the others. However, if shown one small and three large pins, the same child will not be able to identify the small one as unlike the others and instead may persistently select the large pins, one after the other. The child is selecting large pins because they have the quality of being "big," rather than responding to oddity. Another child might do the opposite, always selecting the small size. Similar difficulties arise when oddity involves dimensions of shape, function, color, orientation, texture, or any other dimension (Pasnak, 1987; Pasnak et al., 1986). The difficulty is not a communication problem and cannot be resolved without extensive instruction (Chalmers & Halford, 2003). It arises from an immature stage in the cognitive development of all children. The child does not adequately recognize the relation between the objects in the group and instead searches for an absolute quality like "big" or "little" to govern choices. This is pervasive across all dimensions and is difficult to overcome. Initial progress depends upon identifying the item that differs most from the others—a response rule that is only very slowly replaced by purely relational responding (Chalmers & Halford, 2003). A child who has learned to apply the oddity principle to one dimension will have substantial difficulty in applying it to different dimensions and to different types of problems within dimensions (Pasnak et al., 1986). Much like the preschoolers studied by Zelazo and Frye (1998), children who are in the process of developing their understanding of the oddity rule need a great deal of instruction before they can abstract oddity in a new dimension.

Unidimensional Seriation

Unidimensional seriation is arranging things in order by size or some other ordinal dimension. This is a very fundamental form of reasoning that is expressed in several different ways at different levels of cognitive development and has long been thought to be important (Inhelder & Piaget, 1959/1964; Leiser & Gillieron, 1990).

Many preschool children develop the ability to form a series of objects in the natural course of maturation and experience. However, inserting new interior items into an already constructed series is much more difficult. When confronted with this task, nearly all 3-year-olds and many 4-year-olds make the error of placing the new object at one end of the series or the other and are unable to find the appropriate place for it in the middle of the series by relating it to neighboring objects (Leiser & Gillieron, 1990; Malabonga, Pasnak, & Hendricks, 1994; Southard & Pasnak, 1997; Young, 1976). Progress in recognizing that misplacement of the object is an error and making corrections to such errors is not very predictable (Southard & Pasnak, 1997). It is necessary to comprehend clearly the relations between objects in a series in order to make accurate insertions. This necessity for relational thinking is the reason for the difficulty in making insertions and the importance of being able to do so easily. It marks the transition from perceptually based thinking to early abstract thought. This is the transition that is needed for a child to deal with such concepts as the number line and ordinality, which are among the earliest understandings of numeracy presented in kindergarten and early elementary school, and nearly all 5-year-olds have made it (Malabonga et al., 1994). Children who cannot make an ordered series with concrete objects to compare lack the understanding necessary to put abstract numerical symbols like 6, 9, 11, 15 in order; they do not understand that there is anything inappropriate in an ordering like 6, 7, 10, 8, 9. Memorization, rather than understanding, is the only way a child can deal with such problems, and the child's efforts are as unsuccessful as they are uninsightful.

Conservation

Children who grasp number conservation understand that the number of items in a group cannot change unless one or more is added or subtracted. Many school curricula offer some instruction in some early forms of conservation. However, the instruction is too brief to benefit a child whose understanding of conservation is not already emergent. This is unfortunate because the concepts involved in conservation are critical to an understanding of numeracy. A kindergartner who does not understand conservation does not understand that adding one object is necessary to increase the number of objects in a group from 13 to 14, even though the addition, or lack of it, is easily observed with concrete objects that can be counted. Such a child cannot be expected to understand that adding the numerical symbols 13 and 1 in response to a + symbol produces 14. Lessons involving addition and subtraction become trials of memorization rather than paths to understanding because the child lacks the basic understanding of what addition and subtraction accomplish.

Early forms of conservation develop somewhat later than the oddity principle or insertions into series, and kindergartners diverge more in their understanding of this important form of early abstract thought. The central issue in number conservation is no longer the relation between objects in a group or series. To conserve number, a child must comprehend how the number of items in a group can be changed. Piaget (1941/1952) described conservation of discontinuous quantities by 5-year-olds and their recognition that rows of items retain equivalency in number irrespective of the arrangement of their elements. This understanding of reversibility and equivalence, which Piaget defined as mathematical operations, is not shared by other children of the same age. Piaget reported that it might not arise until age 6 or 7, and more recent research has supported his conclusion (Pasnak, 1987; Pasnak et al., 1991). Children who are unable to conserve are unable to maintain their intuitive grasp of equivalence in the face of a perceptual conflict. They do not recognize that numbers cannot change in the absence of an addition or subtraction. Yet, these children are required to master addition and sometimes subtraction via numerals and plus and minus signs in kindergarten curricula, even though they lack the basic concept that addition and subtraction change quantities. Addition and subtraction often are taught to kindergartners through the understanding that sets of units and tens are increased or decreased by addition or subtraction. To benefit from such instruction, nonconserving children must be helped to learn more accurate rules for identifying number and the appropriate rule to solve each problem (Siegler, 1996).

Relations to School Achievement

There are modest but statistically significant relations between kindergartners' mastery of conservation, seriation, and classification (which subsumes the oddity principle) and early literacy and numeracy as measured by the Metropolitan Achievement Test at the end of kindergarten (Silliphant, 1983). Silliphant's longitudinal study demonstrated that relations between the children's mastery of these thinking abilities at age 5 and their subsequent achievement in reading and mathematics at the end of second grade, as measured by the Iowa Tests of Basic Skills, and at the end of third grade, as measured by the California Achievement Test, were still statistically significant, although somewhat diminished. Thus, *timely* mastery of these thinking abilities has measurable effects on both early and future school achievement.

Consider the impact that differences in these key cognitive abilities can have in kindergarten. The thinking of children who have not mastered conservation, oddity, and insertions into series is often closely tied to one or two perceptual properties of whatever they are considering or studying. As a result, they have trouble conceptualizing similarities and differences that have a nonperceptual, abstract basis, and they make poor decisions. They, too, often fail to abstract. Instead, they take advantage of simple relations based on size, shape, orientation, function, or type. A child may think that numbers that are written smaller on a blackboard refer to smaller quantities than those that are written large, or that the largest letters on a written page are the most important and that the rest do not mean much. These deficiencies interfere with children's problem solving (Malabonga, Pasnak, Hendricks, Southard, & Lacey, 1995; Malabonga et al., 1994; Pasnak, 1987; Pasnak, Hansbarger, et al., 1996).

Failure to understand conservation indicates a lack of understanding that numbers cannot be changed unless addition or subtraction occurs. In addition to not recognizing the significance of the addition and subtraction operations, nonconservers fail to understand that reversing a change restores the original number and do not fully grasp the principle of reciprocity. They attempt to solve number problems on a perceptual basis (i.e., by appearances). Such children may attempt to solve conservation problems by counting. Progress through their kindergarten numeracy lessons depends mainly on memorization, as conceptualization is greatly hindered by their basic misunderstanding of how and why quantities can be increased or decreased. This poor foundation coupled with the inevitable failures of memory result in poorer retention by nonconservers than conservers (Dudek, Strobel, & Thomas, 1987; Pasnak, Hansbarger, et al., 1996; Pasnak et al., 1995).

The Learning Set Approach

Teaching children to think more abstractly than they have ever done before is no small task. Piaget (1941/1952) argued that it was impossible. Subsequent research showed that students can improve their level of cognitive development, but that improvement in abstract thinking is especially difficult (Gelman, 1969; Smedslund, 1961). This study coupled the content of the instruction—principles of abstraction that recent research (Pasnak, Greene, Ferguson, & Levit, 2006; Pasnak et al., 2007; Pasnak, Maccubbin, & Ferral-Like, 2007) has shown are fundamental to cognitive growth—with an educational psychology based teaching method that has been amply proven to help any learner effectively master principles of abstraction.

The learning set method (Harlow, 1949, 1959) produces progress in mastery of abstract principles of thought even by learners who lacked many prerequisite abilities. The term *learning set* sometimes refers to the problems used in instruction and sometimes refers to what is in the child's head. In any event, it is clear that abstractions can be taught to children lacking most or all prerequisite abilities via a collection of a very large number of problems that embody the abstract principle concretely. The power of this approach to induce concept formation is a function of the number and variety of problems presented to the child, rather than the skill of the adult in presenting them (Gelman, 1969; Harlow, 1949; Pasnak 1987; Pasnak, Hansbarger, et al., 1996; Pasnak et al., 2007). Therefore, this technique has great potential applicability in school settings.

Through the use of numerous and variable concrete problems, learning sets teach children that the relation between objects in different problems is always fundamentally the same. The children develop an understanding of this relation and transfer it to new problems by analogical mapping. This form of abstraction depends upon relational knowledge and is "a case of transfer based on structure mapping par excellence" (Halford, 1993, p. 223).

This approach has been used effectively to teach oddity, seriation, and conservation to children who are developing atypically due to blindness (Friedman & Pasnak, 1973; Lebron-Rodriguez & Pasnak, 1977; Lopata & Pasnak, 1976) or mental disabilities (Campbell, McCutcheon, Perry, & Pasnak, 1988; Pasnak, Campbell, Perry, & McCormick 1989; Perry, Pasnak, & Holt 1992). It succeeds more easily with typically developing children (Gelman, 1969; Kingsley & Hall, 1967; Pasnak, 1987; Pasnak, Hansbarger, et al., 1996; Pasnak et al., 1991). The instructor represents any principle concretely in scores of problems that vary widely in appearance and particulars and assists the learner to solve each problem in turn. Solutions gradually become automatic and generalize very broadly.

Implications for Education

Strengthening key reasoning abilities early in the educational process allows children to bring processes of attention, decision-making, and memory to bear on academic problems more efficiently. The ability to abstract relations between target objects, events, and occurrences develops first in the arenas of classification and seriation and emerges at the onset of formal schooling. Kindergarten occurs during the time when children's thinking begins a great shift from responses based on perceptual and absolute qualities of the things they observe to the apprehension of and attention to relational qualities between those things. At this transition in the development of thinking and reasoning, the ability to classify on one dimension by the oddity principle and the ability to accurately understand the relations between items into a unidimensionally increasing or decreasing series are especially important. They are the earliest forms of incontestably abstract thinking, the hallmark of the increasingly elaborated application of abstract thought to concrete situations, a skill that is needed in elementary school.

Despite the growing evidence that a focus on developing children's reasoning abilities may promote cognitive thinking and ultimately academic progress, modern educational curricula often deal only briefly with simpler forms of hierarchical categorization and seriation, rather than oddity and insertions into series. They do not typically take advantage of theoretical and empirical advances that suggest that applications of the oddity principle, conservation, and insertions into series may result in improved academic performance (Pasnak, Hansbarger, et al., 1996; Pasnak et al., 1991). A preliminary effort (Pasnak et al., 1991) to teach these three concepts in 17 classrooms from 5 schools produced significant gains on the Otis-Lennon School Ability Test (OLSAT), a standardized test of cognitive ability that is used as a predictor of school performance. Four months later, these cognitive gains were followed by significant gains on the Stanford Early School Achievement Test (SESAT) in verbal comprehension and mathematics concepts. These children's gains persisted in first grade (Pasnak, Madden, Malabonga, Martin, & Holt, 1996).

A second effort (Pasnak, Hansbarger, et al., 1996) extended these findings. One lesson emerged from these two studies: Gains in school achievement are likely to surface only after improvements in thinking have been in place for a while, rather than immediately. It takes 3 to 4 months of classroom instruction and educational exercises to lead to great enough improvements in overall educational achievement to appear on standardized tests. Recognizing the potential implications of these studies, this study focused on developing key reasoning abilities that are critical at the outset of elementary school. The central research question was whether investing resources in teaching the oddity principle, insertions into serial orders, and conservation to kindergarten children *who did not already possess these reasoning abilities* produced improvement not only in these abilities, but also in mathematical skills and concepts as well as in cognitive reasoning abilities. More specifically, would instruction in these concepts develop early numeracy more effectively, less effectively, or as effectively as an equal amount of instruction invested directly in numeracy? This issue had not been addressed in any previous research. Therefore, this study compared the effects of instruction focused on developing the cognitive abilities of insertions into series, oddity, and number conservation with mathematics instruction based on the state standards of learning. In this research, art instruction was used as a comparison procedure, designed to provide children with equal amounts of constructive activity and contact with researchers, but that would not have a direct impact on mathematics.

Research Questions

This study was designed to compare the effects of providing kindergarten children with instruction that focused on teaching the oddity principle, insertions into serial orders, and conservation with the effects of numeracy and art instruction on kindergartners' mathematics achievement as well as on their cognitive abilities. Specifically, the study addressed the following research questions:

1. Are there differences between the cognitive and numeracy groups, the cognitive and arts groups, and/or the numeracy and art groups on the numeracy measure?
2. Are there differences between the cognitive and numeracy groups, the cognitive and art groups, and/or the numeracy and art groups on the cognitive measures?

Method

Participants and Setting

Initially, 516 kindergarten students in 26 classrooms in a culturally and socioeconomically diverse, metropolitan school district were screened using 12 oddity test problems and 10 seriation test problems. Children who scored an overall total of 18 problems or more correct on the 22-problem oddity and seriation screening test were excluded because they appeared to possess the first two cognitive abilities targeted by the intervention. There was no screening on conservation, for efficiency's sake and because Waiss and Pasnak (1993) found that children deficient only in conservation had little benefit from the intervention. Hence, the remaining 102 kindergartners comprised approximately the lowest fifth of the kindergarten population, according to the oddity and insertion measures, in the schools where the study was conducted. They were likely to be at-risk academically because they lagged behind their classmates in the development of these thinking skills. The 102 children were formed into trios matched on their screening scores. The members of each trio were randomly assigned to one of three types of instruction conducted in sessions of equal duration and frequency, as explained in the procedure section. During the study, 8 children moved away from the school. Because the children were in matched trios, when a member of the trio moved, the other 2 children in the trio could not remain in the study. As a result, the final sample included 78 kindergartners, including 32 girls and 46 boys. Of these kindergarten students, 7 in the group that received the cognitive intervention were Hispanic/Latino, 7 were African American, 6 were White European American, and 6 were Middle Eastern. In the first (numeracy) comparison group, 6 were Hispanic/Latino, 9 were African American, 5 were White European American, and 6 were Middle Eastern. In the second (art) comparison group, 7 were Hispanic/Latino, 6 were African American, 8 were White European American, and 5 were Middle Eastern. The differences in ethnic composition did not approach statistical significance, $\chi^2(4) = 1.60, p > .05$. All students were 5 years old by the end of September of the kindergarten year.

Materials

Instruction. Sixty sets of objects found in homes, nature, and various stores were used to teach the oddity principle. Each set had three objects that were identical and one that differed in one dimension. For the first 20 games, the odd object differed only in form (e.g., three squares and a triangle, or three round beads and an oval bead). The next 20 games each had one object that differed from the other three only in size (e.g., three large paper clips and a small one or three small buttons and a large one). In 10 cases, the odd object was larger than the others and in 10 cases it was smaller. For the 20 orientation oddity games, the four objects in each set were identical, but three were presented horizontally and one vertically (or vice versa), three were slanted 45 degrees one way and one 45 degrees the other (or vice versa), or three faced left and the other right (or vice versa).

Insertions into series were taught with 65 sets of everyday objects ranging from Band-Aids® and beans to toy animals and washers. Fifteen of the sets had three objects, 20 had four, 15 had five, 5 had six, 5 had seven, and 5 had eight. Increasing the number of objects in the sets, with correspondingly more places where an object could be inserted, was intended to require and produce increasing mastery of insertions by the learners. In each set, the objects, whether similar or different in shape, differed progressively in size. Fifteen conservation problems used for instruction had 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 31, 33, and 35 ordinary objects (e.g., bobby pins, brads, hex nuts) that could be placed in rows of 3–14 objects. The increasing number of objects in each set was intended to produce increasing mastery of conservation, by decreasing the ease of assessing number at a glance or by counting. Numeracy instruction was carried out with sets of foam numbers, many items such as stars, disks, and cubes that could be counted, and cardboard replicas of coins.

The art group worked in pencil, crayon, paint, plasticine media, construction paper, and art worksheets.

Measures. Measures included oddity, insertion, and conservation tests consisting of new objects, the classification scale from the Otis-Lennon School Ability Test (OLSAT) Primary I, and the Woodcock-Johnson III (W-J III) Problem Solving (quantitative) scale. The 12-problem oddity test had four form oddity problems, four size oddity problems, and four orientation oddity problems. These were not the same objects as those used in screening, but they were comparable in the way they embodied the oddity principle in form, size, and orientation. This was a test of whether the children could apply the oddity principle to problems that had new objects, but were similar in format to those used in instruction. Likewise, a 10-problem insertion test measured whether the children could seriate new sets of objects. Two problems had three objects, two had four objects, two required inserting a fourth object into a series of three, two required inserting a fifth object into a series of four, and two required inserting a sixth object into a series of five.

Each problem in the 10-problem conservation test had two rows of 3 to 10 objects. The number of objects in each row would be equal or different. A row would be expanded or contracted while the child watched, and the child would be asked whether the number of objects in the rows was still equal or whether the row that initially had more objects still had more. The child was then asked to explain why. For the last eight problems, an object would be added to or subtracted from a row, which might make the number of objects in a row more different or might equalize them. The test systematically varied the terms used in the questions asked of the children and the order in which the rows were referenced.

The OLSAT classification scale has 12 rows of five drawings. The drawings in any row are all different, but four are similar in some way and the other differed. The nature of the oddity relation in each row is very variable: one involves four children playing with water while one plays with snow, one has four quartets of geometric forms and one trio, one has four children who are missing teeth and one who is not, and so forth. This is a measure of very broad generalization of the oddity principle because the children must apply it to drawings rather than objects. The drawings in each set vary in many irrelevant details, and overall form, size, or orientation never are relevant to the solutions.

Although designed for first graders, the OLSAT Primary I can be given to kindergartners. Previous experience (Pasnak et al., 1991) indicated that the easiest (classification) scale was appropriate for children like those involved in the present research, but the more demanding Analogies and Omnibus scales were too difficult; hence, the latter scales were not used. The manual reports Kuder-Richardson (K-R 20) kindergarten reliabilities of .88–.90 for the test as a whole, but does not provide separate data for the classification scale.

This study utilized the first 28 problems of the Woodcock-Johnson III Applied Problems (quantitative) scale. This scale starts with, "Show me just one finger," and proceeds through problems such as, "If you took three of these balloons away how many would be left?" to "How much money is this?" The problems refer to pictures of the objects in question. Testing is terminated when a child misses four consecutive problems. The manual gives reliability coefficients of .79 at age 4 and .86 at age 5 for the quantitative scale, with standard errors of 4.6 and 3.7 respectively. There is no information concerning the reliability of the first 28 problems of the scale.

Procedures

At the beginning of the study, researchers trained to conduct the screening assessment met individually with all kindergarten students in the 26 classrooms to administer the preassessments. The kindergartners who scored fewer than 18 problems correct on the 22 problem (12 oddity, 10 insertion) screening test were divided into trios matched as closely as possible. The members of each trio were randomly assigned to one of three groups: the experimental group (cognitive intervention), and two comparison groups (numeracy instruction or art instruction). These matched trios were "yoked" in the sense that the number of instructional sessions the members of each trio received was governed by the number of sessions the child in the cognitive group received. When that child completed all of the cognitive instruction, the instruction of all three children in the trio ended. Thus, some trios had many more sessions than others (M = 43.2, SD = 8.11). Because children in the cognitive group developed the oddity principle, insertions into series, and number conservation at different rates and instruction was paced to match each child's individual rate of development, there was variability in the number of sessions among the trios. It also should be noted that terminating instruction when a child had completed the cognitive intervention disadvantaged the children in the other groups. Had the duration of instruction for each trio been based on when the numeracy child mastered the numeracy objectives, the outcomes might have been different. Although all children in a trio received the same number of sessions, the level of mastery they had achieved in their respective domains presumably differed.

The assignment of comparison group children to receive constructive instruction is an advanced design recommended by Pasnak and Howe (1993). In many types of sociobiological research, the effect of a high or representative value of an independent variable is evaluated by contrasting it with a low or zero value of the variable. That procedure is problematic in educational settings. Educators, school administrators, and parents have legitimate objections to assigning some children to receive no special assistance while classmates receive a great deal of it. Even if no one objected, good control in an educational setting is not really served by such a control group. The control group would be shortchanged if it did not receive equal investment of instructional resources, time, and attention, so as to engender equal expectations of success. Absent such equalizations, the experiment would be confounded. The advantage of the experimental group might arise from the advantages of attention and expectations—a type of Hawthorne effect. Hence, comparison groups should involve instruction that is equivalent to that of the experimental group except for the outcome it is designed to produce. Instructing children in such a way that they will not benefit at all is not usually acceptable and not likely to match the experimental instruction adequately if it were attempted. Hence, beneficial instruction using a different technique, or aimed at some other domain, is a better form of control (Pasnak & Howe, 1993). The preschool curriculum comparison study conducted by Schweinhart, Weikart, and Larner (1986), the studies of preschoolers by Ciancio et al. (2001) and Pasnak et al. (2006), and a study of kindergartners by Pasnak, Kidd, et al. (2007) are examples of the employment of comparison groups that received equal and constructive instruction of a different nature than that employed with the experimental group.

The children receiving art instruction served as a comparison group that had equal contact with the researchers and equal investment of time and resources in constructive activities. Although art involves elements of abstraction, these activities cannot be reasonably expected to have as much of an effect on the dependent variables as the activities in which the other two groups participated. This is not to suggest that art is not a cognitive activity and that children do not benefit from engagement in art activities. Rather, it is to make the point that the art activities engaged children differently from the cognitive intervention and the numeracy instruction. The children receiving numeracy instruction also had equal time, attention, and resources, but these were invested in activities known to directly affect one of the dependent variables: numeracy. Hence, this group provided a test of whether the effect of the cognitive instruction on numeracy was greater than, less than, or equal to that of providing instruction designed to increase numeracy achievement.

The experimental instruction was carried out by 75 college students who received academic credit for spending one morning per week in the kindergarten classrooms. Hence, each classroom had three college students (researchers) with each visiting one morning per week. Each researcher met with each group of children—cognitive, numeracy, and art—for approximately 10 to 15 minutes, so that each group of children was scheduled for three sessions of experimental instruction per week. These sessions were conducted back-to-back during center time in the morning. The researchers were instructed to rotate which group was instructed first. Hence, the children who were matched had equal numbers of sessions at equivalent times with equal number of interruptions for fire drills, special programs, and other special events. The number of children in each group varied from one to three, depending on how many children in a classroom had been randomly assigned to each of the three instructional conditions (cognitive, numeracy, or art). If a child from a matched trio moved away, the instruction of that trio was discontinued.

The children in the cognitive group were instructed in oddity, insertions into series, and conservation. To keep the children interested and motivated to learn concepts that were over their heads, it was necessary for the children to enjoy the learning experience. Enjoyment of what they are doing may be important when children are trying to learn thinking processes more abstract than those they currently possess: "in play a child is always above his average age, above his daily behavior, in play it is as though he were a head taller than himself" (Vygotsky, 1978, p. 129). Hence, the researchers did not maintain a rigid structure to the instruction. Rather, they kept it playful and allowed the children to direct it, so long as the focus remained on learning the concept being taught.

The researchers, as well as the children, held toy ponies or dinosaurs and the adult's animal would ask a child's animal if it could help solve a problem. The adult phrased the questions as, "My pony (dinosaur) is hungry, can you feed him the thing that is different?" when playing the 60 different oddity games. The first oddity games were oddity by form. These problems consisted of three objects that were the same but different colors and a fourth object that was different in shape and color from the other three. After the children were able to select the different object by form, then they progressed to playing games that consisted of oddity by size problems. The adults frequently asked the children to help them find the different one or asked that they show their pony which one was different. If the child's pony was correct, the adult's pony greeted the choice with enthusiastic neighing, head-bobbing, tail-swishing, or high fives. If the child's pony chose incorrectly, the adult's pony refused the choice, pushing the object away, or shaking its head from side to side, and asking for another choice.

The sequence of the oddity by size problems was particularly important. If in one problem the different object would be a small object grouped with three large objects, then in the next problem the different object would be a large object grouped with three small ones.

The last type of oddity game consisted of oddity by orientation problems. At the beginning, the objects were oriented such that three were placed vertically and one was placed horizontally. The position of the odd object was changed each time a problem was given to the child. After the children consistently answered these problems correctly, the orientation of the objects changed so that they were slanted in one direction at a 45-degree angle and the different object was slanted in the opposite direction at a 45-degree angle. After children demonstrated proficiency at this type of task, they were given problems in which three objects faced left and the different object faced right.

In the 65 insertion games, the children were asked to line up the items from small to big. The adult began by demonstrating what was to be done by dumping three objects on the table and saying, "Look what my pony can do. He can line these things up from little to big." After the adult had lined up the objects using his pony, the objects were passed to a child and the adult's pony would say, "Can your pony do that?" After the first problem was completed, a child was then given the objects for the second problem and the adult's pony would prompt the child's pony, "Let's line these up from little to big." This procedure was followed for all 15 three-object problems and repeated for the 20 four-object problems. Then the four-object problems were repeated. This time, the children were given three of the objects in a set, but one of the midsized objects was held back. The child's pony was asked to put the objects in order from small to big. Upon completion of this task, the adult's pony then told them, "Oops! I forgot to give you this one. Can you show me where it goes?" When the children reached proficiency with those tasks they were moved to five-object problems. Again, one of the midsized objects was withheld and the child's pony was asked to order the remaining four objects and then insert the missing object into the sequence. If the child put the objects in an incorrect sequence, the adult used his pony to gently nudge the objects into the correct order and reminded the child's pony that the objects went from little to big. Insertion problems with 6, 7, and 8 objects followed.

In the conservation games, the adult constructed two rows of three or more equally spaced items. The adult's pony asked the child's pony, "Do you have more, do I have more, or do we have the same number of things?" After getting the child to agree that the two rows were equal, the adult then expanded the child's row and contracted their own row. Then the child would be asked, "Are there still the same number of hearts (or stars, etc.) in my row and your row or are there more in your row or more in my row?" As the child's understanding increased, the number of objects in the rows was increased, additions and subtractions of objects from one or both rows were introduced, and the rows might be equal or unequal to start with, so that the addition or subtraction might make the number of objects in them equal or unequal. The phrasing and order of the questions the adult asked were varied systematically.

Children receiving instruction in numbers also were engaged in a playful and self-directed approach to learning. Using the ponies to facilitate instruction, the children were first taught the numbers 1–10 using foam numbers. Their ponies were asked to select a specific number from several numbers displayed. They were then asked to verbally identify the numbers selected by the adult's pony. When the child recognized the numbers 1–10, the adult selected a number and gave the child some small blocks. The adult's pony then asked the child's pony to help him count out that number of blocks. In the beginning, if children were asked to count out 10 blocks, they would only be given 11 or

12 blocks. As they became more proficient, more extra blocks were added. After learning numbers 1–10, the children then went on to learn additional numbers in groups of three to five at a time. New numbers were always mixed in with the old numbers. For example, a child who had learned numbers 1–10 might be presented with 17 or 18 blocks, and the child's pony asked to count out 7, 10, 11, 12, or 13 of them. In order to keep the children interested, they periodically played a version of bingo that encompassed the numbers they knew. Children were given bingo cards with numbers 1–5, 1–10, 1–15, and 1–20 on them. After learning numbers up through 30, the children were taught to count by fives using the blocks. Finally the children's ponies were asked to give the adult's pony a number that was more than the number being held by the pony. For example, the adult would ask, "Can you give me a number that is more than 10?" The same type of questioning was used to teach the concept of "less than."

The comparison group participating in art instruction worked in pencil, crayon, paint, and plasticine media representing persons, specific animals and plants, self-portraits, and inanimate objects. They identified colors, shapes, and patterns and developed motor skills in pasting, gluing, folding, cutting, modeling, printing, and stamping. All activities for the numeracy and art groups were designed to meet specific outcomes from the state standards of learning.

When children in the cognitive group mastered insertions into series, oddity, and number conservation, their instruction and that of the yoked numeracy and art children was terminated. In late May and the first week of June, postassessments were conducted by testers blind to the groups to which the children had been assigned. All 78 participants whose instruction had been completed were assessed on oddity, insertion, and conservation. They were administered the Woodcock-Johnson III Applied Problems scale (numeracy) and the oddity scale from the Otis-Lennon Ability Tests (OLSAT). A priori comparisons on the cognitive measures were planned between the children receiving the cognitive instruction and those receiving the other forms of instruction. In addition, comparisons were planned between all three groups on the Woodcock-Johnson III mathematics measure. The comparison between the children receiving the cognitive and numeracy instruction was central to the research. Additionally, previous research (Pasnak, Hansbarger, et al. 1996; Pasnak et al., 1991) had shown a difference between the cognitive group and a control group on another standardized measure of kindergarten mathematics (Stanford Early School Achievement Test).

Results

Initially the sample consisted of 1 to 9 (M = 3.92, SD = 2.31) children in each class. Instruction of all members of a matched trio, who might be in different classes, was discontinued if a member moved away, leaving 1 to 6 children in each class for the final sample (M = 3.00, SD = 2.11). This procedure was designed to produce equal attrition for each experimental condition and to ensure that the children lost from each condition were children who had been matched in initial ability. Hence, the final sample consisted of 26 children from the experimental group and 26 from each of the comparison groups. The mean number of children within each class receiving each form of instruction was 1, but the range was 0 to 3 (SD = .77).

Mean scores on the oddity screening measure were 6.99 (SD = 2.49) for the children who were later randomly assigned to cognitive instruction. The matched children who were randomly assigned to numeracy instruction averaged 6.77 (SD = 2.51), and those assigned to art instruction averaged 7.13 (SD = 2.51). These means do not differ significantly, F (2, 75) = 0.97, p =.48. Screening scores on the seriation measure averaged 3.90 (SD = 2.59) for the cognitive group, 3.93 (SD = 2.71) for the numeracy group, and 3.74 for the art group (SD = 2.72). These means also did not differ significantly, F (2, 75) = .35, p = .71.

Descriptive analyses of the data also were performed to determine the mean and standard deviation of each group (cog-

Table 1

Means and Standard Deviations for the Dependent Variables

Group	Cognitive	Numeracy	Art
	Oddity Scores (12 possible)		
Mean	11.00	9.81	10.11
SD	.96	1.73	1.34
	Seriation Scores (10 possible)		
Mean	8.56	6.96	6.67
SD	2.42	2.63	3.02
	Conservation Scores (10 possible)		
Mean	7.07	4.77	5.15
SD	3.75	2.82	3.91
	Numeracy (Woodcock-Johnson III) Scores		
Mean	20.25	18.56	17.48
SD	3.75	2.82	3.91
	O-LSAT Oddity Scores (12 possible)		
Mean	8.00	6.70	6.78
SD	2.57	2.97	2.81

nitive, numeracy, and art) in June on the oddity, seriation, and conservation tests as well as on the W-J III Applied Problems scale and the oddity scale on the OLSAT (see Table 1), and the correlations between these measures (see Table 2). All outcome measures were correlated; the smallest correlation was between the oddity and seriation measures.

Because there were five dependent variables, a MANOVA, shown in Table 3, was conducted. MANOVA guards against the inflated p resulting from multiple comparisons. For example, an ANOVA indicated a significant overall difference on the oddity measure, whereas the more conservative MANOVA did not. The lack of a significant difference reflects in part that only one of the three groups could be expected to have an advantage from instruction, and in part that the group means in June were approaching the ceiling for this test (the cognitive group averaged only one error). The overall F for the MANOVA was

Table 2

Pearson Product-Moment Correlations Between Scores on Five Dependent Variables

	Seriation	Conservation	Woodcock-Johnson III	OLSAT
Oddity	.234*	.397**	.311**	.461**
Seriation		.357**	.367**	.406**
Conservation			.445**	.456**
Woodcock-Johnson III				.395**

*** Correlation is significant at the .05 level. ** Correlation is significant at the .01 level.**

significant, and there were significant overall differences among the three groups on the seriation, conservation, and Woodcock-Johnson III measures.

Although the overall difference in oddity scores was not statistically significant, according to Winer (1962, p. 85), a priori (planned comparisons) are always justified whether or not an overall F is statistically significant. Accordingly, Least Significant Difference (LSD) analyses were conducted (see Table 4). It is important to note that due to the number of statistical tests run at alpha = .05, there is a heightened possibility of making a Type I error. Scheffé (1959) suggested that an alpha value of .033 be employed for testing independent comparisons between three groups, which is the case here. (To the extent the groups were not independent, the alpha value would be increased). All tests are two-tailed, although one-tailed tests would be justified for those researchers who employ them.

The LSD analyses showed that there were statistically significant differences between the cognitive intervention group and the numeracy instruction group on the oddity (d = .88), insertions (d = .63), and conservation (d = .82) tests. The effect size on the insertion test is medium, a difference in the groups of scores likely to be visible to the naked eye of a careful observer and approximately the average size of observed effects in various

Table 3

MANOVA for Five Dependent Variables

Multivariate Tests						
Effect		Value	*F*	Hypothesis *df*	Error *df*	*p*
Intercept Wilks' Lambda		.02	773.30	5	70	<.001
Intervention Wilks' Lambda		.76	2.03	10	140	.034

Tests for Differences Between Groups					
Source		*df*	*MS*	*F*	*p*
Corrected Model	Oddity	2	9.34	2.78	.068
	Seriation	2	49.41	7.63	.001
	Conservation	2	52.94	5.85	.004
	Woodcock-Johnson III	2	37.57	3.12	.049
	O-LSAT	2	14.77	1.95	.149
Intercept	Oddity	1	8595.08	2557.05	<.001
	Seriation	1	4782.39	738.63	<.001
	Conservation	1	3029.08	334.56	<.001
	Woodcock-Johnson III	1	30678.42	2551.02	<.001
	O-LSAT	1	4415.20	581.72	<.001
Intervention	Oddity	2	9.34	2.78	.068
	Seriation	2	49.41	7.63	.001
	Conservation	2	52.94	5.85	.004
	Woodcock-Johnson III	2	37.57	3.12	.049
	O-LSAT	2	37.57	3.12	.049
Error	Oddity	73	3.36		
	Seriation	73	6.48		
	Conservation	73	9.05		
	Woodcock-Johnson III	73	12.03		
	O-LSAT	73	7.59		

Note. **Oddity Adjusted R^2 = .06, Seriation Adjusted R^2 = .14, Conservation Adjusted R^2 = .11, W-J III Adjusted R^2 = .05, O-LSAT Adjusted R^2 = .02**

fields according to effect size surveys (Cohen, 1992). The effects on oddity and conservation are large, representing approximately four fifths of the standard deviations on the oddity and conservation tests. On each, the cognitive group outscored the numeracy group, with the latter making roughly twice as many errors, a difference that was statistically significant. No statistically significant differences existed between these two groups on the Woodcock-Johnson III Applied Problems scale (d = .51) or on the OLSAT (d = .47).

There were statistically significant differences between the cognitive and art groups on the oddity (d = .77), seriation (d = .69), and conservation (d = .65) tests. These are medium effects. Again, the cognitive group made about half as many errors on each of these tests as the control group. The cognitive group was better than this control group on the Woodcock-Johnson III Applied Problems scale (d = .72), a difference that was statistically significant. The effect size was again medium. The cognitive group's average score at the end of kindergarten is a bit above that made by children in the first month of first grade (1.0) on the national norms for this test; the comparison group's score is at the K-6 level. The difference between these groups on the OLSAT was not statistically significant (d = .45).

There were no statistically significant differences between the numeracy and art groups on any of the dependent variables (see Table 4).

Discussion

When students have not developed key reasoning abilities that are critical upon entering kindergarten, instruction in oddity, insertions into series, and number conservation appear to promote early abstract thought and also to enhance early numeracy, as measured by the Woodcock-Johnson III. The results of this study support earlier research that suggested that cognitive functioning can be enhanced through instruction on classification, number conservation, and insertions into series (Pasnak, Hansbarger, et

Table 4

Post Hoc Multiple Comparisons of Groups
(Least Significant Differences)

Groups	Variable	Difference	*SD*	*p*	*d*
Cognitive vs. Numeracy	Oddity	11.19	.383	.003	.88
	Seriation	1.60	.719	.025	.63
	Conservation	3.00	.767	.031	.82
	Woodcock-Johnson III	1.69	.899	.065	.51
	O-LSAT	1.30	.760	.093	.47
Cognitive vs. Art	Oddity	.89	.318	.007	.77
	Seriation	1.89	.747	.014	.69
	Conservation	1.92	.814	.022	.65
	Woodcock-Johnson III	2.77	1.037	.010	.72
	O-LSAT	1.22	.731	.101	.45
Numeracy vs. Art	Oddity	-.70	.429	.485	.20
	Seriation	.29	.763	.703	.10
	Conservation	-.38	.809	.660	.12
	Woodcock-Johnson III	1.08	.931	.252	.32
	O-LSAT	-.08	.758	.791	.03

Note. Exact *p* values are given to avoid the use of statistical symbols for greater than or less than. An alpha value of .033 was used for null hypothesis tests.

al., 1996; Pasnak, Madden, et al., 1996). The difference between the cognitive group and art group on the oddity, insertions into series, and number conversation tests indicates that instruction in these areas can promote early abstract thought. The effect sizes indicate that the effects of such instruction are appreciable.

In addition, the difference between the cognitive and art groups on the Woodcock-Johnson III Applied Problems scale offers evidence that instruction on oddity, insertions into series, and number conservation can promote achievement on numeracy tasks. In this case, the difference due to the cognitive instruction amounted to about a 3-month gain on a standardized scale of the development of early numeracy. The Woodcock-Johnson III Applied Problems scale does not contain any oddity, insertions into series, or conservation items; therefore, this difference cannot be attributed to a direct transfer effect. Instead, it is reasonable to conclude that the higher achievement of the cognitive group is due to the cognitive intervention designed to promote the kindergarten students' reasoning ability. All children received year-long group mathematics instruction from their classroom teacher as part of the Program of Studies mandated by the local school system. This instruction presumably accounted for most of the mathematics learning by all students, and dwarfed, in duration and content, the brief sessions conducted as part of the present research project. The probable mechanism for the enhanced performance of the cognitive group is that cognitive improvement led to the enhanced understanding of the classroom instruction, which resulted in better performance on the Woodcock-Johnson III numeracy measure. It also is likely that this effect is limited to children who do not employ these key reasoning abilities that most kindergarten students possess. These are the earliest purely abstract concepts that children develop and may be essential to understanding some aspects of kindergarten curricula.

As expected, there were differences between the cognitive and numeracy groups on the oddity, seriation, and conservation tests. The cognitive group outscored the numeracy and art groups on each measure; the latter two groups did not differ on any of these measures. This verifies that the cognitive instruction produced gains in the cognitions taught. The art and numeracy groups were not expected to differ in this respect because neither was taught oddity, seriation, or conservation.

Although it would be reasonable to expect the numeracy group to score higher than the cognitive group on the Woodcock-Johnson III, because the oddity principle has no obvious connection to numeracy and seriation problems only involved the dimension of size, this was not the case. Instead, there was no difference between the cognitive and numeracy groups. This finding indicates that developing the abstract thinking involved in oddity, insertions into series, and conservation has some importance when learning kindergarten mathematics. It might reflect the difficulty children have when learning mathematics without these supporting concepts.

It also is important to note that the Woodcock-Johnson III scores for the numeracy and art groups were not significantly different. It would be reasonable to expect the numeracy group to outscore the art group on an assessment that measured numeracy. The fact that this did not occur may be a second indication of the difficulty children whose cognitive abilities are not fully developed have learning mathematical concepts when they interact with the mathematics curriculum. An important caveat, however, is that other tests of mathematics may have shown differences in understanding mathematics that the Woodcock-Johnson III did not reveal. It also is possible that the difference between the two groups would have been greater had all children in the numeracy group completely mastered the numeracy instruction, instead of receiving the same number of sessions that children in the cognitive group required to master the cognitive instruction. The same limitation applies to interpretation of the nonstatistically significant difference between the cognitive and numeracy group.

Implications

The results of this study make a case for providing kindergarten children who do not possess key reasoning abilities with systematic instruction on the oddity principle, insertions into series, and number conservation. If teachers spent as little as 10 minutes a day, three times a week on cognitive interventions, it is likely that kindergartners involved in these activities would increase their reasoning abilities. These enhanced abilities would enable them to take advantage of the regular classroom instruction in mathematics, a task that might otherwise be beyond them because they do not possess the cognitive skills needed to fully benefit from what is being taught. As their cognitive abilities are enhanced, their opportunity to learn from classroom instruction would most likely be increased, and they might benefit from instruction in ways similar to those kindergartners who already possessed the cognitive abilities to succeed in school.

It is important to note that the cognitive intervention did not involve teaching to a specific test, improving children's test-taking abilities, or teaching the kind of content that is assessed via tests in general. Instead, it was a child-friendly form of guided play that is developmentally appropriate for those children who may be less mature than peers at the outset of their first contact with formal schooling. The abilities strengthened were those that children develop naturally in the course of their normal daily activities and investigation of the world around them, both inside and outside the classroom. Because these are normal thinking abilities that children eventually develop and continually strengthen, rather than artificial problem-solving or test-taking techniques imposed by adults, there is, as Pasnak, Madden, et al. (1996) pointed out, no reason for children to lose them in the months or years ahead.

It also is important to note that, just as when the children participated in the typical classroom mathematics instruction, they would have engaged in classroom activities that promoted abstract reasoning. This cognitive intervention does not take the place of a rich classroom environment that promotes children's inquiry into the world around them and enhances problem solving in an authentic environment. Rather, the implementation of this targeted instruction assumes that children are engaged in a variety of learning experiences both at home and at school that promote cognitive growth and academic achievement. The research shows that including 10 minutes three times a week of playful, child-directed instruction focused on enhancing the oddity principle, insertions into series, and number conservation provided supplemental instruction that led to increased mathematical achievement for children who did not initially possess these key reasoning abilities. It does not, however, clearly define the mechanism between the abilities taught and the mathematics achievement that results. Transfer is implicit, but the linkages are not directly tested.

Limitations

A major limitation of the research is that the effectiveness of the intervention depends on its being a good match for the cognitive development of the children who receive it. Pasnak, Hansbarger, et al. (1996) reported differential effects when a more limited version of the intervention was attempted in an urban and a suburban school. Most children in the urban school did not master seriation or conservation and consequently did not show gains on standardized achievement tests, while the children from the suburban school had much better results. Waiss and Pasnak (1993) reported that a preliminary attempt to apply the intervention in a suburban school was ineffective because most of the children were too advanced for it, having already mastered form and size oddity and seriation. These two studies indicate that children can be too advanced cognitively, or not advanced enough, to be aided by this intervention. Income and family issues combine with individual differences play significant roles in how children develop cognitively and how well they do in school. Children who are more cognitively advanced should be offered the intervention at an earlier age, those not as cognitively advanced at a later age, for the intervention to have a positive impact on numeracy.

Another limitation is that the length of the interventions was determined by when the children in the cognitive group had mastered all of their objectives. There is a possibility the results would have been different if instruction had been concluded based on the progress of the numeracy or art groups rather than the cognitive group. Although matched as carefully as possible, it is possible that a child in the trio may progress through the activities at a slower or faster rate than the other children in the trio.

Future Research

These findings also point out the need for further investigation into the effect of instruction designed to improve cognitive functioning on other kinds of academic achievement. If the students who received instruction in oddity, insertions into series, and conservation reached a similar level of mathematics achievement as those who received numeracy instruction, it is possible that the cognitive group may also perform literacy tasks better than those engaged in art activities and perhaps as well as a group receiving literacy instruction. Preliminary research (Pasnak, Hansbarger, et al. 1996; Pasnak et al., 1991) supports the idea that children receiving the cognitive instruction develop better verbal comprehension than control children who do not receive any special instruction. However, comparisons with those who receive literacy instruction have not yet been made. Such comparisons might provide additional insight into the potential effect of providing systematic instruction on key reasoning abilities.

Conclusion

The implications of the current research are significant. Teaching oddity, insertions into series, and conservation to kindergarten students who do not possess these reasoning skills is a promising approach to promoting early abstract thought and mathematical achievement. The results indicate that it is possible to provide instruction that will help children gain the early abstract thought needed to be successful in school. In addition, there is evidence to suggest that children who are helped to develop early abstract thought perform as well as or better than those who are provided extra numeracy instruction and better than those engaged in art activities. Implementing cognitive instruction with those who need it may be the key to ensuring that children who do not possess key early reasoning abilities when they enter kindergarten can benefit from their daily classroom instruction and increase their mathematical achievement.

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