Developing number sense
through
mathematical diary writing

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provides insights into the

teaching of mathematics

in Taiwan through

his use of

mathematical diaries

as a strategy for

developing

number sense.

Many documents and reports (Australian Education Council (AEC), 1991; National Council of Teachers of Mathematics (NCTM), 1989, 2000) emphasise that communication is a major component of mathematics education and highlight that writing should be considered as an essential communication skill in learning mathematics. For example, the *Principles and Standards for School Mathematics* (NCTM, 2000) states that "written communication should be nurtured" (p. 62). It also emphasises that "communication is an essential feature as students express the results of their thinking orally and in writing" (NCTM, 2000, p. 268). Furthermore, *A National Statement on Mathematics for Australian Schools* (AEC, 1990) highlights that "mathematical communication skills are needed in order to understand, assess and convey ideas which involve mathematical concepts" (p. 13).

Writing can be a useful catalyst because it not only supplies students with an opportunity to describe their feelings, thinking, and ideas clearly, but it also serves as a means of communicating with other people (Baxter, Woodward, Olson & Robyns, 2002; Liedtke & Sales, 2001; NCTM, 2000). Writing "about mathematics, such as describing how a problem was solved, also helps students clarify their thinking and develop deeper understanding" (NCTM, 1989, p. 26).

Even though there is an important math curriculum reform effort under way in Taiwan, the new curriculum pays little attention to mathematical diary

writing. Writing about mathematics is a new experience to many students and teachers in Taiwan because it is not considered part of mathematics learning and teaching in the Taiwanese mathematics curriculum.

Can an emphasis on mathematical diary writing help children develop number sense?

NCTM (2000) and Liedtke and Sales (2001) believe that writing should play an important role in mathematics teaching and learning because it can help children develop conceptual under-standing. This stimulated my research topic: can mathematical diary writing help children develop number sense? One of my graduate students, a mathematics teacher with five years teaching experience, asked her third grade students to write a mathematics diary in one class period. The task was to "write a short paper describing the number 12". One of her third grade students wrote:

> Ming is a 12 year-old boy. His class has 12 students total. They are all from 12 different fami-lies. Ming celebrated his birthday on December 12. On the day of the party, Ming's mother bought a birthday cake and cut it into 12 pieces for them. His mother also bought 12 cans of soda for these kids. The party began at 12 o'clock noon on time.

In this example, the number "12" represents different meanings. This student separately provided the number "12" with multiple meanings and vivid life, showing she could make sense of her understanding of numbers and connect her understandings to real-life situations. As a matter of fact, a number is not an obvious and formal entity but is an abstract symbol. We create the significance and the value of a number. This episode encouraged me to study the impact of diary writing further and was the stimulus for this article.

Why ask students to write mathematical diaries?

Leung and Wu (2000) believe that posing and solving problems at home through mathematical diary writing is useful. They also claim that this is "practical because it makes mathematics learning continuous and promotes teacher-child-parent relationships" (p. 29). Using mathematical diary writing appropriately is helpful in promoting the teaching and learning of number sense. Teachers also can get feedback and check children's learning through the mathematical diary. Diary writing has numerous benefits. The following examples will support several major reasons why mathematical diary writing can help nurture chil-dren's number sense particularly in regard to fractions.

Mathematical diary writing can help children make sense of fractions

Reys (1994) states that "having students summarise their thinking in written form is an effective method for helping students nurture their sense of number" (p. 117). The diaries of numerous students confirm this claim. For example, one grade 6 student wrote:

> I used to hate math because the computations were always complex. Now, I like the number sense classes because they make me become smarter. Math is not so difficult as I thought. These activities help me develop useful and meaningful ways to find the answers. For example: Decide whether 31/32 or

26/27 is larger. My answer is 31/32. Suppose I have two cakes of the same size.

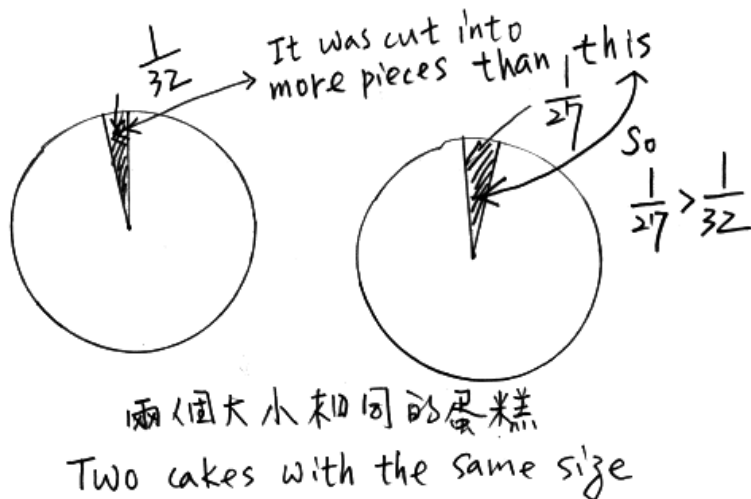One cake was cut into 27 pieces and I got 26 pieces, so I have 26/27.

Figure 1. Pictorial representation of fractions (26/27 and 31/32) with different numerators and denominators.

The remaining part is 1/27. The other cake was cut into 32 pieces and I took 31 pieces, so I have 31/32. The remaining part is 1/32. I only need to compare 1/27 and 1/32. Because 1/27 is larger, the remaining part of the fraction is smaller. Therefore, 31/32 is larger.

Diary entries such as this one show that mathematical diary writing not only helps children make sense of fractions but also help them reflect upon their thinking and ideas about fractions. This confirms the statement that "writing in mathematics can also help students consolidate their thinking because it requires them to reflect on their work and clarify their thoughts about the ideas developed in the lesson" (NCTM, 2000, p. 61)

Teachers can get feedback about their teaching through mathematical diary writing

Integrating number sense activities into these sixth graders' mathematics curriculum was a new experience for both the students and the teachers. The teachers were able to observe the depth of students' ideas to assess whether or not these activities helped them develop number sense. Math diaries provide an unobtrusive way of better understanding what students feel about what happens in math class. This helps students develop better understanding of number sense. In a review of the diaries, over 90% of students wrote that the teaching of number sense was a very useful catalyst because it challenged their idea that "mathematics is computation". These activities not only helped them understand that there are different ways to solve problems besides paper-and-pencil computation but also nurtured their ability to think and reason. For example, one student wrote in her diary:

I used to believe that written computation was the only method to find the exact answer. These number sense activities help me to know that estimation and mental computation are also useful in finding the answer. The teaching of number sense causes me to think, estimate, and decide whether or not the answer is reasonable. Now I know I can find the answer without using

the paper-and-pencil method. I am sure I will do better in mathematics in the future.

Another student wrote:

> Before the instruction, I never knew how to use benchmarks or estimation to find answers. Now, I know that benchmarks are very useful in solving estimation problems.

As a result of these diaries, not only did the researcher discover that the number sense project had a very positive influence on students' learning, but teachers also saw that their teaching was effective. For example, the diaries show that children can use benchmarks flexibly, confirming the statement of Reys, Kim and Bay (1999) that the use of benchmarks help children develop better understanding of fractions.

Mathematical diaries can provide insight into what students learned in the activities and can help teachers to evaluate students' understanding

Writing diaries can not only can nurture students' abilities in using the various components of number sense, but can also help teachers to investigate whether or not they really understand these different components. Many students' diaries confirmed that they have better understanding of number concepts after engaging in the number sense activities. For example, one student wrote:

> The use of benchmarks is a new experience for me. Our mathematics classes never introduced how to develop benchmarks. Estimation is also a new topic for me. Benchmarks and estimation are useful tools in solving problems. The homework was very easy: For example, in finding the decimal place: $638.5 \times 0.254 = 162479$, I knew the answer was (3)162.179 because 0.254 is about 1/4 and 638 is about 600, then $600 \times 1/4$ is about 150. Hence, (3)162.179 is the best answer. Now, I know that estimation and benchmarks are powerful tools in solving mathematical problems. I like number sense classes.

This student did not have any knowledge of estimation strategies such as the use of benchmarks before the class. However, through the diaries we can see that now she has a good understanding of these tools. For example, this student used 1/4 (0.254 is about 1/4) as a benchmark to help her solve problem reasonably. Another student said:

> After I heard Santy's explanations (Figure. 2), I thought her opinions made sense. She used the left parts (1/17 and 1/19) to compare 16/17 and 18/19. Since 1/19 is smaller, the fraction 18/19 is larger. This is easy and makes sense. This gave me a different way to think. For example, in comparing 3/4 and 2/3 (Figure 3), we can consider two cakes of the same size. One cake is cut into four pieces, I ate three pieces, so the remainder is 1/4. The other cake is cut into three pieces, Daddy ate two pieces of the cake, so 1/3 is left. Since 1/4 is less than 1/3, 3/4 is larger. I used to compare fractions by finding the common denominators. Now I know other ways that help me do it in my head.

These mathematical diaries demonstrate that students can improve at using number sense tools, such as benchmarks, estimations, and recognising the relative magnitude of numbers with explicit instant in the use of these strategies.
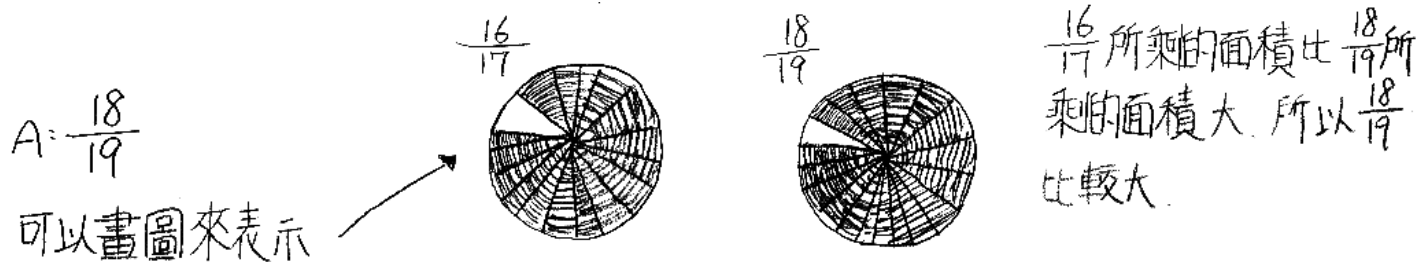
Figure 2. Pictorial representation of fractions (16/17 and 18/92) with different numerators and denominators.
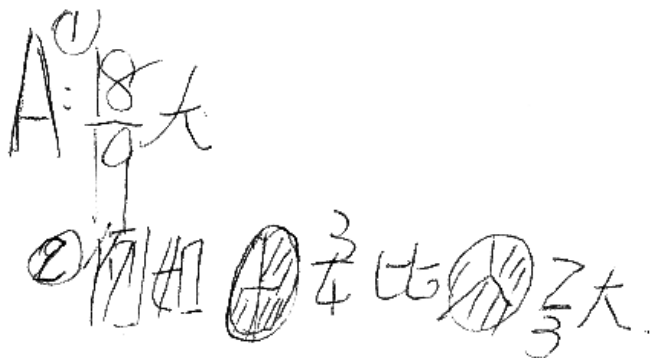

Figure 3. Student's explanations by using specific example.

Mathematical diary writing gives children a good forum to create, formulate, and extend their thinking

The study of Reys & Yang (2003) and Yang (2003) demonstrated that ordering numbers that include decimals and fractions is very difficult for sixth- and fifth-grade Taiwanese students. However, after the instruction, students' reflection in their diaries confirmed they are comfortable using benchmarks to compare such numbers. For example, one student described how she found the benefits in using the benchmarks to help her solve fractional problems.

I usually thought this kind of question (e.g., without using calculation, which sum is more than 1? (1) 5/11 + 3/7 (2) 7/15 + 5/12 (3) 1/2 + 4/9 (4) 5/9 + 8/15) is very boring and tedious. After the class discussion, I found that the effective use of benchmarks helps me a lot. For example, I knew that both 5/9 and 8/15 are over 1/2, hence their sum is over 1. I don't need to do a lot of complex computation to decide the answer. I like mathematics more than before.

The entry in her mathematical diary demonstrates her understanding of the use of benchmarks and how they help her make sense of fractional numbers.

Even though their thinking was not public, the students found a way to enjoy their problem solving through writing. This is a different way to represent their thinking. These mathematical diaries confirm that diary writing is "particularly useful because it allows a child who is uncomfortable in oral situations to express understanding in a less public forum" (NCTM, p. 28).

Suggestions for using mathematical diaries

Many teachers in Taiwan do not believe that middle grade students have the ability to write mathematical diaries. However, I agree with the statement that: "Teachers should not underestimate what young students can learn" (NCTM, 2000, p.79). Teachers should challenge their students to write diaries as part of their instruction. Asking students to reflect in writing on a lesson can provide insight into what they have learned. In fact, children can exceed our expectations as shown in this article. I strongly suggest that teachers give students writing tasks each week that include:

1. Asking them to reflect and write down their feelings about a particular activity, their ideas, thinking processes, and problem solving procedures;
2. Posing similar questions to further enhance and deepen their mathematical concepts.

Conclusion

These mathematical diaries show that children could develop and use the methods learned in number sense class, especially benchmarks (1, 1/2 and 1/4), to help them make sense of fractions. Reys, Kim and Bay (1999) suggest that the use of benchmarks may be helpful in developing students' number sense related to fractions. Through the diary writing described in this article we can see this is possible.

Mathematical diary writing is not only a good way for students to represent their thinking through pictures, language, or symbols freely and privately, but also a useful channel for children to communicate with themselves and with their teachers. This study demonstrates that diary writing has many benefits for teaching and learning. It helps nurture number sense, helps teachers investigate children's understanding and feelings about lessons, and gives children a way to formulate their thinking.

Writing mathematical diaries is a process that can develop students in a deeper understanding of mathematics and encourage the use of reflection. Children need to use their own ways to explain what they learn in class. Mathematical diary writing also promotes children's problem solving ability through explaining their ideas. Overall, writing in mathematical diaries can have a very positive effect on teaching and their development of number sense.

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Acknowledgement

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