Why teachers matter

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Wondering whether we are really making a difference to young people's mathematics learning is a question that most teachers have probably wrestled with at some stage of their careers. However, evidence from a multitude of research studies shows that students' mathematics learning and their dispositions towards mathematics are indeed influenced — for better or for worse — by the teaching that they experience at school (see Mewborn, 2003, for a review of this research). In other words, teachers *do* matter. It is difficult for researchers to specify exactly how different types of teaching and teacher qualities affect student achievement because this would require untangling the complicated relationships that exist between teacher characteristics, teaching practices, and student learning. Nevertheless, the general trends in these relationships are clear. In this article I want to illustrate some of these trends by drawing on my experiences in working with pre-service and practising mathematics teachers and their students, and in doing so to propose three reasons why teachers matter. I will then give some examples of *how* teachers can matter to their students in a more practical sense.

Beliefs about the nature of mathematics

One reason why teachers matter is related to the vision of mathematics we bring to our students. Whether we are aware of it or not, all of us have our own beliefs about what mathematics is and why it is important, and we enact these beliefs through our teaching practices. At the start of each year I find out about the mathematical beliefs of my pre-service students (prospective secondary mathematics teachers) by asking them to write their answers to the question "What is mathematics?". Their responses always fall into several distinct categories. The first category reflects a fairly limited view of mathematics as *number*, with mathematics described as being about "working with numbers and symbols", "the understanding of numbers", or "rules that apply to numbers". A more pragmatic type of response refers to mathematics as a *tool* that is important in our everyday and working lives; for example, it is "a skill needed for many jobs, "the base for science, accounting, engineering", and it "provides the fundamentals for everyday living".

Some pre-service teachers interpreted mathematics as *logical thinking*, because it involves "learning how to think logically" and is "a way to think and solve real life problems". Others thought of mathematics as a *language*, describing it as "a creative language" or "a language of numbers and symbols" that provides "a means of communication". The idea of mathematics as patterns was evident in responses that claimed mathematics is "the science of patterns and relationships" or is "about numbers and patterns".

The final two categories of responses communicate the sense of pleasure and wonder that some people gain from mathematics. Understanding mathematics as a way of interpreting the world means that, for some pre-service teachers, it is "a daily life experience — you see it everywhere!" or "a way of describing everything in life". Those who appreciated mathematics as *beauty* believed that mathematics is "inescapable, fun, exciting, awe inspiring", and even that "mathematics and love is [sic] the best thing that can happen to anyone".

As well as challenging these prospective teachers to articulate their own beliefs about the nature of mathematics, I also wanted them to find out what their students think about learning mathematics at school. A more subtle way of doing this than asking a direct question involves using metaphors for mathematics, such as:

- If mathematics was a food, what kind of food would it be?
- If mathematics was a colour, what colour would it be?
- If mathematics was music, what kind of music would it be?

(See Frid, 2001, for more ways of using metaphors for mathematics.)

Pre-service teachers who tried this activity with their junior secondary students during a practice teaching session were surprised, and somewhat disturbed, by the results. If mathematics was a food, most students agreed that it would be a green vegetable such as broccoli, brussels sprouts, or zucchini. According to them, these vegetables taste terrible but we have to eat them because they are good for us, thus implying that mathematics is a necessary but unpleasant part of their school diet. Others who were more favourably disposed towards mathematics compared it with bread (a staple food), fruit salad (because it contains a variety of ingredients), or lasagne (different layers are revealed as you eat it). Students thought that if mathematics was a colour it would be either black (depressing, evil), red (the colour of anger and pain), or brown (boring). The few who admitted to liking mathematics often said it would be blue because this colour is associated with intelligence or feelings of calm and peacefulness. There was more variety in metaphors for mathematics as music. Many students said that mathematics was like classical music because they found it difficult to understand; some likened it to heavy metal music because "it hurts your brain"; while one responded that it was like the theme from the movie *Jaws* — because "it creeps up on you". Writing in her practice teaching journal, one pre-service teacher lamented that "there was not one person in the class who admitted to liking maths and compared it with McDonald's or Guy Sebastian!"

Comparing their students' generally negative views about mathematics with their own very positive beliefs led the pre-service teachers to reflect on the role of teachers in enriching or limiting students' perspectives on the nature and value of mathematics, and to consider how students' dispositions towards mathematics might be shaped by their experiences in school mathematics classrooms. The first reason why teachers matter, then, is because *through our words and actions we communicate our beliefs about what mathematics is to the students we teach.*

Perceptions of mathematics teachers

Through their daily experiences in classrooms, students develop long lasting perceptions about mathematics and mathematics teachers. Some of these perceptions involve memories about particular teachers while others are more stereotypical, arising from students' experiences over time in many different mathematics classrooms. To find out about my pre-service students' perceptions of their own mathematics teachers I ask them to write their personal mathematical life history, in which they describe their experiences of learning mathematics at school and at university and recall the influence of good and bad teachers they may have encountered. Almost always there is one teacher who stands out in their memory, occasionally as someone whose ridicule or harsh words caused feelings of shame and discomfort, but most often as a teacher fondly remembered for inspiring a love of mathematics through their patience, enthusiasm, and willingness to help students outside of class time.

To further emphasise the key role that teachers play in influencing students' dispositions towards mathematics, I also ask the pre-service teachers to explore their own students' perceptions by inviting a junior secondary class to draw a typical mathematics teacher. One pre-service teacher tackled this task by drawing a stick figure on the whiteboard and asking the class to give her instructions on what additional features to include. The finished drawing, complete with

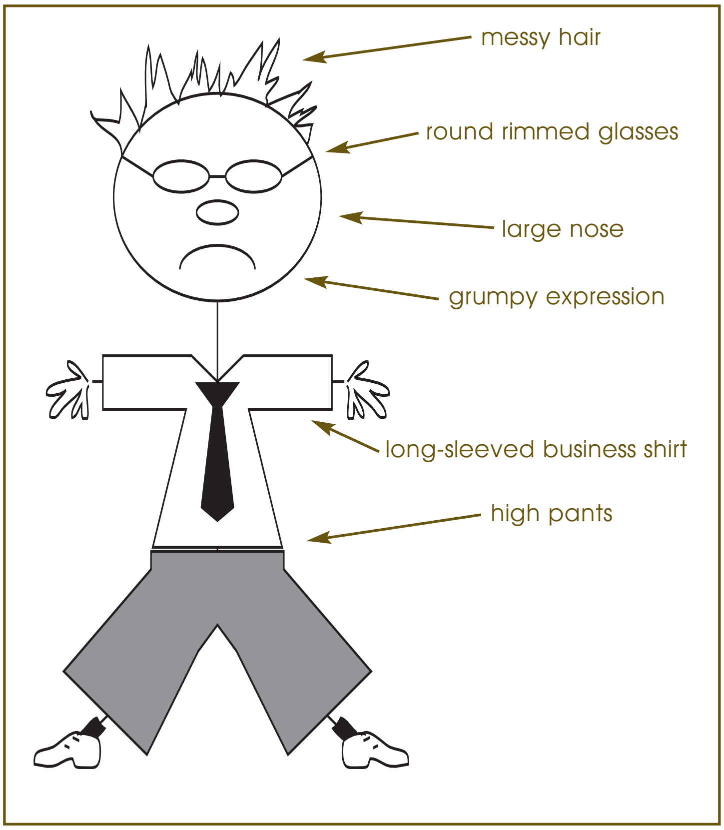

Figure 1. Secondary school students' drawing of a typical mathematics teacher.

annotations provided by the class, is reproduced in Figure 1.

The school students also commented on aspects of a typical mathematics teacher's personality, using words such as "boring", "old", "depressing", "cranky", and "ugly". Other pre-service teachers found that their students produced very similar drawings and described mathematics teachers in much the same way. Likewise, local and international studies of students' images of mathematicians have identified themes such as the foolish mathematician (lacking common sense or fashion sense), the mathematician who cannot teach (does not know the material or cannot control the classroom), or even mathematics as coercion (mathematicians as teachers who use intimidation or threats) (Picker, & Berry, 2001; see also Grootenboer, 2001; Ryan, 1992). While you may not recognise yourself in these drawings or descriptions, the clear message here is that teachers matter because *we have the power to engage or alienate students in ways they will remember for the rest of their lives.*

Knowing and doing mathematics

A third reason why teachers matter is related to what it means to know and do mathematics. Most teachers would agree that we want students to learn mathematics with understanding, but how do you know when you understand something in mathematics? This is a question I have put to several hundred school students and teachers over the past few years, either in writing in a students questionnaire or orally in professional development workshops.

Let us look at students' perceptions of mathematical understanding. I analysed the written responses of over 300 secondary school mathematics students and grouped these into the categories shown in Table 1. The majority of students considered they understood something in mathematics if they could do the associated problems and get the correct answer. A few described understanding in affective terms; that is, understanding was accompanied by feelings of increased confidence or enjoyment or excitement. Only a small proportion of students associated understanding with knowing why something worked or made sense, and even fewer referred to the ability to apply their knowledge to unfamiliar problems as evidence of understanding. Perhaps the most sophisticated kind of response came from students who knew they understood something when they could explain it to someone else.

I recorded teachers' oral responses to this question during in-service workshops and found they could be categorised similarly, but the most common answers always referred to aspects of understanding rarely mentioned by students. Teachers know they understand something in mathematics when they can apply it to a real life situation to get a meaningful answer, when it makes sense, and when they feel comfortable explaining it to another person. These comments demonstrate the importance of giving students opportunities to know and do mathematics in the ways we as teachers value so highly — by using mathematics to explain and make sense of the world around them. Thus teachers matter because through our own curiosity about the world we can demonstrate what it means to know and do mathematics.

Table 1. Evidence of understanding for secondary mathematics students (n=329)

Response category	Sample responses	Frequency	Proportion
I Correct answer	When I get it right. You can do heaps of them without mistakes.	234	0.71
II Affective response	I get interested. I feel confident when doing it.	35	0.11
III Makes sense	It fits in with my previous knowledge. You realise why you use the formula, what reasons.	52	0.16
IV Application/transfer	When I can apply it to something else outside school. When I can understand a complex problem and do all the related problems.	27	0.08
V Explain to others	When I can explain it to other people without confusing myself. I can explain theory to other students.	24	0.07

How can teachers matter? Some practical suggestions for the classroom

Students' beliefs about the nature of mathematics and how it is taught and learned are strongly influenced by their experiences in mathematics classrooms. We can help students develop positive mathematical beliefs by planning learning experiences around a problem to be solved, a question to be answered, a significant task to be completed, or an issue to be explored. Sometimes this might mean starting with a real world situation where the mathematics that might explain the situation is not obvious — and neither is the "problem" to be "solved". The following two examples illustrate this approach.

Cars around the moon

Some years ago I saw in a local newspaper the following notice proclaiming the effectiveness of their classified advertising service.

So many cars have been advertised in the Courier-Mail Classifieds, lined up they would circle the moon.

What an intriguing claim! I decided to investigate whether it makes sense.

Questions that came to mind included:

1. How far is it around the moon?
2. How many cars does this represent?
3. How long would it take to advertise this number of cars?

(Notice that the initial situation was not presented as a “problem”; it was up to me to pose the problem and work out how to solve it.)

1. I found out via an Internet search that the diameter of the moon is 3445 km.
   ∴ circumference of the moon
   = $\pi \times 3445$ km = 10 822 km

2. Number of cars
   = 10 822 × 1000 ÷ (average length of one car in metres)
   This presented me with a new problem: working out the average length of a car. Some possibilities are to consult manufacturers’ specifications for various makes and models, or take some measurements of real cars and find the average. I looked up the specifications for the car I owned at the time and used this figure (approximately 4 metres) in my calculation. This gave a figure of around 2.7 million cars.

3. To estimate how long it would take to advertise this number of cars, I could find out how many cars are advertised, on average, each day or week in the newspaper in question. Again, I needed to devise a suitable method for this task, which might include resolving questions such as “What counts as a ‘car’?”. Does this include utilities, four-wheel-drive vehicles and so on? For a time the newspaper listed the number of vehicles advertised in its Saturday edition, so I used a sample of this data (2700 vehicles in one edition) to calculate that 2.7 million cars would be advertised in 1000 weeks, or 19.2 years. Finally, I was in a position to ask the most important question of all, “Does my answer make sense?” I suspect that the newspaper has been advertising cars for sale for longer than 19 years, but perhaps I now need to assess whether some of my assumptions would hold true across this time span (e.g., the average car was probably larger in the past, and fewer might have been advertised when the population was smaller).

A shady pergola

The outdoor environment can be a rich source of problems, questions, tasks and issues for investigation. When designing a mathematics trail around the university campus, my pre-service students became interested in a pergola that provides a shady place to sit during the day. They wondered how the amount of sunlight penetrating between the pergola slats varied with the time of day, and at what times the area beneath the pergola would be in full shade.

After measuring the size of the slats and their spacing (see Figure 2), they realised it would be helpful to make some simplifying assumptions:

1. The slats are positioned lengthwise in a north/south direction.
2. The sun is at its highest point in the sky at noon.
3. There are 12 hours between sunrise and sunset.

From these assumptions it follows that the sun moves through 15 degrees every hour (180 degrees in 12 hours). From Figure 2, the amount of sunlight that falls on people beneath the pergola can be expressed as the ratio of distance DC (width of the sunbeam that penetrates for each repeating unit) to distance EB (width of the repeating unit). This represents the proportion of the area under the pergola in sunlight.

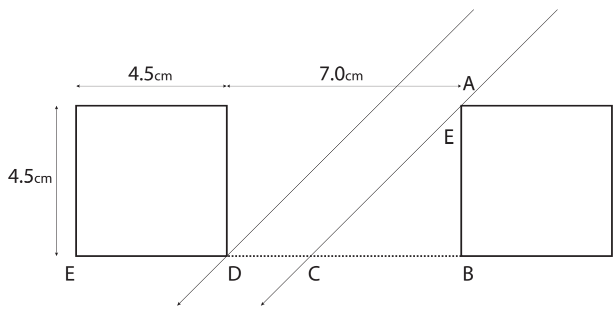

Figure 2. Cross sectional view of pergola.

The pre-service teachers decided to calculate the sunlight proportion for every hour. For example, at 11 am and 1 pm, α = 15 degrees and BC = 4.5 tan α = 1.2 cm, so the sunlight proportion is

$$\frac{7-1.2}{11.5}=50\%$$

However, this is equivalent to calculating a table of values for the function

$$y=\frac{7-4.5\tan x}{11.5}$$

where y is the sunlight proportion (in decimal form) and x the angle of inclination of the sun from the vertical. We can use a graphics calculator or spreadsheet to plot this function and see how the sunlight proportion varies with the sun's angle and hence time of day (as in Figure 3).

Alternatively we can calculate the time exactly by solving the equation 7 – 4.5 tan α = 0 (for values of α between 0 and 90 degrees), which gives α = 57.3 degrees. Since we assume the sun moves through 15 degrees every hour, the time frame of interest is 57.3/15 = 3.82 hours before and after midday. Thus people can sit beneath the pergola in full shade before 8.11 am and after 3.49 pm.

I have used both of these tasks with many groups of teachers and their curiosity is invariably piqued as they attempt to make sense of a real world situation. In discussing their ideas and approaches with colleagues they formulate questions to be answered, suggest ingenious means of gathering the necessary data, and identify similar situations to investigate with their students. Through learning experiences such as these we can communicate a belief that mathematics is a tool for thinking with, a unique and concise language, a way of investigating patterns and relationships, a part of our everyday lives. Our task as mathematics teachers is to help students make sense of this world and their experiences in it, in ways that engage them with powerful mathematical ideas and leave them feeling confident about their ability to learn. No syllabus, textbook, or worksheet can do that on its own, because it takes the creativity and vitality of teachers to bring mathematics to life.

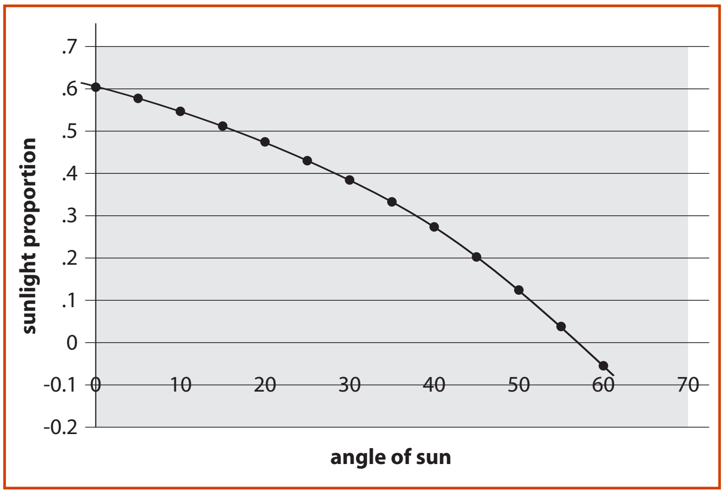

Figure 3. Relationship between sunlight proportion and angle of sun from vertical.

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