π round and round

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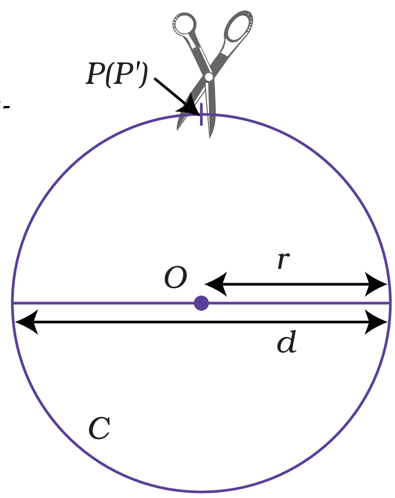
The number π

One of the best known numbers in mathematics is the number denoted by the symbol π (pi). It is most commonly associated with the circle, but our students (and perhaps even we!) may be surprised to learn that it occurs in all sorts of unexpected places. We shall investigate these various occurrences throughout this year, but for starters, let us see how we might encourage our students to explore the circle connection.

Diameter and circumference

The circle is one of the simplest of geometric figures, yet it took mathematicians many centuries to unravel its secrets.

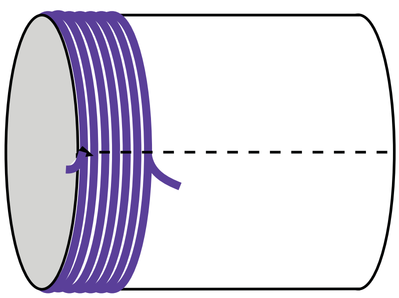
It is easy to define a circle, and to define its *centre* O, *radius* r and *diameter* $d = 2r$. Then the circumference is "the distance round the circle." If we think of the circle being cut at some point P, and then laid out straight to form a segment PP', the circumference C is the length of this segment. The historical question then was: If we know the diameter of a circle, can we find the circumference?



Class investigations

1. Carefully measure the diameter of the wheel of a bicycle, including the tyre. Place the wheel on a straight line in the playground, and mark with chalk the tyre and the ground at the point of contact. Now carefully wheel the bicycle along the line until the wheel has made three complete revolutions. Mark the ground again at the point where the chalk mark on the tyre comes in contact with the ground. You should now be able to find the circumference C of the wheel by measuring the distance between the chalk marks on the ground. Measuring $3C$ and d to the nearest centimeter, now calculate C/d to two decimal places.
   Repeat the exercise using bicycle wheels of different sizes. What do you notice about the ratio C/d?
   You might expect more accurate results by allowing the wheel to make more than three revolutions (and adjusting the calculation!).

2. Take an empty fruit tin of circular cross-section and measure its diameter d. Cut a small notch in the rim and carefully draw a straight line along the length of the tin, starting from the notch. Tie a knot in the end of a piece of string, catch it in the



notch, and wind ten complete turns around the tin, making sure that successive turns are close together and not overlapping. You can now estimate the circumference C by measuring the length of the thread used. Measure $10C$ and d in millimeters and calculate C/d to two decimal places.
Do other tins give the same value of C/d?

Conclusions

By now your class should have discovered that

- the ratio C/d appears to be constant for all circles measured;
- $C/d \approx 3.14$.

We denote the exact value of C/d by π. Thus $C = \pi d$.

Further investigations

It is always good to be able to make use of facts that we discover. Here are some interesting explorations and applications.

1. Express 22/7 as a decimal correct to two decimal places. How does this decimal compare with our experimental results for π? (A good enough approximation for most practical purposes.)

2. (a) The symbol π occurs outside mathematics. What is it?
   (b) Is there a circle with circumference π? Describe it.
   (c) What is another word for "circumference"?
   (d) Can you suggest why the Greek letter π was chosen to represent the ratio C/d?

3. Suppose a metal band is wrapped tightly around the earth's equator. An electric current is passed through the band causing an expansion of 11 m. How high will the band lift off the surface of the earth? (The circumference of the earth is about 40 000 km. Do you need this information?)

4. The distance from the earth to the moon is about 380 000 km. How far does the moon travel in a revolution around the earth?

5. The gear wheels of a bicycle have radii 80 cm, 30 cm. If the smaller wheel has 21 teeth, how many teeth has the larger wheel?

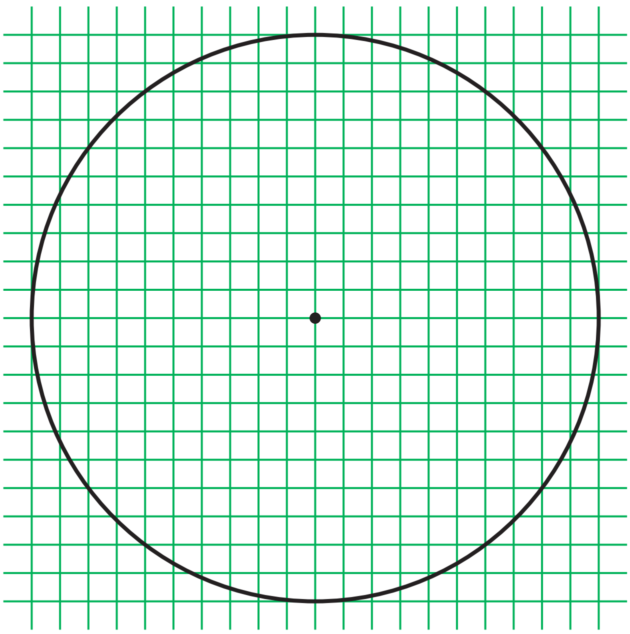
Answers: 1. (a) $22/7 = 3.1428...$; 2. (a) a Greek letter (b) diameter 1 (c) perimeter (d) π is a Greek "p"; 3. 1.75 m; 4. 2 386 400 km; 5. 56.

Counting and folding

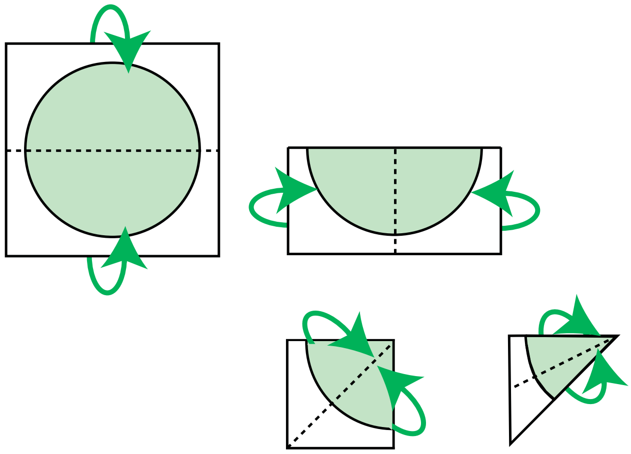
One way of deciding the size of a given circle is to measure its *area* A.

Class investigations

1. The illustrated circle has radius 10 units. Estimate its area as closely as you can by counting the little squares. Determine the ratio A/r^2. Repeat the experiment by drawing circles of different radii on graph paper. Do you get the same values of the ratio A/r^2? Can you make a conjecture (guess) about the value of this ratio?



2. Draw a large circle on a sheet of paper, mark the centre, and shade the circular region. Fold the circle repeatedly in half like this:



(a) The portion of the shaded region now showing should look almost like an isosceles triangle. If the circle has radius r, what is the approximate altitude of this "triangle"? If the base of this "triangle" has length x, what is its area?

(b) Now unfold your paper. How many shaded "triangles" are there? Add their areas to obtain an expression for A. What is the relation between x and the circumference C? Express A in terms of C and r.

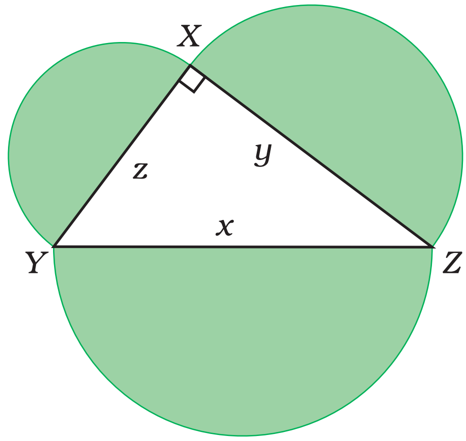
Conclusions

From our second class investigation it appears that the area A and the circumference C of a circle are related by the expression $A = \frac{1}{2}Cr$. Since $C = 2\pi r$, your class should have deduced the well-known formula for the area of a circle: $A = \pi r^2$.

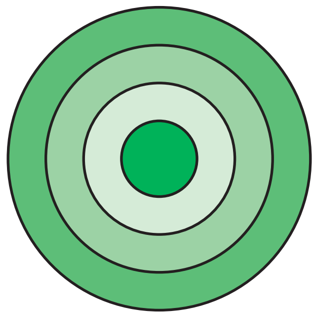
Further investigations

1. If the diameter of a circle is doubled, what happens to its area?

2. In the figure below, semicircles have as diameters the sides of the right-angled triangle, ΔXYZ. A well-known theorem of Pythagoras tells us that the side lengths x, y, z of the triangle satisfy $x^2 + y^2 = z^2$. What can you say about the areas of the semi-circles?



3. See if you can construct a circular target on which an amateur can expect to hit the bulls-eye as often as he hits any one of the rings.



Answers: 1. The area increases by a factor of 4; 2. $\frac{1}{2}.\pi z^2 = \frac{1}{2}.\pi x^2 + \frac{1}{2}.\pi y^2$; 3. If the smallest radius is 1, the next radius is $\sqrt{2}$.

Bibliography

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The topic pi, http://42explore.com/pi.htm.