Fractions from concrete to abstract using Playdough Mathematics

Rosemaree Caswell guides us through an approach to developing a sound understanding in fractions by linking concrete models and written notations.

A few years ago, I found myself in a school where all students in my year level had been asked to purchase a bucket of playdough, in four different colours. When I considered the problem of what to do with all of this playdough, I began a journey of discovery and learning that has engaged the interest of students and teachers in mathematics in the middle years (ages 9–12 year old) through a series of activities I began referring to as "playdough maths".

In these middle years of primary schooling, the teaching of common fractions frequently involves written activities which use abstract representations of numbers, symbols and images. Many students, however, still need the benefit of concrete materials and sensory motor experiences to enhance their understanding of the concepts associated with common fractions. Sowell (1989) and von Glasersfeld (2002) have argued for the continuing engagement of students with concrete materials. They claim this builds deeper understanding of mathematical concepts. Bastick (1993) has also argued strongly for the need to develop deeper understandings in this transition phase of learning. My experiences with 'playdough maths' provide evidence of effectively engaging learners in building bridges from concrete to abstract understanding in mathematics.

The following article presents some simple ideas for engaging students in working with common fractions and moving their understanding from concrete to abstract with ease and enjoyment. Common fractions are usually introduced in the early years of learning and associated with concrete materials. In the following years, students are often expected to work with numerical values or visual drawings of fractions.

Let's begin with representing simple fractions

Initially in a classroom context, I engage students in making associations between fractions and real life. Most often they think of pizzas which are commonly produced as circular patterns of eighths. This association is particularly beneficial in discussing the concept of fractions as equal parts of a whole. Pizza slices are well known to be cut into unequal sections and students are very familiar with the problems this can cause (particularly if you are left with the smallest slice). Thus the need for fractions to be represented in equal parts is established very early. Students are asked to make a playdough pizza, which is then cut into two halves (Figure 1). From this point the class begins experimenting and suggesting other ways of dividing the pizza. Fourths are commonly chosen, then eighths.

Figure 1: Cutting halves, then fourths.

As the teacher, I begin by modelling the numerical values, and then engage the students in writing the values themselves as they construct the fractions.

Multiple parts of a fraction are soon being created, described and written using fraction notation (Figure 2).

Figure 2: Written notation for multiple parts

Very soon, students begin to observe that the denominators are all even numbers, so the suggestion is made to create pizzas where the denominator is an odd number. This creates a challenge because the circular form of the pizzas does not cut easily into fractions with an odd number of parts. As students engage with the problem, they use reasoning and explanation to identify whether the shapes they are creating are equal and represent the chosen fraction.

At this point, the class is further challenged to use different shapes, such as rectangles, triangles and so on. The students usually become completely engaged in this activity and the discussions around the plausibility of using each shape becomes an interesting class conversation. When using rectangles particularly, students also engage in measurement as the shapes become very defined by length and width to accommodate equal partitioning into the fractional parts.

Mixed numbers and improper fractions

After the initial work with fractions and parts of a whole, the activities begin to engage students in creating and representing improper fractions and mixed numerals. Initially, the class is asked to show 3 halves using playdough. Many students begin by creating thirds. Students engage in a class discussion debating whether this actual shows thirds or halves and what the difference would be. Soon students realize that two of the halves must fit to make a whole shape with the remaining portion clearly showing half of the whole. Once the solution is gained, the challenge starts to create a diverse range of improper fractions and mixed numbers using playdough. Any shapes can be represented and students frequently move around the classroom to view each others ideas and share their own thinking.

At the same time, the written form of improper fractions is continually modelled for the students along with the associated mixed number. Students are encouraged to share in this activity and soon engage in writing the equations themselves on the board (Figure 3). They are further encouraged to experiment with their own improper fractions and share these with their partners.

Figure 3: Modelling mixednumerals and improper fraction notation.

As students become familiar with this process, they are lead to search for the pattern that underpins the process. To do this, several mixed numerals and associated improper fractions are displayed and the students engage in observing and analysing the patterns that they can see. Students very quickly describe the processes of division, multiplication, basic facts and factorisation. The combination of mathematical concepts and skills enhances their understanding and makes the mathematics more interesting.

Equivalent fractions

Equivalent fractions are often a vague concept for students to comprehend. To engage students with this concept using playdough, they are first asked to create the simplest fraction–two halves. From here, students swap one half with a person who has a different colour of playdough. This results in every student having a whole shape made up of two halves, each a different colour. Students are then asked to cut the parts in half to create fourths and describe how many fourths can be seen in each colour. The transition is modelled on the board using numerical values to show the change from a half to two fourths. Students copy this written notation and then divide their shapes again to create eighths (Figure 4).

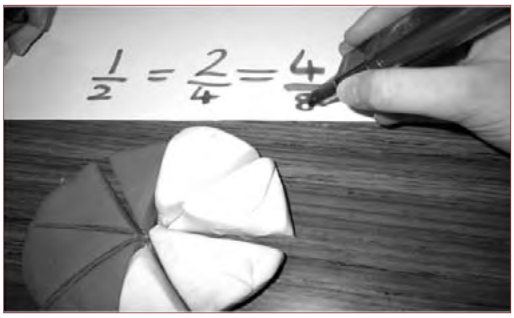

Figure 4: Modelling and expressing equivalent fractions

This time students continue the written pattern themselves. Very soon, eighths become sixteenths, although by now, most students have ceased to cut their playdough and simply make the changes numerically. At this point, students are further challenged to consider the pattern that is occurring (Figure 5).

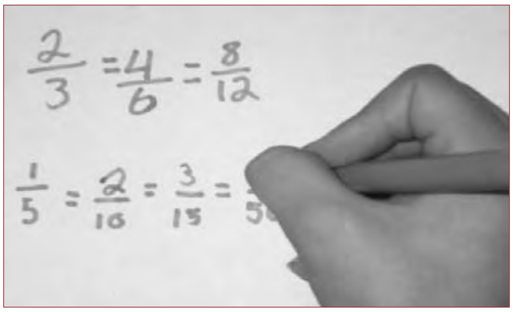

Figure 5: Building sequences of equivalent fractions

Once students believe they have identified the mathematical process, an abstract denominator such as fiftieths is added for an extension of the process. Students enjoy this challenge and employ their reasoning to see if they can solve the fraction. Following on from this level of understanding, an odd denominator is employed and the process continues. Many students do not bother to use the playdough now, they simply move to abstract computations.

Computations involving fractions

Addition and subtraction of fractions is presented initially using like denominators. Students find no difficulty at all in manipulating the playdough to represent these operations.

Therefore the challenge is created by using fractions that create more than a whole such as 3/4 + 3/4. (Figure 6).

Students soon realise that there are too many parts for one whole shape and the result will create a whole and a part–a mixed number. The previous work on mixed numbers and equivalent fractions is an essential foundation for effective understanding of computations involving fractions.

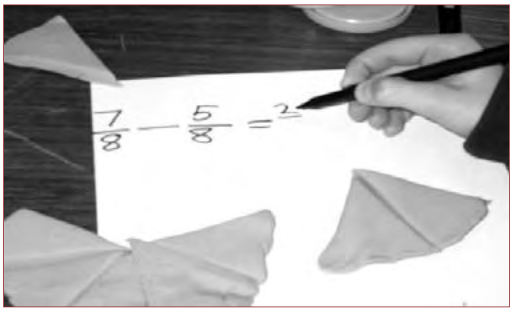

Figure 6: Beginning operations with fractions.

After students have confidently engaged with this early stage of computations, addition and subtraction using unlike denominators is introduced. Initially, a simple problem involving 1/2 and 1/3 is presented to the students. Once again, the students discuss, predict and use their reasoning to solve the problem. At first, they manipulate the shapes to create equal parts (Figure 7). At all times, the process is initially modelled for the students using numerical values, after which, the students are engaged in recording the process numerically by themselves.

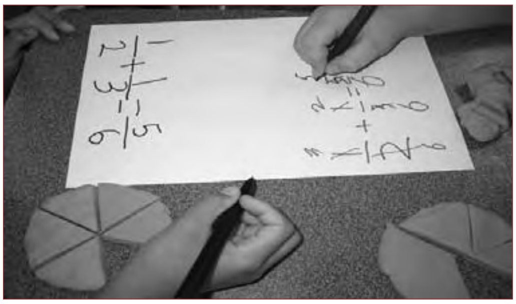

Figure 7: Working with unlike denominators.

As they become more independent, they automatically use both equations and playdough. As they engage further with the more abstract processes, the students participate in extended discussions and explanation of their thinking. This engages students in describing and defining the patterns and functions that are used to create common denominators (Figure. 8).

Figure 8:
Demonstrating multiplication with a fraction

There are many more activities that are employed to support students' understanding and development of the concepts related to common fractions. Most amazing is that all of the above activities have been used, and continue to be used, in classrooms with students ten years and younger. When teachers in upper primary and lower secondary levels see the results, hear the discussions and experience for themselves playdough maths and fractions, they are amazed at the levels of deep understanding these students have attained. Students thoroughly enjoy the experience of working with playdough, which stimulates all learners, provides for the needs of a wide range of learning styles, and can be used to support most conceptual learning in mathematics in these middle primary years.

References:

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