Using paper folding, fraction walls, and number lines

to develop understanding of fractions for students from Years 5–8

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Abstract

Several researchers have noted how children's whole number schemes can interfere with their efforts to learn fractions. An Australian study found that children who were successful with the solution of rational number tasks exhibited greater whole number knowledge and more flexible solution strategies. Behr and Post (1988) indicated that children needed to be competent in the four operations of whole numbers, along with an understanding of measurement, for them to understand rational numbers. This paper describes a "hands on" approach developed by researchers that focuses on the use of paper folding, fraction walls and number lines to develop an understanding of fractions using a measurement model.

Introduction

Over the past 20 years, research on rational number learning has focused on the development of basic fraction concepts, including partitioning of a whole into fractional parts, naming of fractional parts, and order and equivalence. Kieren (1976) distinguished seven interpretations of rational number which were necessary to enable the learner to acquire sound rational number knowledge, but subsequently condensed these into five: whole–part relations, ratios, quotients, measures, and operators (Kieren, 1980; 1988). Kieren suggested that children have to develop the appropriate images, actions and language to precede the formal work with fractions. Saenz-Ludlow (1994) maintained that students needed to conceptualise fractions as quantities before being introduced to standard fractional symbolic computational algorithms. Streefland (1984) discussed the importance of students developing their own understanding of fractions by constructing the procedures of the operations, rules and language of fractions.

Behr and Post (1988) suggested that children needed to be competent in the four operations of whole numbers, along with an understanding of measurement, for them to understand rational numbers. They suggested that rational numbers are the first set of numbers experienced by children that are not dependent on a counting algorithm. Steffe and Olive (1990, 1993) showed that concepts and operations represented by children's natural language are used in their construction of fraction knowledge.

An Australian research project (Hunting, Davis & Pearn, 1996) was designed to investigate the extent to which children's thinking processes might be associated with qualitative differences in their whole number knowledge when solving rational number tasks. This research revealed the vast difference in the children's mathematical knowledge and the type of whole number strategies they used. Children who used a variety of strategies to solve whole number tasks were more successful, and used superior strategies, when solving rational number tasks. Children, who relied on rules and procedures to solve whole number tasks, were less successful with rational number tasks. They experienced some success with partitioning and ratio tasks but little or no success with fraction tasks set in various contexts. This study raises several questions about conventional approaches to teaching fractions. Most children studied had difficulty understanding the language of fractions. While most students were successful with tasks involving one half, very few understood, or were successful, with tasks involving other unit fractions.

As there appears to be an interdependence between the development of rational number knowledge and whole number knowledge, fraction tasks need to be given that allow children to develop numerical relationships and strategies flexible enough to be used in various contexts. Success with the ratio tasks indicated that problems and tasks developed in the context of sharing discrete items would be a good starting point for the teaching of fractions in the early years of schooling with emphasis on the introduction of appropriate language. This emphasis on the appropriate language needs to be continued into the middle years of schooling (Years 5–8).

This paper focuses on the measurement model which is designed to assist teachers to identify the mathematical understanding of the students they teach and to develop activities to help all students to progress at their relative level of understanding (see for example, Pearn, Stephens & Lewis, 2003; Stephens & Pearn, 2003; Pearn & Stephens, 2007). This approach to teaching fractions for understanding is being used in many Victorian primary and secondary schools: state, catholic, and independent.

Introduction to fractions

Students are given various lengths of paper strips or pieces of paper streamers. Ask the students to fold their paper strips into halves and ask a question such as: "How do you know you have folded your strip into halves?" Ask students to compare their half strips with those of other students. Students are then shown other students' attempts to show one half of a rectangle (Figure 1).

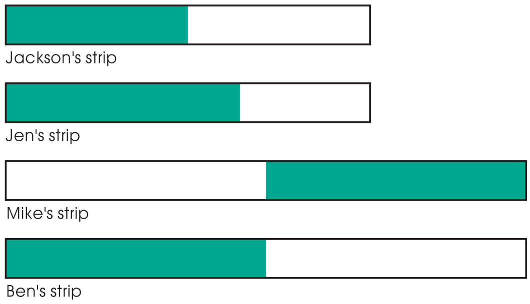

Figure 1. Students' attempts at shading one half of a rectangle.

Ask questions such as:

- Which of these students have successfully shaded their rectangles to shows one half? (Some students will not recognise that Mike's rectangle is showing one half as they think the left hand side is one half and the right hand side is two halves.)
- Why is Jackson's half different to Mike's half?
- Why do you think Jen has shaded her rectangle how she has?

Comparison of half of a square

Students are handed two squares of paper and asked to fold each square in half. Once students have folded one square in half, ask them to fold the other in a different way.

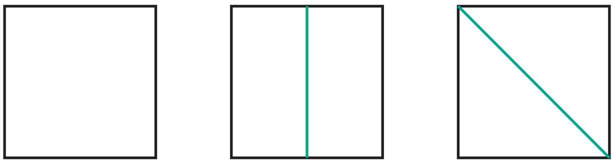

Figure 2

Questions to ask students include:

- Which half is larger: the triangle or the rectangle or are they both the same?
- How do you know?
- Prove it. (Show me.)

Students enjoy proving that the triangular half is the same size as the rectangular half.

Representations of a fraction other than one half

Write the fraction one third ($\frac{1}{3}$) on the board and ask students to draw a picture or a diagram to show what this symbol means. Previous experience has shown that most students will draw a circle divided into three parts not necessarily equal.

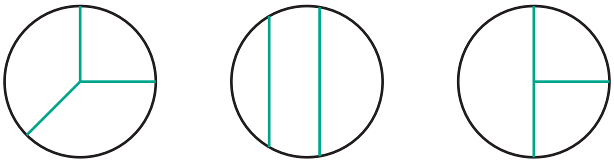

Figure 3

Take particular notice of which of the three pieces students colour. Is it always the left hand piece or is it the right hand piece? Ask students how they decided which piece to colour.

Students are then asked to draw a different representation of one third. We would expect students to draw a variety of 2D shapes such as squares, triangles, as well as groups of objects. Do they attempt to make equal divisions?

If students colour one of three objects to represent one third do they colour the third object in the row? If so, ask if they can show you a different one third of the group.

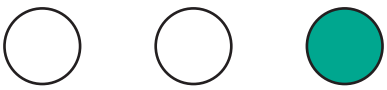

Figure 4

If students colour one of three objects ask them to show you one third of six objects, 12 objects and ask how they decided on how many to colour.

Some students try to replicate the format of the fraction one third ($\frac{1}{3}$) and colour one of four objects as shown in Figure 5.

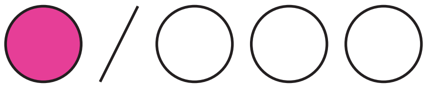

Figure 5

If students draw nine objects and colour three, ask them to show you one third of three or six objects. They may think that one third will always be three regardless of the number of objects they started with.

The measurement model

Working with teachers and students has highlighted the relevance of the measurement model where students fold paper strips into fractional parts then use these strips to mark fraction walls and number lines. Fraction walls highlight equivalent fractions and allow students to compare and order fractions between zero and one. Number lines allow students to experience the density of the number system.

Folding paper strips

Students are given a paper strip that is 20 cm long and asked to fold it into two equal pieces. Discussion includes questions such as:

- How many parts are there?
- How many folds are there?
- What do we call each part?
- Show me one half of the paper strip. Show me a different half.
- How many halves are there in a whole?

Students are then asked to fold their halves of paper strip in half. Before opening their paper they are asked:

- How many parts will there be?
- How many folds will there be?
- What do we call each part?
- Show me one quarter of the paper strip. Show me a different quarter.
- Show me two quarters. What is another name for two quarters?
- Which is larger: one half or one quarter? How do you know?

Students are then asked to use their paper folding to show: three quarters, four quarters, one half and one whole. After folding their paper streamer in eighths students will be asked questions that involve equivalence, showing fractions that are larger and smaller than given fractions, and questions such as: Show me a fraction that is larger than one eighth but smaller than one half?

If using paper folding for the first time then just fold halves, quarters and eighths. If students have used paper folding before another paper strip will be folded into thirds, sixths, ninths and similar questions asked as for the halves, quarters and eighths. Strips can then be folded into fifths and tenths. Students should be challenged to fold a paper strip into sevenths.

Fraction wall

A fraction wall can be constructed using the Insert Table command from Word. The table is formatted so that it is 20 cm wide and contains ten rows (see Figure 6). Using their folded paper strips students are asked to complete the fraction wall to show a whole, halves, thirds, quarters… tenths.

Ask students to mark each of the parts with the appropriate unit fraction. See Figure 7 for the correct way to mark the Fraction Wall as compared to Figure 8 that shows one way (incorrect) found in some textbooks.

Ask students to show you several representations that will demonstrate three fifths. Students need to realise that any three pieces marked $\frac{1}{5}$ will be $\frac{3}{5}$ and that the three pieces do not necessarily need to be adjacent.

Students should be asked similar questions about the fraction walls as those asked for the folded paper strips; that is, questions related to equivalence and fractions that are smaller and larger than given fractions.

For example, ask students to find fractions that are equivalent to:

- one third;
- two fifths.

Ask students to find fractions that are smaller than:

- one half;
- three fifths.

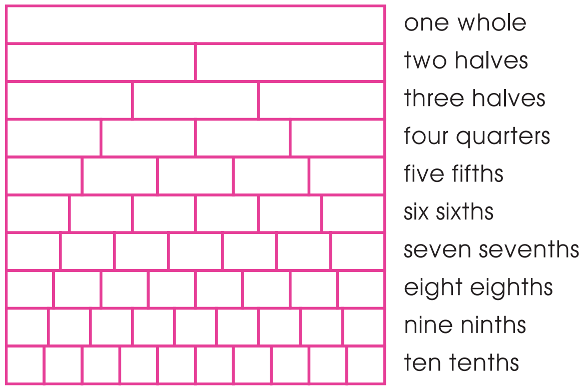

Figure 6. Blank fraction wall.

$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$

Figure 7. Correct marking of the fraction wall with fifths.

$\frac{1}{5}$	$\frac{2}{5}$	$\frac{3}{5}$	$\frac{4}{5}$	$\frac{5}{5}$

Figure 8. Incorrect marking of the fraction wall for fifths.

After constructing their fraction walls using their folded paper strips students can use Microsoft Word to construct their own fraction walls using the Split Cells instruction. Ensure that students are able to refer to the fraction walls when working with fractions.

Many teachers like to use pizzas, pies or circles for fractions. Many worksheets in textbooks focus on students shading circles. Usually the divisions have been marked on these circles and students are asked to shade the required number of pieces. To highlight the difficulty of using pizzas, pies, or circles ask students to divide circles without the use of compass or protractor. Another dilemma in the use of pizzas is that students know that pizzas already come cut into pieces not necessarily equal!

Fractions as numbers

Given a number line marked 0 to 1, students are asked to estimate and mark given fractions. For example: “Mark your number line where you think the number one third would be.” After marking their line, students check using their folded paper strips. Repeat this procedure with other fractions. After using the number line marked 0 to 1, give students number lines marked 0 and 2; 1 and 2; 2 and 3; 0 and $\frac{1}{2}$. Students need to justify why they have placed their fractions where they have.

The language of fractions

All sessions should emphasise the language of fractions. To reinforce the language of fractions, ask students to write in words (everyday English) some expressions such as:

$\frac{1}{2} \times \frac{1}{2}$ (one half of a half: a quarter)

$\frac{1}{3} - \frac{1}{6}$ (the difference between one third and one sixth: *one sixth*)
$\frac{1}{3} \div \frac{1}{6}$ (How many sixths are there in one third? *two*)
$\frac{1}{6} \div 2$ (share one sixth between two: *one twelfth*)

Students also need to move between the words and the symbols. For example, ask students to write the symbols for:

- one third and one quarter
- the difference between one half and one third
- one half of 12
- How many quarters in one half?
- How many sixths in one third?
- How many thirds in two wholes?

Conclusion

This model uses the folding of paper strips to complete the fraction wall and then mark the appropriate places for fractions on number lines. These activities assist students to develop the understanding of fractions rather than rely on rules and procedures without understanding. This “hands-on” approach highlights the need to develop fractional language and the ability to move between everyday language and fractional symbols.

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