

WHEN NEAR ENOUGH IS GOOD ENOUGH

8 principles for enhancing the value of measurement estimation experiences for students

Everyday, adults and children encounter situations where they have to make judgements about “how much” or “how long” or “how many”. The significance of estimation as an ordinary, everyday and natural aspect of measurement needs to be conveyed to students through their mathematical experiences (Department of Education and the Arts, Tasmania, 1994). Many students, however, tend to view estimation as a difficult technique where success is dependent upon how close the student’s estimate is to the teacher’s estimate rather than a useful and practical experience. Teachers and students need to realise, however, that estimation is not simply guessing, but rather an informed judgement. Research has shown that estimation employs mental computation, rewards flexible thinking and helps dispel the “one-right-answer” syndrome often associated with exact computation (McIntosh, Reys, Reys & Hope, 1997). Based upon my reading of recent research and experience as a classroom teacher, I have devised the following eight principles of estimation which may assist teachers with making estimation a more purposeful and enjoyable experience for students.

1. Estimation is useful

Fundamentally, measurement is about “making comparisons of one thing with another according to a selected attribute” (Department of Education and the



TRACEY MUIR outlines eight guiding principles for helping children understand the importance of estimation when measuring at work, in school and in our everyday lives.

Arts, Tasmania, 1994). Estimation is an integral part of everyday life as many measurement situations do not require exact measurements (Hodgson, Simonsen, Luebeck & Anderson, 2003). I recently asked a class of Grade 4/5 students to “brainstorm” a list of the times where they had used estimation, rather than a more “exact” measurement as a result of using a formal measuring instrument¹. After initially focusing on school-based mathematics lessons, such as estimating how long the tennis court was, they eventually generated a list which included experiences such as judging how much cordial to place in their drink bottle before adding water, whether or not they had enough money to buy something at the canteen, which queue to join at the canteen counter and how much paint to place in the paint palette. This highlighted how much estimation was a part of their everyday life and that often approximations or estimates are “near enough”. Out of the discussion arose an observation that estimates are used frequently in cooking and that “my mother never measures anything — she just adds what she thinks”. This was an opportunity to discuss when exact measurements are necessary (see Principle 7) and also provided a springboard into discussing which occupations use estimations.

1. While no measurement is ever “exact”, the term is used hereafter to refer to situations when a formal measuring device is used rather than relying on estimation alone.

2. Estimation should be related to real life

Many professionals rely on making appropriate estimates in order to carry out their work successfully (Adams & Horrell, 2003). Adams & Horrell (2003) interviewed a number of professionals about how and why they use estimation in their work and their findings provide a good starting point for generating student discussion in this area. Students were able to identify, for example, that umpires and referees use estimates of time and length in many sports, a chef uses taste and touch to develop estimates for a “pinch of salt” and the assistant at the delicatessen has developed the ability to estimate the approximate amount of ham asked for by the customer and then adjusts accordingly. It was agreed that it would be rather frustrating for the customer to have to wait while the ham was painstakingly measured out; the assistant also has the ability to estimate according to whether it is a request for a “kilogram” or \$4.00 worth of ham.

While estimation saves time for many professionals, for others, such as police officers, it helps to validate measuring tools and methods (Adams & Horrell, 2003). Professionals also estimate when more exact measures are not required. For example, customers who employ builders or landscape gardeners will often request a quote on how much the task will cost and how long it will take, rather than how many bricks will be needed and how much cement needs to be mixed. Teachers could consider asking professionals to visit their classrooms and talk to the students about how they incorporate estimation into their occupations.

3. Estimation saves time

To engage students in considering this aspect, the teacher may pose the question, “What would the world be like if everything had to be measured exactly?” Students could draw on ideas generated by previous discussions to illustrate how much longer, for example, it would take to do your shopping, build a house, paint a wall, cook your tea, etc. While such a discussion reinforces the practical application of estimation, it also serves to raise the issue that it only saves time if people have developed a fairly accurate

way of estimating and can be confident that their estimates are close approximations. One strategy for helping students learn to estimate is to consider the outside limits within which a measure may lie (Department of Education and the Arts, Tasmania, 1994). In their book *Think Mathematically*, McIntosh, DeNardi and Swan (2000) describe an activity entitled “Within Limits”. Students are encouraged to agree on a higher and lower limit within which they are certain that the actual measurement lies. This strategy could be used as the basis of an estimate for finding the length of the classroom, desk or book, the area of the floor or a piece of paper or the number of beans in a jar. Students should be encouraged to estimate within increasingly narrow limits (Department of Education and the Arts, Tasmania, 1994) and to make statements about the confidence they hold in their estimates.

4. Estimation experiences should be purposeful and relevant

To many students, estimation is equated with being a guess — an answer to be placed at the teacher’s request before the “real” measuring activity is undertaken. This view seems also to be reinforced by the way many textbooks and worksheets present meas-

uring activities (see Figure 1). In many situations an exact measurement is not necessary or even desired, so when students are asked to estimate, it should not always be followed with, “Now measure exactly. How close were you?” Teachers need to emphasise that an estimate which happens to be more exact is no more “correct” than other reasonable estimates — the goal is an approximate, not an exact number (Department of Education and the Arts, Tasmania, 1994).

Teachers also need to consider the validity behind some of the tasks we ask students to do. I am sure we have all been guilty in the past of asking students to measure lengths of corridors, netball courts, doorways without providing students with a reason for why we need to know these lengths. If students were training for a cross country event, for example, then it would be legitimate to find out the perimeter of the netball court to work out how many times they would need to run around it as part of their training.

As previously stated, sometimes an exact answer is also not required. Consider the following question:

You need to buy paint to cover a wall which is 20 square metres in area. According to the tin, a litre of paint covers 16 square metres and you would need to do two coats. How much paint would you need to buy?

Estimate length, then use a ruler to measure the actual length of each object to the nearest centimetre					
Object	Estimate	Actual length	Object	Estimate	Actual length
a. pen			d. height of desk		
b. width of calculator			e. length of shoe		
c. length of lunchbox			f. width of maths book		

Figure 1. Example of “traditional” textbook estimate then measure exercise (from *Step Ahead with Maths*, 5).

It could be argued that this question traditionally, would be posed to elicit the response of 2.5 L, whereas an answer of three one-litre tins or one four-litre tin would be more appropriate and embedded in a real-world context. It is also further recommended that this context should be relevant to the student's real-world, rather than the adult's (McIntosh, 2003). A preparation for a class party, for example, would bring in many opportunities for estimating including how much drink or food would be needed, how long it would take to play the games, how many decorations would be required and so on.

5. The ability to estimate is enhanced through practice and over time

Students should be given ample opportunities to practice and refine their estimation skills. The butcher's ability to quickly cut steaks from a slab of meat, or the delicatessen assistant's apportioning of \$4.00 worth of pizza ham has been developed over a long period of time. Likewise, a fisherman's seemingly uncanny ability to estimate the mass of his catch has also been based on years of experience. For this reason, a simple task like estimating how many jelly beans are in a jar, is not a wasted task as a beginning estimation activity (Adams & Harrell, 2003). There are many simple tasks that students can engage in which foster estimation and also serve to develop the

concepts of the attributes of measurement. A simple task to develop the concept of time, for example, is to ask students to work in pairs and to nominate a specific period of time, such as one minute. With one person timing and the other estimating this length of time, gauge whether students' estimates improve after three trials.

Students also need to be encouraged to use the result of measuring with a unit to improve their next estimates with that unit. It is also important for students to explain how they made their estimates and to discuss the benefits and disadvantages of their methods (Department of Education and the Arts, Tasmania, 1994).

6. Personal benchmarks are useful referents for estimation

Personal benchmarks can enhance the meaningfulness of standard units of measure and are often recommended by maths educators as an important component of instruction on measurement and measurement estimation (Joram, 2003; McIntosh, Reys & Reys, 1992).

Benchmarks typically consist of nonstandard units whose lengths are used to represent the lengths of standard units, but should not be confused with manipulatives (Joram, 2003). While some benchmarks may start off as a manipulative, for example, a paper clip, students would need to be able to represent it mentally for it to be considered a benchmark (Joram, 2003). Many adults use personal benchmarks to assist with estimation (e.g., using their height to determine whether the new fridge will fit), however, these are not always appropriate for children as they tend to grow and therefore the benchmarks change. Teacher imposed benchmarks are not always appropriate either, as they may not be particularly memorable for children. Instead Joram (2003) recommends that where possible, children should generate their own "personal benchmarks". These may include objects from home and can vary according to the attribute. One benchmark I personally use for mass is a litre carton of milk for a kilogram — I know what this mass feels like and can use it as a referent to gauge how

heavy the fruit and vegetables I select at the supermarket are and therefore work out approximately how much they will cost. Students can be encouraged to make use of referents through the posing of open-ended questions such as the following:

Julie knows the mass of one marble. How does this help her to estimate the number she would want to carry in her pocket at one time? (Department of Education and the Arts, Tasmania, 1994).

What are some things your hand will cover so that they cannot be seen? What are some things you cannot cover completely with your hand? (Sullivan & Lilburn, 2002).

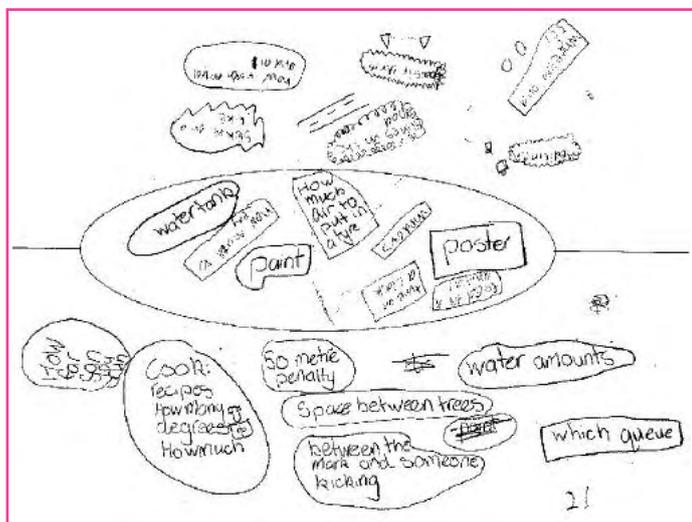


Figure 2. Placemat.

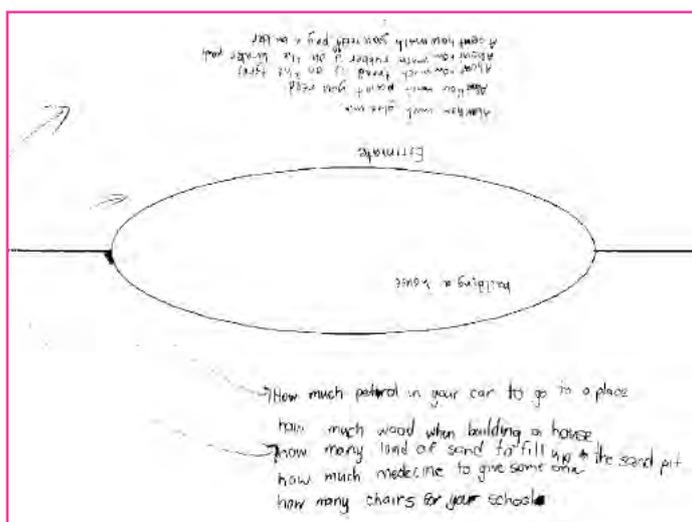


Figure 3. Placemat.

I recently observed a Grade 4/5 class working on some measuring and estimation activities involving some routine tasks, such as measuring the length of the corridor, tennis court and height of the door. While most estimates of the door were appropriate (two metres), when asked how they decided upon their estimates, most students responded that they just looked, just guessed or “don’t know”. One student did reveal that he had a good mental image of what a metre looked like by stating, “I looked at it in the middle and I thought it would be one metre on each side.”

From observing students participating in the task, it was apparent that students were inclined to “guess” rather than estimate. When estimating the length of the tennis court and corridor, some students elected to use their feet or paces to make an estimate, while others chose to simply “look and guess”. The follow-up discussion focused on establishing referents that may assist them with making more valid estimations. Many students thought it would be useful to use a pace or step to approximate a metre when measuring horizontal lengths. The notion of using benchmarks was introduced and some possible suggestions were brainstormed. In a subsequent estimation activity, one student used her school bag as a benchmark “because I knew it was about half a metre.”

Benchmarks can be effective tools for helping students develop meaningful, accurate mental representations of measurement units (Joram, 2003). Teachers can assist by developing students’ repertoire of personal benchmarks that are eventually represented as mental images (Joram, 2003).

7. Sometimes it is better to overestimate or underestimate, depending upon the context

As stated previously, estimations are often enough, but there are many situations when exact measurements are required. It is important for students to develop an understanding of this and of situations when overestimating or underestimating is more appropriate. To illustrate this I asked some students to devise a “placemat” where one side was to be devoted to situ-

ations where estimation was sufficient, the other side was to be documented with situations where exact measurements were necessary and in the circle in the middle they should place examples of when estimation and measurement should go hand in hand (see Figures 2 and 3). Some of the estimation examples they produced included how much paint to use, how much tread is left on the tyres, how much petrol to travel to somewhere, hair cuts and clothing sizes. Examples of when exact measurements were required were fewer, but included measuring medicine, giving change and some recipes. Students were then asked to look again at their estimation responses and indicate whether it would be appropriate to underestimate or overestimate. Students concluded that it depended upon the context: while a landscape gardener may be out of pocket if he underestimated the cost of a job, a chef on the other hand, should underestimate if adding chilli to a dish!

8. Estimation is used to validate measuring tools and methods

The ability to make accurate estimations of temperature and mass for example, can serve as a “check” as to whether the thermometer or scales are working properly or that they have been “read” or interpreted correctly. This principle also becomes very important for later measuring experiences where students are required to calculate rates and work with formulae to calculate measurements. The ability to reliably “predict” a reasonable answer also encourages reflective thinking and problem solving behaviours, especially when the two answers are disparate and the student has to decide where the error has been made.

Conclusions

Many students appear to approach estimation as a guess to be made before doing the “real” activity and fail to see it as a valuable aid in itself. As teachers, I believe we can help foster an appreciation of estimation by incorporating the aforementioned principles into our practice and providing ample opportunities for students to refine and reflect upon their estimation skills.

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