Taking a closer look at young students' images of area measurement

JOANNE MULLIGAN, **ANNE PRESCOTT**, **MIKE MITCHELMORE** and **LYNNE OUTHRED** explore children's responses to a task that requires them to represent square units in a grid pattern. Their article once again highlights the importance to a child's mathematical development of recognising pattern and structure.

In a recent article in this journal (Mulligan, Prescott & Mitchelmore, 2003), we described young students' imagery of a triangular pattern of six circles. In this article, we explore their imagery associated with area measurement.

Pattern and structure

So much of mathematics involves pattern and structure. Right from the early years, patterns occur in number (e.g., addition and multiplication tables), space (e.g., geometrical shapes) and measurement (e.g., repeating units). Learning mathematics is greatly facilitated when students recognise the structures that these patterns reveal (Mulligan, Prescott, & Mitchelmore, 2004).

A structure that is important in area measurement is the grid that results when you divide a rectangle into square units. For young children, it is not easy to construct this grid (Outhred & Mitchelmore, 2000, 2001). They do not seem to recognise the pattern formed when a rectangle is tiled using equal sized squares. Recognition of this structure is a necessary step in understanding how area is measured using square units and, in particular, how area measurement is related to multiplication.

Our research

In an early numeracy project involving 109 Year 1 students from nine NSW Department of Education and Training schools, we explored students' use of pattern and structure across several different strands of the mathematics curriculum (Mulligan, Prescott, & Mitchelmore, 2004). One of these tasks investigated students' imagery associated with the square grid pattern. We were interested in the following questions:

- Do students understand the importance of equal sized units?
- Do students recognise the equal number of squares in each row and column?
- Do students notice the use of horizontal and vertical lines to form a grid?

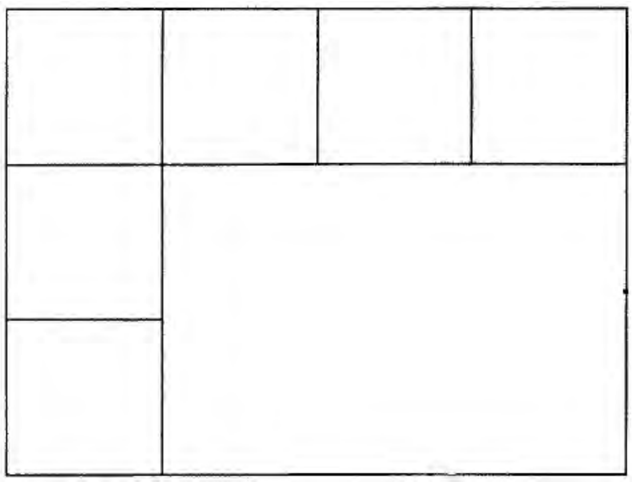

Figure 1. Area task.

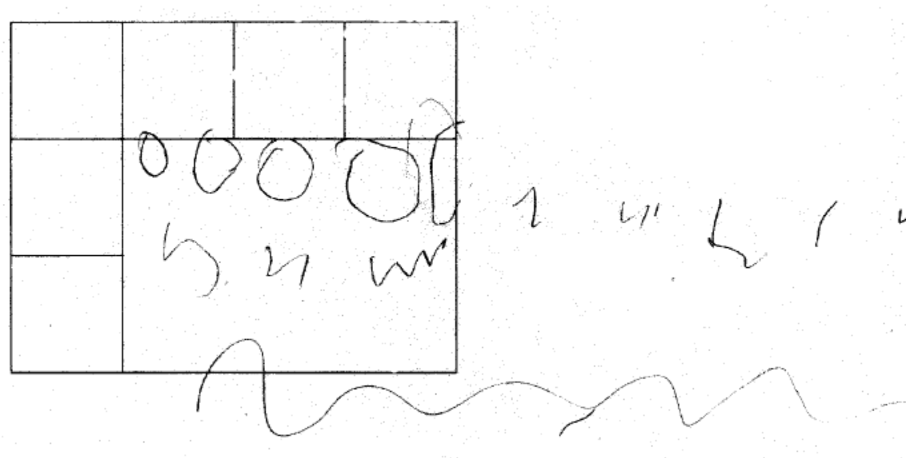

Figure 2. Pre-structural response.

The grid task

The grid task required students to "tile" a rectangular shape by drawing equal sized square units (Figure 1). The students were instructed to complete the rectangle by drawing squares exactly the same as those that had been provided.

This task provided clear evidence of students' difficulties in recognising the importance of equal sized unit squares and the row-column structure of the grid. Drawings ranged from those showing pre-structural (Figure 2) or emergent structure (Figures 3 and 4) to those indicating partial (Figures 5 and 6) or complete structure (Figures 7 and 8).

Figure 2 gives an example of a pre-structural response: the student shows no awareness of the use of unit squares or the purpose of the grid. He completes the task by filling the space with circles, explaining that "I like to draw circles".

Figure 3 is considered an emergent response because the student attempts to draw squares to fill the space; there seems to be an understanding that unit squares will complete the task, even though these are drawn in a disorganised manner. There is some attempt initially to match the size of the squares; however there is no apparent awareness of the structure of the grid.

Figure 4 also shows emergent structure in that the student has used unit squares to complete a border pattern but has not attempted to keep them the same size. In comparison with Figure 4,

Figure 5 shows a definite advance and is considered to indicate partial structure. The student recognises the number of squares to complete the border and has drawn them approximately equal in size. However, the structure is not complete because the student has apparently not recognised how the square tiles fit together.

Figure 6 also shows evidence of partial structure because the student draws the correct number of squares and correctly aligns them. But this student has also not fully understood what happens when equally sized squares are placed next to each other.

Figures 7 and 8 show that the students have recognised the structure of the grid and have accordingly drawn the correct number and size of square units to complete the pattern. The only difference between the two responses is that the student who drew Figure 8 recognises that the squares do not have to be drawn individually but can be constructed using continuous vertical and horizontal lines.

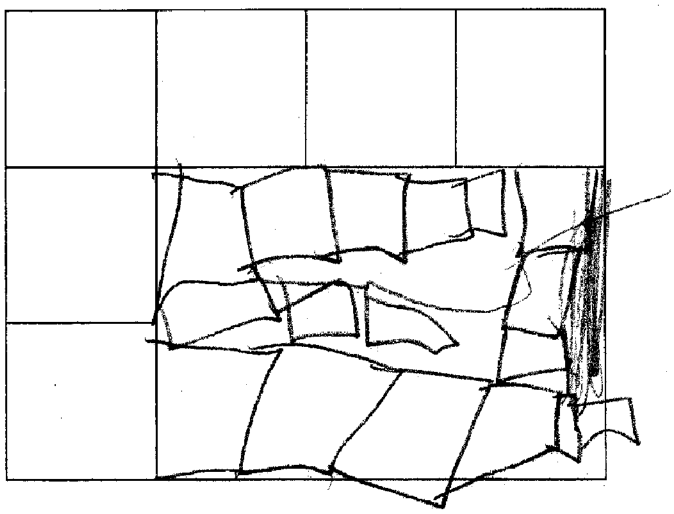

Figure 3. Emergent structural response.

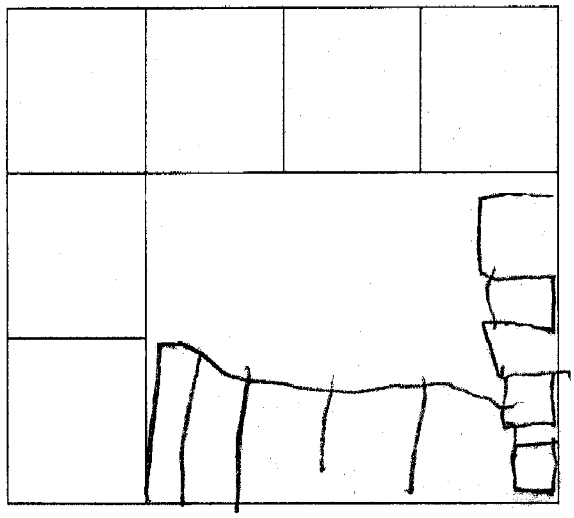

Figure 4. Emergent structural response.

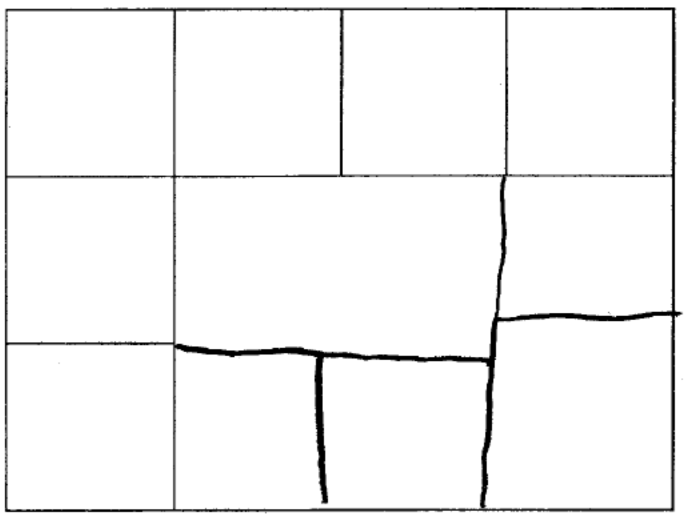

Figure 5. Partial structural response.

Results

As with the previous imagery task, there was a wide range of responses. Only a small number of students made drawings showing pre-structural or emergent structure, with 40% of responses showing some partial structure and 44% indicating complete structure. These are encouraging results and suggest that most students had experienced measurement activities that included making tiling patterns.

Of the 44% of students classified as giving structural responses, the majority drew individual squares (as in Figure 7). Many of these further explained the "short cut" of using continuous lines (as in Figure 8). Very few students drew the row-column structure immediately.

Teaching implications

In our study, several students who told us about the short cut of drawing vertical and horizontal lines had apparently become aware of the row-column structure of the grid while completing the task. There were

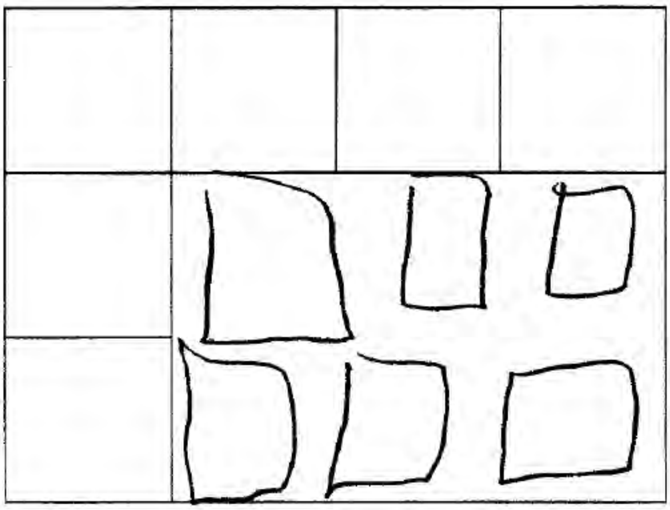

Figure 6. Partial structural response.

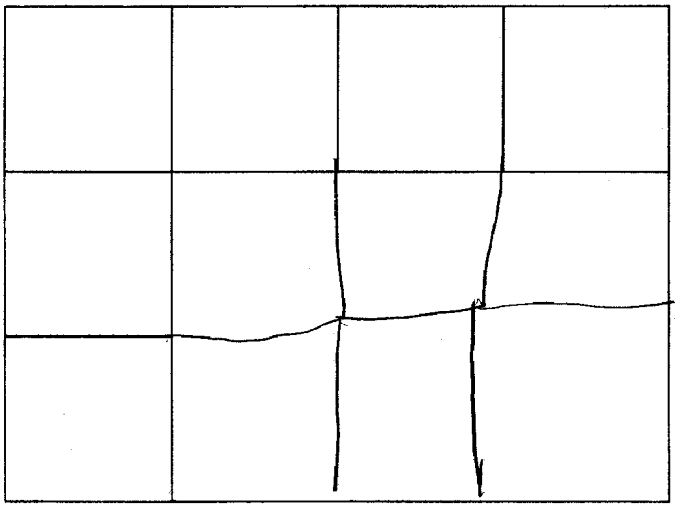

Figure 7. Structural response.

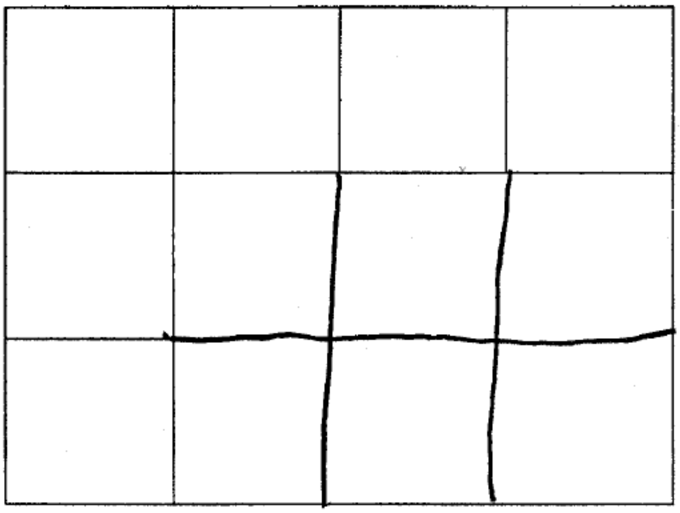

Figure 8. Structural response.

other cases where guided reflection might easily have led students to recognise the grid structure (see, for example, Figures 5 and 6). Even students whose drawings showed emergent structure (Figures 3 and 4) might have advanced their understanding by, for example, comparing their drawings with a physical model.

There is ample evidence that students in the middle years (Years 5–8) confuse the concepts of area and perimeter, even though they may give correct answers to standard assessment questions requiring the use of formulae (Kidman & Cooper, 1997). Teaching that focuses on the recognition of the structure of an area grid is likely to lay the foundation for a deeper understanding of area measurement that would avoid such confusion. Of course, some students may need to focus separately on single structural aspects (shape, size, arrangement or quantity) before they can integrate them into one mathematical representation.

McPhail (2004) describes the effectiveness of a related professional development program for Years 1 and 2 teachers. The teachers designed carefully structured lessons built upon the early multiplication and division skills of grouping, counting by rows or columns, skip counting and counting by multiples. The lessons were successful in assisting students to identify the grid pattern:

> An understanding of this concept together with knowledge of the attribute of area will assist students to understand how area is calculated when formal units are introduced. The lessons demonstrated that a sequential area program which also relates to whole class activities of repeated addition and multiplication can be successfully implemented across a range of classrooms. (p. 365)

The responses from students in our study supported McPhail's findings in that students need to develop grouping, partitioning and unitising skills. Students develop understanding of area by progressing from single squares to the use of an array. At the same time they need to understand alignment and the structure of a square tessellation. These skills can be developed in a carefully sequenced program that must include the development of multiplication skills. Further ideas for teaching area measurement are included in the NSW Department of Education and Training's *Count Me into Measurement* program (NSW DET, 2003a; Outhred, Mitchelmore, McPhail, & Gould, 2003), which is accompanied by detailed teaching resources (NSW DET 2003b, 2004).

References

Kidman, G. & Cooper, T. J. (1997). Area integration rules for Grades 4, 6 and 8 students. In E. Pehkonen (Ed.), *Proceedings of the 21st Annual Conference of the International Group for the Psychology of Mathematics Education, Vol. 3* (pp. 132–143). Lahti, Finland: University of Helsinki.

McPhail, D. (2004). Professional learning in the teaching of area. In I. Putt, R. Faragher & M. McLean (Eds), *Mathematics Education for the Third Millennium: Towards 2010* (Proceedings of the 27th Annual Conference of the Mathematics Research Group of Australasia, pp. 359–367). Townsville: MERGA.

Mulligan, J. T., Prescott, A. & Mitchelmore, M. C. (2003). Taking a closer look at young students' visual imagery. *Australian Primary Mathematics Classroom, 8*(4), 23–27.

Mulligan, J. T., Prescott, A. & Mitchelmore, M. C. (2004). Children's development of structure in early mathematics. In M. Høines & A. Fuglestad (Eds), *Proceedings of the 28th Annual Conference of the International Group for the Psychology of Mathematics Education, Vol. 3* (pp. 393–401). Bergen, Norway: Bergen University College.

NSW Department of Education & Training (2003a). *Count Me In To Measurement.* Sydney: NSW DET.

NSW Department of Education & Training (2003b). *Teaching Measurement: Early Stage 1 and Stage 1.* Sydney: NSW DET.

NSW Department of Education & Training (2004). *Teaching Measurement: Stage 2.* Sydney: NSW DET.

Outhred, L. & Mitchelmore, M. C. (2000). Young children's intuitive understanding of rectangular area measurement. *Journal for Research in Mathematics Education, 31*, 144–168.

Outhred, L. & Mitchelmore, M. C. (2001). Across the great divide: From process to structure in students' representations of rectangular arrays. In R. Nata (Ed.), *Progress in Education, Vol. 5* (pp. 51–66). Hauppauge, NY: Nova Science.

Outhred, L., Mitchelmore, M. C., McPhail, D. & Gould, P. (2003). Count Me Into Measurement: A program for the early elementary school. In D. H. Clements & G. Bright (Eds), *Learning and Teaching Measurement* (pp. 81–99). Reston, VA: National Council of Teachers of Mathematics.

Acknowledgement

A Macquarie University Research Development Grant funded the project on which this article is based. Our thanks to the NSW Department of Education and Training and the teachers and students who gave their time and support.

Joanne Mulligan, Mike Mitchelmore and Lynne Outhred are teachers and researchers at the Australian Centre for Educational Studies at Macquarie University. Anne Prescott is a teacher and researcher at the University of Technology, Sydney.
<joanne.mulligan@mq.edu.au>
<mike.mitchelmore@mq.edu.au>
<lynne.outhred@mq.edu.au>
<anne.prescott@uts.edu.au>

APMC