Investigating functions using real-world data

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The possibilities for using graphic calculators to enhance the teaching and learning of mathematics are great. However, the boundaries explode when these powerful tools for learning are connected to data logging devices: a whole new approach to mathematics learning becomes possible. Using real-world data to introduce the main functions (which are the bread-and-butter of high school mathematics) invites an experimental approach to the subject and encourages students to engage actively in their learning, as participants rather than as passive spectators.

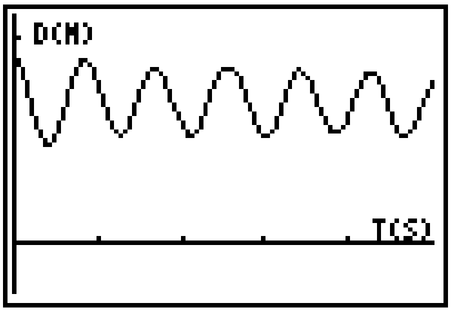
The activity described here offers four different methods for producing a sinusoidal curve and so provides a suitable introduction to or consolidation of trigonometric functions. The emphasis is upon students actively modelling real world data and manually fitting functions (often using trial-and-error techniques) rather than using the inbuilt regression facilities. Far more may be learned by students "getting their hands dirty" than by standing back and letting the technology do all the work.

The activities chosen do not involve complicated equipment or procedures and, although described here using Texas Instruments equipment, will work easily with most data logging devices. It should be noted that the motion detector should be a mathematics teacher's best friend. It can be introduced from a very early age and used to enliven classes at any level. After introducing the main functions described here, there is value in having students attempt to simulate the various functions in turn using their own motion. Much will be gained as they experience for themselves the physical reality of each.

Harmony all around: Trigonometric functions

Using a plastic "slinky" students model a simple harmonic motion to introduce trigonometric functions. (If a slinky is not available, swinging a handbag or any object suspended from a string will work). Initially, students should observe the motion of the slinky and attempt to draw the graph for themselves.

This activity is very simple: set the motion detector to 5 seconds, detach and lay it on the floor. While one student operates the slinky (more than 0.5 metres above the device) another activates the trigger when the motion is stable.

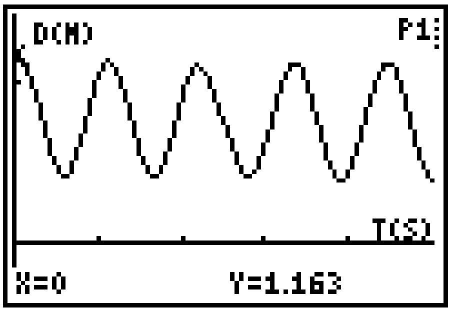


Much will be gained by students using trial-and-error techniques to match a curve as closely as possible to the data. When the class is satisfied with their efforts, then the SinReg option (STAT->CALC) will reinforce and assist in the process.

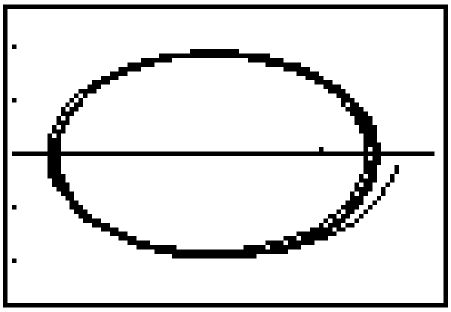
Another simple motion detector activity relevant here involves students on swings — a wonderful excursion for seniors, who will physically feel the motion: when the velocity is zero, and when it is at a maximum — even when the force (or acceleration) is acting (downwards!).

Three ways to test how good a trigonometric function you have collected:

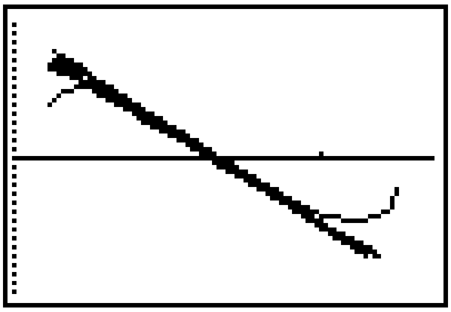
1. Check out the velocity–time graph for the motion. It should be regular and clear, as in the first graph below. (The acceleration–time graph should also retain these features, but usually tends to be limited in its accuracy from such data).
   Why? What are the properties of trigonometric functions at play here?



2. Plot Distance (L2) against Velocity (L3) and see how close to a circle your motion is. This one is a bit surprising and very interesting for students to discuss — hence the name “circular functions”!
   Why does this happen? Some consideration of parametric functions is timely at this point!



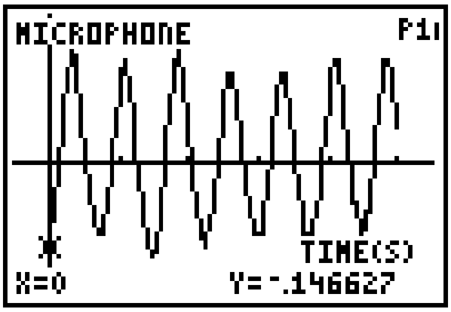
3. Finally, plot Distance (L2) against Acceleration (L4) and observe how close the graph is to a straight line with negative gradient! Once again, it is of benefit for students to attempt to match the function to this graph, and perhaps to derive the defining equation of simple harmonic motion, linking acceleration to distance from the source.
What is the defining equation of simple harmonic motion? What is the gradient of this line representing?

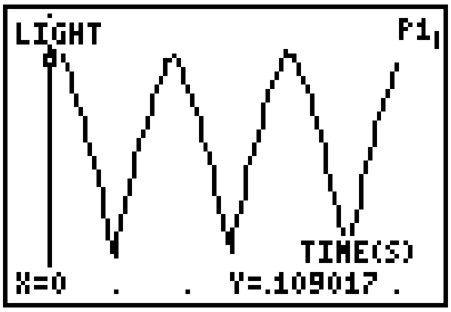


Once again, I would strongly recommend having one or more students use the motion detector to physically model every different function encountered. We assume so much on the part of our students in terms of concept development and deep understanding, especially at senior level. But whether your students are twelve or twenty, there is no more powerful learning experience than physical involvement. Trying to emulate a function offers deeper perception and understanding of the function than any purely "academic" treatment can afford!

The challenge now becomes for students to find other sources of trigonometric data around them! The two sinusoidal curves shown below are derived from two entirely different sources. The data for the first is collected using a microphone probe, with someone singing a clear steady note. The experiment involves collecting 200 samples at intervals of 10^{-4} seconds (10 000 samples per second), hence the data collection only lasts for 0.02 seconds.

The second set of data is generated by holding a light probe underneath a fluorescent light, and works effectively in any classroom. Interestingly, this curve involves the absolute value of a sine (or cosine) function, as the data logger collects, not a smooth continuous effect, but the flickering on and off of the light source at the standard rate of 50 cycles per second (settings are 100 samples, with 0.0003 seconds between samples).





Again, students may attempt to match the curves manually, working in pairs or small groups to encourage verbalisation: they need to put their thoughts into words and to share these with others to get the most out of such activities.

Students need, too, to attempt to relate the real world situation to the features of the mathematical function, with particular focus upon the key concepts of domain and range, and those characteristics that define each of the function types studied.

Mathematics was never taught like this when I was at school! If such an approach blurs the lines a little between mathematics and science, then so be it. Mathematics may be an activity of the mind, but the marvellous ways in which it models our real world are too many and too important not to be used to advantage. Activities like those described here, if used well, build both deep understanding and skills at the same time.

Consider the possibilities.