

Connecting mathematics to studies of society and the environment

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Background

This article reports on the teaching of a unit of lessons which integrates mathematics with studies of society and the environment. The unit entitled "Population Growth Rates" was taught to a double class of Year 6 students by a team of three teachers. The objectives of the unit were (i) to provide students with a realworld context in which to study the mathematical concepts and processes associated with large numbers and percents, (ii) to increase students' awareness of the social effects of population growth rates, and (iii) to provide students with an opportunity to explore real-world data, examine the features and trends in the data, and make reasonable conclusions.

Many calls have been made for students to be given opportunities to make connections between mathematics and other curriculum areas, and be taught mathematics in real world contexts. The National Statement on Mathematics for Australian Schools (Australian Education Council, 1990) states that students should investigate a range of applications of mathematics to problems of concern to Australians. It claims that mathematical investigations help students develop mathematical concepts and provide them with experience of some of the processes through which mathematical ideas are generated and The US tested. curriculum document for Principles and Standards School Mathematics (National Council of Teachers of Mathematics, 2000) states that mathematics programs should emphasise connections between mathematics and other discipline

areas so that all students recognise, use and learn about mathematics in contexts outside of mathematics. According to Schwartz and Curcio (1995), activities which integrate mathematics with other subjects help bring mathematics alive for children through the use of meaningful contexts. The new Years 1 to 10 Syllabus in Mathematics for Queensland Schools (2004) states that through engagement in mathematical investigations, students develop understandings about mathematical relationships and transfer thinking and reasoning to new situations.

Likewise, the new Years 1 to 10 Syllabus for Studies of Society and Environment for Queensland Schools (SOSE) (2004) states that in this key learning area, cross-curricular links with numeracy, literacy, life-skills and a futures perspective are essential priorities. The nature of this key learning area involves "investigations of controversial and challenging issues and promotes critical thinking in the development of optimistic future visions" (p. 1). Students are encouraged to become active participants in their world and to develop (through metacognitive reflection) a deeper awareness, understanding and appreciation of the many concepts, processes and key values associated with it. This appreciation, in turn, can assist students to develop positive dispositions towards their place and role in society, and help to contribute to the valued attributes of the lifelong learner.

By making connections with numeracy, students use the particular skills involved in collecting, organising, analysing and critiquing data to solve problems related to their social environment. As this double classroom focus involved a study related to "our world and its people" an extensive period of context building had been previously established. The topic of population growth rates was chosen in response to a desire by the authors to investigate the relationship between school mathematics and Education for Sustainable Development (ESD). Population growth rates indicate one factor which impacts significantly on issues of resource consumption and the threat to global sustainability. It is one topic for which some teaching resources have been developed and published on the Web (The World Bank Group, 2004). The development of an integrated unit on this topic would satisfy the mathematics curriculum by dealing with large numbers, percentages, and handling data, and the SOSE curriculum by engaging with the theme "our world and its people".

The integrated unit

The authors planned the unit jointly by considering the resources provided in the module on "Population Growth Rates" (The World Bank Group, 2004), analysing the mathematical concepts required for understanding the material and making calculations with large numbers and percents, and analysing the studies of society and environment (SOSE) content and values issues raised by the material which were appropriate for Year 6 students.

The mathematical topics identified were place value, representing and handling large numbers such as thousands, millions and billions, rates and calculating with percents, and analysing and interpreting graphs and tables. The concepts identified in SOSE were those of population, birth rate, death rate, migration, countries of the world, developed, developing and undeveloped countries, and analysing and interpreting maps. It was also recognised that the unit would include aspects of values education, namely awareness and understanding of living conditions in other parts of the world.

A student workbook was prepared containing discussion questions, calculation questions, information about population and population growth rates in tables and graphs, world maps, photos, a glossary of terms, and blank pages for student calculations and reflections.

The first lesson was used to introduce the students to the topic of population growth rates, and to discuss the concepts of population, birth rate, and death rate. The teaching strategy of "think, pair, share" was useful in that it let the teachers observe the range of understandings (and misconceptions) held by the students. During the "think, pair, share" segment, the students first spent two minutes in silence thinking about the meaning of the terms "population", "birth rate", and "death rate". Then the students paired up to discuss the terms and come to an agreement about their meanings. After approximately two or three minutes, the students were then called to order, so that their ideas could be shared in the class as a whole. This process revealed that, although the concept of population was well understood and articulated, the concepts of birth rate and death rate were not. Some students had not heard of these terms (understandably for Year 6 students), and concluded that birth rate referred to how fast a baby was breathing just after birth. The class discussion, under guidance of the teachers, eventually brought out the correct meanings of the terms.

The second lesson was a set of activities, planned to assist with students' comprehension and facility with large numbers - those that are used in describing populations of countries and the world, namely, millions and billions. The students had previously encountered such large numbers, but this lesson was an opportunity to consolidate understanding and reinforce the basic principle of place value in interesting real-world contexts. Number cards and place value charts were used to represent numbers symbolically and orally, starting with thousands, and extending the patterns to millions and billions. A key activity was "How big is a million?" in which different materials were used to demonstrate the space required to contain large numbers of them. Firstly, Base 10 blocks (MAB) were used to represent ones, tens, hundreds, and thousands (the basic ones blocks measure 1 cm^3). Students were then challenged to conceive of the space required for 10 thousand blocks, 100 thousand blocks, and 1 million blocks. A cubic metre made of wooden doweling was

then assembled to demonstrate the concept of 1 million blocks.

Tiny beads (3 mm diameter) obtained from a craft shop were used next to assist in conceiving large numbers. A small plastic bag of 200 beads was emptied into a medicine glass, and was shown to occupy 10 mL (i.e., 10 cm^3). Students were challenged to use proportional reasoning to estimate how many cubic centimetres were needed to contain 1000 beads (50) and 1 million beads? (50 000), how many beads would fit in a cubic metre (20 million), how many cubic metres are needed to fit a billion beads (50), how big that is (5 m × 5 m × 2 m); and how it compares to the classroom size.

A similar activity was conducted with mini M&Ms. Packets containing 60 M&Ms were purchased with a packet size = $4.5 \text{ cm} \times 3 \text{ cm} \times 2 \text{ cm}$, and volume = 27 cm^3 . The challenge questions for the students were:

- 1. How many packets would contain a million M&Ms (16 667)
- 2. how much space is needed to fit a million M&Ms (about half a cubic metre)
- 3. how many packets are needed to make a billion M&Ms (16 666 667) and
- 4. how much space is needed to fit a billion M&Ms (about 450 cubic metres, which is $10 \text{ m} \times 9 \text{ m} \times 5 \text{ m}$).

The next lesson dealt with changes in population for Queensland, Australia and the world. Students were encouraged to refer to the extended place value chart (up to billions) as printed in the student workbook to assist them in saying and writing the numbers if they were not confident.

The students were challenged to complete the following calculations on population growth in the student workbook. Students had previously encountered the concept of "average" and were allowed to use calculators for the calculation if they wished. Queensland's population was 3 265 100 in 1995 and 3 612 300 in 2001. Calculate the increase in population over the six-year period. Now calculate the average yearly increase for that period.

Australia's population was 18 071 000 in 1995 and 19 277 100 in 2001. Calculate the increase in population for the six-year period. Calculate the average yearly increase for that 6-year period.

Australia's population was 20 213 500 on 11 November 2004. Calculate the average yearly increase for the three-year period 2001 to 2004.

The world's population was estimated to be 5 897 000 000 in 1998, and 6 373 000 000 in 2004 (July). Calculate the growth in world population in the six years from 1998 to 2004. Also, Calculate the average yearly growth over that period.

According to the International Programs Center, US Bureau of the Census, the total population of the world was projected by the "PopClock" to be 6 399 548 137 on 11 November 2004 at 3:26:32 GMT (the actual day of this lesson; students were fascinated with this information, and that there was such a thing as a PopClock). Students were then asked to use the following world population projections to calculate the growth in world population for each month.

Date	Population:	Monthly growth:
01/07/04	$6\ 372\ 797\ 742$	
01/08/04	6 379 026 080	
01/09/04	$6\ 385\ 254\ 418$	
01/10/04	$6\ 391\ 281\ 842$	
01/11/04	6 397 510 180	
01/12/04	6 403 537 604	

Which month had the greatest growth?

billions	hundred millions	ten millions	millions	hundred thousands	ten thousands	thousands	hundreds	tens	ones



World population is estimated to be 7 113 000 000 in 2015. Calculate the growth in population from 2004 to 2015. Calculate the average yearly growth in that period.

In order to deal with percentage growth rates, the next lesson focussed on the concept of percents and calculating with percents. As with large numbers, the students had encountered percents previously, but once again, this lesson was an opportunity to consolidate understanding and reinforce the concept of percents and how to perform calculations in interesting real-world contexts.

The language of percents was discussed initially: "per" meaning "for every" — a common example being 60 km per hour meaning 60 km travelled for every hour, and "cent" meaning a hundred — a common example being 100 cents in a dollar. So putting the two words together, 21 percent means 21 for every hundred. Materials used to illustrate the concept were "per cent squares" — multiple grids of 10×10 squares as shown below. Figure 1 below shows the model 21% of 400.

The "percent squares" model was also used to demonstrate the link between percent and decimal fractions (21% = 21 hundredths = 0.21).

The model was then used to develop the process for calculating population growth rates, namely:

- increase in population ÷ population at start of year × 100
 = annual population growth rate
- birth rate death rate
 population growth rate (assuming zero migration).

Students were then given practice at calculating population growth rates for various countries using information from the student

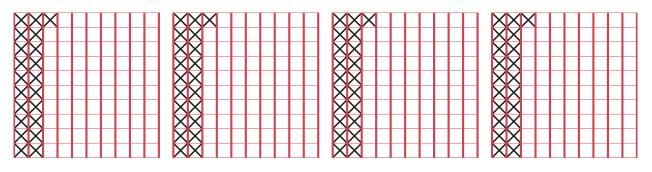


Figure 1: The 'per cents squares' model showing 21% of 400.

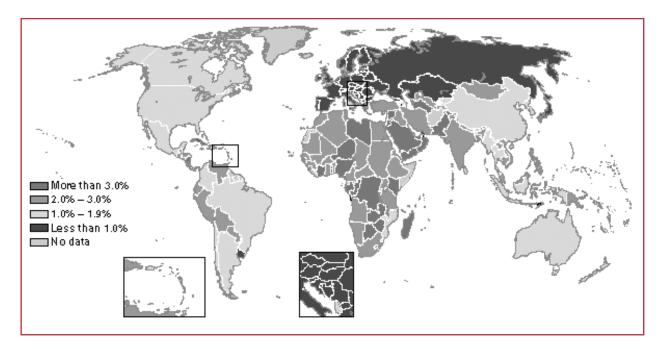


Figure 2. World map showing population growth rates.

workbook. They were then shown a world map (projected onto a large screen from the computer; see Figure 2) which indicated the population growth rates for all countries by colour code. There were five categories: more than 3%, between 2% and 3%, between 1% and 1.9%, less than 1%, and no data.

Students formed groups of two or three and were given discussion questions to assist them to analyse and interpret the map in terms of population growth rates.

- Where are the countries with the highest percentage growth rates?
- Where are the countries with the lowest percentage growth rates?
- Which continent has the largest number of countries with higher PGR?
- Name some countries with a high PGR, and some with a low PGR.

The next lesson focussed on the growth in world population and the variation in growth rates (PGR) among countries of differing income. Figure 3 shows the total world population by country income group for the years 1980, 1998, and 2015 (projected).

Between 1980 and 2000 total world population grew from 4.4 billion to 6 billion. Based on population projections, by 2015 at least another billion people will be added for a total of more than 7 billion. Most of this growth will take place in low- and middle-income countries.

The students found the population figures for each country group and year in the student workbook and calculated their respective percentages of the world population to examine the trends. They soon discovered that the low-income countries were growing in terms of the percentage of the world's total population (60% in 1998 to 62% projected in 2015), middle-income countries were static (25%) and high-income countries were decreasing in terms of the percentage of the world's total population (15% in 1998 to 13% in 2015).

The next lesson focussed on changes in growth rates for each country group over the period 1998 to 2015 (see Figure 4).

Population growth rates have started to decline in many countries, but the absolute numbers will continue to increase over the next several decades because the population base is larger. Growth rates tend to be higher in low- and middle-income countries than in high-income countries.

Discussion groups were again formed to consider the information contained in the chart, trends in growth rates and population sizes for each of the income groups. Students then calculated annual population changes for three developing countries — Ethiopia, Pakistan and Ukraine — and were encouraged to suggest reasons for these trends, and the effect the trends would have on the three countries' economy and environment.

The following lesson was an interesting activity which allowed students to view pictures of children in the "photo gallery" and explore living conditions in various countries.

The final lesson dealt with the population age profiles of high-income, middle-income and low-income countries (see Figures 5 and 6).

In many low-income countries, the large difference between the percentage of people of childbearing age and more elderly adults causes population momentum which keeps population growth rates high even when fertility rates drop. In many high-income countries, where fertility rates are below replacement level and the largest segments of

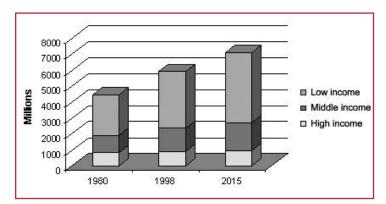


Figure 3. Total world population by country income group, 1980, 1998, 2015.

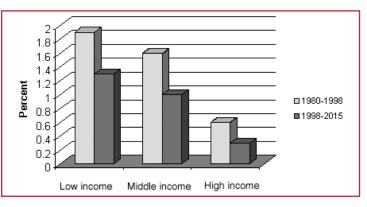


Figure 4. Average annual growth rates by country income group, 1980–2015

the population are older, populations will increase much less. To see the projected composition of population for low- and highincome economies in 2030, see Figure 6.

In many low-income countries, the large difference between the percentage of people of childbearing age and more elderly adults causes population momentum which keeps population growth rates high even when fertility rates drop. In many high-income countries, where fertility rates are below replacement level and the largest segments of the population are older, populations will increase much less. To see the composition of for lowand high-income population economies in 2000, see Figure 5.

The students were asked to analyse the data represented in Figures 5 and 6, describe the data and suggest reasons for the differences between the groups of countries. Most students were able to see low-income countries had greater percentages of young people than older people and were able to deduce that factors such as disease and poverty prevented people from reaching old age. They were able to see the contrasting profile in high-income

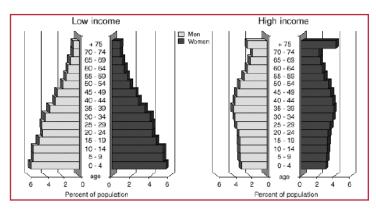


Figure 5. Composition of population in low- and high-income economies, 2000.

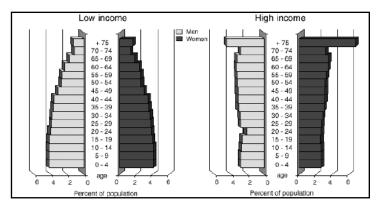


Figure 6. Composition of population in low- and high-income economies, 2030

countries, where good health services and comfortable living conditions assisted people in reaching old age.

At the conclusion of the unit of work students were asked to consider the future ramifications of population growth rates worldwide. They produced two visual representations from a futures perspective. The first focuses on concerns using a 'futures wheel' in order to define an ever-increasing spiral of environmental, economic and political needs. By way of contrast a "futures timeline" which details the past history of world population growth but then separates to forecast both a "probable future" and a "preferred future" provides an alternative view. This approach enables students to understand a future, which if current trends in world population growth were to remain constant, would probably progress. However, it also encourages students to project a positive alternative future — should certain global trends in population growth rates alter significantly from their current dominant trends. See Figures 7 and 8 for samples of students' 'futures' perspectives.

Assessment of the unit

The students were provided with many opportunities to demonstrate their developing understandings of the nominated concepts outlined for this unit of work. To keep assessment authentic, to encourage links to their classroom mathematics program, and foster meaningful application of newly acquired skills and processes gained from the mathematical components of the unit, the classroom teachers devised a set of similar mathematical tasks based on the world population data and large numbers evident in this unit. In addition to these, an investigative project comprising a comparative study of two diverse countries' wealth and opportunity were considered from both a mathematical and socio/environmental perspective.

Consistent with objective (iii) of this unit, students were to determine features and trends from real-world data and make reasonable conclusions and extrapolations from them. Looking ahead and predicting change encourages children to think of causes and consequences. The futures wheel, when correctly applied, clearly displays children's understandings of immediate, first order and second order consequences (Slaughter, 1996). Students should ask themselves the question, "If this occurs then what will happen?". What should be produced is a pattern of possible outcomes originating with the initial assumption.

The futures timeline encourages students to look ahead, predict the probable future based on the trends gained from the data, whereas the preferable future (the ideal, though unlikely) scenario should represent goals and aspirations that use positive assumptions. The futures timeline should contrast markedly on the probable and preferable axes. It is this tension between the two that creates a focus for further discussion and investigation by the students and classroom teachers.

Student reflections

Students were required (as is the normal expectation in this particular classroom) to write reflections on what they had learned and their response to the lessons and activities. The process of student reflection is one strategy developing metacognitive processes and self assessment which can have a significant positive impact on learning (Fernandez, Hadaway, & Wilson, 1994; Trotman, 1998). The students' comments covered a wide range of themes and issues from the cognitive and affective domains, including:

- what they had learned in mathematics: I learned to find percentages and averages and how to find birth and death rates; Before, I found it hard to read large numbers but when I put them in groups of three I found it lot easier; I thought a billion had 6 zeros until now; I learned how to read graphs.
- what they had learned in studies of society and environment:

I learned why different countries grow rapidly; I learnt a lot from the graphs; Calculating how long a country took to double its population interested me; I found that different places had different amounts of people in certain ages.

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Figure 7. Futures wheel example.

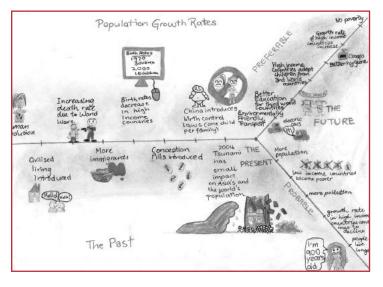


Figure 8. Futures timeline example.

• and in associated key values:

The low-income countries are very badly off and should get a lot more from the highincome countries; I also learnt that some countries have more kids than adults because their medical sources might be poor, and so the older people might die more easily.

• their awareness of issues and amazement at some findings:

It's weird how so few people live in a highincome country; I am really amazed at how many M&Ms fit in a metre (cubed) box; I was amazed to find that 1000 cubic metres can fit one billion base 10 blocks; The thing I liked and learnt was death and birth rates – I never knew there was such a thing; It's wonderfully amazing how mathematics works. • their enjoyment and interest in the overall topic:

I enjoyed learning something new and exciting; I liked searching for information in the back of the book; my favourite activity was calculating how long it took for a country to double its population; Some days I wouldn't want to go to school but the teachers changed that and made it exciting; I had fun doing the comparison table of developed and developing countries.

- their confidence in the topics:
 I discovered that I was better than I thought at the subject of "population"; I feel more confident about percentages now and I also feel smarter; I am amazed at what I can do; The teachers didn't just teach about birth rates and death rates, but also about myself ...They showed me that I am capable of being able to read large numbers, calculate population percentages and other GREAT things.
- their appreciation of the student resource booklet:

The actual contents of the book were amazing in my opinion, even to have a green cover; It was my first time having a booklet for writing in as we were receiving lessons; I wonder how long it took to put this book together, and how much money was used for 3990 pages with ink?

• their overall response: The teachers taught some pretty cool stuff that I thought might have been too sufisticated (sic) for me. But in the end I really did understand.

Teacher evaluation of the unit

Teacher observations noted a definite increase in confidence and application when working with large numbers, percents and analysis of statistical data tables. The most pleasing aspect to note was that transfer was obvious and ongoing, as students continue to apply their new understandings in practical and mathematical ways.

From the teachers' point of view, the unit was successful in achieving its objectives. Students responded positively to the realworld context of population growth rates in order to study the large numbers and percentages, and explore graphs and tables of real-world data. It definitely increased the students' awareness of developed, developing and undeveloped countries, and the social effects of population growth rates. It also increased students' awareness and understanding of living conditions in other parts of world.

Importantly, this unit of lessons provided a suitable way of developing a link between school mathematics and Education for Sustainable Development (ESD). It has also demonstrated the claim (Schwartz & Curcio, 1995) that the integrated model is one of the most promising vehicles for engaging students' interests, stimulating mathematics learning in context, and meeting schools' concerns for covering the curriculum.

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