The power of CONCEPT FIELDS

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Introduction

The ability to discover, explore, describe and mathematise relationships between different concepts is at the heart of scientific work of professional mathematicians and scientists. At school level, however, helping students to link, differentiate or investigate the nature of relationships between mathematics concepts remains in the shadow of skills development and the practice of routine problems.

The vertical hierarchy of mathematics curricula provides educationally rich environments for developing students' abilities to relate concepts and to explore the nature of the relationships between them. Therefore, it is vital that teachers are able to design curricula that nurture students' understanding of the similarities and differences between mathematics concepts, and to present the subject knowledge in a range of simple ways.

The use of visual representations, such as diagrams, tables or graphs, for showing relationships between mathematics concepts, is a common practice in classrooms. Visual representations are often used for illustrating classifications, algorithms or solutions; an example of visual representation in the form of a diagram is shown in Figure 1.

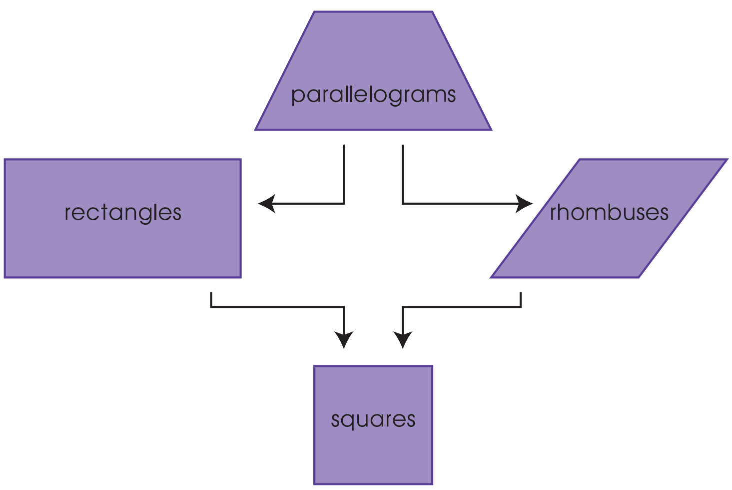


Figure 1. Types of parallelograms.

The diagram shown in Figure 1 could be used to help students understand and distinguish similarities and differences between the four types of parallelograms. For example, the rectangles could be viewed as parallelograms with a right angle, while rhombuses could be viewed as parallelograms with equal adjacent sides. Alternatively, squares could both be viewed as rectangles

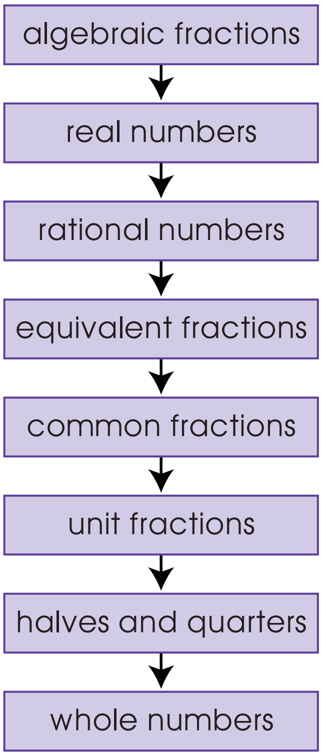


Figure 2. An example of the concept hierarchy of the notion of fraction.

with equal adjacent sides or as rhombuses with a right angle.

Another example of a visual representation is shown in Figure 2.

It is an attempt to illustrate the vertical hierarchy in the development of the concept of fraction. It shows that knowledge of whole numbers serves as a basis for the introduction of the concepts of "a half" and "a quarter." The concept of fraction is then extended to include unit fractions, followed by readily-visualised (common) fractions. At the highest level, the introduction of algebraic fraction is based on an understanding and use of real numbers.

It is obvious that the visual representations have several advantages. Firstly, one could without difficulty recognise the visual symbols. Secondly, the minimum use of text makes it easier to scan and grasp the general idea. And more importantly, very often the visual representation shows the relationships between the elements of the graph.

The aim of this paper is to introduce a specific visual representation in a tabular form, termed as a concept field, and illustrate the use of this as a supporting tool for program development, topic planning and assessment design within the mathematics learning area during the years of schooling.

Concept fields

In this paper a type of visually represented structure that blends both the vertical hierarchy of a particular concept development to the skills and understandings associated with key knowledge and applications will be used. The term of the concept field is defined as a visual representation in a tabular form with vertical links to key levels in concept development and horizontal links to some skills and understandings associated with each level.

The broad definition is deliberate, it allows for addressing a range of factors relating to the complexity of teaching and learning a particular concept during the years of schooling.

Structure of a concept field

The concept field shown in Figure 3 has a structure addressing the hierarchy of the concept development and key skills and understandings associated with teaching and learning of the concept at each particular level.

The left column traces the stages in the vertical development of the concept's hierarchy. The key words used, for example such as "unit fractions" or "algebraic fractions," summarise the nature of the cognitive complexity of the concept at a particular level. The notion of fraction is introduced in the context of halving and repeated halving and then extended at other levels to include: unit fractions, readily-visualised fractions, equivalent fractions, rational numbers, real numbers and the algebraic fractions at the highest level.

Key words	Key understandings
Algebraic fractions	• Problem solving using algebraic fractions • Operations with algebraic fractions • Simplifying algebraic fractions • Considerable skills with factorising algebraic expressions
Algebraic expressions (real numbers)	• Calculating with fractions using fractional indices (e.g., $x^2/3$) • Simplifying and factorising algebraic expressions
Rational numbers	• Written methods to +, −, ×, ÷ rational numbers • Rational numbers (m/n, where m and n whole numbers, n ≠0) • Considerable skills with common fractions
Equivalent fractions	• Understanding the links between fractions and decimals and ratios and fractions • Using the language of ratio to represent a fraction • Adding and subtracting well known fractions by using written methods • Comparing and ordering fractions by using a common denominator
Common fractions (easily-visualised)	• Begin to link ratio and fractions • Begin to link multiplication and division with the use of fractions • Converting fraction to decimals • Positioning fraction on the number line • Using symbols to present a fraction • Comparing fractions visually • Visualising a range of common fractions
Unit fractions	• Comparing unit fractions • Using symbols and words to write a unit fraction • Partitioning a whole to equivalent parts (unit fraction) with diagrams
“Halves” and “quarters”	• Using repeated halving • Using different methods for halving • Halving a whole (discrete quantities, symmetrical objects, amounts) and naming each part

Figure 3. An example of a concept field for the notion of fraction.

In the second column some of the key skills and understandings associated with understanding and using fractions at each particular level are listed. For example, at the lowest level students learn to halve small discrete quantities, symmetrical objects and small amounts and to name the two equal parts. Then they learn different ways for halving a particular whole, which could be a symmetrical shape, a small amount or a quantity. The use of repeated halving and naming each part is used as a natural step for linking halves and quarters. At the highest level students can use different techniques to simplify algebraic expressions and algebraic fractions. They use also the four operations with algebraic fractions and solve questions involving algebraic fractions in a range of practical or abstract contexts.

Classroom applications

The goal of this paper is to illustrate how a specific concept field could be used to help teachers plan and design mathematics curricula addressing teaching, learning and assessing the notion of a fraction at a particular key stage of the concept's developmental hierarchy.

In particular, this paper will illustrate the use of a concept field as a tool for developing: (a) key elements of the vertical curriculum; (b) topic plans; and (c) specific assessment items.

Vertical curriculum

Designing a vertical curriculum to show how a specific concept develops over the years of schooling is not a trivial exercise. Therefore it is expected that the use of a specific concept field would make it easier for teachers to design appropriate curricula across different year levels. An example of lesson topics addressing half and halving is shown in Figure 4. Depending on classroom environments and students' background, teachers may use three, four or more lesson periods to address the topics. For example, students could spend one period for halving symmetrical shapes, up to two periods for halving different amounts, and up to two periods for using the concept in different, but familiar, contexts. In some cases introductory lessons could help students revise or acquire skills such as counting by twos, doubling and halving small discrete quantities.

"Topic" planning

A topic plan is an overview of teaching, learning and developmental outcomes addressing a group of interrelated lesson periods. The example of a topic plan in Figure 5 lists the main teaching, learning and developmental outcomes associated with the introduction of the notion of half. An example of a topic plan for the first part of the table above, "What is a half and using halving," is shown in Figure 4.

What is a half and using halving
Recognising a "half" in symmetrical pictures Halving shapes: shading halves Halving quantities (e.g., discrete quantities) Halving an amount (e.g., sharing a drink) Interpreting and using a half, halving and doubling in a range of contexts
Repeated halving and what is a quarter
Introducing repeated halving and recognising a "quarter" Using repeated halving to find a quarter from different wholes Interpreting and using a quarter, repeated halving, doubling and repeated doubling in a range of contexts

Figure 4. A sequence of lesson topics addressing half and quarter.

The table above presents a concise structure of the key teaching objectives, associated learning outcomes that need to be observed in students' performance, and the nature of the developmental growth in students' understanding of a half and halving. Two important comments need to be made. Firstly, each of the lists could be expanded considerably, depending on the background knowledge that students have. For example, teachers could stress the range of contexts in which halving could be used and on the ways students could represent their responses. Secondly, the nature of the developmental outcomes is crucial. They draw on both the depth of students' understanding via the nature of applications and important relationships between previous and current knowledge.

Teaching outcomes	Learning outcomes	Developmental outcomes
Introducing a "half" and different techniques for halving a whole in familiar contexts	Students should: • develop an understanding of a "half" • learn techniques for halving in familiar contexts by using: – symmetrical shapes; – discrete quantities and amounts.	Students develop an under-standing: • that halves could appear in different ways • that some quantities (objects or amounts) could be halved in different ways • of the relationship between a half and a whole (e.g., a half is less than a whole, 2 halves make a whole, 4 halves make 2 wholes, etc.)

Figure 5. An example of a topic plan for understanding a half and halving.

Assessment design

Before planning particular lessons, detailed planning of the intended assessments should be done. The assessment questions need to reflect the nature of the learning and developmental outcomes and they should not be different by nature from the questions used in the classrooms. For example, students should have an experience in using the concept of a half in "inverse" situations for finding the whole, if the half amount is known. Examples of some assessment items reflecting the nature of the developmental outcomes are given in Figure 6. It should be noted that the list could be extended to include easier or more difficult items, depending on the expected range of students' performances.

The performance profile shown in Figure 7 reflects the skills and understandings assessed by the assessment items shown above. Again, it could be expanded to reflect the specific skills and understanding assessed in other formal and informal assessments used in the classroom. For communicating the quality of student's responses teachers could use percentages, grades, word descriptors or a diagram, depending on the nature of communication used in the classroom. An important element of reporting is communicating clearly some specific misconceptions or areas of concern in student's understanding or performance.

The next step in teacher preparation is to develop a detailed plan for each lesson and to address the teaching, learning and developmental outcomes outlined in Figure 5. Depending on students' backgrounds and knowledge, the teacher should choose appropriate learning content, contexts and plan a sequence of classroom activities and teaching strategies for each lesson.

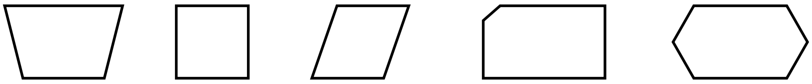
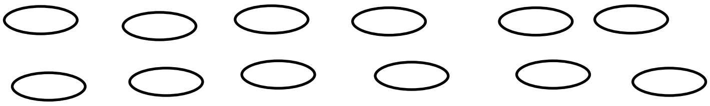
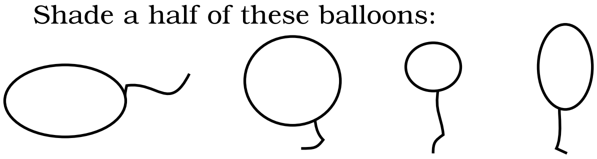
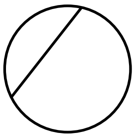
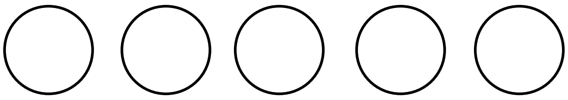
Question	**Development outcome**
1. Here is a biscuit. It has a rectangular form. Halve the biscuit in different ways, as many as possible. 	The student halves a rectangular shape in 2 or more different ways.
2. Put a tick on the shape that would be difficult to halve. 	The student links halving with a line of symmetry of a 2D shape.
3. Helen wants to share 12 Smarties with a friend. Show how she should divide them into 2 halves. 	The student halves an even number of objects in different ways.
4. Shade a half of these balloons: 	The student halves small even quantities in familiar contexts.
5. Kathryn tried to cut this pizza into halves. Did she halve the pizza? Explain why. 	The student justifies why a symmetrical object has not been halved.
6. Peter's pizza has 2 pieces. He gave 1 piece to George. Did he give George a half of his pizza? Explain why.	The student under-stands that halving gives two equal pieces
7. Here are 5 pizzas of the same size. Divide the pizzas equally between 2 people. 	The student halves an odd number of symmetrical objects.
8. Mrs Smith halved her pizza and gave one piece to Nick. Nick halved his pizza and gave a piece to his friend Ned. Ned shared his pizza with George. Who got the biggest slice of pizza. Show why.	The student links repeated halving and a half.
9. Emma and Kelly shared a bag of lollies equally. Then Emma gave a lolly to her friend Jasmine. Now Emma has 3 lollies. How many lollies did she have initially? Show why.	The student uses halving and working backwards in a familiar context.

Figure 6

PERFORMANCE PROFILE: Understanding half and using halving Name:	
SKILLS AND UNDERSTANDINGS OBSERVED	ACHIEVEMENT
The student uses halving and working backwards in familiar contexts Q9).	
The student links repeated halving and a half (Q8).	
The student halves an odd number of symmetrical objects (Q7).	
The student halves an even number of objects in different ways (Q3).	
The student justifies why a symmetrical object has not been halved (Q5).	
The student halves small even quantities in familiar contexts (Q4).	
The student links halving with a line of symmetry of a 2D shape (Q2).	
The student understands that halving gives two equal pieces (Q6).	
The student halves a rectangular shape in 2 or more different ways (Q1).	
The student halves a SYMMETRICAL shape (Q1).	
Comments for further improvement:	

Figure 7. Performance profile.

Concluding remarks

I believe that mathematics is the easiest school subject. The challenge for teachers, however, is to be able to present the subject knowledge in a range of simple ways. In this paper, via the use of the concept field, I tried to make transparent the links in a specific concept's hierarchy that will be covered during different stages of learning.

I hope that the message was clear: we need to help our students to understand numerical fractions in the early years of schooling. This is not because we want to challenge their thinking, but because the operations underlying numerical fractions are the same as those that underlie algebraic fractions — skills that are the heart of learning and using mathematics at the highest school level.