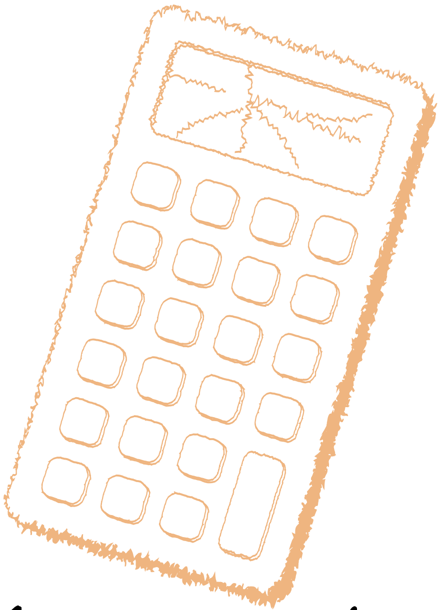
If it is broken, how can you fix it?



GARRY CLARK

describes the use of some simple software as a device to challenge children's calculation strategies.

Calculators can be used in primary schools in a number of situations. They are most beneficial when working with large numbers, dealing with real data that leads to complex calculations, performing repetitive calculations, developing concepts, estimating and checking, problem solving and looking for patterns and/or relationships. But what if the calculator is broken? This article describes the mathematics that children learnt and a teacher's awareness of children's mathematical understanding, when a "broken calculator" activity was undertaken regularly over two terms.

The background

For the last four years I have been using the Microworlds (2000) software to write Mathematical Activities (2002) for primary children. Information regarding all 126 activities can be accessed at www.northnet.com.au/~mathsactivities/mathact.htm. One of the activities provides students with a calculator where it is possible to "break" up to 9 keys. It was decided to use this activity with children in a composite 4/5 class with the aim of investigating issues surrounding the integration of it into the regular classroom. Each week in Term 2, 2005, the teacher gave the children 5–10 minutes to find solutions to a challenge. They had to imagine that they had a calculator with particular keys broken. The students were required to write solutions to the challenge in their books. In Term 3 students were introduced to the Broken Calculator software and asked to use the virtual calculator to find solutions to similar challenges.

The challenges

The weekly challenges are listed in Table 1. Initially the tasks were quite straightforward. The first challenge of making the number 56 on an imaginary calculator revealed students' levels of mathematical understanding in a number of respects. Most students used simple additions (50 + 6, 40 + 16, 56 + 0, 50 + 3 + 3), while some used more complex combinations such as (28 + 28, 27 + 29 and 100 – 44). There were also a few that used multiplication and a combination of operations (7 × 8 and 5 × 11 + 1). There were a number of students that demonstrated an understanding of the commutative property of numbers (7 × 8 and 8 × 7, 30 + 26 and 26 + 30). Others presented patterns (60 – 4, 70 – 14, 100 – 44, 200 – 144). Place value was understood by most (5 × 10 + 6). Three students used division (112 ÷ 2) one even seeing a pattern (112 ÷ 2, 224 ÷ 4).

The students also revealed a number of misconceptions. The most common occurred when subtraction was used (60 – 3, 100 – 34, 70 – 24, 76 – 30), though some students used zero inappropriately (51 + 05). There were also a number of mistakes presented (9 × 6, 4 ÷ 224, 70 – 41, 6 × 5, 8 ÷ 7, 11 × 11, 90 – 40 + 16, 17 + 45, 72 – 25). For some, the reasoning is hard to understand.

Students were also asked to present the most complex way they could make 56. Interesting ideas of complex were revealed. Some used larger numbers (1000 – 944, 506 – 450), some used more operations (3 + 3 + 3 + 3 – 2 + 40 + 8 – 2) and yet others used more obscure ways of making the number (106 – 50, 120 – 64, 85 – 29, 14 × 4, 224 ÷ 4) or combinations of the above (200 – 100 – 50 + 6).

Before the next challenge was given to the children, one or two points of interest were discussed. For example, after the first challenge, the incorrect use of zero was mentioned and children were reminded, through examples, to take care with subtraction because

Table 1

Pen and paper challenges	**Broken Calculator software challenges**
Make the number 56	Make 63: 6 and + keys broken
Make 56: Have to use "×" or "÷"	Make 150: 1, 5 and + keys broken
Make 56: Have to use "×" or "÷"	Calculate 25 × 6: 5 key broken, use parentheses
Make 56: 5 key is broken	Create your own challenge but you must have at least one solution
Make 56: 5 and 6 keys broken	Make 8: 8, 1, + and × keys are broken, must use ÷
Make 56: 5, 6 and + keys broken	Make 4: use ÷, – and × to perform a division and find a remainder of 4
Make 56: 5, 6, + and × keys broken	Make 5: 5, +, – and × keys broken, they had to describe any patterns discovered
Make 777: 7 key broken	
How would you calculate 23 × 4 if the 4 key was broken?	

it is very easy to be one or ten off a correct calculation. These discussions were very beneficial. There was virtually no incorrect use of zero from that point on and the subtraction mistakes reduced considerably over the following weeks. This may have been a different story had a number other than 56 been chosen.

Gradually the challenges were made more difficult. Very few students used multiplication and division in the first week, so it was required that one of these operations be used in the second week. The task for the third week was the same because many students forgot the requirement or could not use these operations. Table 1 indicates how an extra key was broken in subsequent weeks. The last two weeks before using the computer software were quite different to the previous weeks as the students were gaining in confidence and a change from making the number 56 seemed necessary. This provided insight into whether the students could transfer skills to different situations.

When presented with the challenge for the week, initially the students thought they could not do it. After a little thought they realised that they could find a solution and, once they found one, they could usually find more. The challenges made them think and the students started to look forward to the next challenge. One student commented that "it was hard to keep track when you have to miss numbers" because of the broken keys.

The mathematics

A number of important ideas in mathematics were highlighted through the challenges provided. At the beginning of Week Three, a discussion was undertaken on the order of operations. A number of students were writing $2 \div 112$ when they meant $112 \div 2$. In Week Three, a number of students wrote $2 + 3 \times 10 + 6$, thinking that a calculator would perform the calculation from left to right doing the addition first and then the multiplication and the final addition. This provided a context for discussion on the order in which a calculator performs operations and the use of parentheses to change the order of operations. In Week Six, when the + key was broken a number of students wrote $2 \times 2 \times 2 \times 2 \ldots \times 2$. This provided a context to discuss the difference between 28×2 and 2^{28}. In Weeks Six and Seven, when addition could not be used, one student used division instead ($(2 \times 3) \div 140 - 90$, $(10 - 2) - 2 \div (140 - 90)$). This provided a context for discussing the four main operations and when they are used.

Another key idea was the use of "=". In the early weeks many students would record calculations in the form $2 \times 25 = 50 + 6 = 56$ and $50 \div 2 = 25 + 31 = 56$. The difference between the use of the equal sign to mean "gives the answer" and its use to indicate the equivalence of two expressions was discussed. Students were encouraged to write the complete expression with an equal sign at the end rather than use the equal sign for each step in the calculation.

While some of these ideas are quite sophisticated for children of this age, generally there was noticeable improvement in their use of operations, parentheses and the equal sign. As the weeks progressed, their expressions also became more sophisticated. One student, however, seemed preoccupied with writing as many expressions as he could in the time available and many students made no attempt to check whether the expression would really produce the desired result.

The software

Figure 1 is a screenshot of the software presented to the students in Term 3. The image shows a situation at Level_4 where two numerical keys and two operation keys are broken. For the challenges presented each week the students used the computers in the computer laboratory. They used the Create_Challenge button to enter the required target number and to break the appropriate keys. The students were still required to record their calculations in their work-book. The Show_Calculations button on the left of the screen was used to reveal their calculations for recording.

The challenges with the software started at a simpler level compared with the final challenges of the previous term. This allowed the students to recall the processes they had been using and gave them time to become familiar with the software. Initially, students seemed to use less complex calculations than they had been using at the end of the previous term.

It was observed that there was a major difference in the approach taken by the students. With pen and paper, the students often wrote expressions in their books without checking the correctness; now there was more experimentation, allowing the calculator to do the work, until they found a solution. Quite often they would type what they thought would work and then adjust the result to achieve the target number. For example, when trying to make 63 some students entered 100 – 47 =. To their surprise the calculator produced 53 so they simply pressed + 10 = and recorded 100 – 47 + 10 =.

Students used the equal key far more frequently than necessary. To make 150 when the 5 key was broken, they would enter 7 – 2 = and then × 30 =. Again the use of parentheses was discussed and students were shown how the number could be achieved using the equal key once, namely (7 – 2) × 30 =. The discussion was reinforced with the challenge of calculating 25 × 6 with the 5 key broken and the requirement that they must use parentheses.

The software's Create_Challenge button was utilised in the fourth week when the students were allowed to create a challenge for a friend. The proviso that there was at least one solution stopped the students from breaking nine keys and creating impossible challenges. There were some interesting challenges created, for example: "Make 123 when all the number keys except 0 and 1 are broken", "Make 333 when the 3, 6 and ÷ keys are broken" and "Make 10 when the 5, ×, +, – and 1 keys are broken". Many students felt that creating large numbers made the challenge more difficult and a large proportion of the class chose to break the ÷ key as they were not confident with this operation. The challenge of making 8 with the requirement of having to use the ÷ key forced the children to

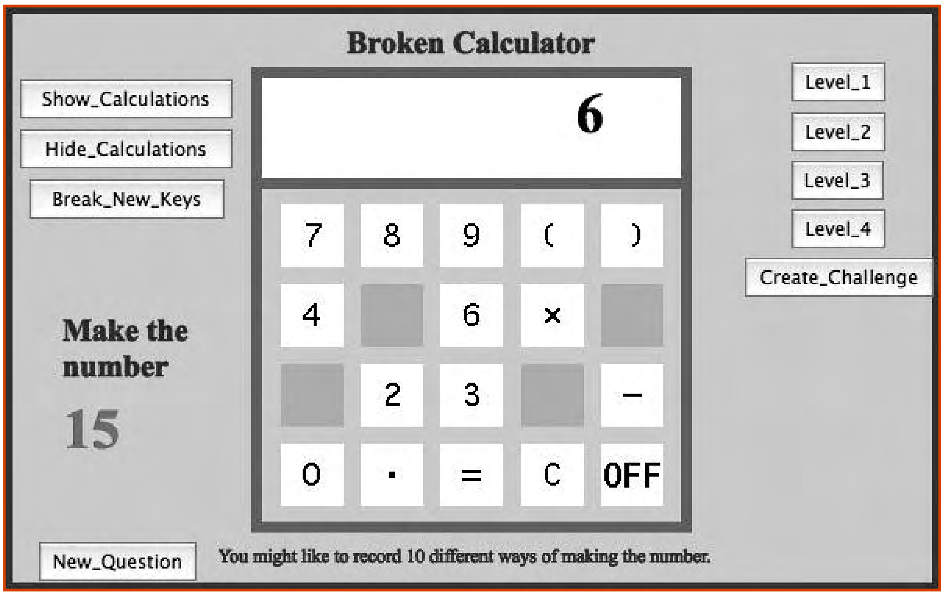


Figure 1

use this operation in the following week.

Division was the focus of other mathematics lessons at this time. Students had been discussing remainders in class, so the idea of finding the remainder as a whole number when the calculator produced a decimal appealed to them. It probably meant they could use their calculators to do their homework, but it forced them to think of what number would have a remainder of 4 when a division was performed. It did not take long for students to realise that the number to divide by had to be larger than 4, but many students had trouble reversing the standard question of finding the remainder when dividing. After a few simple examples were provided (12 ÷ 8 = produces 1.5, – 1 produces 0.5 and × 8 = produces the remainder of 4) students started to find many others (40 ÷ 6, 53 ÷ 7).

The task of making 5 with the 5 key broken and only the division operation available also provided a challenge that complemented work undertaken in other mathematics lessons at the time. Many students discovered patterns for achieving the number but found it quite difficult to communicate the pattern in writing in their workbooks (I went up by 10 was a common response). The teacher became aware that communicating mathematical thinking was an area that needed some attention in the near future.

The teacher found it quite easy to create a meaningful challenge each week, thereby making the integration of this software a useful component of the mathematics curriculum. She also learnt about the students' understanding of number and was able to provide relevant discussion, in context, to remedy some misconceptions. The students enjoyed the challenges. It made them think. Challenges with both the pen and paper technology and the computer technology were seen as worthwhile.

References

Clark, G. J. (2002). *Mathematical Activities* [CD-ROM]. Armidale, NSW.

Logo Computer Systems Inc. (2000), Microworlds 2.05, Montreal, Canada.

Garry Clark
University of New England, NSW
<gclark@une.edu.au>

APMC