Teaching children to understand operations in early childhood

In WA's Kimberley, educators provide focused teaching in the early childhood years by using the *First Steps in Mathematics* materials. As a result students come to "understand the meaning, use and connections between addition, multiplication, subtraction and division." (WA Curriculum Framework)

Kununurra is isolated: it is hundreds of kilometres from other settlements in the far north of WA. Around 5000 residents live in and around the town. Kununurra District High School (KDHS) with a student population (K–12) of approximately 800 students is the largest school in the remote East Kimberley. It has some unique challenges:

- a high proportion of transient students;
- forty percent of students are Aboriginal and many speak English as a second language;
- Aboriginal students consistently perform less well in the West Australian Literacy and Numeracy Assessment (WALNA) for numeracy than the general student population;
- teachers experience professional isolation;
- there are high financial costs, time commitments and logistical difficulties involved in getting personnel to central meeting places.

The Western Australian State Government recognised the need for greater parity of literacy and numeracy achievement across all groups of students but especially for Aboriginal students with a language background other than English, boys and students in rural and remote locations. It funded the *Getting It*

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shares effective

teaching strategies

that emerged from

a rather special situation

in remote Western Australia,

but should be useful for

teachers in all schools.

Right Literacy and Numeracy Strategy (GiR-LNS) implemented through the Department of Education and Training of Western Australia. This strategy with a numeracy focus was introduced at Kununurra District High School in 2003. It provides a targeted and coordinated program of additional specialist teaching personnel (of whom I am one), professional development derived from the *First Steps in Mathematics* materials, and support from the Central Office team who provide the professional development.

As the appointed *Getting it Right* specialist teacher I have been involved in 25 days of professional development which has heightened my understanding that classroom teachers make the difference to numeracy outcomes achieved by students; deepened my knowledge of the *Western Australian Curriculum Framework Mathematics* outcomes; enhanced my repertoire of successful numeracy teaching strategies particularly for those students not making sufficient progress; enabled me to collect and analyse credible diagnostic and summative student performance data to inform the planning and teaching cycle; helped me to participate in cohesive, data driven, whole school planning for numeracy; and to engender and participate in two way home school collaboration and communication in support of numeracy development.

I now have planning time out of the classroom and work "shoulder to shoulder" in the classroom with my Kindergarten to Year 3 teaching

colleagues. I support the diagnosis and monitoring of student learning and model teaching strategies that are designed to help children achieve the outcome particularly in the area of number and measurement. The focus of my work with each teacher in each class is to find out what the children know, decide what mathematics the children need to learn next and then together with the teacher find the most effective ways to teach that mathematics. One area of mathematics that we have found particularly challenging is that of having children understand operations. This article seeks to discuss and describe the distinct difference between Operations and Calculate and will outline the approach taken to teach children in the early years to enable them to achieve the Understand Operations outcome. Although the focus of this paper will be mainly on addition and subtraction, a lot of work has also been done by the teachers on multiplication and division.

The difference between operating and calculating

The *Western Australian Curriculum Framework* Mathematics Student Outcome Statements for Number is divided into four outcomes: Understand Numbers, Understand Operations, Calculate, and Reason about Number Patterns. When teaching Understanding Operations teachers develop curricula to help students "understand the meaning, use and connections between addition, multiplication, subtraction and division… as distinct from how to carry out the calculations associated with them. They decide which operation is needed in contexts where no obvious verbal cues indicate which operation is expected" (Willis, Devlin, Jacob, Tomazos, Treacy & Powell, 2004). Children learn to recognise a wide range of problem types, discern which of the four operations apply, learn to see the relationships and connections between the four operations and develop flexibility in their thinking when it comes to solving self generated, orally presented or written problems. The emphasis is on choosing the correct operation and being able to justify the thinking involved in the choice so that when the calculation is done, students arrive at the right answer for the right reasons.

The Calculate outcome states that students need to learn to "choose and use a repertoire of mental, paper and calculator computational strategies, meeting needed levels of accuracy and judging the reasonableness of results" (Education Department of Western Australia, 1998) for each operation. Before students can employ a mathematical process to arrive at the correct answer they need to be "confident of their capacity to deal, correctly and efficiently, with everyday counting and computational situations" (Willis et al., 2004). Students learn to "count a collection one-to-one, recognise skip counting in two and threes as more efficient, and combine collections using strategies such as counting on" (Willis et al., 2004). The emphasis is on correctly using the numbers presented in a problem so that the right answer is calculated.

Where we started
— Diagnosing students' understandings

To develop an understanding of how to move the children on in their understanding of operations, we needed to first find out what the students knew about numbers and operations. This required the collection of different information for different year levels:

In the Kindy (4 year olds) we needed to know whether the children understood the meaning of the words "more", "less", and "the same as". (Aboriginal children talk about "biggest mob", "littlest mob" and "same".) This was done through a diagnostic task in the form of a game children play with a partner using subitising cards displaying up to six items. Each child turns over a card at the same time and the child that has "more" dots on their card wins the two cards. If both children turn up the same amount on their cards, they turn over another pair of cards until a winner is found. We also used these cards to see if children could subitise small quantities.

We also needed to know whether the children had acquired some basic counting principles such as touching each object only once as they count; saying the numbers in the correct order as they count; and, knowing that the last number they arrive at tells them the size of the whole collection not the last item

counted (Willis et al., 2004).

We needed to listen to the children count and hear how well they could recite the number string.

We also needed to know whether they were able to do a more complex counting task where they are asked to get a number of blocks, say five, from a basket of blocks (Willis et al., 2004).

In Pre-Primary (5 year olds) we used the same tasks described above to find out what the children knew about subitising and counting. We also exposed the children to a variety of story problem types. To begin with, we were interested to find out which problem types the children could solve and how they went about finding the solutions. Traditionally, result unknown addition and subtraction questions have been asked of students (e.g., Sal had 3 marbles and Tess gave her 4 more. How many marbles does Sal have now?). When this type of problem is written it looks like: 3 + 4 = ? The 3 in this problem is the start, the 4 is the change and the result is unknown. Children were very challenged when trying to solve start unknown (Tom had some marbles. John gave him 5 more. Now he has 12. How many did he have in the beginning?) and change unknown problems (Tom had 7 marbles. John gave him some more. Now Tom has 12 marbles. How many did John give him?). They were also challenged by *compare* problems (e.g. Dan had 5 bananas and Sam has 3 more than Dan. How many

bananas does Sam have?) (Willis et al., 2004).

In Year 1 we used the same tasks again but with larger quantities to find out what the children knew about subitising and counting. We were also interested to see how sophisticated the children's thinking was when solving story problems. We introduced more problem types. One is called *combine* (Tom had 3 red marbles and 4 blue marbles. How many marbles did he have altogether?) and the other is called *equalise* (Tom had 7 red marbles and 4 blue marbles. If he paired each red marble with a blue marble, how many red marbles will not get a partner?) (Willis et al., 2004). I expected the children to begin writing number sentences to communicate how they thought about these addition and subtraction number story problems. These children were also exposed to some different types of Repeating Equal Quantities multiplication and division number story problems to see how they could solve them.

In Year 2 we initially needed to find out what kinds of number story problems the children could solve and for which types of problems they could write number sentences using numbers up to 20. They also had exposure to the part-part-whole approach to solving number story problems.

In Year 3 we wanted to find out which types of number story problems the children could solve for all four operations and how well they communicated their mathematical understandings through number sentences. The numbers used were slightly larger again (as high as 50). A greater emphasis was placed on finding out if they understood that if the whole quantity is unknown, addition is required, and if only one of the parts is unknown subtraction will be required. A diagnostic task was used. It asked the children to demonstrate their understanding of inverse relationships by getting them to write a number sentence from a number story problem so that a calculator could be used to find the solution to the problem. The emphasis in this task was not on solving the problem, but rather on writing the correct number sentence.

What we found out

Each week I met each classroom teacher and their Aboriginal and Islander Education Officer (AIEO) for a collaborative planning session. Our starting point was to work out what the students knew. We used the FSiM Diagnostic Map to make judgements about the phases of mathematical thinking the students were demonstrating in their work samples. We then planned activities for two teaching sessions in the following week that would address the needs of the different ability groups. For each class the teachers and I discovered different weaknesses, but for each year level a pattern of missing links in knowledge began to emerge.

The Kindy children were able to solve addition and subtraction number story problems if they were given the opportunity to dramatise the story or use concrete materials. For these children understanding operations could only be done with adult support and only with very small numbers. For Aboriginal children these sessions were not only great fun, but rich in language and helped them to develop an understanding of specific mathematical words.

The Pre-Primary children solved number story problems by dramatising them, using concrete materials and by drawing a picture to show their understanding. Aboriginal children again derived great benefit from these experiences not only because it gave them the opportunity to develop their fine motor skills when "drawing their thinking", but also because

they provided a rich language learning environment. As expected the children did not use number sentences and many still needed help to solve the problems.

Figure 1 shows that this Pre-Primary child has a strong understanding of what the story problem is about and has chosen the correct operation to get the answer. Without the use of numbers he has shown the solution.

The Year 1 children used dramatisation with adult support. For example I presented the problem: "Some mangos were hanging on a tree. The bats ate four of them and now there are only thirteen left. How many mangos were there before the bats came?" To solve this problem thirteen children came to the front of the class and pretended to be mangos hanging on a tree. Then four other children pretended to be bats eating a delicious mango. The children solved the problem by discussing as a class, the relationship of the bats to the mangos and what had happened in the story. For many problems they could use concrete materials to solve number story problems without adult intervention. For example "Dad went fishing on the weekend and caught 12 bait fish. His friend caught 5 Silver Cobbler. How many fish did they catch altogether?" They were able to draw pictures of what they thought was happening in a number story and used numbers to help explain their thinking, but were reluctant to write number sentences. The children demonstrated an ability to verbally explain their thinking, and with the help of a scribe were able to put their ideas into numbers and symbols.

The Year 2 children occasionally used dramatisation to understand what was happening in number story problems, but they mainly used concrete materials and drawings. They used number sentences to explain their thinking but sometimes used the incorrect mathematical symbol to explain their choice of operation. (Willis et al., 2004).

Figure 2 shows that this Year 1 child has chosen the correct operation to successfully solve this problem, however she has chosen the incorrect mathematical symbol to explain her thinking.

The Year 3 children mainly communicated their problem

Dan had 8 bananas. Tracy had 3 more than Dan. How many bananas did Tracy have?

Figure 1

Sally 11 marbles and gave Anna some. Now she only has 8. How many did Sally give to Anna?

11+8=3
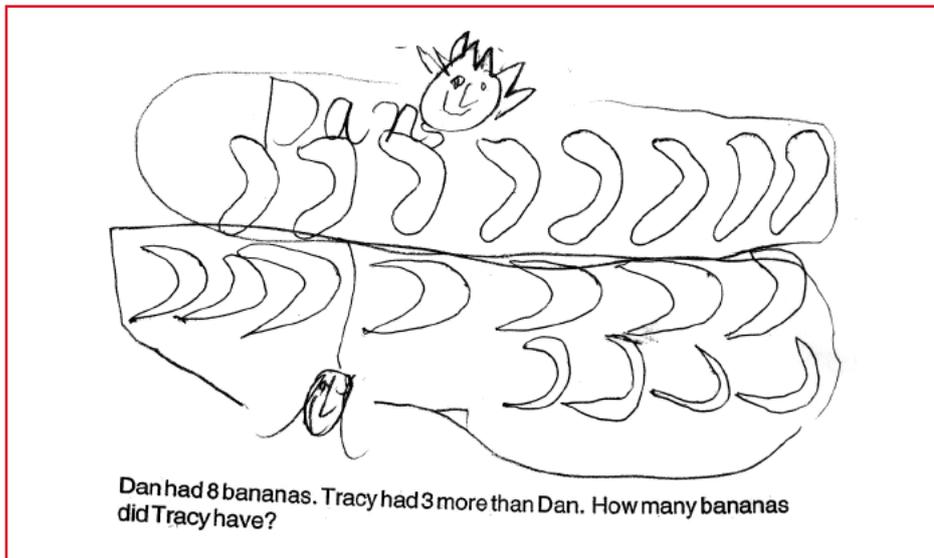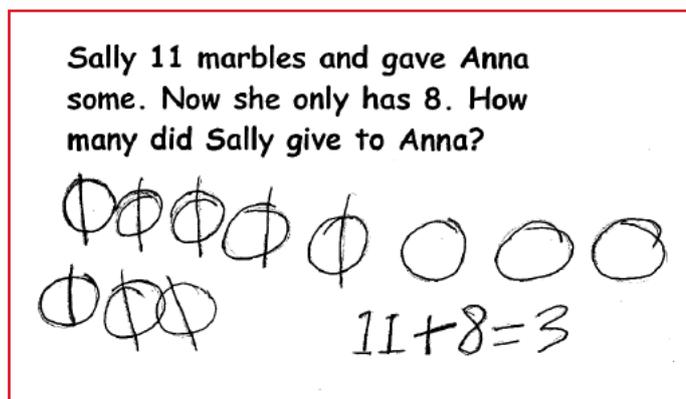
Figure 2

solving strategies using numbers and number sentences although some children occasionally used drawings. They used mathematical symbols with greater confidence and had developed a number of different strategies to solve number story problems.

What we did

By now the classroom teachers had developed an in-depth knowledge of the mathematics their students knew and used, their preferred ways of learning, and how confident they felt about learning mathematics. Kindy to Year 3 children were given "play time" with calculators allowing them to discover interesting facts about numbers and "short cuts". Throughout all year levels formal teaching sessions took place. Special care was taken to integrate story problems that had relevance to the student's social and cultural contexts so that their interest was sparked and they become engaged in problem solving (The Australian Association of Mathematics Teachers, 2002).

The Kindy children were exposed to operations through the use of *story boxes* — a brightly coloured gift box containing a calculator, a storybook, and other concrete materials with which the children could "play". The story box was either set up as a learning centre in the classroom, became a small group activity with an adult facilitator, or was used for a whole class experience led by the teacher. For example, the teacher

Figure 3

would read the story from the box (e.g., *Ten In The Bed* by Penny Dale) and ask the children, "If there were six children out of the bed, how many were still in the bed?". To solve the problem, the children would first act out the story problem using props, then the teacher would model the use of concrete materials, the drawing of a picture and the writing of a number sentence. Later, during activity time, the children may choose to draw their own picture and write numbers.

We gave the Pre-Primary children a lot of experiences solving number story problems in a variety of contexts. Children worked in small groups, or as part of a whole class. Story problems were generated to integrate with all learning areas in a variety of social contexts (AAMT, 2002). By the end of the year, some children were able to compose their own story problems.

Figure 3 shows the children writing number stories based on the numbers they derived from the two liquorice allsorts they found in each little paper bag. Children counted the number of layers or colours in the lollies then wrote a number story about any subject of their choosing.

We needed to teach the Year 1–3 children the language of operations before they could successfully decode number story problems independently. Key addition and subtraction words were presented on large strips of paper to each student as part of a whole class activity. The word's meaning was explained by putting it into a number story. (We found that it was best to work on add and subtract first.) When everyone

seemed to have an understanding of the word, it was then glued onto a class poster under the appropriate heading "+" or "−". Sometimes this activity required two or three sessions before the poster was complete. Year 2 and 3 students were challenged to make their own A4-sized copy of the class poster without any help, giving them the opportunity to demonstrate their reading and comprehension abilities.

Figure 4 shows an Addition and Subtraction poster made by a Year 2/3 class.

With a better understanding of the meaning of the words being used, the children were ready to solve different types of number story problems. We had children in small groups or individually use Think Boards — pieces of paper divided into four sections — to help them make sense of what was happening in the story problem. First they write the problem down, then use concrete objects to explain what is happening in the story problem. Next they draw a picture, and finally they write a number sentence to explain the mathematical thinking that has taken place.

The children were given many opportunities to orally compose and solve their own number story problems which showed us that they had moved to a higher level of thinking.

Another powerful strategy was the "How Did You Do It" lessons. Either as a short session at the beginning of a new lesson, or as a whole lesson, each child

Figure 4

was given the opportunity to solve a problem in his or her own way. A class discussion then took place and every student explained how he or she solved the problem. Children saw that there are many different approaches to getting the same answer.

The outcome

Two significant groups were affected by teaching young children to understand operations using the strategies mentioned: the teachers and the children.

The teachers used a wider range of pedagogies in their classroom. They became more diagnostic in their approach to teaching the operations and used successful learning experiences to initiate purposeful mathematical dialogue, arouse curiosity, challenge thinking, and actively engage students in learning (AAMT, 2002). They noticed the weaknesses the children were exhibiting, listened to the kinds of questions they were asking, and implemented learning experiences that addressed their needs. As a group the teachers became deeply informed about the mathematics appropriate to the student levels they taught, and also developed a broader understanding of the mathematics in the Student Outcome Statements. They exposed children to a wider range of addition and subtraction problems even from the earliest years. They began to use more mathematical language and participated in collegial discussions across year
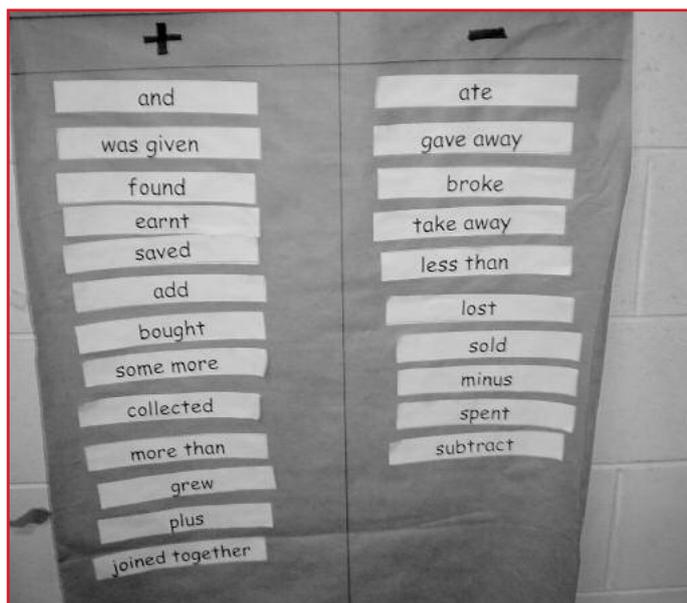
levels at every opportunity. Indirectly they were professionally developing each other and promoting an enjoyment of learning and positive attitudes towards mathematics. Where possible they encouraged and enabled parental involvement (AAMT, 2002). The AIEOs played a vital role in helping Aboriginal students understand mathematical language. They communicated the underlying processes and gave Aboriginal students the support needed to experience success when solving number story problems. They also supported non-Aboriginal students and teachers in the two-way language process by helping them understand the differences between the world-views of some Aboriginal students.

As a result of being taught a wide range of problem types to which all four basic operations apply, children developed flexibility in their thinking and use of numbers. They built "connections between dramatic, physical, diagrammatic and verbal forms of problems and the symbolic representation of them" (Willis et al., 2004) and solved number story problems. Because children were developing the language necessary to communicate their thoughts, they were empowered with a greater capacity to make sense of their world in mathematical terms and viewed solving mathematical problems as stimulating, interesting and fun (AAMT, 2002).

Conclusion

Over the two years I have been a Getting It Right Specialist Teacher of Numeracy I have monitored the results of a cohort children who in my first year were in Pre-Primary, Year 1 and Year 2. I have seen:

- Pre-Primary children develop from being unable to count out collections to represent amounts in number stories to being able to solve change, combine, equalise and compare problems and write number sentences to explain their thinking by the end of year 1;
- Year 1 children who were unable to use numbers or symbols to explain mathematical thinking being able by the end of Year 2 to write number sentences that demonstrate their understanding of the links between addition and subtraction by solving part-part-whole problems; and,

- Year 2 children who were unable to solve a wide variety of addition and subtraction number story problems being able by the end of Year 3 to solve number story problems involving all four operations.

Teaching in the East Kimberley is both very challenging and rewarding. The remoteness affects the amount of professional collaboration teachers can have with other schools and some children have unique challenges because of their language and cultural backgrounds. The transiency of the population creates both challenges and opportunities. Whilst it is time consuming introducing new teaching staff and students to the innovative approaches of the *First Steps in Mathematics* materials, when this population moves away they take with them higher levels of understandings that affect the way they teach or communicate mathematical understandings.

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